

The background of the cover is a light blue gradient. On the left side, there is a stylized illustration of a tree or branching structure. The branches are thin, dark lines, and the leaves are solid red. The tree starts from the bottom left and branches out towards the top right. The authors' names are printed in a dark, sans-serif font in the top right corner.

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INTRODUCTION TO

ALGORITHMS

THIRD EDITION

Introduction to Algorithms

Third Edition

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Ronald L. Rivest
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Introduction to Algorithms
Third Edition

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