Assignment Maths

Date.....................

(i) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=3$

$Ax=B$ $\quad A=\begin{pmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{pmatrix}$, $x=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$, $B=\begin{bmatrix}5\\ 13\\ 3\end{bmatrix}$

$(A:B)=\begin{vmatrix}2 & -3 & 7 & 5\\ 3 & 1 & -3 & 13\\ 2 & 19 & -47 & 32\end{vmatrix}$ $\quad R_2\to R_2-\frac{3}{2}R_1$

$A:B=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & \frac{11}{2} & -\frac{27}{2} & \frac{11}{2}\\ 2 & 19 & -47 & 32\end{pmatrix}$ $\quad R_3\to R_3-R_1$

$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & \frac{11}{2} & -\frac{27}{2} & \frac{11}{2}\\ 0 & 22 & -54 & 27\end{pmatrix}$ $\quad R_3\to R_3-4R_2$

$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & \frac{11}{2} & -\frac{27}{2} & \frac{11}{2}\\ 0 & 22 & -54 & 27\end{pmatrix}$ $\quad R_3\to R_3-4R_2$

$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & \frac{11}{2} & -\frac{27}{2} & \frac{11}{2}\\ 0 & 0 & 0 & 5\end{pmatrix}$ $\quad A=\begin{pmatrix}2 & -3 & 7\\ 0 & \frac{11}{2} & -\frac{37}{2}\\ 0 & 0 & 0\end{pmatrix}$ $\quad C=\begin{pmatrix}5\\ \frac{11}{2}\\ 5\end{pmatrix}$

$\rho(A:B)\neq\rho(A)$

No solution exist (inconsistent)

Date..........

(ii) $2x-y+3z=8$, $-x+2y+z=4$, $3x+y-4z=0$

$Ax=B$ $\quad A:B=\begin{pmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{pmatrix}$ $R_2 \to 2R_2$, $R_2 \to R_2+R_1$

$A:B=\begin{pmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 3 & 1 & -4 & 0 \end{pmatrix}$ $R_3 \to R_3 - \frac{3}{2}R_1$, $R_3 \to 2R_3$, $R_3 \to R_3 - \frac{5}{3}R_2$

$A:B=\begin{pmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -\frac{76}{3} & -\frac{152}{3} \end{pmatrix}$ $\rho(A:B)=\rho(A)=3$

$Ax=B$, $\begin{pmatrix} 2 & -1 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -\frac{76}{3} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 8 \\ 16 \\ -\frac{152}{3} \end{pmatrix}$

$2x-y+3z=8$, $3y+5z=16$, $\cancel{-\frac{76}{3}z=-\frac{152}{3}}$ 2

$x=2$, $y=2$, $z=2$

$(x,y,z)=(2,2,2)$

(iii) $4x-y=12$, $-x+5y-2z=0$, $-2x+4z=$

$\begin{pmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 12 \\ 0 \\ 8 \end{pmatrix}$

Date.....................

$A:D = \begin{pmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{pmatrix}$ $R_2 \to 4R_2 + R_1$
$R_3 \to 2R_3 + R_1$

$A:B = \begin{pmatrix} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ 0 & -1 & 8 & -4 \end{pmatrix}$ $R_3 \to 19R_3 + R_2$

$A:D = \begin{pmatrix} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & 144 & -64 \end{pmatrix}$ $P(A:B) = P(A)$, so consistent
$= 3$ unique sol.

$\begin{pmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 0 & 144 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ -64 \end{pmatrix}$

$144z = -64,\ z = \frac{-64}{144} \cdot \frac{4}{9},\ 19y - 8z = 12,\ y = \frac{4}{9}$

$4x - y = 12 = x = \frac{28}{9},\ y = \frac{4}{9},\ z = \frac{-4}{9}$

~~(b) $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z = \mu$~~

(i) $A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{pmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$ $\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ & 1 & 2 & \lambda & \mu \end{pmatrix} \to R_3 \to R_3 - R_2$

$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{pmatrix}$

No this echleonform
$P(A:B) = P(A)$
$\lambda - 3 = 0,\ \lambda = 3$
$\mu - 10 \neq 0,\ \mu \neq 10$

Spiral

Date..................

(ii) For ~~unique sol~~, $\rho(A:B) = \rho(A) = 3$

$$A:B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

Since the sys is ~~consistent~~ $\rho(A:B) \neq \rho(A)$

$R_2 \to R_2 - R_1$

(iii) For infinitely many sol^n, $\rho(A:B) = \rho(A) < 3$

$$A:B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$\lambda - 3 = 0$, $\mu - 10 = 0$

$\lambda = 3$, $\mu = 10$

(iv) $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$

$$A:B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$A:B \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$A:B = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{array}\right]$$

$\rho(A:B) = 3$

$\rho(A) = 2$

$\lambda^2 - 3\lambda + 2 = 0 \to (\lambda-1)(\lambda-2) = 0$

$\lambda = 1$, $\lambda = 2$

(v) $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=$

$AX = 0$, Since the eq is Homogeneous

Date.....................

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 4 \end{pmatrix} \quad R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - R_1$$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 1 & -14 & 16 \end{pmatrix} \quad R_3 \to R_3 - 2R_2$$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{pmatrix}$$

Since in echlon form no of eq > no of variable so, z is free variable

$$x + 3y - 2z = 0$$
$$-7y + 8z = 0$$

Let us $z = 7$, then, $7y = 56$, $y = 8$

$$x + 3(8) - 2(7) = 0, \quad x = -10$$

Linear Algebra – Tuisem – II Assignment

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix} \quad \begin{matrix} R_4 \to R_4 - 2R_3 \\ R_3 \to R_3 - 3R_1 \\ R_2 \to R_2 - 2R_1 \end{matrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{pmatrix} \quad \begin{matrix} R_4 \to R_4 + R_3 \\ R_2 \leftrightarrow R_3 \end{matrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & -3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{pmatrix} \xrightarrow{R_4 \to R_4 - R_3} \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho(A) = 3$$

Spiral

Date.........

2. Vector space of 2x2 ~~skew symmetric matrix~~

$W = \{ A$ where $A^T = -A \}$

$T : W \to P_2$ where $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-d)x + (c-a)x^2$

rank$(T) = \dim(\text{Im}\,T)$ and nullity$(T) = \dim(\ker T)$

$w \in W \quad w = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to$ symmetric matrix

$W = W^T, \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \to b = c$

And hence $\dim W = 3$

Now if we find the rank(T)

Standard basis:- $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$

$T \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 1 + 0x + (-1)x^2 = 1 - x^2$

$T \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 + 0x + 1x^2 = -1 + x^2$

$T \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 + 0x + 0x^2 = 0$

$M = \begin{vmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow M \begin{vmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ rank$(M) = 1 \to$

rank$(T) = 1 = \dim(\text{Im}\,T) = $ rank (M)

nullity$(T) = 2$, [illegible]

[illegible]

Date..................

1. $A = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \rightarrow |A - \lambda I| = 0$

$\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$

$(2-\lambda)^2 - 1 = 0$

$4 + \lambda^2 - 4\lambda + 1$, $\lambda^2 - 4\lambda + 3 = 0$

~~Eigen values = 3, 1~~ , $\lambda = 3, \lambda = 1$

$\lambda = \frac{1}{3}, 1$

$\text{Adj } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ so, $A^{-1} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{3} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \rightarrow A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$(A^{-1} - \lambda I) \; X^{-1} = 0$

$\lambda = \frac{1}{3} \rightarrow \left[\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} - \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \right) X^{-1} = 0$

$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$\frac{1}{3} \begin{pmatrix} x+y \\ x+y \end{pmatrix} = 0$ $\quad x + y = 0$

$x = -y$

$\begin{bmatrix} -k \\ k \end{bmatrix} = -k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad x = -k$

$y = k$

Spiral

Date..................

$\lambda = 1, (A^{-1} - I)x^{-1} = 0$

$$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x^{-1} = 0$$

$$\begin{pmatrix} \frac{-1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad \begin{pmatrix} -\frac{1}{3}x + \frac{1}{3}y \\ \frac{1}{3}x - \frac{1}{3}y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-x + y = 0, \; x - y = 0$

$x = y, \; x = y = k$

So, $v_2 = \begin{pmatrix} k \\ k \end{pmatrix}$

eigen vector $\lambda = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Now eigen value of $A + 4I = \lambda + 4$

$\lambda = 7, 5$

$$B = A + 4I = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix}$$

$\lambda = 7, \quad (B - 7I)\bar{x} = \bar{0}$

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0, \quad x + y = 0, \; x = -y$$

So eigen vector $= (-1, 1)$

$\lambda = 5, \; (B - 5I)\bar{x} = \bar{0} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$x - y = 0$

$x = y$ vector $= (1, 1)$

Spiral

Date.....................

4. $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

Ist itera: $y=0, z=0$

$3x = 7.85$, $x = 2.61$

$0.1x - 0.3z + 19.3 = 7y$

$$y = \frac{0.1 \times 2.61 + 19.3}{-7} \rightarrow y = -2.79$$

$$z = \frac{71.4 + 0.2y - 0.3x}{10} = \frac{71.4 + 0.2(-2.79) - 0.3 \times 2.61}{10}$$

$z = 7.00$

$x = 2.61, y = -2.79, z = 7.00$

2nd iteration $\rightarrow$ $y = -2.79, z = 7.00$

$$x = \frac{7.85 + 0.2z + 0.1y}{3} = 2.99$$

$$y = \frac{0.1x - 0.3z + 19.3}{-7} = -2.49$$

$$z = \frac{71.4 + 0.2y - 0.3x}{10} = 7.0005 = 7$$

$x = 2.99$
$y = -2.49$
$z = 7$

Spiral

Date.....................

3rd iteration $x = \frac{7.85 + 0.2z + 0.1y}{3} = 3.00$

$y = \frac{0.1x - 0.3z + 19.3}{-7}$ $z = 7$, $x = 3$

$z = \frac{71.4 + 0.2y - 0.3x}{10}$ $y = -2.5$

$x = 3, y = -2.5, z = 7$

5. $x + 3y + 2z = 0$
$2x - y + 3z = 0$
$3x - 5y + 4z = 0$

$A = \begin{bmatrix} 1 & 17 & 4 \\ 3 & -5 & 4 \\ 3 & 2 & 5 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 17 & 4 \\ 3 & -5 & 4 \\ 0 & 7 & +1 \end{bmatrix}$

$A - \begin{vmatrix} 1 & 17 & 4 \\ 0 & -56 & -8 \\ 0 & 7 & +1 \end{vmatrix} = \begin{vmatrix} 1 & 17 & 4 \\ 0 & -56 & -8 \\ 0 & 0 & 0 \end{vmatrix}$

rank $= 2 < 3$

So, the consistent sols have infinite solutions

$x + 17y + 4z = 0$, $-56y - 8z = 0$
$7y + z = 0$

$y = k$, $z = -7k$

$x + 7k - 18k = 0$

$x = 11k$

Spiral

$x = 11k,\ y = k,\ z = -7k$

6. $T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

(i) $T(P_1) + T(P_2) = T(P_1 + P_2)$

$T(a_1 + b_1x + c_1x^2) + T(a_2 + b_2x + c_2x^2)$

$= (a_1 + 1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$

$= (a_1 + a_2 + 2) + (b_1 + b_2 + 2)x + (c_1 + c_2 + 2)x^2$

$T(P_1 + P_2) = T((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2)$

$= (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$

$T(P_1) + T(P_2) \neq T(P_1 + P_2)$

So, T is not a linear Transformation

Q-7 $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$

For S to be basis of R^3, the vectors must be a linear independent

$x(1,2,3) + y(3,1,0) + z(-2,1,3) = (0,0,0)$

$(x + 3y - 2z,\ 2x + y + z,\ 3x + 3z) = (0,0,0)$

$A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{pmatrix} \Rightarrow A = \begin{vmatrix} 1 & 3 & -2 \\ 3 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -2 \\ 0 & -9 & 9 \\ 0 & -5 & \end{vmatrix}$

Spiral

Date.....................

$D = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -9 & 9 \\ 0 & 0 & 0 \end{pmatrix}$ → infinite soluti

So, vecto of S are linearly depended

Subspace of S = $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -9 & -9 \end{bmatrix}$ → The two vecto are basis of subspace spanning

dimension of subspace is 2

8. $3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + 7z = 6$

First $x = \frac{23 + 6y - 2z}{3} = 7.66$

$y = -15 + z + 4x = 15.67$

$z = \frac{16 + 3y - x}{7} = 6.46$

$x = 7.66,\ y = 15.67,\ z = 6.46$

Second iterate $x = \frac{23 + 6y - 2z}{3} = 34.64$

$y = 15.64$

$z = 1.46$

Date........

$y = 130.02,\ x = 34.64$

$z = \frac{16 + 34 - x}{7} = 53.06$

$x = 34.64,\ y = 130.02,\ z = 53.06$

Thus $x = \frac{23 + 6y - 2z}{3} \rightarrow y = 130.02,\ z = 53.06$

$x = 232.33$

$x = 232.33,\ z = 53.06,\ y = 4x + z - 11$
$= 967.38$

$y = 967.38,\ x = 232.33$

$z = \frac{10 + 3y - x}{7} = 383.63$

$x = 232.33,\ y = 967.38,\ z = 383.63$

9. Matrix operation extremely used in image processing, Like transpose of matrix is used to rotate the image in various direction and the blur matrix is used to blur certain area of image

Apart from this, image are made up of matrix itself. Images are made up of p[illegible] which are arranged in grid to produce image

Date.....................

$y = 130.02,\ x = 34.64$

$z = \frac{16+34-x}{7} = 53.06$

$x = 34.64,\ y = 130.02,\ z = 53.06$

Thus $x = \frac{23+6y-2z}{3} \rightarrow y = 130.02,\ z = 53.06$

$x = 232.33$

$x = 232.33,\ z = 53.06,\ y = 4x+z-11$
$= 967.38$

$y = 967.38,\ x = 232.33$

$z = \frac{10+3y-x}{7} = 383.68$

$x = 232.33,\ y = 967.38,\ z = 383.68$

9. Matrix operation extremely used in image processing, Like transpose of matrix is used to rotate the image in various direction and the blur matrix is used to blur certain area of image

Apart from this, image are made up of matrix itself. Images are made up of pixels which are arranged in grid to produce image

Date.....................

10 Linear transformation plays very important role in computer vision. Linear transformation is extremely used in manipulating image to various purpose.

One example is rotating image by θ angle about z-axis.

For this purpose we use famous rotation matrix in LD to do this transformation.

Here, $T: V \to W$

where $T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

If we have to rotate (x, y) about θ, then new x and y are:-

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this way we perform this basic operation for each pixel of the image and find the rotated image. This transformation is also used in image registration, object detection and image alignment.