Faculty of Computer Science, Heinrich-Heine-University Düsseldorf

Deep Learning

Winter term 25/26 – Exercise Sheet 9

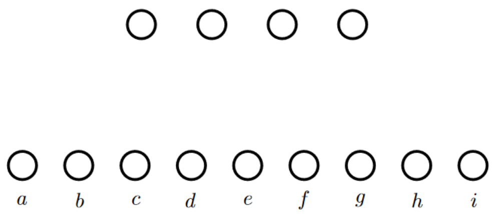
Submission Deadline: Monday, December 15, 2025, 2:00 PM

1. **Filters and weight matrices (3P)** This exercise aims at illustrating the connection between filters and weight matrices in CNNs. Lets look at the 3×3 dimensional input and a 2×2 dimensional filter:

a	b	c
d	e	f
g	h	i

w_{11}	w_{12}
w_{21}	w_{22}

   A 'valid' convolution (with stride$= 1$) results in a 2×2 dimensional feature map corresponding to 4 hidden neurons in the next hidden layer. Infer the resulting convolutional layer by drawing the connections and their weights into the following image and write down the weight matrix.



2. **Convolution (4P)**
   Let f and g be two discrete, two-dimensional functions. The *convolution* of f with

g is defined by

$$h(i,j) = (f * g)(i,j) = \sum_x \sum_y f(x,y)g(i-x, j-y).$$

Let us consider the input of the hidden neurons $z_{in,i,j}$ of a convolutional layer (without bias). Let I be the two-dimensional input and, and $W \in \mathbb{R}^{M \times M}$ the corresponding filter, then

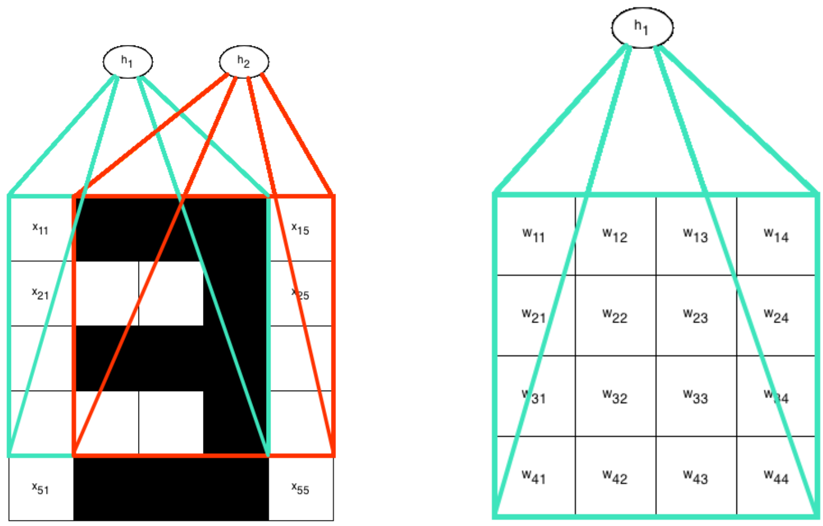
$$z_{in,i,j} = \sum_{m=1}^{M} \sum_{n=1}^{M} I_{i+m-1,j+n-1} w_{m,n} \quad .$$

Interpret this equation in terms of the mathematical definition of a convolution. That is, define f and g such that $z_{in,i,j} = h(i,j)$.

3. **Feature Matching (3P**

A convolutional neural network is a type of neural network well suited for image recognition. It combines weight sharing with an optimized network topology, that can exploit the 2D-structure of the input data.

Consider a convolutional neural network for images of 5 by 5 pixels. Imagine that for some training case, we have an input image where each of the black pixels in the top diagram has value 1, and each of the white ones has value 0.



Notice that the image shows a "3" in pixels. The network has no biases. The hidden units are linear. The weights of the network are given as follows:

$$\begin{array}{llll}
w_{11} = 1 & w_{12} = 1 & w_{13} = 1 & w_{14} = 0 \\
w_{21} = 0 & w_{22} = 0 & w_{23} = 1 & w_{24} = 0 \\
w_{31} = 1 & w_{32} = 1 & w_{33} = 1 & w_{34} = 0 \\
w_{41} = 0 & w_{42} = 0 & w_{43} = 1 & w_{44} = 0
\end{array}$$

For the training case with that "3" input image, what is the output of each of the four hidden units?