Exercise 2.

Malik Mardan

1)

Let $\pi(x, \theta) = P(y=1 | x, \theta) = \frac{e^{\theta^T x}}{1+e^{\theta^T x}} = \sigma(\theta^T x)$

likelihood:

$$L(\theta) = \prod_{i=1}^{n} [\pi(x^{(i)}, \theta)]^{y^i} [1-\pi(x^i, \theta)]^{1-y^i}$$

log-likelihood:

$$l(\theta) = \sum_{i=1}^{n} \left(y^i \log \pi(x^i, \theta) + (1-y^i) \log(1 - \pi(x^i, \theta)\right)$$

$\pi(x, \theta) = \sigma(\theta^T x)$:

$$l(\theta) = \sum_{i=1}^{n} \left(y^i \theta^T x^i - \log(1 + e^{\theta^T x^i})\right)$$

2)

$$y(x) = 1[wx+b \geq 0], \quad w=1 \quad b=-t_0$$

Then:

$$wx+b = x - t_0, \quad \text{so } y(x) = 1 \quad \text{if } x_0 \geq t_0$$

We need one neuron an we use the threshold activation.

$\sigma =$

3)

Goal: $\text{softmax}(z_1, z_2) = \sigma(z_1 - z_2)$

$$\text{softmax}(z_1, z_2) = \left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}}, \frac{e^{z_2}}{e^{z_1}+e^{z_2}} \right)$$

Take first probability p_1:

$$p_1 = \frac{e^{z_1}}{e^{z_1}+e^{z_2}} = \frac{e^{z_1-z_2}}{e^{z_1-z_2}+1} = \sigma(z_1-z_2)$$

$$p_2 = 1 - p_1 = 1 - \sigma(z_1 - z_2)$$

$$\text{softmax}(z_1, z_2) = (\sigma(z_1 - z_2), 1-\sigma(z_1-z_2))$$