

2.

Forward:

$$s_1 = \omega_1 x, s_2 = \omega_2 x$$

$$z_1 = \max(0, s_1), z_2 = \max(0, s_2)$$

$$d_1 = z_1 \cdot u_{1,1} + z_2 \cdot u_{2,1}, d_2 = z_1 \cdot u_{1,2} + z_2 \cdot u_{2,2}$$

$$f_j = \frac{e^{d_j}}{e^{d_1} + e^{d_2}} \text{ for } j=1,2$$

$$L = \frac{1}{2} [(f_1 - y_1)^2 + (f_2 - y_2)^2]$$

$$\text{Let } e_j = f_j - y_j$$

Backward:

1)

$$\frac{\partial L}{\partial f_j} = e_j$$

2)

$$\text{softmax Jacobian: } \frac{\partial f_i}{\partial s_j} = f_i (s_j - f_j)$$

in vector form with $f = [f_1, f_2]^T, e = [e_1, e_2]^T$:

$$g_2 = \frac{\partial L}{\partial \alpha} = (\text{diag}(f) - f f^T) e$$

For 2 classes:

$$g_{2,1} = c_1 \ell_1 (1 - \ell_1) - c_2 \ell_1 \ell_2$$

$$g_{2,2} = c_2 \ell_2 (1 - \ell_2) - c_1 \ell_1 \ell_2$$

3)

$$\boxed{\frac{\partial L}{\partial v_{k,j}} = z_k g_{k,j}}$$

$$\frac{\partial L}{\partial v_{1,1}} = z_1 g_{2,1}$$

$$\frac{\partial L}{\partial v_{1,2}} = z_1 g_{2,2}$$

$$\frac{\partial L}{\partial v_{2,1}} = z_2 g_{2,1}$$

$$\frac{\partial L}{\partial v_{2,2}} = z_2 g_{2,2}$$

4)

$$\frac{\partial L}{\partial z_k} = v_{k,1} g_{2,1} + v_{k,2} g_{2,2}$$

5)

$$\text{ReLU: } 1[\bar{z}_k > 0]. \quad \frac{\partial \bar{z}_k}{\partial w_k} = x$$

$$\frac{\partial L}{\partial w_k} = (v_{k,1} g_{2,1} + v_{k,2} g_{2,2}) 1[\bar{w}_k x > 0] x$$

$$\text{So: } \frac{\partial L}{\partial w_1} = (v_{1,1} g_{2,1} + v_{1,2} g_{2,2}) 1[\bar{w}_1 x > 0] x$$

$$\frac{\partial L}{\partial w_2} = (v_{2,1} g_{2,1} + v_{2,2} g_{2,2}) 1[\bar{w}_2 x > 0] x$$