1)

$$\tau(z_{in}) = \frac{1}{1+e^{-2z_{in}}} = \sigma(2z_{in})$$

$$\hookrightarrow \tau(z_{in}) = \sigma(2\cdot(w^T x + b))$$

To watch the second neuron with the first we need:

$$\tau(z_{in}) = z'_{out} = \sigma(w'^T x + b')$$

now we can say:

$$\sigma(w'^T x + b') = \sigma(2\cdot(w^T \cdot x + b))$$

we can watch the inside because sigmoid is strictly increasing, therefore if $\sigma(a) = \sigma(b)$ then is $a = b$.

$$w'^T x + b' = 2(w^T x + b)$$

$$w'^T x + b' = 2w^T x + 2b$$

$$\hookrightarrow w' = 2w \qquad b' = 2b$$

$g = RELU$

a) $f_{in} = U \cdot g(Wx+b) + c$

$$Wx = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 2 & 2 \end{pmatrix}$$

$$Wx + b = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} = z$$

$$RELU(z) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} = z_R$$

$$U \cdot z_R = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = z_{RU}$$

$$z_{RU} + c = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + (-0{,}5) = \begin{pmatrix} -0{,}5 \\ 0{,}5 \\ 0{,}5 \\ -0{,}5 \end{pmatrix} = f_{in}$$

$J(f_{in})$ will return the expected results for XOR modelling.

b) change v to $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ to model OR and change v to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to model AND. c can stay $-0{,}5$ for both scenarios.