

2.

Forward:

$$s_1 = w_1 x, s_2 = w_2 x$$

$$z_1 = \max(0, s_1), z_2 = \max(0, s_2)$$

$$a_1 = z_1 \cdot u_{1,1} + z_2 \cdot u_{2,1}, a_2 = z_1 \cdot u_{1,2} + z_2 \cdot u_{2,2}$$

$$f_j = \frac{e^{a_j}}{e^{a_1} + e^{a_2}} \quad \text{for } j=1,2$$

$$L = \frac{1}{2} [(f_1 - y_1)^2 + (f_2 - y_2)^2]$$

$$\text{Let } e_j = f_j - y_j$$

Backward:

1)

$$\frac{\partial L}{\partial f_j} = e_j$$

2)

$$\text{softmax Jacobian: } \frac{\partial f_i}{\partial a_j} = f_i (\delta_{ij} - f_j)$$

$$\text{do vector form with } f = [f_1, f_2]^T, e = [e_1, e_2]^T:$$

$$g_a = \frac{\partial L}{\partial a} = (\text{diag}(f) - f f^T) e$$

For 2 classes:

$$g_{21} = c_1 t_1 (1 - t_1) - c_2 t_1 t_2$$

$$g_{22} = c_2 t_2 (1 - t_2) - c_1 t_1 t_2$$

3)

$$\left| \frac{\partial \mathcal{L}}{\partial u_{kj}} = z_k g_{2j} \right|$$

$$\frac{\partial \mathcal{L}}{\partial u_{11}} = z_1 g_{21}$$

$$\frac{\partial \mathcal{L}}{\partial u_{12}} = z_1 g_{22}$$

$$\frac{\partial \mathcal{L}}{\partial u_{21}} = z_2 g_{21}$$

$$\frac{\partial \mathcal{L}}{\partial u_{22}} = z_2 g_{22}$$

4)

$$\frac{\partial \mathcal{L}}{\partial z_k} = u_{k1} g_{21} + u_{k2} g_{22}$$

5)

$$\text{ReLU: } 1[\bar{w}_k > 0]. \quad \frac{\partial \bar{w}_k}{\partial w_k} = x$$

$$\frac{\partial \mathcal{L}}{\partial w_k} = (u_{k1} g_{21} + u_{k2} g_{22}) 1[\bar{w}_k > 0] x$$

$$\text{So } \frac{\partial \mathcal{L}}{\partial w_1} = (u_{11} g_{21} + u_{12} g_{22}) 1[\bar{w}_1 > 0] x$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = (u_{21} g_{21} + u_{22} g_{22}) 1[\bar{w}_2 > 0] x$$