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Frattini subgroup of a finite group is nilpotent, the

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The Frattini subgroup of a finite group is http://planetmath.org/NilpotentGroupnilpotent.

Proof. Let $\Phi(G)$ denote the Frattini subgroup of a finite group G. Let S be a Sylow subgroup of $\Phi(G)$. Then by the Frattini argument, $G = \Phi(G)N_G(S) = \langle \Phi(G) \cup N_G(S) \rangle$. But the Frattini subgroup is finite and formed of non-generators, so it follows that $G = \langle N_G(S) \rangle = N_G(S)$. Thus S is normal in G, and therefore normal in $\Phi(G)$. The result now follows, as http://planetmath.org/ClassificationOfFiniteNilpotentGroupsany finite group whose Sylow subgroups are all normal is nilpotent. $\square$

In fact, the same proof shows that for any group G, if $\Phi(G)$ is finite then $\Phi(G)$ is nilpotent.