a semilattice is a commutative band

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This note explains how a semilattice is the same as a commutative band.

Let S be a semilattice, with partial order $<$ and each pair of elements x and y having a greatest lower bound $x \wedge y$. Then it is easy to see that the operation $\wedge$ defines a binary operation on S which makes it a commutative semigroup, and that every element is idempotent since $x \wedge x = x$.

Conversely, if S is such a semigroup, define $x \leq y$ iff $x = xy$. Again, it is easy to see that this defines a partial order on S, and that greatest lower bounds exist with respect to this partial order, and that in fact $x \wedge y = xy$.