proof of the converse of Lagrange's theorem for finite cyclic groups

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The following is a proof that, if G is a finite cyclic group and n is a nonnegative integer that is a divisor of $|G|$, then G has a subgroup of order n.

Proof. Let g be a generator of G. Then $|g| = |\langle g \rangle| = |G|$. Let $z \in \mathbb{Z}$ such that $nz = |G| = |g|$. Consider $\langle g^z \rangle$. Since $g \in G$, then $g^z \in G$. Thus, $\langle g^z \rangle \leq G$. Since $|\langle g^z \rangle| = |g^z| = \dfrac{|g|}{\gcd(z, |g|)} = \dfrac{nz}{\gcd(z, nz)} = \dfrac{nz}{z} = n$, it follows that $\langle g^z \rangle$ is a subgroup of G of order n. $\square$