# symmetric group is generated by adjacent transpositions

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**Theorem 1.** *The symmetric group on* $\{1, 2, \ldots, n\}$ *is generated by the permutations*

$$(1, 2), (2, 3), \ldots, (n - 1, n).$$

*Proof.* We proceed by induction on $n$. If $n = 2$, the theorem is trivially true because the the group only consists of the identity and a single transposition.

Suppose, then, that we know permutations of $n$ numbers are generated by transpositions of successive numbers. Let $\phi$ be a permutation of $\{1, 2, \ldots, n+1\}$. If $\phi(n+1) = n+1$, then the restriction of $\phi$ to $\{1, 2, \ldots, n\}$ is a permutation of $n$ numbers, hence, by hypothesis, it can be expressed as a product of transpositions.

Suppose that, in addition, $\phi(n + 1) = m$ with $m \neq n + 1$. Consider the following product of transpositions:

$$(nn + 1)(n - 1n) \cdots (m + 1m + 1)(mm + 1)$$

It is easy to see that acting upon $m$ with this product of transpositions produces $+1$. Therefore, acting upon $n + 1$ with the permutation

$$(nn + 1)(n - 1n) \cdots (m + 1m + 1)(mm + 1)\phi$$

produces $n + 1$. Hence, the restriction of this permutation to $\{1, 2, \ldots, n\}$ is a permutation of $n$ numbers, so, by hypothesis, it can be expressed as a product of transpositions. Since a transposition is its own inverse, it follows that $\phi$ may also be expressed as a product of transpositions. $\square$