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natural projection

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Proposition. If H is a normal subgroup of a group G , then the mapping

$$\varphi: G \rightarrow G/H \quad \text{where} \quad \varphi(x) = xH \quad \forall x \in G$$

is a surjective homomorphism whose kernel is H .

Proof. Because every coset appears as image, the mapping φ is surjective. It is also homomorphic, since for all elements x, y of G , one has

$$\varphi(xy) = (xy)H = xH \cdot yH = \varphi(x)\varphi(y).$$

The identity element of the factor group G/H is the coset $eH = H$, whence

$$\ker(\varphi) = \{x \in G : \varphi(x) = H\} = \{x \in G : xH = H\} = H.$$

The mapping φ in the proposition is called *natural projection* or *canonical homomorphism*.