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descending series

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Defines	hypoabelian
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Let G be a group.

A *descending series* of G is a family $(H_\alpha)_{\alpha \leq \beta}$ of subgroups of G , where β is an ordinal, such that $H_0 = G$ and $H_\beta = \{1\}$, and $H_{\alpha+1} \trianglelefteq H_\alpha$ for all $\alpha < \beta$, and

$$\bigcap_{\alpha < \delta} H_\alpha = H_\delta$$

whenever $\delta \leq \beta$ is a limit ordinal.

Note that this is a generalization of the concept of a subnormal series. Compare also the dual concept of an ascending series.

Given a descending series $(H_\alpha)_{\alpha \leq \beta}$, the subgroups H_α are called the *terms* of the series and the <http://planetmath.org/QuotientGroupquotients> $H_\alpha/H_{\alpha+1}$ are called the *factors* of the series.

A subgroup of G that is a term of some descending series of G is called a *descendant subgroup* of G .

A descending series of G in which all terms are normal in G is called a *descending normal series*.

Let \mathfrak{X} be a property of groups. A group is said to be *hypo- \mathfrak{X}* if it has a descending normal series whose factors all have property \mathfrak{X} . So, for example, a *hypoabelian group* is a group that has a descending normal series with abelian factors. Hypoabelian groups are sometimes called *SD-groups*; they are precisely the groups that have no non-trivial perfect subgroups, and they are also precisely the groups in which the transfinite derived series eventually reaches $\{1\}$.