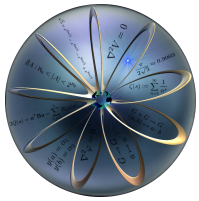

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order of products

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If a and b are elements of a group, then both ab and ba have always the same order.

Proof. Let e be the indentity element of the group. For $n > 1$, we have the `http://planetmath.org/Equivalent3`equivalent conditions

$$e \;=\; (ab)^n \;=\; \underbrace{(ab)(ab)\cdots(ab)}_{n} \;=\; a(ba)^{n-1}b,$$

$$a^{-1}b^{-1} \;=\; (ba)^{n-1},$$

$$(ba)^{-1} \;=\; (ba)^{n-1},$$

$$e \;=\; (ba)^n.$$

As for the infinite order, it makes the conditions false.

Note. More generally, all elements of any conjugacy class have the same order.