orthogonal group

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Defines	orthogonal transformation

Let Q be a non-degenerate symmetric bilinear form over the real vector space $\mathbb{R}^n$. A linear transformation $T\colon V \to V$ is said to *preserve* Q if $Q(Tx, Ty) = Q(x, y)$ for all vectors $x, y \in V$. The subgroup of the general linear group $\mathrm{GL}(V)$ consisting of all linear transformations that preserve Q is called the *orthogonal group* with respect to Q, and denoted $\mathrm{O}(n, Q)$.

If Q is also positive definite (i.e., Q is an inner product), then $\mathrm{O}(n, Q)$ is equivalent to the group of invertible linear transformations that preserve the standard inner product on $\mathbb{R}^n$, and in this case the group $\mathrm{O}(n, Q)$ is usually denoted $\mathrm{O}(n)$.

Elements of $\mathrm{O}(n)$ are called *orthogonal transformations*. One can show that a linear transformation T is an orthogonal transformation if and only if $T^{-1} = T^{\mathrm{T}}$ (i.e., the inverse of T equals the transpose of T).