unity of subring

Canonical name	UnityOfSubring
Date of creation	2013-03-22 14:49:40
Last modified on	2013-03-22 14:49:40
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	7
Author	pahio (2872)
Entry type	Theorem
Classification	msc 20-00
Classification	msc 16-00
Classification	msc 13-00
Related topic	UnitiesOfRingAndSubring
Related topic	CornerOfARing

Theorem. Let S be a proper subring of the ring R. If S has a non-zero unity u which is not unity of R, then u is a zero divisor of R.

Proof. Because u is not unity of R, there exists an element r of R such that $ru \neq r$. Then we have $(ru)u = r(uu) = ru$, which implies that $0 = (ru)u - ru = (ru - r) \cdot u$. Since neither $ru - r$ nor u is 0, the element u is a zero divisor in R.