generating set of a group

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Let G be a group.

A subset $X \subseteq G$ is said to *generate* G (or to be a *generating set* of G) if no proper subgroup of G contains X.

A subset $X \subseteq G$ generates G if and only if every element of G can be expressed as a product of elements of X and inverses of elements of X (taking the empty product to be the identity element). A subset $X \subseteq G$ is said to be *closed under inverses* if $x^{-1} \in X$ whenever $x \in X$; if a generating set X of G is closed under inverses, then every element of G is a product of elements of X.

A group that has a generating set with only one element is called a cyclic group. A group that has a generating set with only finitely many elements is called a finitely generated group.

If X is an arbitrary subset of G, then the subgroup of G *generated by* X, denoted by $\langle X \rangle$, is the smallest subgroup of G that contains X.

The *generating rank* of G is the minimum cardinality of a generating set of G. (This is sometimes just called the *rank* of G, but this can cause confusion with other meanings of the term *rank*.) If G is uncountable, then its generating rank is simply $|G|$.