# ideal of elements with finite order

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**Theorem.** The set of all elements of a ring, which have a finite order in the additive group of the ring, is a (two-sided) ideal of the ring.

*Proof.* Let $S$ be the set of the elements with finite order in the ring $R$. Denote by $o(x)$ the order of $x$. Take arbitrary elements $a$, $b$ of the set $S$.

If $\operatorname{lcm}(o(a), o(b)) = n = ko(a) = lo(b)$, then

$$n(a-b) = na - nb = ko(a)a - lo(b)b = k \cdot 0 - l \cdot 0 = 0 - 0 = 0.$$

Thus $o(a-b) \leqq n < \infty$ and so $a - b \in S$.

For any element $r$ of $R$ we have

$$o(a)(ra) = \underbrace{ra + ra + \ldots + ra}_{o(a)} = r(\underbrace{a + a + \ldots + a}_{o(a)}) = r(o(a)a) = r \cdot 0 = 0.$$

Therefore, $o(ra) \leqq o(a) < \infty$ and $ra \in S$. Similarly, $ar \in S$.

Since $S$ satisfies the conditions for an ideal, the theorem has been proven.