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normalizer

Canonical name	Normalizer
Date of creation	2013-03-22 12:36:53
Last modified on	2013-03-22 12:36:53
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	15
Author	yark (2760)
Entry type	Definition
Classification	msc 20A05
Synonym	normaliser
Related topic	Centralizer
Related topic	NormalSubgroup
Related topic	NormalClosure2
Defines	self-normalizing

Definitions

Let G be a group, and let $H \subseteq G$. The *normalizer* of H in G, written $N_G(H)$, is the set
$$\{g \in G \mid gHg^{-1} = H\}.$$

A subgroup H of G is said to be *self-normalizing* if $N_G(H) = H$.

Properties

$N_G(H)$ is always a subgroup of G, as it is the stabilizer of H under the action $(g, H) \mapsto gHg^{-1}$ of G on the set of all subsets of G (or on the set of all subgroups of G, if H is a subgroup).

If H is a subgroup of G, then $H \leq N_G(H)$.

If H is a subgroup of G, then H is a normal subgroup of $N_G(H)$; in fact, $N_G(H)$ is the largest subgroup of G of which H is a normal subgroup. In particular, if H is a subgroup of G, then H is normal in G if and only if $N_G(H) = G$.