idempotency

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If $(S, *)$ is a magma, then an element $x \in S$ is said to be *idempotent* if $x * x = x$. For example, every identity element is idempotent, and in a group this is the only idempotent element. An idempotent element is often just called an idempotent.

If every element of the magma $(S, *)$ is idempotent, then the binary operation $*$ (or the magma itself) is said to be idempotent. For example, the $\wedge$ and $\vee$ operations in a `http://planetmath.org/Lattice`lattice are idempotent, because $x \wedge x = x$ and $x \vee x = x$ for all x in the lattice.

A function $f \colon D \to D$ is said to be idempotent if $f \circ f = f$. (This is just a special case of the first definition above, the magma in question being $(D^D, \circ)$, the monoid of all functions from D to D with the operation of function composition.) In other words, f is idempotent if and only if repeated application of f has the same effect as a single application: $f(f(x)) = f(x)$ for all $x \in D$. An idempotent linear transformation from a vector space to itself is called a projection.