cyclic group

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Defines	infinite cyclic subgroup

A group is said to be *cyclic* if it is generated by a single element.

Suppose G is a cyclic group generated by $x \in G$. Then every element of G is equal to x^k for some $k \in \mathbb{Z}$. If G is infinite, then these x^k are all distinct, and G is isomorphic to the group $\mathbb{Z}$. If G has `http://planetmath.org/OrderGroup`finite order n, then every element of G can be expressed as x^k with $k \in \{0, \ldots, n-1\}$, and G is isomorphic to the quotient group $\mathbb{Z}/n\mathbb{Z}$.

Note that the isomorphisms mentioned in the previous paragraph imply that all cyclic groups of the same order are isomorphic to one another. The infinite cyclic group is sometimes written C_∞, and the finite cyclic group of order n is sometimes written C_n. However, when the cyclic groups are written additively, they are commonly represented by $\mathbb{Z}$ and $\mathbb{Z}/n\mathbb{Z}$.

While a cyclic group can, by definition, be generated by a single element, there are often a number of different elements that can be used as the generator: an infinite cyclic group has 2 generators, and a finite cyclic group of order n has $\phi(n)$ generators, where ϕ is the Euler totient function.

Some basic facts about cyclic groups:

- Every cyclic group is abelian.
- Every subgroup of a cyclic group is cyclic.
- Every quotient of a cyclic group is cyclic.
- Every group of prime order is cyclic. (This follows immediately from Lagrange's Theorem.)
- Every finite subgroup of the multiplicative group of a field is cyclic.