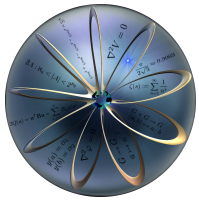

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triangle groups

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Consider the following group presentation:

$$\Delta(l,m,n) = \langle a,b,c : a^2, b^2, c^2, (ab)^l, (bc)^n, (ca)^m \rangle$$

where $l,m,n \in \mathbb{N}$.

A group with this presentation corresponds to a triangle; roughly, the generators are reflections in its sides and its angles are $\pi/l, \pi/m, \pi/n$.

Denote by $D(l,m,n)$ the subgroup of `http://planetmath.org/Coset`index 2 in $\Delta(l,m,n)$, corresponding to preservation of of the triangle.

The $D(l,m,n)$ are defined by the following presentation:

$$D(l,m,n) = \langle x,y : x^l, y^m, (xy)^n \rangle$$

Note that $D(l,m,n) \cong D(m,l,n) \cong D(n,m,l)$, so $D(l,m,n)$ is of the l,m,n.

Arising from the geometrical nature of these groups,

$$1/l + 1/m + 1/n > 1$$

is called the *spherical case*,

$$1/l + 1/m + 1/n = 1$$

is called the *Euclidean case*, and

$$1/l + 1/m + 1/n < 1$$

is called the *hyperbolic case*

Groups either of the form $\Delta(l,m,n)$ or $D(l,m,n)$ are referred to as *triangle groups*; groups of the form $D(l,m,n)$ are sometimes refered to as *von Dyck groups*.