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normal subgroup

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A subgroup H of a group G is *normal* if $aH = Ha$ for all $a \in G$. Equivalently, $H \subset G$ is normal if and only if $aHa^{-1} = H$ for all $a \in G$, i.e., if and only if each conjugacy class of G is either entirely inside H or entirely outside H.

The notation $H \trianglelefteq G$ or $H \triangleleft G$ is often used to denote that H is a normal subgroup of G.

The kernel $\ker(f)$ of any group homomorphism $f : G \longrightarrow G'$ is a normal subgroup of G. More surprisingly, the converse is also true: any normal subgroup $H \subset G$ is the kernel of some homomorphism (one of these being the projection map $\rho : G \longrightarrow G/H$, where G/H is the quotient group).