uniqueness of additive inverse in a ring

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Lemma. *Let R be a ring, and let a be any element of R. There exists a unique element b of R such that $a+b=0$, i.e. there is a unique* `http://planetmath.org/Ring`*additi inverse for a.*

Proof. Let a be an element of R. By definition of ring, there exists at least one `http://planetmath.org/Ring`additive inverse of a, call it b_1, so that $a+b_1=0$. Now, suppose b_2 is another additive inverse of a, i.e. another element of R such that

$$a + b_2 = 0$$

where 0 is the `http://planetmath.org/Ring`zero element of R. Let us show that $b_1 = b_2$. Using properties for a ring and the above equations for b_1 and b_2 yields

$$\begin{aligned}
b_1 &= b_1 + 0 \quad \text{(definition of zero)} \\
&= b_1 + (a + b_2) \quad (b_2 \text{ is an additive inverse of } a) \\
&= (b_1 + a) + b_2 \quad (\text{associativity in } R) \\
&= 0 + b_2 \quad (b_1 \text{ is an additive inverse of } a) \\
&= b_2 \quad \text{(definition of zero).}
\end{aligned}$$

Therefore, there is a unique additive inverse for a. □