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characterization of finite nilpotent groups

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Let G be a finite group. The following are equivalent:

1. G is nilpotent.
2. Every <http://planetmath.org/Subgroup> of G is subnormal.
3. Every proper subgroup of G is properly contained in its normalizer.
4. Every maximal subgroup of G is normal.
5. Every Sylow subgroup of G is normal.
6. G is a <http://planetmath.org/DirectProductAndRestrictedDirectProductOfGroups> direct product of <http://planetmath.org/PGroup> p -groups.

For proofs, see the article on finite nilpotent groups.

Condition 3 above is the normalizer condition.