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outer automorphism group

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The *outer automorphism group* of a group is the <http://planetmath.org/QuotientGroupquotient> of its automorphism group by its inner automorphism group:

$$\text{Out}(G) = \text{Aut}(G)/\text{Inn}(G).$$

There is some variance in terminology about “an outer automorphism.” Some authors define an outer automorphism as any automorphism $\phi : G \rightarrow G$ which is not an inner automorphism. In this way an outer automorphism still permutes the group G . However, an equally common definition is to declare an outer automorphism as an element of $\text{Out}(G)$ and consequently the elements are cosets of $\text{Inn}(G)\phi$, and not a map $\phi : G \rightarrow G$. In this definition it is not generally possible to treat the element as a permutation of G . In particular, the outer automorphism group of a general group G does not act on the group G in a natural way. An exception is when G is abelian so that $\text{Inn}(G) = 1$; thus, the elements of $\text{Out}(G)$ are canonically identified with those of $\text{Aut}(G)$ so we can speak of the action by outer automorphisms.