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double coset

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Let H and K be subgroups of a group G. An (H, K)-*double coset* is a set of the form HxK for some $x \in G$. Here HxK is defined in the obvious way as

$$HxK = \{hxk \mid h \in H \text{ and } k \in K\}.$$

Note that the $(H, \{1\})$-double cosets are just the right cosets of H, and the $(\{1\}, K)$-double cosets are just the left cosets of K. In general, every (H, K)-double coset is a union of right cosets of H, and also a union of left cosets of K.

The set of all (H, K)-double cosets is denoted $H \backslash G / K$. It is straightforward to show that $H \backslash G / K$ is a `http://planetmath.org/Partition`partition of G, that is, every element of G lies in exactly one (H, K)-double coset.

In contrast to the situation with ordinary `http://planetmath.org/Coset`cosets, the (H, K)-double cosets need not all be of the same cardinality. For example, if G is the `http://planetmath.org/SymmetricGroup`symmetric group S_3, and $H = \langle (1, 2) \rangle$ and $K = \langle (1, 3) \rangle$, then the two (H, K)-double cosets are $\{e, (1, 2), (1, 3), (1, 3, 2)\}$ and $\{(2, 3), (1, 2, 3)\}$.