conjugate stabilizer subgroups

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Let $\cdot$ be a right group action of G on a set M. Then

$$G_{\alpha \cdot g} = g^{-1} G_\alpha g$$

for any $\alpha \in M$ and $g \in G$. [1]

Proof:

$$x \in G_{\alpha \cdot g} \leftrightarrow \alpha \cdot (gx) = \alpha \cdot g \leftrightarrow \alpha \cdot (gxg^{-1}) = \alpha \leftrightarrow gxg^{-1} \in G_\alpha \leftrightarrow x \in g^{-1} \alpha g$$

and therefore $G_{\alpha \cdot g} = g^{-1} G_\alpha g$.

Thus all stabilizer subgroups for elements of the orbit $G(\alpha)$ of α are conjugate to G_α.

[1] G_α is the stabilizer subgroup of $\alpha \in M$.