quotient representations

Canonical name	QuotientRepresentations
Date of creation	2013-03-22 16:37:59
Last modified on	2013-03-22 16:37:59
Owner	rm50 (10146)
Last modified by	rm50 (10146)
Numerical id	6
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Entry type	Definition
Classification	msc 20C99

We assume that all representations (G-modules) are finite-dimensional.

Definition 1 *If N_1 and N_2 are G-modules over a field k (i.e. representations of G in N_1 and N_2), then a map $\varphi : N_1 \to N_2$ is a* G-map *if φ is k-linear and preserves the G-action, i.e. if*

$$\varphi(\sigma \cdot x) = \sigma \cdot \varphi(x)$$

G-maps have subrepresentations, also called G-submodules, as their kernel and image. To see this, let $\varphi : N_1 \to N_2$ be a G-map; let $M_1 \subset N_1$ and $M_2 \subset N_2$ be the kernel and image respectively of φ. M_1 is a submodule of N_1 if it is stable under the action of G, but

$$x \in M_1 \Rightarrow \varphi(\sigma \cdot x) = \sigma \cdot \varphi(x) = 0 \Rightarrow \sigma \cdot x \in M_1$$

M_2 is a submodule of N_2 if it is stable under the action of G, but

$$y = \varphi(x) \in M_2 \Rightarrow \sigma \cdot y = \sigma \cdot \varphi(x) = \varphi(\sigma \cdot x) \Rightarrow \sigma \cdot y \in M_2$$

Finally, we define the intuitive concept of a quotient G-module. Suppose $N' \subset N$ is a G-submodule. Then N/N' is a finite-dimensional vector space. We can define an action of G on N/N' via $\sigma(n + N') = \sigma(n) + \sigma(N') = \sigma(n) + N'$, so that $n + N'$ is well-defined under the action and N/N' is a G-module.