transfinite derived series

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The *transfinite derived series* of a group is an extension of its derived series, defined as follows. Let G be a group and let $G^{(0)}=G$. For each ordinal α let $G^{(\alpha+1)}$ be the derived subgroup of $G^{(\alpha)}$. For each limit ordinal δ let $G^{(\delta)}=\bigcap_{\alpha\in\delta}G^{(\alpha)}$.

Every member of the transfinite derived series of G is a fully invariant subgroup of G.

The transfinite derived series eventually terminates, that is, there is some ordinal α such that $G^{(\alpha+1)}=G^{(\alpha)}$. All remaining terms of the series are then equal to $G^{(\alpha)}$, which is called the *perfect radical* or *maximum perfect subgroup* of G, and is denoted $\mathcal{P}G$. As the name suggests, $\mathcal{P}G$ is perfect, and every perfect http://planetmath.org/Subgroupsubgroup of G is a subgroup of $\mathcal{P}G$. A group in which the perfect radical is trivial (that is, a group without any non-trivial perfect subgroups) is called a hypoabelian group. For any group G, the http://planetmath.org/QuotientGroupquotient $G/\mathcal{P}G$ is hypoabelian, and is sometimes called the *hypoabelianization* of G (by analogy with the abelianization).

A group G for which $G^{(n)}$ is trivial for some finite n is called a solvable group. A group G for which $G^{(\omega)}$ (the intersection of the derived series) is trivial is called a residually solvable group. http://planetmath.org/FreeGroupFree groups of rank greater than 1 are examples of residually solvable groups that are not solvable.