example of multiply transitive

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Theorem 1. 1. *The general linear group $GL(V)$ acts transitively on the set of points (1-dimensional subspaces) in the projective geometry $PG(V)$.*

2. *$PGL(V)$ is doubly transitive on the set of all of points in $PG(V)$.*

3. *$PGL(V)$ is not 3-transitive on the set of all points in $PG(V)$ if $\dim V \neq 2$.*

Proof. Evidently 2 implies 1. So suppose we have pairs of distinct points (P, Q) and (R, S). Then take $P = \langle x \rangle$, $Q = \langle y \rangle$, $R = \langle z \rangle$ and $S = \langle w \rangle$. As $P \neq Q$, x and y are linearly independent, just as z and w are. Therefore extending $\{x, y\}$ to a basis B and $\{z, w\}$ to a basis C, we know there is a linear transformation $f \in GL(V)$ taking B to C – consider the change of basis matrix. Therefore $GL(V)$ is 2-transitive.

Now suppose $\dim V \geq 2$. Then there exists a linearly indepedent set $\{x, y, z\}$ which gives three distinct non-collinear points (P, Q, R), $P = \langle x \rangle$, $Q = \langle y \rangle$ and $R = \langle z \rangle$. But then we also have three collinear points (P, Q, S) where $S = \langle x + y \rangle$. As $GL(V)$ prevserves the geometry of $PG(V)$, we cannot have a map in $GL(V)$ send (P, Q, R) to (P, Q, S). $\square$

Note that the action of $GL(V)$ on $PG(V)$ is not faithful so we use instead $PGL(V)$.