



Math for the people, by the people.

character

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Let $\rho : G \longrightarrow \text{GL}(V)$ be a finite dimensional representation of a group G (i.e., V is a finite dimensional vector space over its scalar field K). The *character* of ρ is the function $\chi_V : G \longrightarrow K$ defined by

$$\chi_V(g) := \text{Tr}(\rho(g))$$

where Tr is the trace function.

Properties:

- $\chi_V(g) = \chi_V(h)$ if g is conjugate to h in G . (Equivalently, a character is a class function on G .)
- If G is finite, the characters of the irreducible representations of G over the complex numbers form a basis of the vector space of all class functions on G (with pointwise addition and scalar multiplication).
- Over the complex numbers, the characters of the irreducible representations of G are orthonormal under the inner product

$$(\chi_1, \chi_2) := \frac{1}{|G|} \sum_{g \in G} \overline{\chi_1(g)} \chi_2(g)$$