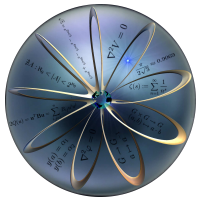

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## $p$-group

| | |
|---|---|
| Canonical name | Pgroup |
| Date of creation | 2013-03-22 14:53:08 |
| Last modified on | 2013-03-22 14:53:08 |
| Owner | yark (2760) |
| Last modified by | yark (2760) |
| Numerical id | 13 |
| Author | yark (2760) |
| Entry type | Definition |
| Classification | msc 20F50 |
| Synonym | p-group |
| Synonym | p-primary group |
| Synonym | primary group |
| Related topic | PGroup |
| Related topic | PExtension |
| Related topic | ProPGroup |
| Related topic | QuasicyclicGroup |
| Related topic | Subgroup |
| Defines | p-subgroup |
| Defines | primary component |
| Defines | p-primary |
| Defines | p-primary subgroup |
| Defines | primary subgroup |

# Primary groups

Let $p$ be a prime number. A *$p$-group* (or *$p$-primary group*) is a group in which the order of every element is a power of $p$. A group that is a $p$-group for some prime $p$ is also called a *primary group*.

Using Lagrange's Theorem and Cauchy's Theorem one may show that a finite group $G$ is a $p$-group if and only if $|G|$ is a power of $p$.

# Primary subgroups

A *$p$-subgroup* (or *$p$-primary subgroup*) of a group $G$ is a `http://planetmath.org/Subgroup`subgroup $H$ of $G$ such that $H$ is also a $p$-group. A group that is a $p$-subgroup for some prime $p$ is also called a *primary subgroup*.

It follows from Zorn's Lemma that every group has a maximal $p$-subgroup, for every prime $p$. The maximal $p$-subgroup need not be unique (though for abelian groups it is always unique, and is called the *$p$-primary component* of the abelian group). A maximal $p$-subgroup may, of course, be trivial. Non-trivial maximal $p$-subgroups of finite groups are called `http://planetmath.org/SylowPSubgroups` $p$-subgroups.