Ihara's theorem

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Let Γ be a discrete, torsion-free subgroup of $\mathrm{SL}_2\mathbb{Q}_p$ (where $\mathbb{Q}_p$ is the field of `http://planetmath.org/PAdicIntegers`p-adic numbers). Then Γ is free.

Proof, or a sketch thereof. There exists a $p+1$ regular tree X on which $\mathrm{SL}_2\mathbb{Q}_p$ acts, with stabilizer $\mathrm{SL}_2\mathbb{Z}_p$ (here, $\mathbb{Z}_p$ denotes the ring of `http://planetmath.org/PAdicIntegers`p-adic integers). Since $\mathbb{Z}_p$ is compact in its profinite topology, so is $\mathrm{SL}_2\mathbb{Z}_p$. Thus, $\mathrm{SL}_2\mathbb{Z}_p \cap \Gamma$ must be compact, discrete and torsion-free. Since compact and discrete implies finite, the only such group is trivial. Thus, Γ acts freely on X. Since groups acting freely on trees are free, Γ is free. $\square$