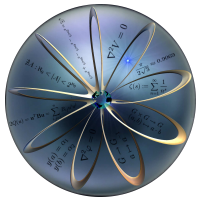

planetmath.org

Math for the people, by the people.

growth

Canonical name	Growth
Date of creation	2013-03-22 14:36:09
Last modified on	2013-03-22 14:36:09
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	8
Author	mathcam (2727)
Entry type	Definition
Classification	msc 20F99
Classification	msc 20E99
Related topic	GrowthOfExponentialFunction
Defines	polynomial growth
Defines	intermediate growth
Defines	exponential growth

Let G be a finitely generated group with generating set A (closed under inverses).

For $g = a_1 a_2 \dots a_m \in G$, $a_i \in A$, let $l(g)$ be the minimum value of m.

Define

$$\gamma(n) =\mid \{g \in G : l(g) \leq n\} \mid$$

.

The function γ is called the *growth function* for G with generating set A. If γ is either

(a) bounded above by a polynomial function,

(b) bounded below by an exponential function, or

(c) neither,

then this condition is preserved under changing the generating set for G. Respectively, then, G is said to have

(a) *polynomial growth*,

(b) *exponential growth*, or

(c) *intermediate growth*.

For a survey on the topic, see: R. I. Grigorchuk, On growth in group theory, Proceedings of the International Congress of Mathematicians, Kyoto 1990, Volume I, II (Math. Soc. Japan, 1991), pages 325 to 338.

Note that, as the generating set is assumed to be closed under inverses, we need only have G as a semigroup - as such, the above applies equally well in semigroup theory.