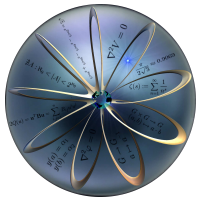

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example of matrix representations

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Sign representation of S_n

Let $G = S_n$ the n-th symmetric group, and consider $X(\sigma) = \text{sign}(\sigma)$ where σ is any permutation in S_n. That is, $\text{sign}(\sigma) = 1$ when σ is an even permutation, and $\text{sign}(\sigma) = -1$ when σ is an odd permutation.

The function X is a group homomorphism between S_n and $GL(\mathbb{C}) = \mathbb{C} \setminus \{0\}$ (that is invertible matrices of size 1×1, which is the set of non-zero complex numbers). And thus we say that $\mathbb{C} \setminus \{0\}$ carries a representation of the symmetric group.

Defining representation of S_n

For each $\sigma \in S_n$, let $X : S_n \to GL_n(\mathbb{C})$ the function given by $X(\sigma) = (a_{ij})_{n \times n}$ where (a_{ij}) is the *permutation matrix* given by

$$a_{ij} = \begin{cases} 1 & \text{if } \sigma(i) = j \\ 0 & \text{if } \sigma(i) \neq j \end{cases}$$

Such matrices are called permutation matrices because they are obtained permuting the colums of the identity matrix. The function so defined is then a group homomorphism, and thus $GL_n(\mathbb{C})$ carries a representation of the symmetric group.