locally nilpotent group

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Definition

A *locally nilpotent group* is a group in which every finitely generated subgroup is nilpotent.

Examples

All nilpotent groups are locally nilpotent, because subgroups of nilpotent groups are nilpotent.

An example of a locally nilpotent group that is not nilpotent is $\mathrm{Dih}(\mathbb{Z}(2^\infty))$, the generalized dihedral group formed from the quasicyclic `http://planetmath.org/PGroup42`-group $\mathbb{Z}(2^\infty)$.

The Fitting subgroup of any group is locally nilpotent.

All N-groups are locally nilpotent. More generally, all Gruenberg groups are locally nilpotent.

Properties

Any subgroup or `http://planetmath.org/QuotientGroup`quotient of a locally nilpotent group is locally nilpotent. Restricted direct products of locally nilpotent groups are locally nilpotent.

For each prime p, the elements of p-power order in a locally nilpotent group form a fully invariant subgroup (the maximal `http://planetmath.org/PGroup4`p-subgroup). The elements of finite order in a locally nilpotent group also form a fully invariant subgroup (the torsion subgroup), which is the restricted direct product of the maximal p-subgroups. (This generalizes the fact that a finite nilpotent group is the direct product of its Sylow subgroups.)

Every group G has a unique maximal locally nilpotent normal subgroup. This subgroup is called the *Hirsch-Plotkin radical*, or *locally nilpotent radical*, and is often denoted $\mathrm{HP}(G)$. If G is finite (or, more generally, satisfies the maximal condition), then the Hirsch-Plotkin radical is the same as the Fitting subgroup, and is nilpotent.