indecomposable group

Canonical name	IndecomposableGroup
Date of creation	2013-03-22 15:23:46
Last modified on	2013-03-22 15:23:46
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	8
Author	CWoo (3771)
Entry type	Definition
Classification	msc 20-00
Synonym	indecomposable
Related topic	KrullSchmidtTheorem
Defines	decomposable
Defines	indecomposable module

By definition, an *indecomposable group* is a nontrivial group that cannot be expressed as the internal direct product of two proper normal subgroups. A group that is not indecomposable is called, predictably enough, *decomposable*.

The analogous concept exists in module theory. An indecomposable module is a nonzero module that cannot be expressed as the direct sum of two nonzero submodules.

The following examples are left as exercises for the reader.

1. Every simple group is indecomposable.

2. If p is prime and n is any positive integer, then the additive group $\mathbb{Z}/p^n\mathbb{Z}$ is indecomposable. Hence, not every indecomposable group is simple.

3. The additive groups $\mathbb{Z}$ and $\mathbb{Q}$ are indecomposable, but the additive group $\mathbb{R}$ is decomposable.

4. If m and n are relatively prime integers (and both greater than one), then the additive group $\mathbb{Z}/mn\mathbb{Z}$ is decomposable.

5. Every finitely generated abelian group can be expressed as the direct sum of finitely many indecomposable groups. These summands are uniquely determined up to isomorphism.

References.

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