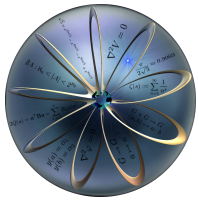

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direct products of homomorphisms

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Assume that $\{f_i : G_i \to H_i\}_{i\in I}$ is a family of homomorphisms between groups. Then we can define the *Cartesian product* (or *unrestricted direct product*) of this family as a homomorphism

$$\prod_{i\in I} f_i : \prod_{i\in I} G_i \to \prod_{i\in I} H_i$$

such that

$$\Big(\prod_{i\in I} f_i\Big)(g)(j) = f_j(g(j))$$

for each $g \in \prod_{i\in I} G_i$ and $j \in I$.

One can easily show that $\prod_{i\in I} f_i$ is a group homomorphism. Moreover it is clear that

$$\Big(\prod_{i\in I} f_i\Big)(\bigoplus_{i\in I} G_i) \subseteq \bigoplus_{i\in I} H_i,$$

so $\prod_{i\in I} f_i$ induces a homomorphism

$$\bigoplus_{i\in I} f_i : \bigoplus_{i\in I} G_i \to \bigoplus_{i\in I} H_i,$$

which is a restriction of $\prod_{i\in I} f_i$ to $\bigoplus_{i\in I} G_i$. This homomorphism is called the *direct product* (or *restricted direct product*) of $\{f_i : G_i \to H_i\}_{i\in I}$.