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Maschke's theorem

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Let G be a finite group, and k a field of characteristic not dividing $|G|$. Then any representation V of G over k is completely reducible.

Proof. We need only show that any subrepresentation has a complement, and the result follows by induction.

Let V be a representation of G and W a subrepresentation. Let $\pi : V \to W$ be an arbitrary projection, and let

$$\pi'(v) = \frac{1}{|G|} \sum_{g \in G} g^{-1} \pi(gv)$$

This map is obviously G-equivariant, and is the identity on W, and its image is contained in W, since W is invariant under G. Thus it is an equivariant projection to W, and its kernel is a complement to W. $\square$