Green's equivalences

Canonical name	GreensEquivalences
Date of creation	2013-03-22 14:23:12
Last modified on	2013-03-22 14:23:12
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	9
Author	mathcam (2727)
Entry type	Definition
Classification	msc 20Mxx
Synonym	Green's relations

Let S be a semigroup. Green's equivalences are five http://planetmath.org/EquivalenceRela
on S: $\mathcal{L}, \mathcal{R}, \mathcal{H}, \mathcal{D}, \mathcal{J}$

For all $x, y \in S$,

$x\mathcal{L}y$ if $S^1x = S^1y$, i.e. $sx = y, ty = x$ for some $s, t \in S^1$

$x\mathcal{R}y$ if $xS^1 = yS^1$, i.e. $xs = y, yt = x$ for some $s, t \in S^1$

$x\mathcal{J}y$ if $S^1xS^1 = S^1yS^1$, i.e. $sxt = y, uyv = x$ for some $s, t, u, v \in S^1$

$x\mathcal{H}y$ if $x\mathcal{L}y$ and $x\mathcal{R}y$, i.e. $\mathcal{H} = \mathcal{L} \cap \mathcal{R}$

$x\mathcal{D}y$ if $\exists z \in S$ such that $x\mathcal{L}z$ and $z\mathcal{R}y$, i.e. $\mathcal{D} = \mathcal{L} \circ \mathcal{R}$

It is clear that $\mathcal{H} \subseteq \mathcal{L}, \mathcal{H} \subseteq \mathcal{R}, \mathcal{L} \subseteq \mathcal{D}, \mathcal{R} \subseteq \mathcal{D}, \mathcal{D} \subseteq \mathcal{J}$

These play a fundamental role in understanding the of semigroups.