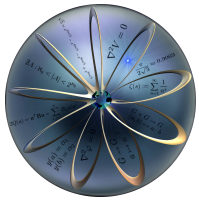

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examples of non-commutative operations

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A standard example of a non-commutative operation is matrix multiplication. Consider the following two integer matrices:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

If we compute AB we get

$$AB = \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix}$$

but if we compute BA we have

$$BA = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}.$$

Since $AB \neq BA$ we conclude that matrix product is not commutative.

Operations do not necessarily have to operate on numbers. Another classic example is function composition. Let f and g be real functions given by

$$f(x) = x^2, \qquad g(x) = 2x.$$

We see that

$$(f \circ g)(x) = f(g(x)) = (2x)^2 = 4x^2,$$

but

$$(g \circ f)(x) = g(f(x)) = 2(x^2) = 2x^2.$$

Since we got different functions, we conclude that function composition is not commutative.