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abelianization

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The *abelianization* of a group G is $G/[G, G]$, the <http://planetmath.org/QuotientGroupquotient> of G by its derived subgroup.

The abelianization of G is the largest abelian quotient of G , in the sense that if N is a normal subgroup of G then G/N is abelian if and only if $[G, G] \subseteq N$. In particular, every abelian quotient of G is a homomorphic image of $G/[G, G]$.

If A is an abelian group and $\phi: G \rightarrow A$ is a <http://planetmath.org/GroupHomomorphismhomomorphism> then there is a unique homomorphism $\psi: G/[G, G] \rightarrow A$ such that $\psi \circ \pi = \phi$, where $\pi: G \rightarrow G/[G, G]$ is the canonical projection.