homogeneous group

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Defines	homomorphism of homogeneous groups

A *homogeneous group* is a set G together with a map $() : G \times G \times G \to G$ satisfying:

i)$(a, a, b) = b$

ii)$(a, b, b) = a$

iii)$((a, b, c), d, e) = (a, b, (c, d, e))$

for all $a, b, c, d, e \in G$.

A map $f : G \to H$ of homogeneous groups is a homomorphism if it $f(a, b, c) = (fa, fb, fc)$, for all $a, b, c \in G$.

A non-empty homogeneous group is essentially a group, as given any $x \in G$, we may define the following product on G:

$ab = (a, x, b)$.

This gives G the of a group with identity x. The choice of x does not affect the isomorphism class of the group obtained.

One may recover a homogeneous group from a group obtained this way, by setting

$(a, b, c) = ab^{-1}c$.

Also, every group may be obtained from a homogeneous group.

Homogeneous groups are homogeneous: Given $a, b \in G$ we have a homomorphism f taking a to b, given by $fx = (x, a, b)$.

In this way homogeneous groups differ from groups, as whilst often used to describe symmetry, groups themselves have a distinct element: the identity.

Also the definition of homogeneous groups is given purely in of identities, and does not exclude the empty set.