proof of Cauchy's theorem in abelian case

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Suppose G is abelian and the order of G is h. Let $g_1, g_2, \ldots, g_h$ be the elements of G, and for $i = 1, \ldots, h$, let a_i be the order of g_i.

Consider the direct sum

$$H = \bigoplus_{i=1}^{h} \mathbb{Z}/a_i\mathbb{Z}.$$

The order of H is obviously $a_1 a_2 \cdots a_h$. We can define a group homomorphism θ from H to G by

$$(x_1, \ldots, x_h) \mapsto g_1^{x_1} \cdots g_h^{x_h}.$$

θ is certainly surjective. So $|H| = |G| \cdot |\ker(\theta)|$. Since p is a prime factor of G, p divides —H—, and therefore must divide one of the a_i's, say a_1. Then $g_1^{a_1/p}$ is an element of order p.