exponentiation

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Related topic	ContinuityOfNaturalPower
Defines	power law
Defines	power of product

- In the entry general associativity, the notion of the *power* a^n for elements a of a set having an associative binary operation "$\cdot$" and for positive integers n as http://planetmath.org/GeneralPowerexponents was defined as a generalisation of the operation. Then the two *power laws*

  $$a^m \cdot a^n \;=\; a^{m+n}, \quad (a^m)^n \;=\; a^{mn}$$

  are . For the validity of the third well-known power law,

  $$(a \cdot b)^n \;=\; a^n \cdot b^n,$$

  the law of *power of product*, the commutativity of the operation is needed.

  Example. In the symmetric group S_3, where the group operation is not commutative, we get different results from

  $$[(123)(13)]^2 \;=\; (23)^2 \;=\; (1)$$

  and

  $$(123)^2(13)^2 \;=\; (132)(1) \;=\; (132)$$

  (note that in these "products", which compositions of mappings, the latter "factor" acts first).

- Extending the power notion for zero and negative integer exponents requires the existence of http://planetmath.org/node/10539neutral and inverse elements (e and a^{-1}):

  $$a^0 \;:=\; e, \qquad a^{-n} \;:=\; (a^{-1})^n$$

  The two first power laws then remain in for all integer exponents, and if the operation is commutative, also the .

When the operation in question is the multiplication of real or complex numbers, the power notion may be extended for other than integer exponents.

- One step is to introduce http://planetmath.org/FractionalNumberfractional exponents by using http://planetmath.org/NthRootroots; see the fraction power.

- The following step would be the irrational exponents, which are in the power functions. The irrational exponents are possible to introduce by utilizing the exponential function and logarithms; another way would be to define a^{ϱ} as limit of a sequence

  $$a^{r_1},\, a^{r_2},\, \ldots$$

  where the limit of the rational number sequence $r_1,\, r_2,\, \ldots$ is ϱ. The sequence $a^{r_1},\, a^{r_2},\, \ldots$ may be shown to be a Cauchy sequence.

- The last step were the imaginary (non-real complex) exponents μ, when also the base of the power may be other than a positive real number; the one gets the so-called *general power*.