Wagner-Preston representation theorem

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Defines	Wagner-Preston representation

Let S be an inverse semigroup and X a set. An inverse semigroup homomorphism $\phi : S \to \mathfrak{I}(X)$, where $\mathfrak{I}(X)$ denotes the symmetric inverse semigroup, is called a *representation* of S by bijective partial maps on X. The representation is said to be *faithful* if ϕ is a monomorphism, i.e. it is injective.

Given $s \in S$, we define $\rho_s \in \mathfrak{I}(S)$ as the bijective partial map with domain

$$\mathrm{dom}(\rho_s) = Ss^{-1} = \left\{ ts^{-1} \,|\, t \in S \right\}$$

and defined by

$$\rho_s(t) = ts, \quad \forall t \in \mathrm{dom}(\rho_s).$$

Then the map $s \mapsto \rho_s$ is a representation called the *Wagner-Preston representation* of S. The following result, due to Wagner and Preston, is analogous to the Cayley representation theorem for groups.

Theorem 1 (Wagner-Preston representation theorem) *The Wagner-Preston representation of an inverse semigroup is faithful.*

References

[1] N. Petrich, *Inverse Semigroups*, Wiley, New York, 1984.

[2] G.B. Preston, *Representation of inverse semi-groups*, J. London Math. Soc. 29 (1954), 411-419.