ideal

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Defines	principal ideal
Defines	principal left ideal
Defines	principal right ideal

Let S be a semigroup. An *ideal* of S is a non-empty subset of S which is closed under multiplication on either side by elements of S. Formally, I is an ideal of S if I is non-empty, and for all $x \in I$ and $s \in S$, we have $sx \in I$ and $xs \in I$.

One-sided ideals are defined similarly. A non-empty subset A of S is a *left ideal* (resp. *right ideal*) of S if for all $a \in A$ and $s \in S$, we have $sa \in A$ (resp. $as \in A$).

A *principal left ideal* of S is a left ideal generated by a single element. If $a \in S$, then the principal left ideal of S generated by a is $S^1 a = Sa \cup \{a\}$. (The notation S^1 is explained `http://planetmath.org/AdjoiningAnIdentityToASemigroup3`here.)

Similarly, the *principal right ideal* generated by a is $aS^1 = aS \cup \{a\}$.

The notation $L(a)$ and $R(a)$ are also common for the principal left and right ideals generated by a respectively.

A *principal ideal* of S is an ideal generated by a single element. The ideal generated by a is

$$S^1 a S^1 = SaS \cup Sa \cup aS \cup \{a\}.$$

The notation $J(a) = S^1 a S^1$ is also common.