



Math for the people, by the people.

(p, q) shuffle

Canonical name	pqShuffle
Date of creation	2013-03-22 13:33:59
Last modified on	2013-03-22 13:33:59
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Last modified by	mathcam (2727)
Numerical id	7
Author	mathcam (2727)
Entry type	Definition
Classification	msc 20B99
Classification	msc 05A05
Synonym	shuffle
Related topic	ShuffleOfLanguages

Definition. Let p and q be positive natural numbers. Further, let $S(k)$ be the set of permutations of the numbers $\{1, \dots, k\}$. A permutation $\tau \in S(p+q)$ is a (p, q) shuffle if

$$\begin{aligned}\tau(1) &< \dots < \tau(p), \\ \tau(p+1) &< \dots < \tau(p+q).\end{aligned}$$

The set of all (p, q) shuffles is denoted by $S(p, q)$.

It is clear that $S(p, q) \subset S(p+q)$. Since a (p, q) shuffle is completely determined by how the p first elements are mapped, the cardinality of $S(p, q)$ is $\binom{p+q}{p}$. The wedge product of a p -form and a q -form can be defined as a sum over (p, q) shuffles.