normal closure

Canonical name	NormalClosure1
Date of creation	2013-03-22 14:41:50
Last modified on	2013-03-22 14:41:50
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	9
Author	yark (2760)
Entry type	Definition
Classification	msc 20A05
Synonym	normal subgroup generated by
Synonym	conjugate closure
Synonym	smallest normal subgroup containing
Related topic	Normalizer
Related topic	CoreOfASubgroup
Defines	nearly normal

Let S be a subset of a group G. The *normal closure* of S in G is the intersection of all normal subgroups of G that contain S, that is

$$\bigcap_{S \subseteq N \trianglelefteq G} N.$$

The normal closure of S is the smallest normal subgroup of G that contains S, and so is also called the *normal subgroup generated by S*.

It is not difficult to show that the normal closure of S is the subgroup generated by all the conjugates of elements of S.

The normal closure of S in G is variously denoted by $\langle S^G \rangle$ or $\langle S \rangle^G$ or S^G.

If H is a subgroup of G, and H is of finite index in its normal closure, then H is said to be *nearly normal*.