Kleene star

Canonical name	KleeneStar
Date of creation	2013-03-22 12:26:58
Last modified on	2013-03-22 12:26:58
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	9
Author	CWoo (3771)
Entry type	Definition
Classification	msc 20M35
Classification	msc 68Q70
Synonym	asterate
Related topic	Alphabet
Related topic	String
Related topic	RegularExpression
Related topic	KleeneAlgebra
Related topic	Language
Related topic	Convolution2
Related topic	WeightStrings
Related topic	WeightEnumerator
Defines	Kleene plus

If Σ is an alphabet (a set of symbols), then the Kleene star of Σ, denoted Σ^*, is the set of all strings of finite length consisting of symbols in Σ, including the empty string λ. * is also called the *asterate*.

If S is a set of strings, then the Kleene star of S, denoted S^*, is the smallest superset of S that contains λ and is closed under the string concatenation operation. That is, S^* is the set of all strings that can be generated by concatenating zero or more strings in S.

The definition of Kleene star can be generalized so that it operates on any monoid $(M, +\!\!+)$, where $+\!\!+$ is a binary operation on the set M. If e is the identity element of $(M, +\!\!+)$ and S is a subset of M, then S^* is the smallest superset of S that contains e and is closed under $+\!\!+$.

Examples

- $\varnothing^* = \{\lambda\}$, since there are no strings of finite length consisting of symbols in $\varnothing$, so λ is the only element in $\varnothing^*$.
- If $E = \{\lambda\}$, then $E^* = E$, since $\lambda a = a\lambda = a$ by definition, so $\lambda\lambda = \lambda$.
- If $A = \{a\}$, then $A^* = \{\lambda, a, aa, aaa, \ldots\}$.
- If $\Sigma = \{a, b\}$, then $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, \ldots\}$
- If $S = \{ab, cd\}$, then $S^* = \{\lambda, ab, cd, abab, abcd, cdab, cdcd, ababab, \ldots\}$

For any set S, S^* is the free monoid generated by S.

Remark. There is an associated operation, called the *Kleene plus*, is defined for any set S, such that S^+ is the smallest set containing S such that S^+ is closed under the concatenation. In other words, $S^+ = S^* - \{\lambda\}$.