subcommutative

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Defines	left subcommutative
Defines	right subcommutative

A semigroup $(S,\,\cdot)$ is said to be *left subcommutative* if for any two of its elements a and b, there exists its element c such that

$$ab = ca. \tag{1}$$

A semigroup $(S,\,\cdot)$ is said to be *right subcommutative* if for any two of its elements a and b, there exists its element d such that

$$ab = bd. \tag{2}$$

If S is both left subcommutative and right subcommutative, it is *subcommutative*.

The commutativity is a special case of all the three kinds of subcommutativity.

Example 1. The following operation table defines a right subcommutative semigroup $\{0,\,1,\,2,\,3\}$ which is not left subcommutative (e.g. $0\cdot 3 = 2 \neq c\cdot 0$):

$\cdot$	0	1	2	3
0	0	0	2	2
1	0	1	2	3
2	0	0	2	2
3	0	1	2	3

Example 2. The group of the square matrices over a field is both left and right subcommutative (but not commutative), since the equations (1) and (2) are satisfied by

$$c \;=\; aba^{-1} \quad \text{and} \quad d \;=\; b^{-1}ab.$$

Remark. One uses the above also for a ring $(S,\,+,\,\cdot)$ if its multiplicative semigroup $(S,\,\cdot)$ satisfies the corresponding requirements.

References

[1] S. Lajos: "On $(m,\,n)$-ideals in subcommutative semigroups". – *Elemente der Mathematik* **24** (1969).

[2] V. P. Elizarov: “Subcommutative Q-rings”. – *Mathematical notes* **2** (1967).