locally finite group

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A group G is *locally finite* if any finitely generated subgroup of G is finite.

A locally finite group is a torsion group. The converse, also known as the Burnside Problem, is not true. Burnside, however, did show that if a matrix group is torsion, then it is locally finite.

(Kaplansky) If G is a group such that for a normal subgroup N of G, N and G/N are locally finite, then G is locally finite.

A solvable torsion group is locally finite. To see this, let $G = G_0 \supset G_1 \supset \cdots \supset G_n = (1)$ be a composition series for G. We have that each G_{i+1} is normal in G_i and the factor group G_i/G_{i+1} is abelian. Because G is a torsion group, so is the factor group G_i/G_{i+1}. Clearly an abelian torsion group is locally finite. By applying the fact in the previous paragraph for each step in the composition series, we see that G must be locally finite.

References

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