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identity element is unique

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Theorem. The identity element of a monoid is unique.

Proof. Let e and e' be identity elements of a monoid (G, \cdot) . Since e is an identity element, one has $e \cdot e' = e'$. Since e' is an identity element, one has also $e \cdot e' = e$. Thus

$$e' = e \cdot e' = e,$$

i.e. both identity elements are the same (in inferring this result from the two first equations, one has used the <http://planetmath.org/Symmetry> and transitivity of the equality relation).

Note. The theorem also proves the uniqueness of e.g. the identity element of a group, the <http://planetmath.org/Ringadditive> identity 0 of a ring or a field, and the <http://planetmath.org/Ringmultiplicative> identity 1 of a field.