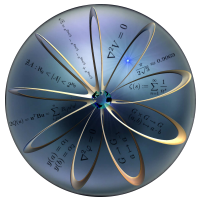

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torsion

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Defines	torsion subgroup
Defines	torsion free

The **torsion** of a group G is the set

$$\mathrm{Tor}(G) = \{g \in G : g^n = e \text{ for some } n \in \mathbb{N}\}.$$

A group is said to be **torsion-free** if $\mathrm{Tor}(G) = \{e\}$, i.e. the torsion consists only of the identity element.

If G is abelian (or, more generally, locally nilpotent) then $\mathrm{Tor}(G)$ is a subgroup (the **torsion subgroup**) of G. Whenever $\mathrm{Tor}(G)$ is a subgroup of G, then it is fully invariant and $G/\mathrm{Tor}(G)$ is torsion-free.

Example 1 (Torsion of a finite group)
For any finite group G, $\mathrm{Tor}(G) = G$.

Example 2 (Torsion of the circle group)
The torsion of the circle group $\mathbb{R}/\mathbb{Z}$ *is* $\mathrm{Tor}(\mathbb{R}/\mathbb{Z}) = \mathbb{Q}/\mathbb{Z}$.