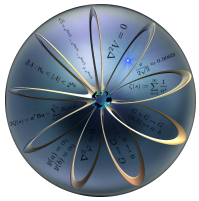

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inverse of inverse in a group

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Let $(G, *)$ be a group. We aim to prove that $(a^{-1})^{-1} = a$ for every $a \in G$. That is, the inverse of the inverse of a group element is the element itself.

By definition $a * a^{-1} = a^{-1} * a = e$, where e is the identity in G. Reinterpreting this equation we can read it as saying that a is the inverse of a^{-1}.

In fact, consider $b = a^{-1}$, the equation can be written $a * b = b * a = e$ and thus a is the inverse of $b = a^{-1}$.