virtually cyclic group

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Defines	virtually cyclic
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Defines	finite-by-cyclic group
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A *virtually cyclic group* is a group that has a cyclic subgroup of finite http://planetmath.org/Cosetindex. Every virtually cyclic group in fact has a normal cyclic subgroup of finite index (namely, the core of any cyclic subgroup of finite index), and virtually cyclic groups are therefore also known as *cyclic-by-finite groups*.

A finite-by-cyclic group (that is, a group G with a finite normal subgroup N such that G/N is cyclic) is always virtually cyclic. To see this, note that a finite-by-cyclic group is either finite, in which case it is certainly virtually cyclic, or it is finite-by-$\mathbb{Z}$, in which case the http://planetmath.org/GroupExtensionextension http://planetmath.org/SemidirectProductOfGroupssplits.

Finite-by-http://planetmath.org/DihedralGroupdihedral groups are also virtually cyclic. In fact, we have the following classification theorem:[?][?]

Theorem. *Groups of the following three types are all virtually cyclic. Moreover, every virtually cyclic group is of exactly one of these three types.*

- *finite*
- *finite-by-(infinite cyclic)*
- *finite-by-(infinite dihedral)*

As an immediate corollary we have the following result:[?]

Corollary. *Every torsion-free virtually cyclic group is either trivial or infinite cyclic.*

References

[1] Lemma 11.4 (pages 102–103) in: John Hempel, *3-Manifolds*, American Mathematical Society, 2004, ISBN 0821836951.

[2] Page 137 of: Alejandro Adem, Jesus Gonzalez, Guillermo Pastor (eds.), Recent developments in algebraic topology — A conference to celebrate Sam Gitler's 70th birthday, San Miguel de Allende, Mexico, December 3–6, 2003.

[3] Lemma 3.2 (pages 225–226) of: Dugald Macpherson, *Permutation Groups Whose Subgroups Have Just Finitely Many Orbits* (pages 221–230 in: W. Charles Holland (ed.) *Ordered Groups and Infinite Permutation Groups*, Kluwer Academic Publishers, 1996).