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## Burnside basis theorem

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**Theorem 1** *If  $G$  is a finite  $p$ -group, then  $\text{Frat } G = G'G^p$ , where  $\text{Frat } G$  is the Frattini subgroup,  $G'$  the commutator subgroup, and  $G^p$  is the subgroup generated by  $p$ -th powers.*

The theorem implies that  $G/\text{Frat } G$  is elementary abelian, and thus has a minimal generating set of exactly  $n$  elements, where  $|G : \text{Frat } G| = p^n$ . Since any lift of such a generating set also generates  $G$  (by the non-generating property of the Frattini subgroup), the smallest generating set of  $G$  also has  $n$  elements.

The theorem also holds for profinite  $p$ -groups (inverse limit of finite  $p$ -groups).