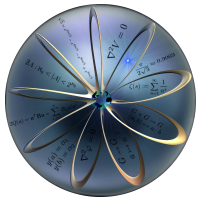

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semilattice

Canonical name	Semilattice
Date of creation	2013-03-22 12:57:23
Last modified on	2013-03-22 12:57:23
Owner	mclase (549)
Last modified by	mclase (549)
Numerical id	6
Author	mclase (549)
Entry type	Definition
Classification	msc 20M99
Classification	msc 06A12
Related topic	Lattice
Related topic	Poset
Related topic	Idempotent2
Related topic	Join
Related topic	Meet
Related topic	CompleteSemilattice
Defines	lower semilattice
Defines	upper semilattice

A *lower semilattice* is a partially ordered set S in which each pair of elements has a greatest lower bound.

A *upper semilattice* is a partially ordered set S in which each pair of elements has a least upper bound.

Note that it is not normally necessary to distinguish lower from upper semilattices, because one may be converted to the other by reversing the partial order. It is normal practise to refer to either structure as a *semilattice* and it should be clear from the context whether greatest lower bounds or least upper bounds exist.

Alternatively, a semilattice can be considered to be a commutative band, that is a semigroup which is commutative, and in which every element is idempotent. In this context, semilattices are important elements of semigroup theory and play a key role in the structure theory of commutative semigroups.

A partially ordered set which is both a lower semilattice and an upper semilattice is a lattice.