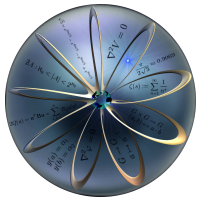

planetmath.org

Math for the people, by the people.

proof of Lagrange's theorem

Canonical name	ProofOfLagrangesTheorem
Date of creation	2013-03-22 12:15:47
Last modified on	2013-03-22 12:15:47
Owner	akrowne (2)
Last modified by	akrowne (2)
Numerical id	6
Author	akrowne (2)
Entry type	Proof
Classification	msc 20D99

We know that the cosets Hg form a partition of G (see the coset entry for proof of this.) Since G is finite, we know it can be completely decomposed into a finite number of cosets. Call this number n. We denote the ith coset by Ha_i and write G as

$$G = Ha_1 \cup Ha_2 \cup \cdots \cup Ha_n$$

since each coset has $|H|$ elements, we have

$$|G| = |H| \cdot n$$

and so $|H|$ divides $|G|$, which proves Lagrange's theorem. $\square$