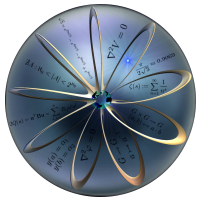

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## projective special linear group

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|---|---|
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**Definition.** *Let $V$ be a vector space over a field $F$ and let $\mathrm{SL}(V)$ be the special linear group. Let $Z$ be the center of $\mathrm{SL}(V)$. The* **projective special linear group** *associated to $V$ is the quotient group $\mathrm{SL}(V)/Z$ and is usually denoted by $\mathrm{PSL}(V)$.*

*When $V$ is a finite dimensional vector space over $F$ (of dimension $n$) then we write $\mathrm{PSL}(n, F)$ or $\mathrm{PSL}_n(F)$. We also identify the linear transformations of $V$ with $n \times n$ matrices, so* PSL *may be regarded as a quotient of the group of matrices $\mathrm{SL}(n, F)$ by its center.*

Note: see the entry on projective space for the origin of the terminology.

**Theorem 1.** *The center $Z$ of $\mathrm{SL}(n, F)$ is the group of all scalar matrices $\lambda \cdot \mathrm{Id}$ where $\lambda$ is an $n$th root of unity in $F$.*

*In particular, for $n = 2$, $Z = \{\pm \mathrm{Id}\}$ and:*

$$\mathrm{PSL}(2, F) = \mathrm{SL}(2, F)/\{\pm \mathrm{Id}\}.$$

As a consequence of the previous theorem, we obtain:

**Theorem 2.** *For $n \geq 3$, $\mathrm{PSL}(n, F)$ is a simple group. Furthermore, if $\mathbb{F}$ is a finite field then the groups*

$$\mathrm{PSL}(n, \mathbb{F}) = \mathrm{SL}(n, \mathbb{F})/Z, \quad n \geq 2$$

*are all finite simple groups, except for $n = 2$ and $\mathbb{F} = \mathbb{F}_2, \mathbb{F}_3$.*

# References


[1] S. Lang, *Algebra*, Springer-Verlag, New York.

[2] D. Dummit, R. Foote, *Abstract Algebra*, Second Edition, Wiley.