semisimple group

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In group theory the use of the phrase *semi-simple group* is used sparingly. Standard texts on group theory including [**?**, **?**] avoid the term altogether. Other texts provide precise definitions which are nevertheless not equivalent [**?**, **?**]. In general it is preferable to use other terms to describe the class of groups being considered as there is no uniform convention. However, below is a list of possible uses of for the phrase *semi-simple group*.

1. A group is *semi-simple* if it has no non-trivial normal abelian subgroups [**?**, p. 89].

2. A group G is *semi-simple* if $G' = G$ and $G/Z(G)$ is a direct product of non-abelian simple groups [**?**, Def. 6.1].

3. A product of simple groups may be called *semi-simple*. Depending on application, the simple groups may be further restricted to finite simple groups and may also exclude the abelian simple groups.

4. A Lie group whose associated Lie algebra is a semi-simple Lie algebra may be called a *semi-simple* group and more specifically, a *semi-simple Lie group*.

Connections with algebra

The use of *semi-simple* in the study of algebras, representation theory, and modules is far more precise owing to the fact that the various possible definitions are generally equivalent.

For example. In a finite dimensional associative algebra A, if A it is a product of simple algebras then the Jacobson radical is trivial. In contrast, if A has trivial Jacobson radical then it is a direct product of simple algebras. Thus A may be called *semi-simple* if either: A is a direct product of simple algebras or A has trivial Jacobson radical.

The analogue fails for groups. For instance. If a group is defined as semi-simple by virtue of having no non-trivial normal abelian subgroups then S_n is semi-simple for all $n > 5$. However, S_n is not a product of simple groups.

References

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