ascending series

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Defines	ascending normal series
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Defines	ascendant
Defines	hyperabelian group
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Let G be a group.

An *ascending series* of G is a family $(H_\alpha)_{\alpha\leq\beta}$ of subgroups of G, where β is an ordinal, such that $H_0 = \{1\}$ and $H_\beta = G$, and $H_\alpha \trianglelefteq H_{\alpha+1}$ for all $\alpha < \beta$, and

$$\bigcup_{\alpha<\delta} H_\alpha = H_\delta$$

whenever $\delta \leq \beta$ is a limit ordinal.

Note that this is a generalization of the concept of a subnormal series. Compare also the dual concept of a descending series.

Given an ascending series $(H_\alpha)_{\alpha\leq\beta}$, the subgroups H_α are called the *terms* of the series and the http://planetmath.org/QuotientGroupquotients $H_{\alpha+1}/H_\alpha$ are called the *factors* of the series.

A subgroup of G that is a term of some ascending series of G is called an *ascendant subgroup* of G. The notation $H \operatorname{asc} G$ is sometimes used to indicate that H is an ascendant subgroup of G.

The groups in which every subgroup is ascendant are precisely the groups that satisfy the normalizer condition. Groups in which every cyclic subgroup is ascendant are called *Gruenberg groups*. It can be shown that in a Gruenberg group, every finitely generated subgroup is ascendant and nilpotent (and so, in particular, Gruenberg groups are locally nilpotent).

An ascending series of G in which all terms are normal in G is called an *ascending normal series*.

Let $\mathfrak{X}$ be a property of groups. A group is said to be *hyper-*$\mathfrak{X}$ if it has an ascending normal series whose factors all have property $\mathfrak{X}$. So, for example, a *hyperabelian group* is a group that has an ascending normal series with abelian factors. Hyperabelian groups are sometimes called *SI^*-groups*.