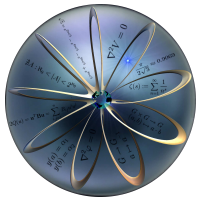

planetmath.org

Math for the people, by the people.

HNN extension

Canonical name	HNNExtension
Date of creation	2013-03-22 16:04:03
Last modified on	2013-03-22 16:04:03
Owner	juanman (12619)
Last modified by	juanman (12619)
Numerical id	8
Author	juanman (12619)
Entry type	Definition
Classification	msc 20E06
Related topic	GroupExtension

The *HNN extension group* G for a group A, is constructed from a pair of isomorphic subgroups $B \overset{\phi}{\cong} C$ in A, according to formula

$$G = \frac{A * \langle t|-\rangle}{N}$$

where $\langle t|-\rangle$ is a cyclic free group, $*$ is the free product and N is the normal closure of $\{tbt^{-1}\phi(b)^{-1} : b \in B\}$.

As an example take a surface bundle $F \subset E \to S^1$, hence the homotopy long exact sequence of this bundle implies that the fundamental group $\pi_1(E)$ is given by

$$\pi_1(E) = \langle x_1, ..., x_k, t|\Pi = 1, tx_it^{-1} = \phi(x_i)\rangle$$

where k is the genus of the surface and the relation Π is $[x_1, x_2][x_3, x_4] \cdots [x_{k-1}, x_k]$ for an orientable surface or $x_1^2 x_2^2 \cdots x_k^2$ is for a non-orientable one. ϕ is an isomorphism induced by a self homeomorphism of F.