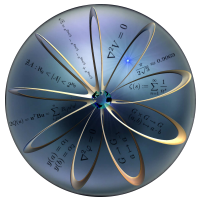

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examples of semidirect products of groups

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Suppose $H = \mathbb{Z}/n\mathbb{Z}$ and let r be a generator for H. Let $Q = \mathbb{Z}/2\mathbb{Z} =< s >$. Define $\theta : Q \to \mathrm{Aut}(H)$ by $\theta(s)(r) = r^{-1}$. Let $G = H \rtimes_\theta Q$. Then in G,

$$srs = srs^{-1} = \theta(s)(r) = r^{-1}$$

by the canonical equivalence of inner and outer semidirect products. So G has $2n$ elements, two generators r, s satisfying

$$r^n = s^2 = 1$$
$$srs = r^{-1}$$

and thus $G = \mathcal{D}_{2n}$, the n^{th} dihedral group.

If instead $H = \mathbb{Z}$, the result is the infinite dihedral group.

As another example, if G is a group, then the holomorph of G is $G \rtimes \mathrm{Aut}(G)$ under the identity map from $\mathrm{Aut}(G)$ to itself.