Baer-Specker group

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Let A be a non-empty set, and G an abelian group. The set K of all functions from A to G is an abelian group, with addition defined elementwise by $(f+g)(x)=f(x)+g(x)$. The zero element is the function that sends all elements of A into 0 of G, and the negative of an element f is a function defined by $(-f)(x)=-(f(x))$.

When $A=\mathbb{N}$, the set of natural numbers, and $G=\mathbb{Z}$, K as defined above is called the *Baer-Specker group*. Any element of K, being a function from $\mathbb{N}$ to $\mathbb{Z}$, can be expressed as an infinite sequence $(x_1,x_2,\ldots,x_n,\ldots)$, and the elementwise addition on K can be realized as componentwise addition on the sequences:

$$(x_1,x_2,\ldots,x_n,\ldots)+(y_1,y_2,\ldots,y_n,\ldots)=(x_1+y_1,x_2+y_2,\ldots,x_n+y_n,\ldots).$$

An alternative characterization of the Baer-Specker group K is that it can be viewed as the countably infinite direct product of copies of $\mathbb{Z}$:

$$K=\mathbb{Z}^{\mathbb{N}}\cong\mathbb{Z}^{\aleph_0}=\prod_{\aleph_0}\mathbb{Z}.$$

The Baer-Specker group is an important example of a torsion-free abelian group whose rank is infinite. It is not a free abelian group, but any of its countable subgroup is free (abelian).

References

[1] P. A. Griffith, *Infinite Abelian Group Theory*, The University of Chicago Press (1970)