automaton over a monoid

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Recall that a semiautomaton A can be visually represented by a directed graph G_A, whose vertices (or nodes) are states of A, and whose edges are labeled by elements from the input alphabet Σ of A. Labeling can be extended to paths of G_A in a natural way: if $(e_1, \ldots, e_n)$ is a path p and each edge e_i is labeled a_i, then the label of p is $a_1 \cdots a_n$. Thus, the labels of paths of G_A are just elements of the monoid Σ^*. The concept can be readily generalized to arbitrary monoids.

Definition. Let M be a monoid. A *semiautomaton* over M is a directed graph G whose edges are labeled by elements of M. An *automaton* over M is a semiautomaton over M, where the vertex set has two designated subsets I and F (not necessarily disjoint), where elements of I are called the start nodes, and elements of F the final nodes.

Note that if $M = \Sigma^*$ for some alphabet Σ, then a semiautomaton G over M according to the definition given above is not necessarily a semiautomaton over Σ under the standard definition of a semiautomaton, since labels of the edges are words over Σ, not elements of Σ. However, G can be "transformed" into a "standard" semiautomaton (over Σ).

Definition. Let A be a finite automaton over a monoid M. An element in M is said to be *accepted by* A if it is the label of a path that begins at an initial node and end at a final node. The set of all elements of M accepted by A is denoted by $L(A)$.

The following is a generalization of Kleene's theorem.

Theorem 1. . *A subset R of a monoid M is a rational set iff $R = L(A)$ for some finite automaton A over M.*

References

[1] S. Eilenberg, *Automata, Languages, and Machines, Vol. A*, Academic Press (1974).