group ring

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Defines	group algebra

For any group G, the *group ring* $\mathbb{Z}[G]$ is defined to be the ring whose additive group is the abelian group of formal integer linear combinations of elements of G, and whose multiplication operation is defined by multiplication in G, extended $\mathbb{Z}$–linearly to $\mathbb{Z}[G]$.

More generally, for any ring R, the *group ring* of G over R is the ring $R[G]$ whose additive group is the abelian group of formal R–linear combinations of elements of G, i.e.:

$$R[G] := \left\{ \sum_{i=1}^{n} r_i g_i \;\middle|\; r_i \in R, \ g_i \in G \right\},$$

and whose multiplication operation is defined by R–linearly extending the group multiplication operation of G. In the case where K is a field, the group ring $K[G]$ is usually called a *group algebra*.