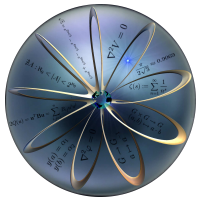

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finitely generated group

Canonical name	FinitelyGeneratedGroup
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Defines	finitely generated
Defines	finitely generated subgroup
Defines	infinitely generated
Defines	infinitely generated group
Defines	infinitely generated subgroup

A *finitely generated group* is a group that has a finite generating set.

Every finite group is obviously finitely generated. Every finitely generated group is countable.

Any http://planetmath.org/QuotientGroupquotient of a finitely generated group is finitely generated. However, a finitely generated group may have subgroups that are not finitely generated. (For example, the free group of rank 2 is generated by just two elements, but its commutator subgroup is not finitely generated.) Nonetheless, a subgroup of finite index in a finitely generated group is necessarily finitely generated; a bound on the number of generators required for the subgroup is given by the http://planetmath.org/ScheierIndexFormul index formula.

The finitely generated groups all of whose subgroups are also finitely generated are precisely the groups satisfying the maximal condition. This includes all finitely generated nilpotent groups and, more generally, all polycyclic groups.

A group that is not finitely generated is sometimes said to be *infinitely generated*.