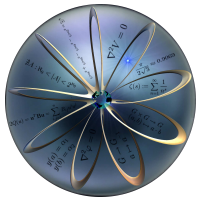

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zero times an element is zero in a ring

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Lemma 1. *Let R be a ring with zero element 0 (i.e. 0 is the additive identity of R). Then for any element $a \in R$ we have $0 \cdot a = a \cdot 0 = 0$.*

Proof.

$$\begin{aligned} 0 \cdot a &= (0+0) \cdot a, && \text{by definition of zero} \\ &= 0 \cdot a + 0 \cdot a, && \text{by the distributive law} \end{aligned}$$

Thus $0 \cdot a = 0 \cdot a + 0 \cdot a$. Let b be the additive inverse of $0 \cdot a \in R$. Hence:

$$\begin{aligned} b + 0 \cdot a &= b + (0 \cdot a + 0 \cdot a) \\ (b + 0 \cdot a) &= (b + 0 \cdot a) + 0 \cdot a \\ 0 &= 0 + 0 \cdot a \\ 0 &= 0 \cdot a \end{aligned}$$

as claimed. The proof of $a \cdot 0 = 0$ is done analogously. □