BN-pair

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Let G be a group. Then G has a BN-pair or a Tits system if the following conditions hold:

1. B and N are subgroups of G such that $G = < B, N >$.
2. $B \cap N = T \triangleleft N$ and $N/T = W$ is a group generated by a set S.
3. $sBw \subseteq BwB \cup BswB$ for all $s \in S$ and $w \in W$.
4. $sBs^{-1} \not\subseteq B$ for all $s \in S$.

Where BwB is a double coset with respect to B. It can be proven that S is in fact made up of elements of order 2, and that W is a Coxeter group.

Example: Let $G = GL_n(\mathbb{K})$ where $\mathbb{K}$ is some field. Then, if we let B be the subgroup of upper triangular matrices and N be the subgroup of monomial matrices (i.e. matrices having one nonzero entry in each row and each column, or more precisely the stabilizer of the lines $\{[e_1], ..., [e_n]\}$). Then, it can be shown that B and N generate G and that T is the subgroup of diagonal matrices. In turn, it follows that W in this case is isomorphic to the symmetric group on n letters, S_n.

For more, consult chapter 5 in the book Buildings, by Kenneth Brown