commutative

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Defines	non-commutative

Let S be a set and $\circ$ a binary operation on it. $\circ$ is said to be *commutative* if

$$a \circ b = b \circ a$$

for all $a, b \in S$.

Viewing $\circ$ as a function from $S \times S$ to S, the commutativity of $\circ$ can be notated as

$$\circ(a, b) = \circ(b, a).$$

Some common examples of commutative operations are

- addition over the integers: $m + n = m + n$ for all integers m, n
- multiplication over the integers: $m \cdot n = m \cdot n$ for all integers m, n
- addition over $n \times n$ matrices, $A + B = B + A$ for all $n \times n$ matrices A, B, and
- multiplication over the reals: $rs = sr$, for all real numbers r, s.

A binary operation that is not commutative is said to be *non-commutative*. A common example of a non-commutative operation is the subtraction over the integers (or more generally the real numbers). This means that, in general,

$$a - b \neq b - a.$$

For instance, $2 - 1 = 1 \neq -1 = 1 - 2$.

Other examples of non-commutative binary operations can be found in the attachment below.

Remark. The notion of commutativity can be generalized to n-ary operations, where $n \geq 2$. An n-ary operation f on a set A is said to be *commutative* if

$$f(a_1, a_2, \ldots, a_n) = f(a_{\pi(1)}, a_{\pi(2)}, \ldots, a_{\pi(n)})$$

for every permutation π on $\{1, 2, \ldots, n\}$, and for every choice of n elements a_i of A.