I-semigroup

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Defines	I-semigroup
Defines	I-monoid

An *I-semigroup* [resp. *I-monoid*] is a semigroup S [resp. a monoid M] with a unary operation $x \mapsto x^{-1}$ defined on S [resp. on M] such that for each $x, y \in S$ [resp. for each $x, y \in M$]

$$(x^{-1})^{-1} = x, \quad x = xx^{-1}x.$$

Notice that
$$x^{-1}xx^{-1} = x^{-1}(x^{-1})^{-1}x^{-1} = x^{-1},$$

so x^{-1} is an inverse of x.

The class of I-semigroups [resp. I-monoids] strictly contains the class of inverse semigroups [resp. inverse monoids]. In fact, the class of inverse semigroups [resp. inverse monoids] is precisely the class of I-semigroups with involution [resp. I-monoids with involution], i.e. the class of I-semigroups [resp. I-monoids] in which the unary operation $^{-1}$ is also an involution.

References

[1] J.M. Howie, *Fundamentals of Semigroup Theory*, Oxford University Press, Oxford, 1991.