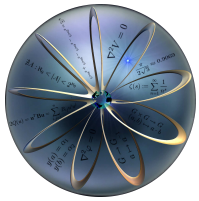

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simply transitive

Canonical name	SimplyTransitive
Date of creation	2013-03-22 14:37:41
Last modified on	2013-03-22 14:37:41
Owner	benjaminfjones (879)
Last modified by	benjaminfjones (879)
Numerical id	7
Author	benjaminfjones (879)
Entry type	Definition
Classification	msc 20M30
Related topic	GroupAction

Let G be a group acting on a set X. The action is said to be **simply transitive** if it is transitive and $\forall x, y \in X$ there is a *unique* $g \in G$ such that $g.x = y$.

Theorem. *A group action is simply transitive if and only if it is free and transitive*

Proof. Necessity follows since $g.x = x$ implies that $g = 1_G$ because $1_G.x = x$ also. Now assume the action is free and transitive and we have elements $g_1, g_2 \in G$ and $x, y \in X$ such that $g_1.x = y$ and $g_2.x = y$. Then $g_1.x = g_2.x \implies g_2^{-1}.g_1.x = (g_2^{-1}g_1).x = x$ hence $g_2^{-1}g_1 = 1_G$ because the action is free. Thus $g_1 = g_2$ and so the action is simply transitive. $\square$