



planetmath.org

Math for the people, by the people.

law of signs under multiplication in a ring

Canonical name	LawOfSignsUnderMultiplicationInARing
Date of creation	2013-03-22 14:14:03
Last modified on	2013-03-22 14:14:03
Owner	alozano (2414)
Last modified by	alozano (2414)
Numerical id	10
Author	alozano (2414)
Entry type	Derivation
Classification	msc 20-00
Classification	msc 16-00
Classification	msc 13-00
Synonym	$(-x) \cdot (-y) = x \cdot y$
Related topic	Ring

Lemma 1. *Let R be a ring with unity, which we denote by 1 . For all $x, y \in R$:*

$$(-x) \cdot (-y) = x \cdot y$$

where $-x$ denotes the additive inverse of x in R .

Proof. Here we use the fact $(-1) \cdot a = -a$ for all $a \in R$. First, we see that:

$$(-1) \cdot (-1) \cdot a = (-1) \cdot ((-1) \cdot a) = (-1) \cdot (-a) = a$$

since, clearly, the additive inverse of $-a$ is a itself.

Hence:

$$(-x) \cdot (-y) = (-1) \cdot x \cdot (-1) \cdot y = (-1) \cdot (-1) \cdot x \cdot y = x \cdot y$$

where we have used several times the associativity of \cdot and the fact that $(-1) \cdot x = x \cdot (-1) = -x$. \square