group homomorphism

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Entry type	Definition
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Synonym	homomorphism
Synonym	homomorphism of groups
Related topic	Group
Related topic	Kernel
Related topic	Subgroup
Related topic	TypesOfHomomorphisms
Related topic	KernelOfAGroupHomomorphism
Related topic	GroupActionsAndHomomorphisms
Related topic	Endomorphism2
Related topic	GroupsOfRealNumbers
Related topic	HomomorphicImageOfGroup
Defines	epimorphism
Defines	monomorphism
Defines	automorphism
Defines	endomorphism
Defines	isomorphism
Defines	isomorphic
Defines	group epimorphism
Defines	group monomorphism
Defines	group automorphism
Defines	group endomorphism
Defines	group isomorphism
Defines	epimorphism of groups
Defines	monomorphism of groups
Defines	automorphism of a group
Defines	endom

Let $(G,*)$ and $(K,\star)$ be two groups. A *group homomorphism* is a function $\phi\colon G \to K$ such that $\phi(s * t) = \phi(s) \star \phi(t)$ for all $s, t \in G$.

A composition of group homomorphisms is again a homomorphism.

Let $\phi\colon G \to K$ a group homomorphism. Then the kernel of ϕ is a normal subgroup of G, and the image of ϕ is a subgroup of K. Also, $\phi(g^n) = \phi(g)^n$ for all $g \in G$ and for all $n \in \mathbb{Z}$. In particular, taking $n = -1$ we have $\phi(g^{-1}) = \phi(g)^{-1}$ for all $g \in G$, and taking $n = 0$ we have $\phi(1_G) = 1_K$, where 1_G and 1_K are the identity elements of G and K, respectively.

Some special homomorphisms have special names. If the homomorphism $\phi\colon G \to K$ is injective, we say that ϕ is a *monomorphism*, and if ϕ is surjective we call it an *epimorphism*. When ϕ is both injective and surjective (that is, bijective) we call it an *isomorphism*. In the latter case we also say that G and K are *isomorphic*, meaning they are basically the same group (have the same structure). A homomorphism from G on itself is called an *endomorphism*, and if it is bijective then it is called an *automorphism*.