non-isomorphic groups of given order

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Theorem. For every positive integer n, there exists only a finite amount of non-isomorphic groups of order n.

This assertion follows from Cayley's theorem, according to which any group of order n is isomorphic with a subgroup of the symmetric group $\mathfrak{S}_n$. The number of non-isomorphic subgroups of $\mathfrak{S}_n$ cannot be greater than

$$\binom{n!-1}{n-1}.$$

The above theorem may be used in proving the following Landau's theorem:

Theorem (Landau). For every positive integer n, there exists only a finite amount of finite non-isomorphic groups which contain exactly n conjugacy classes of elements.

One needs also the

Lemma. If $n \in \mathbb{Z}_+$ and $0 < r \in \mathbb{R}$, then there is at most a finite amount of the vectors $(m_1,\, m_2,\, \ldots,\, m_n)$ consisting of positive integers such that

$$\sum_{j=1}^{n} \frac{1}{m_j} \;=\; r.$$

The lemma is easily proved by induction on n.