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homomorphisms of simple groups

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If a group G is simple, and H is an arbitrary group then any homomorphism of G to H must either map all elements of G to the identity of H or be one-to-one.

The kernel of a homomorphism must be a normal subgroup. Since G is simple, there are only two possibilities: either the kernel is all of G or it consists of the identity. In the former case, the homomorphism will map all elements of G to the identity. In the latter case, we note that a group homomorphism is injective iff the kernel is trivial.

This is important in the context of representation theory. In that case, H is a linear group and this result may be restated as saying that representations of a simple group are either trivial or faithful.