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# absorbing element

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An element $\zeta$ of a groupoid $(G, *)$ is called an *absorbing element* (in French *un élément absorbant*) for the operation "$*$", if it satisfies

$$\zeta * a = a * \zeta = \zeta$$

for all elements $a$ of $G$.

### Examples

- The zero 0 is the absorbing element for multiplication (or *multiplicatively absorbing*) in every ring $(R, +, \cdot)$.

- The zero ideal $(0)$ is absorbing for `http://planetmath.org/IdealMultiplicationLaws`ideal multiplication.

- The zero vector $\vec{0}$ is the absorbing element for the `http://planetmath.org/CrossProduct`vec multiplication "$\times$".

- The empty set $\varnothing$ is the absorbing element for the intersection operation "$\cap$" and also for the Cartesian product "$\times$".

- The "universal set" $E$ is the absorbing element for the union operation "$\cup$":
$$E \cup A = A \cup E = E \quad \forall A \subseteq E.$$

- In an upper semilattice, an element is absorbing iff it is the `http://planetmath.org/Bounded` element. Dually, an element is absorbing iff it is the `http://planetmath.org/BoundedLattic` element in a lower semilattice.

As the examples give reason to believe, the absorbing element for an operation is always unique. Indeed, if in  to $\zeta$ we have in $G$ another absorbing element $\eta$, then we must have $\eta = \zeta * \eta = \zeta$.

Because $\zeta * \zeta = \zeta$, the absorbing element is idempotent.

If a group has an absorbing element, the group is `http://planetmath.org/Subgroup`trivial.