uniqueness of inverse (for groups)

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Lemma Suppose $(G, *)$ is a group. Then every element in G has a unique inverse.

Proof. Suppose $g \in G$. By the group axioms we know that there is an $h \in G$ such that

$$g * h = h * g = e,$$

where e is the identity element in G. If there is also a $h' \in G$ satisfying

$$g * h' = h' * g = e,$$

then

$$h = h * e = h * (g * h') = (h * g) * h' = e * h' = h',$$

so $h = h'$, and g has a unique inverse. $\square$