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elementary abelian group

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Defines	elementary abelian
Defines	Boolean group

An *elementary abelian group* is an abelian group in which every non-trivial element has the same finite order. It is easy to see that the non-trivial elements must in fact be of prime order, so every elementary abelian group is a <http://planetmath.org/PGroup4p>-group for some prime p .

Elementary abelian 2-groups are sometimes called *Boolean groups*. A group in which every non-trivial element has order 2 is necessarily Boolean, because abelianness is automatic: $xy = (xy)^{-1} = y^{-1}x^{-1} = yx$. There is no analogous result for odd primes, because for every odd prime p there is a non-abelian group of order p^3 and exponent p .

Let p be a prime number. Any elementary abelian p -group can be considered as a vector space over the field of order p , and is therefore isomorphic to the direct sum of κ copies of the cyclic group of order p , for some cardinal number κ . Conversely, any such direct sum is obviously an elementary abelian p -group. So, in particular, for every infinite cardinal κ there is, up to isomorphism, exactly one elementary abelian p -group of order κ .