coset

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Defines	index
Defines	left coset
Defines	right coset

Let H be a subgroup of a group G, and let $a \in G$. The *left coset* of a with respect to H in G is defined to be the set

$$aH := \{ah \mid h \in H\}.$$

The *right coset* of a with respect to H in G is defined to be the set

$$Ha := \{ha \mid h \in H\}.$$

Two left cosets aH and bH of H in G are either identical or disjoint. Indeed, if $c \in aH \cap bH$, then $c = ah_1$ and $c = bh_2$ for some $h_1, h_2 \in H$, whence $b^{-1}a = h_2h_1^{-1} \in H$. But then, given any $ah \in aH$, we have $ah = (bb^{-1})ah = b(b^{-1}a)h \in bH$, so $aH \subset bH$, and similarly $bH \subset aH$. Therefore $aH = bH$.

Similarly, any two right cosets Ha and Hb of H in G are either identical or disjoint. Accordingly, the collection of left cosets (or right cosets) partitions the group G; the corresponding equivalence relation for left cosets can be described succintly by the relation $a \sim b$ if $a^{-1}b \in H$, and for right cosets by $a \sim b$ if $ab^{-1} \in H$.

The *index* of H in G, denoted $[G : H]$, is the cardinality of the set G/H of left cosets of H in G.