representation ring vs burnside ring

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Let G be a finite group and let k be any field. If X is a G-set, then we may consider the vector space $V_k(X)$ over k which has X as a basis. In this manner $V_k(X)$ becomes a representation of G via action induced from X and linearly extended to $V_k(X)$. It can be shown that $V_k(X)$ only depends on the isomorphism class of X, so we have a well-defined mapping:

$$[X] \mapsto [V_k(X)]$$

which can be easily extended to the function

$$\beta : \Omega(G) \to R_k(G);$$

$$\beta([X]) = [V_k(X)]$$

where on the left side we have the Burnside ring and on the right side the representation ring. It can be shown, that β is actually a ring homomorphism, but in most cases it neither injective nor surjective. But the following theorem due to Segal gives us some properties of β:

Theorem (Segal). Let $\beta : \Omega(G) \to R_{\mathbb{Q}}(G)$ be defined as above with rationals as the underlying field. If G is a p-group for some prime number p, then β is surjective. Furthermore β is an isomorphism if and only if G is cyclic.