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Simple Groups

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Recall that a group G is simple if it has no normal subgroups except itself and $\{e\}$. Let G be a finite simple group and let p be a prime number.

(a) Suppose G has precisely k Sylow p-subgroups with $k > 1$. Show that G is isomorphic to a subgroup of the symmetric group S_k.

(b) With the same hypothesis, show that G is isomorphic to a subgroup of the alternating group A_k.

(c) Suppose G is a simple group that is a proper subgroup of A_k and $k \geq 5$. Show that the index $[A_k : G] \geq k$.

(d) Prove that if G is a group of order 120 then G is not a simple group. (Parts (b) and (c) may be helpful.)

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