proof that group homomorphisms preserve identity

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Theorem. *A group homomorphism preserves identity elements.*

Proof. Let $\phi : G \to K$ be a group homomorphism. For clarity we use $*$ and $\star$ for the group operations of G and K, respectively. Also, denote the identities by 1_G and 1_H respectively.

By the definition of identity,

$$1_G * 1_G = 1_G. \tag{1}$$

Applying the homomorphism ϕ to (**??**) produces:

$$\phi(1_G) \star \phi(1_G) = \phi(1_G * 1_G) = \phi(1_G). \tag{2}$$

Multiply both sides of (**??**) by the inverse of $\phi(1_G)$ in K, and use the associativity of $\star$ to produce:

$$\phi(1_G) = (\phi(1_G))^{-1} \star \phi(1_G) \star \phi(1_G) = (\phi(1_G))^{-1} \star \phi(1_G) = 1_K. \tag{3}$$

$\square$