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left and right cosets in a double coset

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Let H and K be subgroups of a group G . Every double coset HgK , with $g \in G$, is a union of <http://planetmath.org/Cosetright> or left cosets, since

$$HgK = \bigcup_{k \in K} Hgk = \bigcup_{h \in H} hgK,$$

but these unions need not be disjoint. In particular, from the above equality we cannot say how many right (or left) cosets fit in a double coset.

The following proposition aims to clarify this.

- Let H and K be subgroups of a group G and $g \in G$. We have that

$$HgK = \bigcup_{[k] \in (K \cap g^{-1}Hg) \backslash K} Hgk = \bigcup_{[h] \in H / (H \cap gKg^{-1})} hgK$$

hold as disjoint unions. In particular, the number of right and left cosets in HgK is respectively given by

$$\begin{aligned} \#(H \backslash HgK) &= [K : K \cap g^{-1}Hg] \\ \#(HgK / K) &= [H : H \cap gKg^{-1}] \end{aligned}$$

0.1 Remarks

- The number of right and left cosets in a double coset does not coincide in general, not for double cosets of the form HgH .

References

- [1] A. Krieg, , Mem. Amer. Math. Soc., no. 435, vol. 87, 1990.