proof of third isomorphism theorem

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We'll give a proof of the third isomorphism theorem using the Fundamental homomorphism theorem.

Let G be a group, and let $K \subseteq H$ be normal subgroups of G. Define p, q to be the natural homomorphisms from G to G/H, G/K respectively:

$$p(g) = gH, q(g) = gK \ \forall\ g \in G.$$

K is a subset of $\ker(p)$, so there exists a unique homomorphism $\varphi\colon G/K \to G/H$ so that $\varphi \circ q = p$.

p is surjective, so φ is surjective as well; hence $\operatorname{im} \varphi = G/H$. The kernel of φ is $\ker(p)/K = H/K$. So by the first isomorphism theorem we have

$$(G/K)/\ker(\varphi) = (G/K)/(H/K) \approx \operatorname{im} \varphi = G/H.$$