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inner automorphism

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Defines	conjugation
Defines	outer
Defines	outer automorphism
Defines	automorphism group

Let G be a group. For every $x \in G$, we define a mapping

$$\phi_x : G \to G, \quad y \mapsto xyx^{-1}, \quad y \in G,$$

called conjugation by x. It is easy to show the conjugation map is in fact, a group automorphism.

An automorphism of G that corresponds to conjugation by some $x \in G$ is called *inner*. An automorphism that isn't inner is called an *outer* automorphism.

The composition operation gives the set of all automorphisms of G the structure of a group, $\mathrm{Aut}(G)$. The inner automorphisms also form a group, $\mathrm{Inn}(G)$, which is a normal subgroup of $\mathrm{Aut}(G)$. Indeed, if $\phi_x,\ x \in G$ is an inner automorphism and $\pi : G \to G$ an arbitrary automorphism, then

$$\pi \circ \phi_x \circ \pi^{-1} = \phi_{\pi(x)}.$$

Let us also note that the mapping

$$x \mapsto \phi_x, \quad x \in G$$

is a surjective group homomorphism with kernel $\mathrm{Z}(G)$, the centre subgroup. Consequently, $\mathrm{Inn}(G)$ is naturally isomorphic to the quotient of $G/\mathrm{Z}(G)$.

Note: the above definitions and assertions hold, mutatis mutandi, if we define the conjugation action of $x \in G$ on B to be the right action

$$y \mapsto x^{-1}yx, \quad y \in G,$$

rather than the left action given above.