Proof: The orbit of any element of a group is a subgroup

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Following is a proof that, if G is a group and $g \in G$, then $\langle g \rangle \leq G$. Here $\langle g \rangle$ is the orbit of g and is defined as

$$\langle g \rangle = \{g^n : n \in \mathbb{Z}\}$$

Since $g \in \langle g \rangle$, then $\langle g \rangle$ is nonempty.

Let $a, b \in \langle g \rangle$. Then there exist $x, y \in \mathbb{Z}$ such that $a = g^x$ and $b = g^y$. Since $ab^{-1} = g^x(g^y)^{-1} = g^x g^{-y} = g^{x-y} \in \langle g \rangle$, it follows that $\langle g \rangle \leq G$.