zero elements

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Let S be a semigroup. An element z is called a *right zero* [resp. *left zero*] if $xz = z$ [resp. $zx = z$] for all $x \in S$.

An element which is both a left and a right zero is called a *zero element*.

A semigroup may have many left zeros or right zeros, but if it has at least one of each, then they are necessarily equal, giving a unique (two-sided) zero element.

More generally, these definitions and statements are valid for a groupoid.

It is customary to use the symbol θ for the zero element of a semigroup.

Proposition 1. *If a groupoid has a left zero 0_L and a right zero 0_R, then $0_L = 0_R$.*

Proof. $0_L = 0_L 0_R = 0_R$. $\square$

Proposition 2. *If 0 is a left zero in a semigroup S, then so is $x0$ for every $x \in S$.*

Proof. For any $y \in S$, $(x0)y = x(0y) = x0$. As a result, $x0$ is a left zero of S. $\square$

Proposition 3. *If 0 is the unique left zero in a semigroup S, then it is also the zero element.*

Proof. By assumption and the previous proposition, $x0$ is a left zero for every $x \in S$. But 0 is the unique left zero in S, we must have $x0 = 0$, which means that 0 is a right zero element, and hence a zero element by the first proposition. $\square$