equation

Canonical name	Equation
Date of creation	2013-03-22 15:28:33
Last modified on	2013-03-22 15:28:33
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	30
Author	pahio (2872)
Entry type	Definition
Classification	msc 20N02
Related topic	Equality2
Related topic	AlgebraicEquation
Related topic	DiophantineEquation
Related topic	TrigonometricEquation
Related topic	DifferenceEquation
Related topic	DifferentialEquation
Related topic	IntegralEquation
Related topic	FunctionalEquation
Related topic	HomogeneousEquation
Related topic	ProportionEquation
Related topic	FiniteDifference
Related topic	RecurrenceRelation
Related topic	CharacteristicEquation
Defines	equate
Defines	side
Defines	root
Defines	solution
Defines	root of an equation
Defines	left hand side
Defines	right hand side
Defines	multiplicity of the root
Defines	order of the root
Defines	multiple root

Equation

An *equation* concerns usually elements of a certain set M, where one can say if two elements are equal. In the simplest case, M has one binary operation "$*$" producing as result some elements of M, and these can be compared. Then, an equation in $(M,\,*)$ is a proposition of the form

$$E_1 = E_2, \tag{1}$$

where one has *equated* two expressions E_1 and E_2 formed with "$*$" of the elements or indeterminates of M. We call the expressions E_1 and E_2 respectively the *left hand side* and the *right hand side* of the equation (1).

Example. Let S be a set and 2^S the set of its subsets. In the groupoid $(2^S,\,\smallsetminus)$, where "$\smallsetminus$" is the set difference, we can write the equation

$$(A\smallsetminus B)\smallsetminus B = A\smallsetminus B$$

(which is always true).

Of course, M may be equipped with more operations or be a module with some ring of multipliers — then an equation (1) may them.

But one need not assume any algebraic structure for the set M where the expressions E_1 and E_2 are values or where they elements. Such a situation would occur e.g. if one has a continuous mapping f from a topological space L to another M; then one can consider an equation

$$f(x) = y.$$

A somewhat case is the equation

$$\dim V = 2$$

where V is a certain or a vector space; both elements of the extended real number system.

Root of equation

If an equation (1) in M one indeterminate, say x, then a value of x which satisfies (1), i.e. makes it true, is called a *root* or a *solution* of the equation. Especially, if we have a polynomial equation $f(x) = 0$, we may speak of the or the x_0; it is the multiplicity of the zero x_0 of the polynomial $f(x)$. A *multiple root* has multiplicity greater than 1.

Example. The equation

$$x^2+1 = x$$

in the system $\mathbb{C}$ of the complex numbers has as its roots the numbers

$$x := \frac{1\pm i\sqrt{3}}{2},$$

which, by the way, are the primitive sixth roots of unity. Their multiplicities are 1.