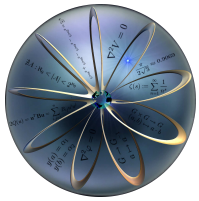

planetmath.org

Math for the people, by the people.

cancellative semigroup

Canonical name	CancellativeSemigroup
Date of creation	2013-03-22 14:25:09
Last modified on	2013-03-22 14:25:09
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	9
Author	yark (2760)
Entry type	Definition
Classification	msc 20M10
Synonym	cancellation semigroup
Related topic	CancellationIdeal
Defines	cancellative
Defines	weakly cancellative
Defines	left cancellative
Defines	right cancellative
Defines	weakly cancellative semigroup
Defines	left cancellative semigroup
Defines	right cancellative semigroup

Let S be a semigroup.

S is *left cancellative* if, for all $a, b, c \in S$, $ab = ac \Rightarrow b = c$
S is *right cancellative* if, for all $a, b, c \in S$, $ba = ca \Rightarrow b = c$

S is *cancellative* if it is both left and right cancellative.

1 Relationship to some other types of semigroup

This is a generalisation of groups, and in fact being cancellative is a necessary condition for a semigroup to be embeddable in a group.

Note that a non-empty semigroup is a group if and only if it is cancellative and regular.

S is *weakly cancellative* if, for all $a, b, c \in S$, $(ab = ac \ \& \ ba = ca) \Rightarrow b = c$

A semigroup is completely simple if and only if it is weakly cancellative and regular.

2 Individual elements

An element $x \in S$ is called *left cancellative* if, for all $b, c \in S$, $xb = xc \Rightarrow b = c$
An element $x \in S$ is called *right cancellative* if, for all $b, c \in S$, $bx = cx \Rightarrow b = c$