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## fourth isomorphism theorem

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Synonym correspondence theorem Synonym lattice isomorphism theorem **Theorem 1 (The Fourth Isomorphism Theorem)** Let G be a group and  $N \subseteq G$ . There is a bijection between G(N), the set of subgroups of G containing N, and the set of subgroups of G/N defined by  $A \to A/N$ . Moreover, for any two subgroups A, B in G(N), we have

- 1.  $A \leq B$  if and only if  $A/N \leq B/N$ ,
- 2.  $A \leq B$  implies |B:A| = |B/N:A/N|,
- 3.  $\langle A, B \rangle / N = \langle A/N, B/N \rangle$ ,
- 4.  $(A \cap B)/N = (A/N) \cap (B/N)$ , and
- 5.  $A \subseteq G$  if and only if  $(A/N) \subseteq (G/N)$ .