center of a group

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The *center* of a group G is the subgroup consisting of those elements that commute with every other element. Formally,

$$\mathrm{Z}(G) = \{x \in G \mid xg = gx \text{ for all } g \in G\}.$$

It can be shown that the center has the following properties:

- It is a normal subgroup (in fact, a characteristic subgroup).
- It consists of those conjugacy classes containing just one element.
- The center of an abelian group is the entire group.
- For every prime p, every non-trivial finite `http://planetmath.org/PGroup4`p-group has a non-trivial center. (`http://planetmath.org/ProofOfANontrivialNormalSubgr` of a stronger version of this theorem.)

A subgroup of the center of a group G is called a *central subgroup* of G. All central subgroups of G are normal in G.

For any group G, the `http://planetmath.org/QuotientGroup`quotient $G/\mathrm{Z}(G)$ is called the *central quotient* of G, and is isomorphic to the inner automorphism group $\mathrm{Inn}(G)$.