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immanent

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Let  $S_n$  denote the symmetric group on  $n$  elements. Let  $\chi : S_n \rightarrow \mathbb{C}$  be a complex character. For any  $n \times n$  matrix  $A = (a_{ij})_{i,j=1}^n$  define the *immanent* of  $A$  as

$$\text{Imm}_\chi(A) = \sum_{\sigma \in S_n} \chi(\sigma) \prod_{j=1}^n A_{j\sigma(j)}.$$

Special cases of immanents are determinants and permanents — in the case where  $\chi$  is the constant character ( $\chi(x) = 1$  for all  $x \in S_n$ ),  $\text{Imm}_\chi(A)$  is the permanent of  $A$ . In the case where  $\chi$  is the sign of the permutation (which is the character of the permutation group associated to the (non-trivial) one-dimensional representation),  $\text{Imm}_\chi(A)$  is the determinant of  $A$ .