semigroup of transformations

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Defines	full transformation semigroup

Let X be a set. A transformation of X is a function from X to X.

If α and β are transformations on X, then their product $\alpha\beta$ is defined (writing functions on the right) by $(x)(\alpha\beta) = ((x)\alpha)\beta$.

With this definition, the set of all transformations on X becomes a semigroup, the *full semigroupf of transformations* on X, denoted $\mathcal{T}_X$.

More generally, a *semigroup of transformations* is any subsemigroup of a full set of transformations.

When X is finite, say $X = \{x_1, x_2, \ldots, x_n\}$, then the transformation α which maps x_i to y_i (with $y_i \in X$, of course) is often written:

$$\alpha = \begin{pmatrix} x_1 & x_2 & \ldots & x_n \\ y_1 & y_2 & \ldots & y_n \end{pmatrix}$$

With this notation it is quite easy to products. For example, if $X = \{1, 2, 3, 4\}$, then

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 2 & 3 \end{pmatrix}$$

When X is infinite, say $X = \{1, 2, 3, \ldots\}$, then this notation is still useful for illustration in cases where the transformation pattern is apparent. For example, if $\alpha \in \mathcal{T}_X$ is given by $\alpha\colon n \mapsto n+1$, we can write

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & \ldots \\ 2 & 3 & 4 & 5 & \ldots \end{pmatrix}$$