proof that every group of prime order is cyclic

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The following is a proof that every group of prime order is cyclic.

Let p be a prime and G be a group such that $|G| = p$. Then G contains more than one element. Let $g \in G$ such that $g \neq e_G$. Then $\langle g \rangle$ contains more than one element. Since $\langle g \rangle \leq G$, by Lagrange's theorem, $|\langle g \rangle|$ divides p. Since $|\langle g \rangle| > 1$ and $|\langle g \rangle|$ divides a prime, $|\langle g \rangle| = p = |G|$. Hence, $\langle g \rangle = G$. It follows that G is cyclic.