# finite subgroup

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**Theorem.** A non-empty finite subset $K$ of a group $G$ is a subgroup of $G$ if and only if

$$xy \in K \quad \text{for all} \quad x,\, y \in K. \tag{1}$$

*Proof.* The condition (1) is apparently true if $K$ is a subgroup. Conversely, suppose that a nonempty finite subset $K$ of the group $G$ satisfies (1). Let $a$ and $b$ be arbitrary elements of $K$. By (1), all () powers of $b$ belong to $K$. Because of the finiteness of $K$, there exist positive integers $r$, $s$ such that

$$b^r \;=\; b^s, \quad r \;>\; s+1.$$

By (1),

$$K \;\ni\; b^{r-s-1} \;=\; b^{r-s}b^{-1} \;=\; eb^{-1} \;=\; b^{-1}.$$

Thus also $ab^{-1} \in K,$ whence, by the theorem of the `http://planetmath.org/node/1045`parent entry, $K$ is a subgroup of $G$.

**Example.** The multiplicative group $G$ of all nonzero complex numbers has the finite multiplicative subset $\{1,\,-1,\,i,\,-i\},$ which has to be a subgroup of $G$.