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maximal subgroup

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Synonym	maximal proper subgroup
Related topic	MaximalElement
Defines	maximal
Defines	maximal normal subgroup
Defines	maximal proper normal subgroup
Defines	simplicity of quotient group

Let G be a group.

A subgroup H of G is said to be a *maximal subgroup* of G if $H \neq G$ and there is no subgroup K of G such that $H < K < G$. Note that a maximal subgroup of G is not <http://planetmath.org/MaximalElement> maximal among all subgroups of G , but only among all proper subgroups of G . For this reason, maximal subgroups are sometimes called *maximal proper subgroups*.

Similarly, a normal subgroup N of G is said to be a *maximal normal subgroup* (or *maximal proper normal subgroup*) of G if $N \neq G$ and there is no normal subgroup K of G such that $N < K < G$. We have the following theorem:

Theorem. *A normal subgroup N of a group G is a maximal normal subgroup if and only if the <http://planetmath.org/QuotientGroup> quotient G/N is <http://planetmath.org/Simple> simple.*