algebra

Canonical name	Algebra
Date of creation	2013-03-22 11:48:37
Last modified on	2013-03-22 11:48:37
Owner	djao (24)
Last modified by	djao (24)
Numerical id	17
Author	djao (24)
Entry type	Definition
Classification	msc 20C99
Classification	msc 16S99
Classification	msc 13B02
Defines	subalgebra

In this definition, all rings are assumed to be rings with identity and all ring homomorphisms are assumed to be unital.

Let R be a ring. An *algebra* over R is a ring A together with a ring homomorphism $f\colon R \to Z(A)$, where $Z(A)$ denotes the center of A. A *subalgebra* of A is a subset of A which is an algebra.

Equivalently, an algebra over a ring R is an R–module A which is a ring and satisfies the property

$$r \cdot (x * y) = (r \cdot x) * y = x * (r \cdot y)$$

for all $r \in R$ and all $x, y \in A$. Here $\cdot$ denotes R-module multiplication and $*$ denotes ring multiplication in A. One passes between the two definitions as follows: given any ring homomorphism $f\colon R \longrightarrow Z(A)$, the scalar multiplication rule

$$r \cdot b := f(r) * b$$

makes A into an R-module in the sense of the second definition. Conversely, if A satisfies the requirements of the second definition, then the function $f\colon R \to A$ defined by $f(r) := r \cdot 1$ is a ring homomorphism from R into $Z(A)$.