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Cayley's theorem

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Let G be a group, then G is isomorphic to a subgroup of the permutation group S_G

If G is finite and of order n, then G is isomorphic to a subgroup of the permutation group S_n

Furthermore, suppose H is a proper subgroup of G. Let $X = \{Hg|g \in G\}$ be the set of right cosets in G. The map $\theta : G \to S_X$ given by $\theta(x)(Hg) = Hgx$ is a homomorphism. The kernel is the largest normal subgroup of H. We note that $|S_X| = [G : H]!$. Consequently if $|G|$ doesn't divide $[G : H]!$ then θ is not an isomorphism so H contains a non-trivial normal subgroup, namely the kernel of θ.