properties of conjugacy

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Defines	normal closure

Let S be a nonempty subset of a group G. When g is an element of G, a conjugate of S is the subset

$$gSg^{-1} \;=\; \{gsg^{-1} \vdots\; s \in S\}.$$

We denote here

$$gSg^{-1} \;:=\; S^g. \tag{1}$$

If T is another nonempty subset and h another element of G, then it's easily verified the formulae

- $(ST)^g \;=\; S^gT^g$
- $(S^g)^h \;=\; S^{gh}$

The conjugates H^g of a subgroup H of G are subgroups of G, since any mapping

$$x \mapsto gxg^{-1}$$

is an automorphism (an inner automorphism) of G and the homomorphic image of group is always a group.

The notation (1) can be extended to

$$\langle S^g \vdots\; g \in G\rangle \;:=\; S^G \tag{2}$$

where the angle parentheses express a generated subgroup. S^G is the least normal subgroup of G containing the subset S, and it is called the *normal closure* of S.

`http://en.wikipedia.org/wiki/Conjugacy`Wiki