presentation of inverse monoids and inverse semigroups

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Let $(X \amalg X^{-1})^*$ be the free monoid with involution on X, and $T \subseteq (X \amalg X^{-1})^* \times (X \amalg X^{-1})^*$ be a binary relation between words. We denote by T^{e} [resp. T^{c}] the equivalence relation [resp. congruence] generated by T.

A *presentation (for an inverse monoid)* is a couple $(X;T)$. We use this couple of objects to define an inverse monoid $\mathrm{Inv}^1 \langle X|T \rangle$. Let ρ_X be the Wagner congruence on X, we define the inverse monoid $\mathrm{Inv}^1 \langle X|T \rangle$ *presented* by $(X;T)$ as

$$\mathrm{Inv}^1 \langle X|T \rangle = \left(X \amalg X^{-1}\right)^* / (T \cup \rho_X)^{\mathrm{c}}.$$

In the previous dicussion, if we replace everywhere $(X \amalg X^{-1})^*$ with $(X \amalg X^{-1})^+$ we obtain a *presentation (for an inverse semigroup)* $(X;T)$ and an inverse semigroup $\mathrm{Inv} \langle X|T \rangle$ *presented* by $(X;T)$.

A trivial but important example is the Free Inverse Monoid [resp. Free Inverse Semigroup] on X, that is usually denoted by $\mathrm{FIM}(X)$ [resp. $\mathrm{FIS}(X)$] and is defined by

$$\mathrm{FIM}(X) = \mathrm{Inv}^1 \langle X|\varnothing \rangle = \left(X \amalg X^{-1}\right)^* / \rho_X, \ \ [\text{resp. } \mathrm{FIS}(X) = \mathrm{Inv} \langle X|\varnothing \rangle = (X \amalg X^{-1})^+ / \rho_X].$$

References

[1] N. Petrich, *Inverse Semigroups*, Wiley, New York, 1984.

[2] J.B. Stephen, *Presentation of inverse monoids*, J. Pure Appl. Algebra 63 (1990) 81-112.