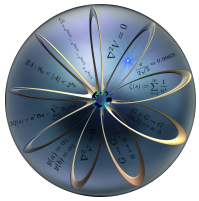

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simple semigroup

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Defines	simple
Defines	zero simple
Defines	right simple
Defines	left simple

Let S be a semigroup. If S has no ideals other than itself, then S is said to be *simple*.

If S has no left ideals [resp. right ideals] other than itself, then S is said to be *left simple* [resp. *right simple*].

Right simple and left simple are stronger conditions than simple.

A semigroup S is left simple if and only if $Sa = S$ for all $a \in S$. A semigroup is both left and right simple if and only if it is a group.

If S has a zero element θ, then $0 = \{\theta\}$ is always an ideal of S, so S is not simple (unless it has only one element). So in studying semigroups with a zero, a slightly weaker definition is required.

Let S be a semigroup with a zero. Then S is *zero simple*, or 0-simple, if the following conditions hold:

- $S^2 \neq 0$
- S has no ideals except 0 and S itself

The condition $S^2 = 0$ really only eliminates one semigroup: the 2-element null semigroup. Excluding this semigroup makes parts of the structure theory of semigroups cleaner.