Feit-Thompson conjecture

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Conjecture (Walter Feit & John Thompson). There are no prime numbers p and q (with $p \neq q$) such that $\dfrac{p^q - 1}{p - 1}$ is divisible by $\dfrac{q^p - 1}{q - 1}$.

Feit and Thompson, in regards to the Feit-Thompson theorem, have said that proving this conjecture would simplify their proof of their theorem, "rendering unnecessary the detailed use of generators and relations." In 1971, Stephens strengthened the conjecture to state that

$$\gcd\left(\frac{p^q - 1}{p - 1}, \frac{q^p - 1}{q - 1}\right) = 1$$

always, and then found the counterexample $p = 17$, $q = 3313$. The numbers $\dfrac{17^{3313} - 1}{17 - 1}$ and $\dfrac{3313^{17} - 1}{3313 - 1}$ do have 112643 as their greatest common divisor, but dividing the former by the latter leaves a remainder of 149073454345008273252753518779212742 No other counterexamples have been found to Stephen's stronger version of the conjecture.

References

[1] N. M. Stephens, "On the Feit-Thompson Conjecture" *Math. of Computation* **25** 115 (1971): 625