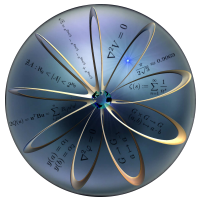

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periodic group

Canonical name	PeriodicGroup
Date of creation	2013-03-22 15:35:50
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Entry type	Definition
Classification	msc 20F50
Synonym	torsion group
Related topic	LocallyFiniteGroup
Related topic	Torsion3
Defines	periodic
Defines	torsion

A group G is said to be *periodic* (or *torsion*) if every element of G is of finite order.

All finite groups are periodic. More generally, all locally finite groups are periodic. Examples of periodic groups that are not locally finite include Tarski groups, and Burnside groups $B(m,n)$ of odd exponent $n \geq 665$ on $m > 1$ generators.

Some easy results on periodic groups:

Theorem 1.

Every http://planetmath.org/Subgroup*subgroup of a periodic group is periodic.*

Theorem 2.

Every http://planetmath.org/QuotientGroup*quotient of a periodic group is periodic.*

Theorem 3.

Every http://planetmath.org/GroupExtension*extension of a periodic group by a periodic group is periodic.*

Theorem 4.

Every restricted direct product of periodic groups is periodic.

Note that (unrestricted) direct products of periodic groups are not necessarily periodic. For example, the direct product of all finite cyclic groups $\mathbb{Z}/n\mathbb{Z}$ is not periodic, as the element that is 1 in every coordinate has infinite order.

Some further results on periodic groups:

Theorem 5. *Every solvable periodic group is locally finite.*

Theorem 6. *Every periodic abelian group is the direct sum of its maximal* http://planetmath.org/PGroup4*p-groups over all primes p.*