subsemiautomaton

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Defines	strongly connected semiautomaton
Defines	subsemiautomata
Defines	subautomaton
Defines	submachine

Just like groups and rings, a semiautomaton can be viewed as an algebraic structure. As such, one may define algebraic constructs such as subalgebras and homomorphisms. In this entry, we will briefly discuss the former.

Definition

A semiautomaton $N = (T, \Gamma, \gamma)$ is said to be a *subsemiautomaton* of a semiautomaton $M = (S, \Sigma, \delta)$ if

$$T \subseteq S, \qquad \Gamma \subseteq \Sigma, \qquad \text{and} \qquad \gamma \subseteq \delta.$$

The last inclusion means the following: $\gamma(s, a) = \delta(s, a)$ for all $(s, a) \in T \times \Gamma$. We write $N \leq M$ when N is a subsemiautomaton of M.

A subsemiautomaton N of M is said to be proper if $N \neq M$, and is written $N < M$.

Examples. Let $M = (S, \Sigma, \delta)$ be a semiautomaton.

- M can be represented by its state diagram, which is just a directed graph. Any strongly connected component of the state diagram represents a subsemiautomaton of M, characterized as a semiautomaton (S', Σ, δ') such that any state in S' can be reached from any other state in S'. In other words, for any $s, t \in S'$, there is a word u over Σ such that either $t \in \delta'(s, u)$, where δ' is the restriction of δ to $S' \times \Sigma$. A semiautomaton whose state diagram is strongly connected is said to be *strongly connected*.

- Suppose M is strongly connected. Then M has no proper subsemiautomaton whose input alphabet is equal to the input alphabet of M. In other words, if $N = (T, \Sigma, \gamma) \leq M$, then $N = M$. However, proper subsemiautomata of M exist if we take $N = (T, \Gamma, \gamma)$ for any proper subset Γ of Σ, provided that $|\Sigma| \geq 2$. In this case, γ is just the restriction of δ to the set $T \times \Gamma$.

- On the other hand, if Σ is a singleton, and M is strongly connected, then no proper subsemiautomata of M exist. Notice that if the transition function δ is single valued, then it is just a permutation on S of order $|S|$.

Specializations to Other Machines

Computing devices derived from semiautomata such as automata and state-output machines may too be considered as algebras. We record the definitions of subalgebras of these objects here.

Note: the following notations are used: given an automaton $A = (S, \Sigma, \delta, I, F)$ and a state-output machine $M = (S, \Sigma, \Delta, \delta, \lambda)$, let A' and M' be the associated semiautomaton (S, Σ, δ). So A and M may be written (A', I, F) and (M', Δ, λ) respectively.

Definition (automaton). $A = (A', I, F)$ is a *subautomaton* of $B = (B', J, G)$ if

$$A' \leq B', \qquad I \subseteq J, \qquad \text{and} \qquad F \subseteq G.$$

Definition (state-output machine). $M = (M', \Delta, \lambda)$ is a *submachine* of $N = (N', \Omega, \pi)$ if

$$M' \leq N', \qquad \Delta \subseteq \Omega, \qquad \text{and} \qquad \lambda \text{ is the restriction of } \pi \text{ to } S \times \Sigma,$$

where S and Σ are the state and input alphabets of M.

References

[1] A. Ginzburg, *Algebraic Theory of Automata*, Academic Press (1968).