Frobenius reciprocity

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Let V be a finite-dimensional representation of a finite group G, and let W be a representation of a subgroup $H \subset G$. Then the characters of V and W satisfy the inner product relation

$$(\chi_{\mathrm{Ind}(W)}, \chi_V) = (\chi_W, \chi_{\mathrm{Res}(V)})$$

where Ind and Res denote the induced representation Ind_H^G and the restriction representation Res_H^G.

The Frobenius reciprocity theorem is often given in the stronger form which states that Res and Ind are adjoint functors between the category of G–modules and the category of H–modules:

$$\mathrm{Hom}_H(W, \mathrm{Res}(V)) = \mathrm{Hom}_G(\mathrm{Ind}(W), V),$$

or, equivalently

$$V \otimes \mathrm{Ind}(W) = \mathrm{Ind}(\mathrm{Res}(V) \otimes W).$$