subgroups with coprime orders

Canonical name	SubgroupsWithCoprimeOrders
Date of creation	2013-03-22 18:55:58
Last modified on	2013-03-22 18:55:58
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	8
Author	pahio (2872)
Entry type	Theorem
Classification	msc 20D99
Related topic	Gcd
Related topic	CycleNotation

If the orders of two subgroups of a group are `http://planetmath.org/Coprime`coprime, the identity element is the only common element of the subgroups.

Proof. Let G and H be such subgroups and $|G|$ and $|H|$ their orders. Then the intersection $G \cap H$ is a subgroup of both G and H. By Lagrange's theorem, $|G \cap H|$ divides both $|G|$ and $|H|$ and consequently it divides also $\gcd(|G|,\, |H|)$ which is 1. Therefore $|G \cap H| = 1,$ whence the intersection contains only the identity element.

Example. All subgroups

$$\{(1),\, (12)\}, \quad \{(1),\, (13)\}, \quad \{(1),\, (23)\}$$

of order 2 of the symmetric group $\mathfrak{S}_3$ have only the identity element (1) common with the sole subgroup

$$\{(1),\, (123),\, (132)\}$$

of order 3.