profinite group

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1 Definition

A topological group G is *profinite* if it is isomorphic to the inverse limit of some projective system of finite groups. In other words, G is profinite if there exists a directed set I, a collection of finite groups $\{H_i\}_{i \in I}$, and homomorphisms $\alpha_{ij} \colon H_j \to H_i$ for each pair $i, j \in I$ with $i \leq j$, satisfying

1. $\alpha_{ii} = 1$ for all $i \in I$,
2. $\alpha_{ij} \circ \alpha_{jk} = \alpha_{ik}$ for all $i, j, k \in I$ with $i \leq j \leq k$,

with the property that:

- G is isomorphic as a group to the projective limit

$$\varprojlim H_i := \left\{ (h_i) \in \prod_{i \in I} H_i \;\middle|\; \alpha_{ij}(h_j) = h_i \ \text{ for all } \ i \leq j \right\}$$

  under componentwise multiplication.

- The isomorphism from G to $\varprojlim H_i$ (considered as a subspace of $\prod H_i$) is a homeomorphism of topological spaces, where each H_i is given the discrete topology and $\prod H_i$ is given the product topology.

The topology on a profinite group is called the *profinite topology*.

2 Properties

One can show that a topological group is profinite if and only if it is compact and totally disconnected. Moreover, every profinite group is residually finite.