planetmath.org

Math for the people, by the people.

inverse of a product

Canonical name	InverseOfAProduct
Date of creation	2015-01-30 21:19:19
Last modified on	2015-01-30 21:19:19
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	17
Author	pahio (2872)
Entry type	Theorem
Classification	msc 20A05
Classification	msc 20-00
Synonym	inverse of a product in group
Synonym	inverse of product
Related topic	InverseOfCompositionOfFunctions
Related topic	GeneralAssociativity
Related topic	Division
Related topic	InverseNumber
Related topic	OrderOfProducts

Theorem. If a and b are arbitrary elements of the group $(G, *)$, then the inverse of $a * b$ is

$$(a * b)^{-1} = b^{-1} * a^{-1}. \tag{1}$$

Proof. Let the neutral element of the group, which may be proved unique, be e. Using only the group postulates we obtain

$$(a*b)*(b^{-1}*a^{-1}) = a*(b*(b^{-1}*a^{-1})) = a*((b*b^{-1})*a^{-1}) = a*(e*a^{-1}) = a*a^{-1} = e,$$

$$(b^{-1}*a^{-1})*(a*b) = b^{-1}*(a^{-1}*(a*b)) = b^{-1}*((a^{-1}*a)*b) = b^{-1}*(e*b) = b^{-1}*b = e,$$

Q.E.D.

Note. The (1) may be by induction extended to the form

$$(a_1 * \cdots * a_n)^{-1} = a_n^{-1} * \cdots * a_1^{-1}.$$