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free submonoid

Canonical name	FreeSubmonoid
Date of creation	2013-03-22 18:21:36
Last modified on	2013-03-22 18:21:36
Owner	Ziosilvio (18733)
Last modified by	Ziosilvio (18733)
Numerical id	5
Author	Ziosilvio (18733)
Entry type	Definition
Classification	msc 20M10
Classification	msc 20M05
Defines	minimal generating set of a submonoid

Let A be an arbitrary set, let A^* be the free monoid on A , and let e be the identity element (empty word) of A^* .

Let M be a submonoid of A^* . The *minimal generating set* of M is

$$\text{mgs}(M) = (M \setminus \{e\}) \setminus (M \setminus \{e\})^2. \quad (1)$$

Shortly, $\text{mgs}(M)$ is the set of all the nontrivial elements of M that cannot be “reconstructed” as products of elements of M . It is straightforward that

1. $(\text{mgs}(M))^* = M$, and
2. if $S \subseteq A^*$ and $M \subseteq S^*$, then $\text{mgs}(M) \subseteq S$.

We say that M is a *free submonoid* of A^* if it is isomorphic (as a monoid) to a free monoid B^* for some set B . A set $K \subseteq A^*$ such that $K = \text{mgs}(M)$ for some free submonoid M of A^* is also called a *code*.