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metabelian group

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Definition

A *metabelian group* is a group G that possesses a normal subgroup N such that N and G/N are both abelian. Equivalently, G is metabelian if and only if the commutator subgroup $[G, G]$ is abelian. Equivalently again, G is metabelian if and only if it is solvable of length at most 2.

(Note that in older literature the term tends to be used in the stronger sense that the central quotient $G/Z(G)$ is abelian. This is equivalent to being nilpotent of class at most 2. We shall not use this sense here.)

Examples

- All abelian groups.
- All generalized dihedral groups.
- All groups of order less than 24.
- All metacyclic groups.

Properties

<http://planetmath.org/SubgroupSubgroups>, <http://planetmath.org/QuotientGroupquotient> and (unrestricted) direct products of metabelian groups are also metabelian.

In other words, metabelian groups form a <http://planetmath.org/VarietyOfGroupsvariety>; they are, in fact, the groups in which $(w^{-1}x^{-1}wx)(y^{-1}z^{-1}yz) = (y^{-1}z^{-1}yz)(w^{-1}x^{-1}wx)$ for all elements w, x, y and z .