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locally cyclic group

Canonical name	LocallyCyclicGroup
Date of creation	2013-03-22 13:47:12
Last modified on	2013-03-22 13:47:12
Owner	yark (2760)
Last modified by	yark (2760)
Numerical id	24
Author	yark (2760)
Entry type	Definition
Classification	msc 20K99
Classification	msc 20E25
Synonym	locally cyclic
Synonym	generalized cyclic group
Synonym	generalized cyclic
Synonym	generalised cyclic
Synonym	generalised cyclic group
Related topic	CyclicGroup
Related topic	AbelianGroup2
Related topic	Group
Related topic	LocallyCalP

Definition

A *locally cyclic* group is a group in which every finite subset generates a cyclic subgroup.

Properties

From the definition we see that every finitely generated locally cyclic group (and, in particular, every finite locally cyclic group) is cyclic.

The following can all be shown to be equivalent for a group G :

- G is locally cyclic.
- For all $a, b \in G$, the <http://planetmath.org/Subgroup> $\langle a, b \rangle$ is cyclic.
- G is the union of a chain of cyclic subgroups.
- The lattice of subgroups of G is <http://planetmath.org/DistributiveLattice> distributive.
- G embeds in \mathbb{Q} or \mathbb{Q}/\mathbb{Z} .
- G is isomorphic to a subgroup of a <http://planetmath.org/QuotientGroup> quotient of \mathbb{Q} .
- G is <http://planetmath.org/SectionOfAGroup> involved in \mathbb{Q} .

From the last of these equivalent properties it is clear that every locally cyclic group is countable and abelian, and that subgroups and quotients of locally cyclic groups are locally cyclic.