symmetric inverse semigroup

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Defines	partial map
Defines	composition of partial maps
Defines	symmetric inverse semigroup

Let X be a set. A *partial map* on X is an application defined from a subset of X into X. We denote by $\mathfrak{F}(X)$ the set of partial map on X. Given $\alpha \in \mathfrak{F}(X)$, we denote by $\mathrm{dom}(\alpha)$ and $\mathrm{ran}(\alpha)$ respectively the domain and the range of α, i.e.

$$\mathrm{dom}(\alpha), \mathrm{ran}\alpha \subseteq X, \quad \alpha : \mathrm{dom}(\alpha) \to X, \quad \alpha(\mathrm{dom}(\alpha)) = \mathrm{ran}(\alpha).$$

We define the composition of two partial map $\alpha, \beta \in \mathfrak{F}(X)$ as the partial map $\alpha \circ \beta \in \mathfrak{F}(X)$ with domain

$$\mathrm{dom}(\alpha \circ \beta) = \beta^{-1}(\mathrm{ran}(\beta) \cap \mathrm{dom}(\alpha)) = \{x \in \mathrm{dom}(\beta) \,|\, \alpha(x) \in \mathrm{dom}(\beta)\}$$

defined by the common rule

$$\alpha \circ \beta(x) = \alpha(\beta(x)), \quad \forall x \in \mathrm{dom}(\alpha \circ \beta).$$

It is easily verified that the $\mathfrak{F}(X)$ with the composition $\circ$ is a semigroup.

A partial map $\alpha \in \mathfrak{F}(X)$ is said *bijective* when it is bijective as a map $\alpha : \mathrm{ran}(\alpha) \to \mathrm{dom}(\alpha)$. It can be proved that the subset $\mathfrak{I}(X) \subseteq \mathfrak{F}(X)$ of the partial bijective maps on X is an inverse semigroup (with the composition $\circ$), that is called *symmetric inverse semigroup* on X. Note that the symmetric group on X is a subgroup of $\mathfrak{I}(X)$.