subring

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Let $(R, +, *)$ a ring. A subring is a subset S of R with the operations $+$ and $*$ of R restricted to S and such that S is a ring by itself.

Notice that the restricted operations inherit the associative and distributive properties of $+$ and $*$, as well as commutativity of $+$. So for $(S, +, *)$ to be a ring by itself, we need that $(S, +)$ be a subgroup of $(R, +)$ and that $(S, *)$ be closed. The subgroup condition is equivalent to S being non-empty and having the property that $x - y \in S$ for all $x, y \in S$.

A subring S is called a left ideal if for all $s \in S$ and all $r \in R$ we have $r * s \in S$. Right ideals are defined similarly, with $s * r$ instead of $r * s$. If S is both a left ideal and a right ideal, then it is called a two-sided ideal. If R is commutative, then all three definitions coincide. In ring theory, ideals are far more important than subrings, as they play a role analogous to normal subgroups in group theory.

Example:

Consider the ring $(\mathbb{Z}, +, \cdot)$. Then $(2\mathbb{Z}, +, \cdot)$ is a subring, since the difference and product of two even numbers is again an even number.