virtually abelian subgroup theorem

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Let us suppose that G is virtually abelian and H is an abelian subgroup of G with a the finite right coset partition

$$G = He \sqcup Hx_2 \sqcup ... \sqcup Hx_q, \qquad (*)$$

so if K is any other subgroup in G we are going to prove:
K is also virtually abelian

Proof: From $(*)$ above we have

$$K = K \cap G = K \cap (He \sqcup Hx_2 \sqcup ... \sqcup Hx_q),$$

$$= (K \cap H) \sqcup (K \cap Hx_2) \sqcup ... \sqcup (K \cap Hx_q). \qquad (**)$$

Here we consider the two cases:
1) $x_i \in K$
2) $x_j \notin K$

In the first case $K = Kx_i$, and then $K \cap Hx_i = Kx_i \cap Hx_i = (K \cap H)x_i$. In the second, find $y_j \in K \cap Hx_j$ hence $K \cap Hx_j = Ky_j \cap Hy_j = (K \cap H)y_j$

So, in the equation $(**)$ above we can replace (reordering subindexation perhaps) to get

$$K = \underbrace{(K \cap H) \sqcup (K \cap H)x_2 \sqcup ... \sqcup (K \cap H)x_s}_{1)} \sqcup \underbrace{(K \cap H)y_{s+1} \sqcup ... \sqcup (K \cap H)y_q}_{2)}$$

relation which shows that the index $[K : K \cap H] \leq [G : H]$.

It could be $<$ since it is posible that $K \cap Hx_r = \emptyset$ for some indexes r $\Box$