generalized dihedral group

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Defines	infinite dihedral

Let A be an abelian group. The *generalized dihedral group* $\mathrm{Dih}(A)$ is the semidirect product $A \rtimes C_2$, where C_2 is the cyclic group of order 2, and the `http://planetmath.org/Generator`generator of C_2 maps elements of A to their inverses.

If A is cyclic, then $\mathrm{Dih}(A)$ is called a dihedral group. The finite dihedral group $\mathrm{Dih}(C_n)$ is commonly denoted by D_n or D_{2n} (the differing conventions being a source of confusion). The infinite dihedral group $\mathrm{Dih}(C_\infty)$ is denoted by D_∞, and is isomorphic to the free product $C_2 * C_2$ of two cyclic groups of order 2.

If A is an elementary abelian 2-group, then so is $\mathrm{Dih}(A)$. If A is not an elementary abelian 2-group, then $\mathrm{Dih}(A)$ is non-abelian.

The subgroup $A \times \{1\}$ of $\mathrm{Dih}(A)$ is of index 2, and every element of $\mathrm{Dih}(A)$ that is not in this subgroup has order 2. This property in fact characterizes generalized dihedral groups, in the sense that if a group G has a subgroup N of index 2 such that all elements of the complement $G \setminus N$ are of order 2, then N is abelian and $G \cong \mathrm{Dih}(N)$.