unimodular matrix

Canonical name	UnimodularMatrix
Date of creation	2013-03-22 14:57:50
Last modified on	2013-03-22 14:57:50
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	13
Author	CWoo (3771)
Entry type	Definition
Classification	msc 20H05
Classification	msc 15A04
Classification	msc 15A09
Related topic	SpecialLinearGroup
Defines	unimodular linear transformation
Defines	unimodular row
Defines	unimodular column
Defines	unimodular group
Defines	unimodular vector

An $n \times n$ square matrix over a field is *unimodular* if its determinant is 1. The set of all $n \times n$ unimodular matrices forms a group under the usual matrix multiplication. This group is known as the special linear group. Any of its subgroup is simply called a *unimodular group*. Furthermore, unimodularity is preserved under similarity transformations: if S any $n \times n$ invertible matrix and U is unimodular, then $S^{-1}US$ is unimodular. In view of the last statement, the special linear group is a normal subgroup of the group of all invertible matrices, known as the general linear group.

A linear transformation T over an n-dimensional vector space V (over a field F) is *unimodular* if it can be represented by a unimodular matrix.

The concept of the unimodularity of a square matrix over a field can be readily extended to that of a square matrix over a commutative ring. Unimodularity in square matrices can even be extended to arbitrary finite-dimensional matrices: suppose R is a commutative ring with 1, and M is an $m \times n$ matrix over R (entries are elements of R) with $m \leq n$. Then M is said to be *unimodular* if it can be "completed" to a $n \times n$ square unimodular matrix N over R. By completion of M to N we mean that m of the n rows in N are exactly the rows of M. Of course, the operation of completion from a matrix to a square matrix can be done via columns too.

Let M is an $m \times n$ matrix and v is any row of M. If M is unimodular, then v is unimodular viewed as a $1 \times n$ matrix. A $1 \times n$ unimodular matrix is called a *unimodular row*, or a *unimodular vector*. A $n \times 1$ *unimodular column* can be defined via a similar procedure. Let $v = (v_1, \ldots, v_n)$ be a $1 \times n$ matrix over R. Then the unimodularity of v means that

$$v_1R + \cdots + v_nR = R.$$

To see this, let U be a completion of v with $\det(U) = 1$. Since det is a multilinear operator over the rows (or columns) of U, we see that

$$1 = \det(U) = v_1r_1 + \cdots + v_nr_n.$$