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subgoups of locally cyclic groups are locally cyclic

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Theorem 1. *A group G is locally cyclic iff every subgroup $H \leq G$ is locally cyclic.*

Proof. Let G be a locally cyclic group and H a subgroup of G. Let S be a finite subset of H. Then the group $\langle S \rangle$ generated by S is a cyclic subgroup of G, by assumption. Since every element a of $\langle S \rangle$ is a product of elements or inverses of elements of S, and S is a subset of group H, $a \in H$. Hence $\langle S \rangle$ is a cyclic subgroup of H, so H is locally cyclic.

Conversely, suppose for every subgroup of G is locally cyclic. Let H be a subgroup generated by a finite subset of G. Since H is locally cyclic, and H itself is finitely generated, H is cyclic, and therefore G is locally cyclic. $\square$