alternative characterization of multiply transitive permutation groups

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This article derives an alternative characterization of n-transitive groups.

Theorem. *For $n > 1$, G is n-transitive on X if and only if for all $x \in X$, G_x is $(n-1)$-transitive on $X - \{x\}$.*

Proof. First assume G is n-transitive on X, and choose $x \in X$. To show G_x is $(n-1)$-transitive on $X - \{x\}$, choose $x_1, \ldots, x_{n-1}, y_1, \ldots, y_{n-1} \in X$. Since G is n-transitive on X, we can choose $\sigma \in G$ such that

$$\sigma \cdot (x_1, \ldots, x_{n-1}, x) = (y_1, \ldots, y_{n-1}, x)$$

But obviously $\sigma \in G_x$, and σ restricted to $X - \{x\}$ is the desired permutation.

To prove the converse, choose $x_1, \ldots, x_n, y_1, \ldots, y_n \in X$. Choose $\sigma_1 \in G_{x_n}$ such that

$$\sigma_1 \cdot (x_1, \ldots, x_{n-1}) = (y_1, \ldots, y_{n-1})$$

and choose $\sigma_2 \in G_{y_1}$ such that

$$\sigma_2 \cdot (y_2, \ldots, y_{n-1}, x_n) = (y_2, \ldots, y_{n-1}, y_n)$$

Then $\sigma_2\sigma_1$ is the desired permutation. $\square$

Note that this definition of n-transitivity affords a straightforward proof of the statement that A_n is $(n-2)$-transitive: by inspection, A_3 is 1-transitive; the result follows by induction using the theorem. (The corresponding statement that S_n is n-transitive is obvious).

Finally, note that the most common cases of n-transitivity are for $n = 1$ (*transitive*), and $n = 2$ (*doubly transitive*).