generated subring

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Defines	subring generated by
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Definition 1 *Let M be a nonempty subset of a ring A. The intersection of all subrings of A that include M is the smallest subring of A that includes M. It is called the* subring generated by M and is denoted by $\langle M \rangle$.

The subring generated by M is formed by finite sums of monomials of the form :
$$a_1 a_2 \cdots a_n, \text{where} \ \ a_1, \ldots, a_n \in M.$$

Of particular interest is the subring generated by a family of subrings $E = \{A_i | \ \ i \in I\}$. It is the ring R formed by finite sums of monomials of the form:

$$a_{i_1} a_{i_2} \ldots a_{i_n}, \text{where} \ \ a_{i_k} \in A_{i_k}.$$

If A, B are rings, the subring generated by $A \cup B$ is also denoted by AB.
In the case when A_i are fields included in a larger field A then the set of all quotients of elements of R (the quotient field of R) is the composite field $\bigvee_{i \in I} A_i$ of the family E. In other words, it is the subfield generated by $\bigcup_{i \in I} A_i$. The notation $\bigvee_{i \in I} A_i$ comes from the fact that the family of all subfields of a field forms a complete lattice.
The of fields is defined only when the respective fields are all included in a larger field.