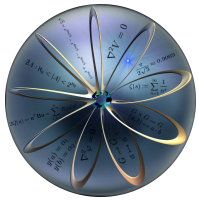

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Minkowski sum

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Definition Suppose A and B are sets in a vector space V over a field K, and suppose $\lambda \in K$. Then

$$\begin{aligned} A + B &= \{a + b \mid a \in A, b \in B\}, \\ A - B &= \{a - b \mid a \in A, b \in B\}, \\ \lambda A &= \{\lambda a \mid a \in A\}, \\ -A &= (-1)A. \end{aligned}$$

The set $A + B$ is called the *Minkowski sum* of A and B. If either A or B is a single point (a singleton), say $B = \{x\}$, then we write $A + x$ instead of $A + \{x\}$. Similarly we define $A - x$, $x - A$ and $x + A$.

Properties

Suppose A,B, V, and λ are as above. Then

- $A + B = B + A$
- $\lambda(A + B) = \lambda A + \lambda B$
- $2A \subseteq A + A$, $3A \subseteq A + A + A$, etc, but in general, $A + A \neq 2A$. (Consider $A = \{(0, 0), (0, 1)\}$ in $\mathbb{R}^2$.)