

1. **Integration:**

$$\int_0^1 x^2 e^x dx$$

2. A coin is tossed twice. The following table shows the probability distribution of the number of tails:

X	0	1	2
$P(X)$	K	$6K$	$9K$

(a) Find the value of K .

(b) Determine whether the coin toss is biased or unbiased, and provide a justification for your answer.

3. Find the general solution for this differential equation:

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

4. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m using integration.

5. Given vectors:

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + \hat{j} - 2\hat{k}, \quad \vec{c} = \hat{i} + 3\hat{j} - \hat{k}$$

and the projection of the vector $\vec{c} + \lambda\vec{b}$ on vector \vec{a} is $2\sqrt{6}$, find the value of λ .

6. (a) **Find:**

$$\int \frac{1}{e^x + 1} dx$$

OR

(b) **Evaluate:**

$$\int_1^4 ||x| + |3-x|| dx$$

7. **Evaluate:**

$$\int_{-3}^3 \frac{x^4}{1+e^x} dx$$

8. (a) Find the particular solution of the differential equation:

$$x \frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$$

given that $y(1) = 0$.

OR

(b) Find the general solution of the differential equation:

$$x(y^3 + x^3) dy = (2y^4 + 5x^3 y) dx$$

9. Find:

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

10. Find the general solution of the following equation:

$$\frac{dy}{dx} = e^x - yx^2 e^{-y}$$

11. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$$

Find the probability distribution of X .

12. (a) **Find:**

$$\int e^x \sin(2x) dx$$

OR

(b) Find:

$$\int \frac{2x}{(x^2 + 1)(x^2 + 2)} dx$$

13. **Evaluate:**

$$\int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}} dx$$

14. (a) Solve the following differential equations:

$$(y - \sin^2 x) dx + \tan(x) dy = 0$$

(b) Find the general solution of the differential equation:

$$(x^3 + y^3) dy = x^2 y dx$$

15. (a) **Find:**

$$\int \frac{1}{\sqrt{12 + 4x - x^2}} dx$$

OR

(b) Find:

$$\int \frac{xe^x}{(x+4)^5} dx$$

16. Find the general solution of the following differential equation:

$$(4 + y^2)(3 + \log x) dx + x dy = 0$$

17. Evaluate:

$$\int_0^{\frac{\pi}{3}} |\cos(3x)| \, dx$$

18. (a) Find the general solution of the following differential equation:

$$2xe^{\frac{y}{x}} dy + (x - 2ye^{\frac{y}{x}}) dx = 0$$

OR

(b) Find the particular solution of the differential equation:

$$(2x^2 + y) \frac{dx}{dy} = x$$

given that $y = 2$ when $x = 1$.

19. Find:

$$\int \frac{x^2 + x + 1}{(x+1)(x^2+4)} \, dx$$

20. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

21.

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}} \, dx$$

is equal to:

- (a) $\frac{\pi}{3}$
- (b) $\frac{\pi}{6}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{2}$

22. Find:

$$\int \frac{(x+2)(x+2\log x)^3}{x} \, dx$$

23.

$$\int_0^{\frac{\pi}{2}} \log(\tan x) \, dx$$

OR

$$\int_{-1}^2 |x| \, dx$$

24. Find:

$$\int x^2 \log x dx$$

25. Find the general solution of the following differential equation :

$$\frac{dy}{dx} = (1+x)(1+y)$$

OR

Find the integrating factor for the following differential equation :

$$\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x (x \neq 0)$$

26. Find:

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

27. Find the following differential equation :

$$x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$$