1. Integration:

$$\int_0^1 x^2 e^x \, dx$$

2. A coin is tossed twice. The following table shows the probability distribution of the number of tails:

X	0	1	2
P(X)	K	6K	9K

   (a) Find the value of K.

   (b) Determine whether the coin toss is biased or unbiased, and provide a justification for your answer.

3. Find the general solution for this differential equation:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

4. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m using integration.

5. Given vectors:

$$\mathbf{a} = 2\hat{i} - \hat{j} + \hat{k}, \quad \mathbf{b} = \hat{i} + \hat{j} - 2\hat{k}, \quad \mathbf{c} = \hat{i} + 3\hat{j} - \hat{k}$$

   and the projection of the vector $\mathbf{c} + \lambda\mathbf{b}$ on vector $\mathbf{a}$ is $2\sqrt{6}$, find the value of λ.

6. Find:

$$\int \frac{1}{e^x + 1} \, dx$$

7. Evaluate:

$$\int_1^4 \{|x| + |3 - x|\} \, dx$$

8. Evaluate:

$$\int_{-3}^{3} \frac{x^4}{1+e^x}\,dx$$

9. Find the particular solution of the differential equation:

$$x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$$

given that $y(1) = 0$.

10. Find the general solution of the differential equation:

$$x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$$

11. Find:

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

12. Find the general solution of the following equation:

$$\frac{dy}{dx} = e^x - yx^2e^{-y}$$

13. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$$

Find the probability distribution of X.

14. Find:

$$\int e^x \sin(2x)\,dx$$

15. Find:

$$\int \frac{2x}{(x^2+1)(x^2+2)}\,dx$$

16. Evaluate:

$$\int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4 - x}}\, dx$$

17. Solve the following differential equations:

$$(y - \sin^2 x)\, dx + \tan(x)\, dy = 0$$

18. Find the general solution of the differential equation:

$$(x^3 + y^3)\, dy = x^2 y\, dx$$

19. Find:

$$\int \frac{1}{\sqrt{12 + 4x - x^2}}\, dx$$

20. Find:

$$\int \frac{xe^x}{(x + 4)^5}\, dx$$

21. Find the general solution of the following differential equation:

$$(4 + y^2)(3 + \log x)\, dx + x\, dy = 0$$

22. Evaluate:

$$\int_0^{\frac{\pi}{3}} \left|\cos(3x)\right|\, dx$$

23. Find the general solution of the following differential equation:

$$2xe^{\frac{y}{x}}\, dy + (x - 2ye^{\frac{y}{x}})\, dx = 0$$

24. Find the particular solution of the differential equation:

$$(2x^2 + y)\frac{dx}{dy} = x$$

given that $y = 2$ when $x = 1$.

25. Find:

$$\int \frac{x^2 + x + 1}{(x + 1)(x^2 + 4)}\, dx$$

26. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

27. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

28. Find:

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}}\, dx$$

is equal to:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

29. Find:

$$\int \frac{(x + 2)(x + 2\log x)^3}{x}\, dx$$

30. Find:

$$\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$$

31. Find:

$$\int_{-1}^{2} |x|\, dx$$

32. Find:

$$\int x^2 log x.dx$$

33. Find the general solution of the following differential equation :

$$\frac{dy}{dx} = (1 + x)(1 + y)$$

34. Find the integrating factor for the following differential equation:

$$\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x (x \neq 0)$$

35. Find:

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

36. Find the following differential equation :

$$x \cos(\frac{y}{x}) \frac{dy}{dx} = y \cos(\frac{y}{x}) + x$$