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A Class of Time Series Urban Water Demand Models With Nonlinear Climatic Effects

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A new class of monthly time series urban water demand model is proposed. The model postulates that water use is made up of base use and seasonal use; and the latter consists of three components: a potential use that is dependent on temperature in the absence of rainfall, a water use reduction due to rainfall occurrences, and a random component. The proposed model utilizes three observations that were established in recent daily water use studies: (1) "hysteresis" temperature effect: under the same temperature, water use has different levels and response rates (to a unit change of temperature) in different seasons, (2) "dynamic" rainfall effect: a rainfall causes a temporary reduction in seasonal use that diminishes over time, and (3) "state-dependent" rainfall effect: the higher the seasonal use level prior to the occurrence of a rainfall, the more significant the effect is expected. Monthly rainfall effects are derived through the time aggregating and averaging of a daily response model. The model so obtained is nonlinear in structure. The performance of the model is compared with conventional linear models using monthly data in Austin, Texas, from 1975 to 1984. The proposed nonlinear models outperform the linear models in describing seasonal water use variations in terms of adjusted R^2, Akaike information criterion value, and ability to estimate the high summer use in dry and wet years.

INTRODUCTION

Water demand modeling plays a key role in urban water resources planning and management, for example, water and wastewater facility scheduling and planning, water conservation program evaluation, and water pricing policy assessment. Since the seminal residential water use study by *Howe and Linaweaver* [1967], a number of water demand models have been proposed in the context of linear multiple regression [e.g., *Hansen and Narayanan*, 1981; *Morgan and Smolen*, 1976] and time series analysis [e.g., *Maidment and Parzen*, 1984]. Water use, together with socioeconomic and climatic variables, which have potential influences on water use, have been collected and examined both in time series and cross-sectional formats. Socioeconomic variables such as population, income, water price, and housing characteristics are postulated to impose long-term changes on water use patterns; while climatic variables such as rainfall and temperature induce short-term seasonal variations.

Much of the attention has been focused on estimating the response of water use to socioeconomic changes, particularly water price: for example, assessing the merits of using marginal price against average price [*Gibbs*, 1978] and identifying short-run and long-run price elasticities [*Agthe and Billings*, 1980; *Carver and Boland*, 1980]. The response to climatic variables, however, did not receive as much consideration, although its close connection to outdoor water use (mainly sprinkling use) has been well perceived. The paper by *Morgan and Smolen* [1976] was one of the very few studies that focused on climatic effects, in which they compared the use of three alternative climatic indicators: rainfall and temperature, effective evapotranspiration, and dummy seasonal binary variables, in a linear multiple regression model. Their study indicated that using different climatic indicators could result in rather different estimates of price elasticity. Therefore, in practice, unless the climatic effects could be properly determined from the water use data and separated from the socioeconomic impacts, analysis results relating socioeconomic variables to water use could be biased.

Due to *Howe and Linaweaver* [1967], many studies have separated water use into two components: nonseasonal (or winter) use and seasonal (or summer) use. Both have traditionally been modeled by either linear or log linear multiple regression models to determine the effects of climatic as well as socioeconomic variables on water use. Though the linear assumption is mainly for simplicity, it is also in part due to the lack of a priori knowledge of the functional relationship between these variables and water use.

Temperature and rainfall were the most commonly used climatic indicators because of their availability. More elaborate indicators reflecting soil moisture conditions such as effective evapotranspiration were also employed in several studies. It is not uncommon, however, that researchers obtained coefficients with unexpected signs for climatic variables from their linear demand models, for example, *Young* [1973] and *Morgan and Smolen* [1976].

A closer examination of the following questions is needed: Are the climatic effects on monthly water use purely physical, as the use of effective evapotranspiration implies? Is it a good assumption that monthly water use is affected by temperature and rainfall "linearly," as the linear regression model assumes? Are the climatic effects adequately accounted for in the traditional linear monthly demand models? This paper is intended to provide better answers to these questions by utilizing some useful observations and evidence of the climatic effects that were established in recent daily urban water use studies.

The main objective of this paper is to propose a new class of mathematical representation (as opposed to conventional linear regression) of the climatic effects, specifically temperature and rainfall effects, on monthly water use. Monthly rainfall effect is derived through the time aggregating and averaging of a daily response model, which is a Box-Jenkins' type of transfer function, proposed in daily urban water use

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studies. It is demonstrated that the class of model so derived is nonlinear in structure. The performance of the proposed class of model is compared with the linear regression models employed in other studies using monthly time series water use, temperature, and rainfall data in Austin, Texas, from 1975 to 1984.

This paper is organized as follows. First, previous studies on the responses of daily water use to temperature and rainfall are reviewed. Second, a class of monthly demand models with nonlinear climatic effects are proposed. Third, the performance of the proposed models is compared with the linear models used in other studies. The final section concludes the study.

PREVIOUS RESEARCH

Recent daily urban water use studies have focused on the characterization of climatic effects and the assessment of model forecasting capabilities. Results from various cities have been reported [e.g., *Miaou*, 1983, 1986, 1987; *Maidment et al.*, 1985; *Maidment and Miaou*, 1986]. This section summarizes some observations of daily climatic effects and their implications from these studies.

Daily Rainfall Effects

The effect of rainfall on daily urban water use was characterized to be both "dynamic" and "state-dependent":

Dynamic. The occurrence of a rainfall causes a temporary reduction in seasonal water use that diminishes over time and eventually becomes negligible. Studies indicated that the significant effect of a day's rainfall could last from one to two weeks.

State-Dependent. Under the same rainfall conditions the higher the seasonal water use level prior to the occurrence of a rainfall, the more the effect can be expected. Figure 1 illustrates this observation, in which the immediate water use reductions due to the occurrence of rainfalls are shown to be approximately proportional to their previous day's seasonal water use level, and the proportionality increases as the rainfall amount increases.

The state-dependent property of rainfall effect has two important implications: (1) people respond more to its occurrence than to its amount; in other words, the effect is more psychological than physical (at least in the short run), and (2) rainfall has relatively no effects when water use approaches its base (or indoor) use level, which is either a result of low temperature in the winter or several days of sustained rainfalls.

Miaou [1983] suggested that the dynamic and state-dependent nature of the rainfall effects could be modeled by the Box-Jenkins transfer function. The response of daily water use to the occurrence of a rainfall at day t_i was modeled by

$$E_t^R = \frac{\omega_0 - \omega_1 B}{1 - \delta B} L_t^i \tag{1}$$

where L_t^i is a pulse input series defined as

$$L_t^i = S_{t_i - 1} \qquad t = t_i$$

$$L_t^i = 0 \qquad \forall t \neq t_i$$

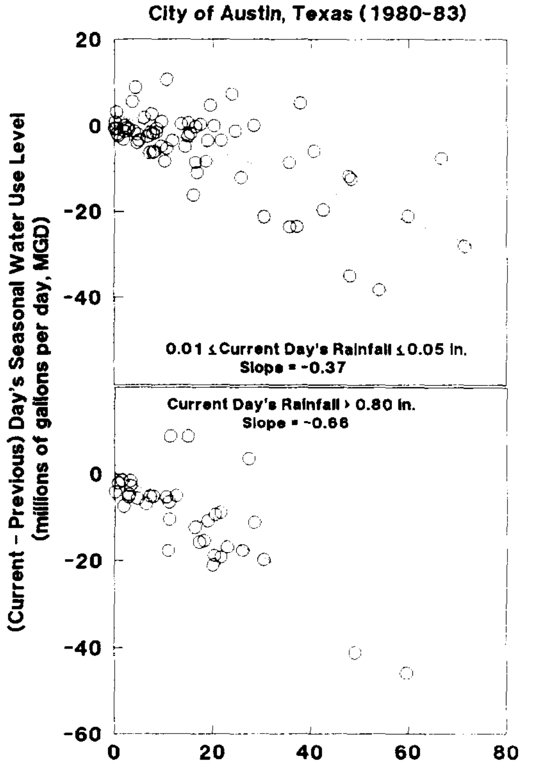

Fig. 1. State-dependent rainfall effect: the higher the seasonal water use level prior to the occurrence of a rainfall, the more the water use reduction can be expected.

t	daily time index ($t = 1, 2, \cdots, t_i, \cdots$);
E_t^R	water use response to rainfall at day t;
S_{t_i-1}	seasonal (or outdoor) water use level at day $t_i - 1$;
B	backshift operator such that $BL_t^i = L_{t-1}^i$;
$\omega_0, \omega_1, \delta$	transfer function coefficients dependent on rainfall amount.

It can be shown that the integrated response of water use through time to a unit pulse input (i.e., $S_{t_i-1} = 1$) in (1) can be computed by replacing the backshift operator B with value 1.0, and as a result we obtain $(\omega_0 - \omega_1)/(1 - \delta)$. *Maidment and Miaou* [1986] obtained two sets of transfer function coefficients for rainfall greater than and less than or equal to 0.05 inches (0.13 cm/d) for the city of Austin using data from 1980 to 1984: $\omega_0 = -0.303, -0.230$, $\omega_1 = 0.148, 0.114$, and $\delta = 0.764, 0.624$. The magnitudes of the integrated response for these two rainfall groups, due to a unit input, are -1.91 (= $(-0.303 - 0.148)/(1 - 0.764)$) and -0.92 (= $(-0.230 - 0.114)/(1 - 0.624)$), respectively. For the time period under their study the number of rains in these two rainfall groups are, respectively, about three quarters and one quarter of the total. Therefore, on the average, the occurrence of a daily rainfall, regardless of its amount, reduces 1.66 (= $-1.91 \times 0.75 - 0.92 \times 0.25$) times its previous day's seasonal water use level over a period of time. The Austin daily rainfall model also implies that a sustained rainfall of 5 days would reduce the seasonal water use level to about 20% of the level prior to the rainfall occurrences.

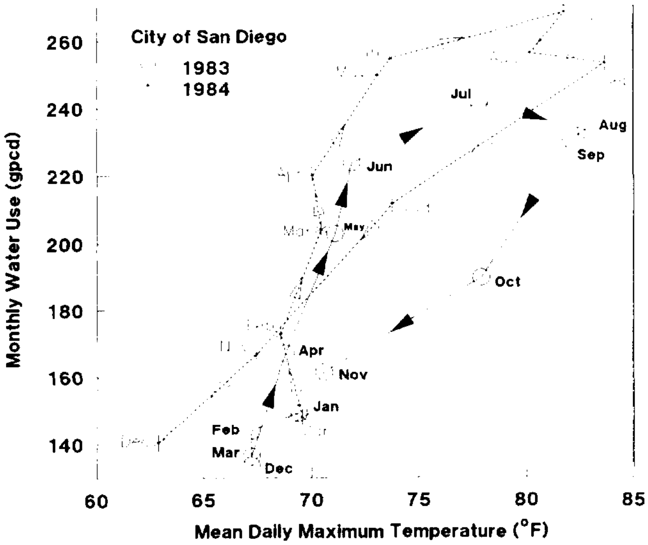

Fig. 2. Hysteresis temperature effect on monthly water use in San Diego, California.

Daily Temperature Effects

Two major concerns about temperature effect in the daily water use study were (1) to determine a break point temperature, called "reference" or "threshold" temperature, below which water use is independent of temperature, and (2) to characterize the seasonal water use variations as a function of temperature. Efforts were made to identify the reference temperature in different climatic areas by *Maidment and Miaou* [1986], in which nine cities within three regions of the United States were studied, and the temperature effect was characterized by a segmented linear model. As to the second concern, *Miaou* [1986] has examined situations where water use may exhibit a pronounced seasonal variation, as that exemplified in Figure 2, in which monthly water use (in units of gallons per capita per day) from the City of San Diego, California, is plotted against mean daily maximum temperature (in °F) of the corresponding month. The "hysteresis" shown in the figure is a result of several possible causes, for example, the calendar of activity of the city's institutions, variations in solar radiation characterized by daylight lengths, or the persistent nature of lawn watering behavior. Under the same temperature the hysteresis effect renders different seasonal water use levels and, possibly, different response rates (to a unit change of temperature) during the spring and fall. In general, applying conventional linear models to water use series with pronounced hysteresis temperature effects, as exhibited in Figure 2, would produce residual series with high correlation.

In order to separate temperature effect from other factors, particularly rainfall, *Miaou* [1986] defined a potential seasonal water use as a function of recent temperature in the absence of rainfall. To model the hysteresis effect, the response rates (or slopes) of the potential seasonal water use to temperature variations were modeled by a Fourier series. An extension of his potential seasonal water use model with both response rate and intercept modeled by Fourier series is adopted in this study and presented in the next section.

THEORY

Monthly water use in a city, W_m, is made up of base use, B_m, and seasonal use, S_m, where m is the monthly time index, both of which may exhibit a trend through time. W_m is collected in units of volume per capita per day. B_m is defined as the portion of water use (mainly indoor use) that is insensitive to climatic conditions and is typically characterized by the winter use from November to March. However, studies indicate that winter use can be quite weather-sensitive in some areas, for example, Florida [*Gibbs*, 1978] and southern California [*Miaou*, 1987]. In this section a more general model framework, which allows both B_m and S_m to be weather-dependent, is proposed.

Base Use

Base use can be represented by a function of socioeconomic and climatic variables:

$$B_m = h_m(T, R, X; \beta) + \nu_m \qquad \forall\ m \in \text{winter months} \tag{2}$$

where

m monthly time index;
$h_m(\ )$ function h at month m;
T monthly maximum air temperature;
R monthly total rainfall;
X a set of socioeconomic variables $(X_1, X_2, \cdots)$;
β parameter vector to be estimated;
ν model residuals.

A special formulation of (2) would be

$$B_m = \beta_0 + \beta_1 H_\tau(T_m) + \beta_2 G_\gamma(R_m) + \sum_i \beta_{i+2} X_{i,m} + \nu_m \tag{3}$$

where $H_\tau(\ )$ and $G_\gamma(\ )$ are the "heat" and "effective rainfall" functions defined as

$$H_\tau(T_m) = T_m - \tau \qquad T_m \geq \tau$$
$$H_\tau(T_m) = 0 \qquad \text{otherwise}$$
$$G_\gamma(R_m) = R_m \qquad R_m \leq \gamma$$
$$G_\gamma(R_m) = \gamma \qquad \text{otherwise}$$

where τ and γ are called "reference" or "threshold" temperature and "reference" or "threshold" rainfall, respectively. A threshold temperature below which water use is independent of temperature is represented by τ, and γ is a threshold rainfall amount whereby excess rainfall would not contribute more to the water use reduction, as a result of saturated soil moisture content or because water use has already been driven to its base level. Depending on the ranges of the available climatic variables, these two thresholds may not be identifiable in practice.

If we can identify τ and γ statistically from the data, then a plausible estimate of base use that is totally independent of climatic effects is to substitute T_m and R_m with τ and γ in (3):

$$\hat{B}_m = \hat{\beta}_0 + \hat{\beta}_2\gamma + \sum_i \hat{\beta}_{i+2} X_{i,m} \tag{4}$$

where $\hat{\beta}$s are the estimated parameters from the regression analysis. The temperature term zeros out because $H_\tau(\tau)$ equals zero.

Seasonal Use

The same model framework as that presented in (3) can be constructed for seasonal water use S_m, $m = 1, \cdots, M$, where M is the total number of monthly observations. However, the variations of S_m are expected to be more influenced by the climatic effects than winter use. Hence a different approach is taken.

Adopting the postulates suggested in the daily water use study by *Maidment et al.* [1985], S_m is hypothesized to consist of three components: (1) a potential seasonal water use that is dependent on temperature in the absence of rainfall, (2) a water use reduction due to rainfall occurrences, and (3) a zero mean random component. Both potential seasonal water use and the response of water use to rainfall could exhibit trends over time. For ease of exposition the following discussions may ignore trend variations where climatic effect is the theme. However, the inclusion of these trends is straightforward.

Potential Seasonal Water Use

If the hysteresis effect, exhibited in Figure 2, is not significant, potential seasonal water use P_m can be modeled as a linear function of the heat function:

$$P_m = \beta_0 + \beta_1 H_\tau(T_m) \tag{5}$$

When the hysteresis effect is deemed significant and a relatively long data set (e.g., more than 3 years) is available, the Fourier series coefficient model proposed by *Miaou* [1986] can be adopted:

$$P_m = c_m + f_m T_m \tag{6}$$

where

$$c_m = a_0 + \sum_{j=1}^{h1} [a_j \cos(\eta j m) + b_j \sin(\eta j m)]$$

$$f_m = \bar{a}_0 + \sum_{j=1}^{h2} [\bar{a}_j \cos(\eta j m) + \bar{b}_j \sin(\eta j m)]$$

m monthly time index, $m = 1$ being January of the first year;
c_m, f_m intercept and response rate at month m;
$h1, h2$ number of harmonics in a year;
$\eta = 2\pi/12$;
a, b Fourier series intercept coefficients;
$\bar{a}, \bar{b}$ Fourier series response rate coefficients.

Models in (5) and (6) can easily be extended to include trend variations, for example, by including the social-economic variables or by allowing the amplitude of the Fourier series response rate f_m in (6) to have an exponent trend: $P_m = c_m + f_m e^{\lambda m} T_m$, where λ is a growth constant to be estimated [*Miaou*, 1987].

Seasonal Water Use Response to Rainfall

The response of monthly water use to rainfall occurrences can be determined through the time aggregating and averaging of the daily response model in (1) as follows.

Let us suppose that there are n rainy days in the month m occurring at days $t_1, t_2, \cdots, t_n$, respectively. Equation (1) suggests that the total response of seasonal water use to these n rainfalls, TE_m^R, can be computed as the sum of each individual effect:

$$TE_m^R = \sum_{i=1}^{n} \sum_{t=t_i}^{\infty} \frac{\omega_0 - \omega_1 B}{1 - \delta B} L_t^i \tag{7}$$

where L_t^i, $i = 1, 2, \cdots, n$ are the ith pulse input series, as defined in (1). By assuming a constant integrated water use response to rainfall and summing each individual effect we have

$$\begin{aligned} TE_m^R &= \sum_{i=1}^{n} \left(\frac{\omega_0 - \omega_1}{1 - \delta}\right) S_{t_i - 1} \\ &= \left(\frac{\omega_0 - \omega_1}{1 - \delta}\right) \sum_{i=1}^{n} S_{t_i - 1} \\ &\approx N_m \left(\frac{\omega_0 - \omega_1}{1 - \delta}\right) S_m \end{aligned} \tag{8}$$

The last step of (8) uses the approximation that $\Sigma_{i=1}^n S_{t_i - 1} \approx nS_m$, where S_m is the per capita daily average of seasonal water use in month m, and the number of rainy days n is replaced by N_m. The integration result shows that the number of rainy days in a month is a key variable in determining the total response to rainfall. Recall that the transfer function coefficients are actually dependent on the rainfall amount. The constant integrated response assumption applied in (8) will be relaxed later in the derivation by letting it be a linear function of rainfall amount.

Since the empirical daily studies indicated that the effect of a day's rainfall can last for about 1–2 weeks, rainfalls that occur at the last two quarters of the month are expected to have some impacts on the following month but not further; therefore TE_m^R can be regarded as the total response of water use at months m and $m + 1$ to the rainfalls that occur in month m. Now, let $\Delta_{m,m+1}$ represent the carryover effect of rainfalls that occur in month m to water use at month $m + 1$. The total water use response at month m to rainfalls is then

$$\Gamma_m = TE_m^R - \Delta_{m,m+1} + \Delta_{m-1,m} \tag{9}$$

Many empirical studies [e.g., *Hansen and Narayanan*, 1981] have suggested that the lagged climatic variables were statistically insignificant, which could be a result of either $\Delta_{m,m+1}$ and $\Delta_{m-1,m}$ both being quite small, or our inability, in fact, to distinguish between $\Delta_{m,m+1}$ and $\Delta_{m-1,m}$ from the data. To simplify the derivations that follow, we assume that $\Delta_{m,m+1}$ and $\Delta_{m-1,m}$ are insignificant and use TE_m^R to represent water use response to the current month's rainfall. Derivations to include these lagged climatic effects are, however, possible (at the cost of more parameters).

With this assumption, the daily average of the response E_m^R can then be obtained by dividing the total monthly response in (8) by the number of calendar days, say 30, of that month:

$$\begin{aligned} E_m^R &= N_m \left(\frac{\omega_0 - \omega_1}{1 - \delta}\right) S_m / 30 \\ &= \alpha N_m S_m \end{aligned} \tag{10}$$

where $\alpha = [(\omega_0 - \omega_1)/(1 - \delta)]/30$. For the Austin example cited in the last section, on the average, $\alpha = -0.055$ (= $-1.66/30$), and by applying a 20% daily rainfall probability, N_m is roughly six per month, therefore αN_m is approximately -0.33.

Seasonal Water Use Model

Combining the potential seasonal water use model in either (5) or (6) with the water use response to rainfall in (10), we have the seasonal water use model

$$S_m = P_m + \alpha N_m S_m + \varepsilon_m \tag{11}$$

where ε_m is a zero mean random component. Rearranging (11), we get

$$S_m = (1 - \alpha N_m)^{-1} P_m + (1 - \alpha N_m)^{-1} \varepsilon_m \tag{12}$$

which is a nonlinear model of temperature and rainfall, and the last term is a new random component dependent on rainfall. To further simplify this model, $(1 - \alpha N_m)^{-1}$ is first approximated by the first-order Taylor series expansion at zero in power of αN_m as $(1 - \alpha N_m)^{-1} \approx 1 + \alpha N_m$. For a good approximation, αN_m has to be small. (Recall that in the Austin example it is approximately -0.33, and therefore the ignored higher-order terms are roughly 0.08.) Second, the new random component is assumed to follow a first-order autoregressive process (AR(1)), which was suggested in many empirical studies. With these two assumptions the simplified model appears to be

$$\begin{aligned} S_m &= (1 + \text{RainEff}) P_m + u_m \\ u_m &= \phi u_{m-1} + \varepsilon_m \end{aligned} \tag{13}$$

where RainEff is equal to αN_m, ϕ is an AR(1) coefficient, u_m is a correlated random component, and ε_m is an independent normal zero mean residual series. The second assumption has, in fact, ignored that the random component in (13) is rainfall-dependent, and as a result, rainfall is expected to be partially responsible for the AR(1) coefficient ϕ in (13). For generality, we can replace N_m in RainEff with $Q_l(N_m)$, a function representing "effective number of rainy days," where l is called "reference" number of rainy days, which is a threshold whereby additional rainy days in a month would not further reduce water use.

Several plausible propositions relating to RainEff in (13) depends on rainfall amounts are possible, for example, assuming that α is linearly related to rainfall amount

$$\text{RainEff} = [\theta_0 + \theta_1 G_\gamma(R_m)] Q_l(N_m) \tag{14}$$

or assuming that RainEff is linearly dependent on both rainfall amount and the number of rainy days in a month:

$$\text{RainEff} = \theta_0 Q_l(N_m) + \theta_1 G_\gamma(R_m) \tag{15}$$

Equation (13) constitutes a class of monthly time series regression urban water demand model with nonlinear climatic effects. The model is a nonlinear regression model with AR(1) residual, in which α (or θ_0, θ_1), ϕ, and other parameters associated with P_m are to be determined by the nonlinear least squares estimation discussed below.

Nonlinear Estimation

Nonlinear least squares estimates of the parameters in (13) are obtained by minimizing the objective function $g(\ )$:

$$g(\Theta) = e_1^2 + \sum_{m=2}^{M} \varepsilon_m^2 \tag{16}$$

where $e_1^2 = (1 - \phi^2)[S_1 - (1 + \alpha N_1)P_1]^2$, which is the unconditional sum of squares of the first observation; and Θ is a vector summarizing all the parameters that need to be estimated in potential seasonal use, rainfall effect, and random component. When the sample size is large and ϕ is not too close to 1, the nonlinear least squares estimates of the parameters are essentially the same as the maximum likelihood estimates [e.g., *Judge et al.*, 1980, p. 186].

RESULTS

The data employed for model performance evaluation are monthly water use (in gallons per capita per day or gpcd), total rainfall (in inches), and mean daily maximum temperature (in °F) in Austin, Texas, from 1975 to 1984. In addition, the number of rainy days in each month for the same time period are also collected for analysis. Rainy days are defined as days having rainfall greater than or equal to 0.01 inches (0.025 cm). Total water use in Austin is composed of approximately 60% residential and 40% of commercial/industrial/public uses.

During 1984, voluntary and mandatory restrictions on outdoor water use were implemented from May to September [*Shaw and Maidment*, 1987]; hence in this study 1984 data are used only for estimating base use. Since the focus of this study is on climatic effects and, in addition, Austin water use exhibits quite smooth trends, time is employed as a substitute for socioeconomic variables in explaining trend variations. The evaluation begins with an estimation of base use. Model comparisons are then performed for the seasonal water use in the summer and subsequently for the whole year.

Model performances are evaluated according to four criteria: (1) residual standard error $\hat{\sigma}$, (2) adjusted R^2, $\bar{R}^2$, (3) F statistics, and (4) Akaike information criterion (AIC) value (computed as AIC $= M \ln \hat{\sigma}^2 + 2p$, where p is the total number of parameters in the model, and M is the total number of observations). A favorable model is the one with high $\bar{R}^2$ and F statistics and low residual standard error and AIC value.

Base Use

Usually, the lowest level of monthly water use in Austin is from December to February, relatively unaffected by climatic conditions. These data are selected to estimate the base use in (3). The base use exhibits a step change in 1980 and a different growth rate before and after 1980. Dummy variables are included in (3) in order to represent these changes. Four models (models B1–B4) using different climatic variables are examined through least squares estimation, and the results are presented in Table 1.

Model B1 includes temperature and total rainfall without thresholds. Temperature is found to be statistically insignificant and is dropped from the regression equation, and the

TABLE 1. Coefficients and Associated Statistics of Alternative Base Water Use Models

Model	Intercept	Trend, 1975–1979	Level Change Dummy	Trend, 1981–1984	Maximum Tempera-ture, °F	Total Rainfall, inches	Number of Rainy Days	Reference Rainfall, inches	Reference Number of Rainy Days	Residual SE	Adjusted R^2	F Statistic	AIC
B1	148.093	0.152	9.030	0.206	···	−2.677				4.84	0.825	32.30	104.61
	(48.38)	(2.42)	(2.33)	(2.08)		(−2.31)							
B2	144.807	0.131	13.412	0.143*			−0.188†			4.94	0.785	25.33	105.84
	(37.93)	(1.81)	(3.47)	(1.32)			(−0.39)						
B3	148.836	0.144	10.804	0.204		−3.023		2.864		4.44	0.820	25.11	101.44
	(42.10)	(2.16)	(2.98)	(1.99)		(−2.15)		(2.34)					
B4	146.605	0.144	12.646	0.153*			−0.514†		9.997†	4.88	0.783	19.98	107.11
	(28.12)	(1.84)	(3.23)	(1.40)			(−0.57)		(1.14)				

Statistics are for the city of Austin, Texas, December–February, 1975–1984 (30 observations). Values in parentheses are t statistics of coefficients above. Data ranges are demands, 140.54–180.20 gpcd; maximum temperature, 48.10–66.00 °F; total rainfall, 0.34–3.54 inches; and number of rainy days per month, 3–14 days. One inch is 2.54 cm and 1 gpcd is 0.003785 m^3/capita/day.

*Values are significantly different from zero at 20% level.

†Values are not different from zero at 20% level. All other coefficients are significantly different from zero at 10% level.

model is reestimated. Model B2 uses the number of rainy days in a month instead of rainfall amount. Models B3 and B4 use effective rainfall and effective number of rainy days, respectively. The results indicate that rainfall is a significant variable in explaining winter use variations, while the number of rainy days is not. Model B1 has the highest F statistics and $\bar{R}^2$, while model B3 has the smallest residual standard error and AIC value. Although reference rainfall is estimable in model B3, the ranges of rainfall data (from 0.34 to 3.54 inches (0.86 to 8.99 cm)) are, however, considered quite limited in identifying the true reference rainfall. Model B1 is therefore employed to represent the base use in the subsequent analysis. The rainfall effect on winter use is statistically significant, suggesting that rainfall reduces water-related recreational and business (e.g., restaurants) activities in the winter.

Model B1 indicates that the trend in base use is 0.152 gpcd/month from 1975 to 1979 and 0.206 gpcd/month beginning in 1981 (1 gpcd = 0.003785 m^3/capital/day). The total increase in base use during 1980 is 9.030 gpcd. Seasonal use S_m is obtained by subtracting the base use from the total use W_m in which base use is estimated by substituting the normal winter rainfall (2.3 inches (5.84 cm)) into model B1. Seasonal water use S_m ranges from −5.05 to 237.45 gpcd per month from 1975 to 1983.

Seasonal Water Use in the Summer

Seasonal water uses from April to October are employed to examine the climatic effects on summer demand. Eight models with different model structures (four linear and four nonlinear) and rainfall variables are investigated using the least squares method. The results are presented in Table 2.

Models SS1–SS4 are linear models using the same rainfall variables as those examined in the winter use. Model SS1 is a conventional linear model examined in many studies [e.g., *Morgan and Smolen*, 1976; *Danielson*, 1979]:

$$S_m = \beta_0 + \beta_1 m + \beta_2 T_m + \beta_3 R_m + u_m \quad (17)$$

$$\forall\ m \in \text{(April–October)}$$

TABLE 2. Coefficients and Associated Statistics of Alternative Seasonal Water Use Models for Summer

	Model	Intercept	Trend	Maximum Tempera-ture, °F	Total Rainfall, inches	Number of Rainy Days	Reference Rainfall, inches	Reference Number of Rainy Days	Residual SE	Adjusted R^2	F Statistics	AIC
Linear	SS1	−426.648	−0.019	5.994	−6.550				25.47	0.787	77.21	415.93
		(−9.24)	(−0.17)	(11.69)	(−5.39)							
	SS2	−331.499	0.012	5.198		−7.008			23.53	0.818	93.86	405.94
		(−6.79)	(0.12)	(10.10)		(−6.65)						
	SS3	−354.183	−0.022	5.468	−22.482		3.066		22.15	0.836	79.93	400.33
		(−8.12)	(−0.23)	(11.74)	(−5.46)		(7.92)					
	SS4	−289.299	0.078	5.098		−15.628		6.767	20.34	0.862	97.53	389.59
		(−6.65)	(0.90)	(11.38)		(−6.67)		(13.47)				
Nonlinear	SS5	−610.537	0.049	8.233	−0.157		3.973		17.63	0.896	134.58	371.57
		(−13.41)	(0.39)	(16.29)	(−11.16)		(9.41)					
	SS6	−686.963	0.069	9.364		−0.080		8.532	16.32	0.911	159.38	361.84
		(−12.70)	(0.47)	(15.32)		(−15.44)		(13.76)				
	SS7	−655.641	0.082	8.903	−0.010	−0.051	4.531	7.063	14.83	0.924	126.26	353.78
		(−13.73)	(0.65)	(16.15)	(−2.75)	(−4.35)	(4.38)	(10.07)				
	SS8	−661.961	0.060	9.040	−0.057	−0.056	4.618	7.804	14.97	0.923	123.79	354.96
		(−13.24)	(0.46)	(15.95)	(−2.64)	(−5.25)	(3.88)	(7.57)				

Statistics are for the city of Austin, Texas, April–October, 1975–1983 (63 observations). Values in parentheses are t statistics of coefficients above. None of the trend coefficients are significantly different from zero at the 30% level; all other coefficients are highly significant at the 5% level. Data ranges are demand, 8.99–237.45 gpcd; maximum temperature, 71.23–100.10 °F; total rainfall, 0.06–14.96 inches; and number of rainy days per month, 1–14 days. See Table 1 for conversions and definitions.

Two important messages can be derived from the performance of these four models: (1) the number of rainy days in a month is a better explanatory variable in describing water use variations in the summer than rainfall amount, and (2) significant improvements in performance are obtained by considering rainfall thresholds in models SS3 and SS4. Comparison of the rainfall coefficients (SS3, −22.482 versus SS1, −6.55 and SS4, −15.628 versus SS2, −7.008) suggest that linear models without identifying rainfall thresholds considerably underestimate the effect of rainfalls with an amount less than the threshold. Note that all the trend coefficients presented in Table 2 are insignificant at the 30% level and are presented only to facilitate comparison. Also, F statistics are all significant at the 0.01 probability level.

Four possible variations of the proposed nonlinear models with rainfall thresholds are considered in models SS5–SS8. They all have the following basic form:

$$S_m = (1 + \text{RainEff})(\beta_0 + \beta_1 m + \beta_2 T_m) + u_m \qquad (18)$$

$$\forall\ m \in \text{(April–October)}$$

where u_m is a random component at month m, and RainEff is the fraction of potential seasonal water use that is reduced by rainfall. Models SS5 and SS6 assume RainEff to be proportional to effective rainfall and the effective number of rainy days, respectively. That is, RainEff = $\alpha G_\gamma(R_m)$ in model SS5 and RainEff = $\alpha Q_l(N_m)$ in model SS6. In model SS5 the least squares estimates of α and γ are −0.157 and 3.973, respectively, indicating that a 15.7% reduction in potential seasonal water use occurs due to 1 inch (2.54 cm) of rainfall, and excess rainfall over 3.973 inches (10.091 cm) would not contribute to more water use reduction. In model SS6, estimates of α and l are −0.08 and 8.532, respectively, suggesting that, on the average, one rainy day reduces 8% of the potential seasonal water use, and the rainfall effect in a month with nine rainy days or more is equivalent in magnitude to a month with 8.532 rainy days. The improvement of SS6 over SS5 in all criteria indicates again that the number of rainy days is a better explanatory variable than rainfall amount, when they are considered alone, to describe water use response to rainfall. Models SS5 and SS6 have better performance in all criteria than their linear counterparts models SS3 and SS4.

Models SS7 and SS8 consider both rainfall amount and the number of rainy days as proposed in (14) and (15), respectively. In model SS7, $\hat{\theta}_0 = -0.051$ and $\hat{\theta}_1 = -0.01$ indicate that one rainy day with 1 inch (2.54 cm) of rainfall would reduce 6.1% of the seasonal water use, and the reduction increases at a rate of 1% for each additional inch of rainfall. Values for $\hat{\gamma}$ and $\hat{l}$ are 4.531 inches (11.509 cm) and 7.063 rainy days, respectively. Model SS8, which has $\hat{\theta}_0 = -0.056$ and $\hat{\theta}_1 = -0.057$, suggests that one rainy day would reduce 5.7% of the seasonal water use and 1 inch of rainfall reduce 5.6%. The values γ and l are estimated to be 4.618 inches (11.730 cm) and 7.804 rainy days, respectively. The higher t statistic of the number of rainy days coefficient over rainfall amount coefficient in both models SS7 and SS8 shows again that the number of rainy days outweighs the rainfall amount in explaining seasonal water use response to rainfall.

It is interesting to note that all the nonlinear models tested indicate that a maximum seasonal water use reduction of approximately 70% can be achieved in Austin when rainfall exceeds the threshold level. This is consistent with the daily observation made in Figure 1, which shows that on average a 66% reduction is attained when daily rainfall exceeds 0.8 inches (2.03 cm). The response rates of water use to temperature obtained in the nonlinear models are higher than their linear counterparts, for example, 9.364 gpcd/°F in model SS6 versus 5.098 gpcd/°F in model SS4. The continual decrease in AIC value from model SS1 to model SS7 indicates that the increase in number of parameters is not statistically unjustified.

Seasonal Water Use of the Whole Year

Seasonal water uses for the whole year are employed in this comparison. Seven models including five linear models, models S1–S5, and two nonlinear models, models S6–S7, are studied. The random component u_m in all the models assumes a first-order autoregressive process with parameter ϕ, adopted in (13). The NLS estimate of the parameters discussed in (16) are presented in Tables 3 and 4. Durbin-Watson statistic is calculated to determine whether the residual series ε_m in each model is uncorrelated, as required.

Model S1 regresses water use with time, temperature, and rainfall amount without separating them into two seasons:

$$S_m = \beta_0 + \beta_1 m + \beta_2 T_m + \beta_3 R_m + u_m \qquad (19)$$

where m is from January 1975 (m = 1) to December 1983 (m = 108). Similar models were examined in the study by *Morgan and Smolen* [1976] and also by *Kher and Sorooshian* [1986].

Model S2 is a model suggested by *Hansen and Narayanan* [1981] in which water use is expressed by two seasons: winter (November–March) and summer (April–October). The model assumes that winter use is not affected by climatic conditions and that summer use is linearly correlated with temperature and rainfall:

$$S_m = \beta_0 + \beta_1 m + \beta_2 D_m + \beta_3 T_m^* + \beta_4 R_m^* + u_m \qquad (20)$$

where D_m is a nonseasonal (or winter months) dummy variable equal to one if month m is a winter month and zero otherwise; and T_m^* and R_m^* are seasonal temperature and rainfall series equal to T_m and R_m, respectively, in the summer months and set to zeros in the winter.

Model S2 shows a substantial improvement over model S1 in every criterion, as presented in Table 3. Also, a lower AR(1) coefficient in model S2 implies that a relatively uncorrelated random component, u_m, is achieved. In addition, a relatively uncorrelated residual series, ε_m, is obtained, as indicated by a Durbin-Watson statistic closer to 2.

Models S3–S5 are the same as model S2 except that R_m^* in (20) is replaced by $G_\gamma(R_m^*)$ in model S3, N_m^* (seasonal number of rainy days) in model S4, and $Q_l(N_m^*)$ in model S5. Again, better performances are attained when rainfall threshold is introduced and when the number of rainy days is considered instead of rainfall amount. An increase of 4.8% in $\bar{R}^2$ and a lower AIC value are obtained by model S5 over Hansen and Narayanan's model S2.

Model S6 is a nonlinear counterpart of model S5:

$$S_m = [1 + \alpha Q_l(N_m^*)](\beta_0 + \beta_1 m + \beta_2 D_m + \beta_3 T_m^*) + u_m \qquad (21)$$

TABLE 3. Coefficients and Associated Statistics of Alternative Seasonal Water Use Models for the Whole Year

	Model	Intercept	Trend	Nonseasonal Dummy	Seasonal Maximum Temperature, °F	Seasonal Total Rainfall, inches	Seasonal Number of Rainy Days	Reference Rainfall, inches	Reference Number of Rains	AR(1) Coefficient	Residual SE	Adjusted R^2	F Statistic	Durbin-Watson Statistic	AIC
Linear	S1	−198.282	0.083		3.316*	−6.619*				0.328	25.71	0.770	90.85	1.783	711.33
		(−9.47)	(0.70)		(12.97)	(−6.34)				(3.42)					
	S2	−419.219	−0.004	425.349	5.892	−6.310				0.128†	19.92	0.860	133.55	1.930	658.21
		(−11.20)	(−0.05)	(11.36)	(14.10)	(−6.67)				(1.29)					
	S3	−349.835	−0.004	356.126	5.390	−21.828		3.042		0.161‡	17.43	0.893	149.02	1.949	631.37
		(−10.04)	(−0.06)	(10.24)	(14.32)	(−6.68)		(9.92)		(1.62)					
	S4	−326.134	0.015	331.628	5.111		−6.721			0.196	18.30	0.883	161.86	1.941	639.89
		(−8.38)	(0.20)	(8.56)	(12.28)		(−8.33)			(1.96)					
	S5	−285.196	0.050	288.847	5.038		−15.070		6.837	0.176‡	16.11	0.908	177.27	1.920	614.36
		(−8.20)	(0.80)	(8.30)	(13.89)		(−8.26)		(17.01)	(1.75)					
Nonlinear	S6	−679.989	0.024	684.956	9.302		−0.080		8.544	0.235	13.09	0.940	276.95	1.934	569.52
		(−15.06)	(0.34)	(15.24)	(18.16)		(−18.89)		(18.08)	(2.34)					

Statistics are for the city of Austin, Texas, January–December, 1975–1983 (108 observations). Values in parentheses are t statistics of coefficients above. None of the trend coefficients are significantly different from zero at the 30% level. All coefficients are significant at 5% level, except those marked with a dagger. Data ranges are demand, −5.05–237.45 gpcd; maximum temperature, 48.10–100.10 °F; total rainfall, 0.06–14.96 inches; and number of rainy days per month, 1–14 days. Seasonal climatic variables (April–October) are used for all models except those marked with an asterisk or a dagger. See Table 1 for conversions and definitions.

*Every month of the year is included.

†Value is significant at the 20% level.

‡Values are significant at the 10% level.

TABLE 4. Coefficients and Associated Statistics of the Fourier Series Coefficient Seasonal Water Use Model

Nonlinear model	Fourier Series Intercept		Trend	Fourier Series Maximum Temperature, °F		Total Rainfall, inches	Number of Rainy Days	Reference Rainfall inches	Reference Number of Rainy Days	AR(1) Coefficient	Residual SE	Adjusted R^2	F Statistic	Durbin-Watson Statistic	AIC
S7	a_0	−491.743	−0.006	$\bar{a}_0$	6.299	−0.010	−0.035	4.216	7.235	0.113*	10.65	0.961	181.89	1.949	542.96
		(−9.79)	(−0.10)		(10.62)	(−3.25)	(−3.35)	(4.32)	(8.38)	(1.14)					
	a_1	580.606		$\bar{a}_1$	−7.116										
		(8.79)			(−9.20)										
	b_1	289.118		$\bar{b}_1$	−3.498										
		(4.34)			(−4.27)										
	a_2	−57.686		$\bar{a}_2$	0.406†										
		(−2.55)			(1.55)										
	b_2	−87.333		$\bar{b}_2$	0.637										
		(−3.36)			(2.19)										

Statistics are for the city of Austin, Texas, January–December, 1975–1983 (108 observations). Values in parentheses are t statistics of coefficients above. Data ranges are demand, −5.05–237.45 gpcd; maximum temperature, 48.10–100.10 °F; total rainfall, 0.06–14.96 inches; and number of rainy days per month, 1–14 days. The trend coefficient is not significantly different from zero at the 30% level. All coefficients are significant at the 5% level except those marked with an asterisk or a dagger. See Table 1 for conversions and definitions.

*Value is significant at the 25% level.

†Value is significant at the 15% level.

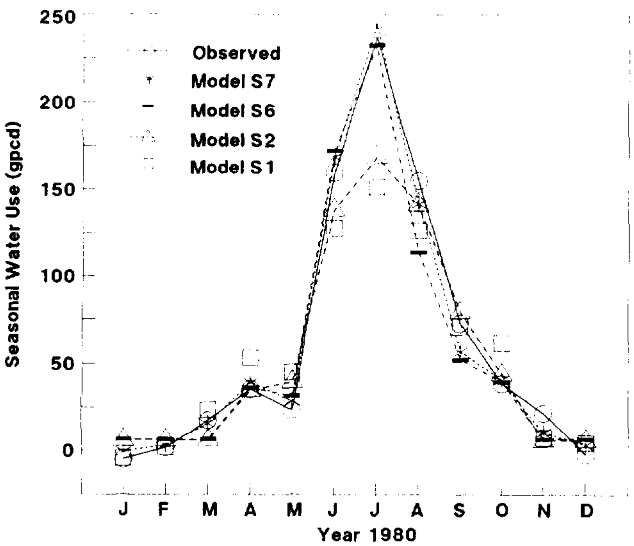

Fig. 3. Comparison of model performance in a dry year. Models S7 and S6 are the proposed nonlinear models and Models S2 and S1 are conventional linear models.

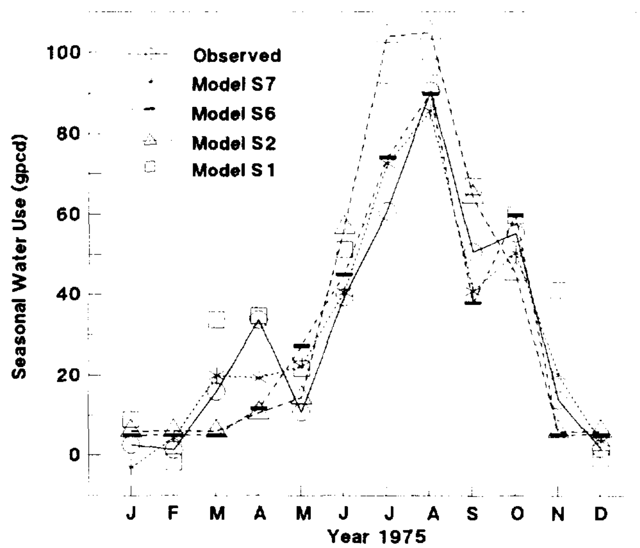

Fig. 4. Comparison of model performance in a wet year. Models S7 and S6 are the proposed nonlinear models and Models S2 and S1 are conventional linear models.

Improvements over model S5 are further obtained by model S6. A more significant temperature effect is again suggested by the nonlinear model, which is indicated by a higher β_3 value of 9.302 gpcd/°F over 5.038 gpcd/°F in model S5. As implied in the nonlinear summer seasonal water use models (models SS5–SS8), model S6 also indicates a maximum rainfall reduction of approximately 70% when the number of rainy days exceeds nine in a month.

Residual standard error is reduced from 25.71 gpcd in model S1 to 19.92 gpcd in model S2 and further to 16.11 gpcd in model S5 and finally to 13.0 gpcd in model S6. F statistics in every model attain a significance level of 0.01. AIC value decreases from 711.33 in model S1 to 569.52 in model S6. The trend is shown to be insignificant at the 30% level in all of the models examined.

Austin water use data does not display a conspicuous hysteresis temperature effect, as exhibited in the case of San Diego. However, for the purposes of demonstration, the Fourier series representation for potential seasonal water use that was introduced in (6) is employed in model S7:

$$S_m = \{1 + [\theta_0 + \theta_1 G_\gamma(R_m)]Q_l(N_m)\}(c_m + \beta_1 m + f_m T_m) + u_m \quad (22)$$

To avoid overparameterizing, the model the number of harmonics for c_m and f_m are both set equal to 2, and the estimation results are presented in Table 4. The estimated parameters are $\hat{\theta}_0 = -0.035$, $\hat{\theta}_1 = -0.01$, $\hat{\gamma} = 4.216$ inches (10.709 cm), and $\hat{l} = 7.235$ days. Model S7 has the highest $\bar{R}^2$ of 0.961, the least residual standard error of 10.65 gpcd, and the smallest AIC value of 542.96 among the tested models.

Figures 3 and 4 show the performance of two proposed nonlinear models, models S7 and S6, and two conventional linear models, models S2 and S1, in a dry year (1980) and a wet year (1975). The estimated water use in the figures is constructed from these models without considering the autoregressive random component u_m. By examining the residual standard errors over different years it is concluded that linear models (S1 and S2) underestimate the response of water use to climatic variables in such a way that their performances are relatively poor, as indicated by high residual standard errors in both dry and wet years. Evidence for this is also shown in Figures 3 and 4, in which linear models underestimate the high summer demand in a dry year while they overestimate it in a wet year. The proposed nonlinear models (S6 and S7), however, are able to adequately reflect water use variations under different climatic conditions and estimate the high summer demand accurately for both dry and wet years.

SUMMARY AND CONCLUSIONS

A new class of monthly time series urban water demand model is proposed. The model follows the conventional hypothesis that water use can be decomposed into a base use and a seasonal use; the former is primarily indoor use and is unaffected by climatic conditions, while the latter is largely outdoor use and dependent on climatic conditions. Seasonal water use is further postulated to consist of three components: a potential use that is dependent on temperature and free from rainfall effect, a water use reduction due to rainfall occurrences, and a random component. The proposed models make use of three useful observations of climatic effects that were established in recent daily water use studies. These include (1) "hysteresis" temperature effect: seasonal water use has different levels and response rates (to a unit change of temperature) in different seasons, (2) "dynamic" rainfall effect: a rainfall causes a temporary reduction in seasonal water use that diminishes over time, and (3) "state-dependent" rainfall effect: the higher the seasonal water use level prior to the occurrence of a rainfall, the more the water use reduction can be expected. The response of monthly water use to rainfall occurrences is derived through the time aggregating and averaging of a daily response model, which is a Box-Jenkins' type of transfer function describing observations 2 and 3. In contrast to the conventional demand model, which is either linear or log linear, the class of model so obtained is nonlinear in structure. The performance of the proposed class of model is compared with conventional linear models using monthly time series data in Austin, Texas, from 1975 to 1984. Major results from the comparison are

1. The number of rainy days in a month is consistently a better explanatory variable, in both linear and nonlinear models, than rainfall amount in describing seasonal water use response to rainfall occurrences. This suggests that the observation established at the daily level that people respond more to rainfall occurrence than to rainfall amount is still valid at the aggregated monthly level.

2. If the ranges of rainfall data permit, it is important for a demand model, linear or nonlinear, to be able to identify the thresholds beyond which excess rainfall or additional numbers of rainy days in a month would not further contribute to water use reduction. Physically, the threshold rainfall indicates that either soil moisture content is saturated or that water use has already been driven to its base level by the rainfall.

3. The proposed nonlinear model outperforms conventional linear models in estimating seasonal water use in terms of adjusted R^2, residual standard error, and AIC value. In the case where seasonal water use in the summer is considered, a nonlinear model has adjusted R^2, residual standard error, and AIC values of 0.924, 14.83 gpcd, and 353.78, while a conventional linear model is 0.787, 25.47 gpcd, and 415.93. When seasonal water use for the whole year is examined, a nonlinear model has an adjusted R^2 of 0.961, a residual standard error of 10.65 gpcd, and an AIC value of 542.96, while the respective figures for a linear model are 0.860, 19.92 gpcd, and 658.21.

4. Conventional linear models underestimate water use response to climatic variables. This is evidenced by their relatively poor performance, as indicated by high residual standard errors, in both dry and wet years. As a result, conventional linear models underestimate the high summer demand in dry years while they overestimate it in wet years. The proposed nonlinear models, however, are able to adequately reflect water use variations and estimate the high summer demand accurately under different climatic conditions.

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