COMP 330 Autumn 2015
Mid-term Examination

School of Computer Science
McGill University

15^{th} October 2015

This examination is closed book. You may have a two-sided sheet of notes in your handwriting or typed using 10pt font. If you are using a result from the notes, just mention it, please do not copy it. You have 90 minutes. There are 4 questions over **two pages**.

Question 1[30 points]

Show, using the pumping lemma, that the following language is not regular. The alphabet is $\Sigma = \{a, b, c\}$.

$$L = \{a^i b^j c^k | i, j, k > 0 \text{ and } i + k > j\}.$$

Question 2[30 points]

Design a DFA that reads words over the alphabet $\{a, b\}$ and only accepts words with the following property: every a must be *immediately* preceded by a b **and** *immediately* followed by a b. If there are no a's at all the string is accepted. Your machine should accept *bababbbab* and *bbbbabbbbbb* and *bab* and *bbb*. It should reject, for example, *baabb*, *ab* and *abbbb*.

I want a DFA not an NFA and any dead states must be shown explicitly. I would like it to be as simple as you can make it. It is fine to have a couple more states than necessary but, if it gets excessively complicated — say 8 states or more — I will take points off, **even if it works**. For 10 or more states I will give you **zero** without checking to see if it works.

Question 3[20 points]

Give a **high-level** algorithm that takes as input the description of an NFA and a DFA and decides whether the language recognized by the NFA is the same as the language recognized by the DFA. Use any algorithms covered in class as basic building blocks; i.e. you do not have to describe them again. For example, you can just say or "minimize the DFA" or "use a cycle detection algorithm" without further explanation. **Please do not write code!**

Question 4[20 points]

Are the following statements true or false? No explanations are required.

1. If L is a regular language then $L \cdot L$ is also regular.
2. If L_1 and L_2 are regular languages then $L_1 \cap L_2$ is also regular.
3. If $L_1 \subseteq L_2$ and L_2 is regular then L_1 could be non-regular.
4. If $L_1 \subseteq L_2$ and *both* are regular then the minimal DFA to recongnize L_2 could be smaller than the minimal DFA to recognize L_1.
5. The Myhill-Nerode theorem can be used to show that a given language is not regular.