COMP 330 Assignment 2

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Question 1:

Give regular expressions for the following languages over $\{a, b\}$:

- $\{w|w$ contains an even number of occurrences of $a\}$
- $\{w|w$ contains an odd number of occurrences of $b\}$
- $\{w|w$ does not contain the substring $ab\}$
- $\{w|w$ does not contain the substring $aba\}$

SOLUTION:
(1):

$$\left(b^*ab^*a\right)^*b^*$$

(2):

$$\left(a^*ba^*b\right)^*a^*ba^*$$

(3): "b" can be followed by any letter but "a" can be only followed by "a"

$$b^*a^*$$

(4): "a" may be followed by either "a" or by "bb"
But when I am ending, I can have "a" followed by "b" as there is no more letters coming

$$b^*\left(a^*bb \quad b^*\right)^*a^*b^*$$

Question 2:

Suppose that you have a DFA $M = (S, \Sigma, s_0, \delta, F)$. Consider two distinct states s_1,s_2 i.e. $s_1 \neq s_2$. Suppose further that for all $a \in \Sigma, \delta(s_1, a) = \delta(s_2, a)$. Show that for any nonempty word w over Σ we have $\delta^*(s_1, w) = \delta^*(s_2, w)$.

Proof. We prove by induction on the length of word $|w|$

- Base Case: we start with word with length 1, which means $|w| = 1$
  Obviously, we have that
  $$\delta^*(s_1, a) = \delta(s_1, a) = \delta(s_2, a) = \delta^*(s_2, a).$$
- Inductive Step: Suppose it is true for word x, which means that
  $$\delta^*(s_1, x) = \delta^*(s_2, x)$$
  Then for $w = xa$, we have
  $$\begin{aligned}\delta^*(s_1, xa) &= \delta\Big(\delta^*(s_1, x), a\Big) \\ &= \delta\Big(\delta^*(s_2, x), a\Big) \textbf{ By induction hypothesis} \\ &= \delta^*(s_2, xa)\end{aligned}$$

□

Question 3:

Show that the following languages are not regular by using the pumping lemma.

- 1. $L = \{a^n b^m a^{n+m} | n, m \geq 0\}$,
- 2. $L = \{x | x = x^R, x \in \Sigma^*\}$, where x^R means x reversed; these strings are called palindromes. An example is $abba$, a non-example is $baba$.

(a):

- Demon pick a random variable p
- I pick $w = a^p b^1 a^{p+1}$, obviously $w \in L$ and $|w| \geq p$
- Then Demon needs to pick x, y, z such that $w = xyz$ and $|xy| \leq p, |y| > 0$

  As $|xy| \leq p$, Demon's y must contain "a" only, which means $y = a^k \quad 0 < k \leq p$
- I will choose $i = 2$, such that $xy^2z = a^{p+k} b^1 a^{p+1}$

  Obviously, $p + k + 1 > p + 0 + 1 = p + 1$

  Thus $xy^2z \notin L$.

  Therefore, the language is not regular.

(b):

- Demon pick a random variable p
- I pick $w = a^p b a^p$, obviously $w \in L$ and $|w| \geq p$
- Then Demon needs to pick x, y, z such that $w = xyz$ and $|xy| \leq p, |y| > 0$

  As $|xy| \leq p$, Demon's y must contain "a" only, which means that $y = a^k \quad 0 < k \leq p$
- I will choose $i = 2$, such that $xy^2z = a^{p+k} b a^p$

  Obviously, $p + k > p + 0 = p$, so $w = xy^2z$ can't be palindrome

  Thus $xy^2z \notin L$

  Therefore, the language is not regular.

Question 4:

Show that the following languages are not regular by using the pumping lemma.

- (a) $L = \{x \in \{a, b, c\}^* || x| \text{ is a square.}\}$ Here $|x|$ means the length of x
- (b) $L = \{a^{2n}b^n\}$

(a):

- Demon pick a random value p
- I pick $w = a^{p^2}$, so $|w| = p^2$ and obviously $|w| \geq p$
- Then Demon needs to pick x, y, z such that $w = xyz$ and $|xy| \leq p, |y| > 0$

  Suppose that: $y = a^k \quad 0 < k \leq p$

- I will choose $i = 2$, such that $xy^2z = a^{p^2+k}$, so we have $|xy^2z| = p^2 + k$
  we have:

$$\begin{aligned} |xy^2z| &= p^2 + k \\ &\leq p^2 + p \\ &< (p+1)^2 \end{aligned}$$

  And we have $|xy^2z| = p^2 + k > p^2 + 0 = p^2$,
  Thus, we have $p^2 < |xy^2z| < (p+1)^2$ so we can say that $|xy^2z|$ is not a square $\implies xy^2z \notin L$.

  Therefore, the language is not regular.

(b):

- Demon pick a random value p
- I pick $w = a^{2p}b^p$ obviously $|w| > p$ and $w \in L$
- Then Demon needs to pick x, y, z such that $w = xyz$ and $|xy| \leq p, |y| < 0$

  As $|xy| \leq p$, Demon's y must contain "a" only, which is $y = a^k$ with $0 < k \leq p$

- I will choose $i = 2$, such that $xy^2z = a^{2p+k}b^p$

$$\frac{2p+k}{p} = 2 + \frac{k}{p} > 2 \implies xy^2z \notin L$$

  Therefore, the language is not regular.

Question 5:

We are using the alphabet $\{0, 1\}$. We have a DFA with 5 states, $S = \{s_0, s_1, s_2, s_3, s_4\}$. The start state is s_0 and the only accepting state is also s_0. The transitions are given by the formula

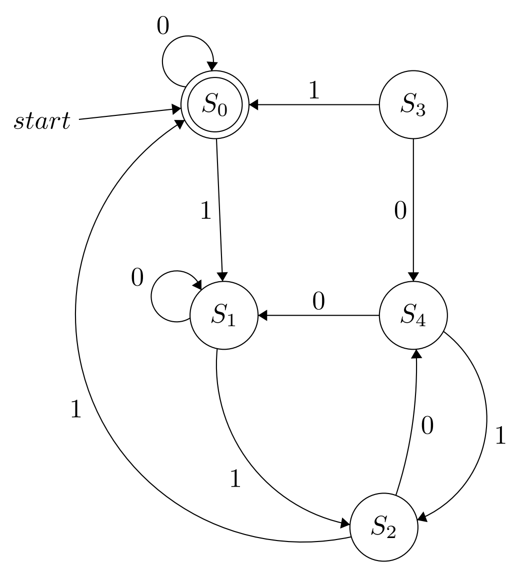
$$\delta(s_i, a) = s_j \textbf{ Where } j = i^2 + a \quad mod 5$$

Draw the table showing which pairs of states are inequivalent and then construct the minimal automaton. Remember to remove useless states right from the start, before you draw the table. I am happy with a drawing of the automaton.

$$\delta(s_0, 0) = s_0 \quad \delta(s_0, 1) = s_1 \quad \delta(s_1, 0) = s_1 \quad \delta(s_1, 1) = s_2$$

$$\delta(s_2, 0) = s_4 \quad \delta(s_2, 1) = s_0 \quad \delta(s_3, 0) = s_4 \quad \delta(s_3, 1) = s_0$$

$$\delta(s_4, 0) = s_1 \quad \delta(s_4, 1) = s_2$$



As we can see from the above automata, S_3 is unreachable. So, when we build the table without S_3

First Step: S_0 is accept state and S_1, S_2, S_4 are reject states, so we set position $(0,1),(0,2),(0,4)$ as 0

S_4				$\times$
S_2			$\times$	
S_1		$\times$		
S_0	$\times$	0	0	0
	S_0	S_1	S_2	S_4

Second Step:

- $\Big(\delta(S_1,1),\delta(S_2,1)\Big) \implies (S_2,S_0)$
- $\Big(\delta(S_1,1),\delta(S_3,1)\Big) \implies (S_2,S_0)$
- $\Big(\delta(S_2,1),\delta(S_4,1)\Big) \implies (S_0,S_2)$

S_4				$\times$
S_2			$\times$	0
S_1		$\times$	0	
S_0	$\times$	0	0	0
	S_0	S_1	S_2	S_4

Third Step: we set the remaining position as 1

S_4				$\times$
S_2			$\times$	0
S_1		$\times$	0	1
S_0	$\times$	0	0	0
	S_0	S_1	S_2	S_4

Thus, we can say that

$$\begin{cases} S_1 \text{ is equivalent to } S_4 \\ S_3 \text{ is unreachable state and has been removed} \end{cases}$$

FINAL RESULT: