COMP 330

Name: Yuhao Wu
ID Number: 260711365

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Oct 22 Lecture 17

Three important facts

- Acceptance only happens at the end of the input
- A PDA can't decide to jam when there are moves possible
- When there are choices, PDA can select any choices it wants

REMARK:

- PDAs are equivalent to CFLs
- DPDAs are equivalent to DCFLs

$$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to (Q \times \Gamma_\varepsilon) \;\; or \;\; \emptyset$$

REMARK:
For every $q \in Q, a \in \Sigma, x \in \Gamma$, **exactly one** of $\delta(q,a,x)$, $\delta(q,a,\varepsilon)$, $\delta(q,\varepsilon,x)$, $\delta(q,\varepsilon,\varepsilon)$ is non-empty

Counter-Example such that the intersection of 2CFL is not CFL

$$\{a^n \; b^n \; c^m | n,m \geq 0\} \cap \{a^m \; b^n \; c^n | n,m \geq 0\} = \{a^n b^n c^n\}$$

Algorithms for CFLs:

$$Is \; L(G) = \emptyset \, ? \quad S \to aS$$

We say that a NT x is generating if $x \xrightarrow{*} w \in \Sigma^* (\; or \; T^*)$

Theorem : $L(G) \neq \emptyset \iff S$ is generating

Algorithm 1 Check if it is generating

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procedure ALGORITHM( )
    initialize GEN = ∅
    Put all terminal symbols in GEN
    while there is still some changes happening do        ▷ repeat until no change happens
        for each rule x → α, check if every symbol of α belongs to GEN*
        if so, add x to GEN
    if S ∈ Gen then
        return L(G) ≠ ∅
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EXAMPLES[1]:
$S \to aS \quad GEN = \{a\}$
Now we have just one rule $S \to \alpha\ where\ \alpha = aS$, as not all of a, S belong to GEN^*, we can now end the loop, and $S \notin GEN$, we can conclude $L(G) = \emptyset$

EXAMPLES[2]:
$S \to bA|aB \qquad A \to bAA|aS|a \qquad B \to aBB|bS|b$
$[1]: \quad GEN = \{a, b\} \qquad [2]: \quad GEN = \{a, b, A, B, S\} \implies L(G) \neq \emptyset$

There are more complex algorithms to check if $L(G) = \infty$

REMARK:

- The complement of CFL may not be a CFL
- The complement of DCFL is a DCFL

OCT.24 Lecture 18
Question: $w \in L(G)$?
We can convert G to Chomsky Normal Form $X(non-T) \to AB \qquad X \to a(terminal)$
Using Dynamic Programming $O(n^3)$

(Cocke-Younger-Kasami Algorithm)

PDA: 2 distinct notions of accpetance

- Accept by accept states
- Accept by empty stack: at the end of the string is the stack empty?
  There are no accept states.

Both of them are equally powerful.

OCT.26 Lecture 19

PUMPING LEMMA:
Let L be a CFL.
$\exists p \geq 0$ such that $\forall s \in L$ with $|s| \geq p$, $\exists u, v, w, x, y \in \Sigma^*$ where $s = uvwxy$ such that:

- $|vx| > 0$ (i.e., v and x can't both be the empty string);
- $|vwx| \leq p$
- $\forall i \geq 0,\ uv^i wx^i y \in L$

As with regular languages, we can use the contrapositive of the pumping lemma to prove that a language is not context-free. Here's a formal statement of the contrapositive:

Fix some L
$\forall p \geq 0$ such that $\exists s \in L$ with $|s| \geq p$, $\forall u, v, w, x, y \in \Sigma^*$ where $s = uvwxy$ such that:

- $|vx| > 0$ (i.e., v and x can't both be the empty string);
- $|vwx| \leq p$
- $\exists i \geq 0,\ uv^i wx^i y \notin L$

Then we can say that L is not a CFL

EXAMPLE [1] $L = \{a^n\ b^n\ a^n \mid n \geq 0\}$

- Demon: choose P
- I choose $s = a^p\ b^p\ a^p \in L$
- Demon chooses any $u, v, w, x, y \in \Sigma^*$ where $s = uvwxy$

$$\underbrace{a, a, \ldots, a}_{p}\ \underbrace{b, b, \ldots, b}_{p}\ \underbrace{a, a, \ldots, a}_{p}$$

- v or x straddles a block boundary. Choose $i = 2$. Then, the a's and b's would be out of order in the resulting string.
- If v and x both contain only a's or b's, then v and x must be in the same block since $|vwx| < p$, choose $i = 2$
- v has a's and x has b's or v has b's and x has a's
  Then we can choose $i = 2$

EXAMPLE [2] $L = \{\ ww | w \in \Sigma^*\ \}$ suppose Σ has 2 letters.
Then, we define a new language

$$\widehat{L} = L \cap a^*b^*a^*b^* = \{a^m b^n a^m b^n | m, n \geq 0\}$$

Suppose that L is CFL then $\widehat{L} = CFL \cap$ regular language $= CFL$

- Demon: choose P
- I choose $s = a^p\ b^p\ a^p\ b^p \in L$
- Demon chooses any $u, v, w, x, y \in \Sigma^*$ where $s = uvwxy$

$$\underbrace{a, a, \ldots, a}_{p} \quad \underbrace{b, b, \ldots, b}_{p} \quad \underbrace{a, a, \ldots, a}_{p} \quad \underbrace{b, b, \ldots, b}_{p}$$

NOTE: $\overline{L}$ is a CFL

$$S \to AB|BA|A|B \qquad A \to CAC|a \qquad B \to CBC|b \qquad C \to a|b$$

$$\underbrace{\ldots\ldots\ldots}_{n} \quad \underbrace{a}_{n+1\ position} \quad \underbrace{\ldots\ldots\ldots}_{n} \quad \underbrace{\ldots\ldots\ldots}_{m} \quad \underbrace{b}_{n+1 \text{ position in the second half}} \quad \underbrace{\ldots\ldots\ldots}_{m}$$

$$L' = \{\ w\ w^{rec}\ | w \in \Sigma^*\} \qquad \textbf{this is CFL}$$

OCT 29 Lecture 20:
REMARK:

- CFL **are NOT** closed under complementation.
  Suppose L_1 and L_2 are context free. $L_1 \cap L_2$ is not necessarily context-free
- $L_1 \cup L_2$ we just need to add one rule $S \to S_1 | S_2$
- $L_1 \cdot L_2$ we just need to add one rule $S \to S_1 S_2$

EXAPMLE [1]: Show $L = \{0^i 1^j \mid j = i^2\}$ is not a CFL

- Demon: choose P
- I choose $s = 0^p\ 1^{p^2} \in L$
- Demon chooses any $u, v, w, x, y \in \Sigma^*$ where $s = uvwxy$

$$\underbrace{0, 0, \ldots, 0}_{p} \quad \underbrace{1, 1, \ldots, 1}_{p^2}$$

- vwx: all zeros or all ones   pick $i \neq 1$
- $v\ or\ x$ overlaps the boundary. I pick $i = 2$
- v : all 0's x: all 1's Either $|v| = 0\ or\ |x| = 0\ pick\ i = 2 \qquad if\ |v|, |u| \neq 0\ \ pick\ i = 2$

Exercise [1]: $L = \{a^p \mid p \text{ is a prime}\}$

Exercise [2]: $L = \{a^i\ b^j\ c^k \mid 0 < i < j < k\}$