COMP 330 Autumn 2018
Assignment 2
Due Date: 5^{th} Oct 2018

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There are **5** questions for credit and one for your spiritual growth (Q6) and one for pure fun (Q7). The homework is due in class at the beginning of the class. There are a couple of alternate questions; you should read them even if you do not plan on answering them. If question 6 makes no sense to you do not let it worry you. Have a look at question 7, it is a pure puzzle. If you don't get it, it does not mean that you do not grasp the material. If you like mathematical puzzles you might enjoy this one.

Question 1[20 points]
Give regular expressions for the following languages over $\{a, b\}$:

1. $\{w|w$ contains an even number of occurrences of $a\}$
2. $\{w|w$ contains an odd number of occurrences of $b\}$
3. $\{w|$ does not contain the substring $ab\}$
4. $\{w|$ does not contain the substring $aba\}$

Try to make your answers as simple as possible. We will deduct marks if your solution is *excessively* complicted.

Question 2[20 points]
Suppose that you have a DFA $M = (S, \Sigma, s_0, \delta, F)$. Consider two distinct states s_1, s_2 *i.e.* $s_1 \neq s_2$. Suppose further that *for all* $a \in \Sigma$ $\delta(s_1, a) = \delta(s_2, a)$. Show that for any *nonempty* word w over Σ we have $\delta^*(s_1, w) = \delta^*(s_2, w)$.

Alternate Question 2[20 points]
Let M be any finite monoid and let $h : \Sigma^* \to M$ be a monoid homomorphism. Let $F \subseteq M$ be any *subset* (not necessarily a submonoid) of M. Show that the set $h^{-1}(F)$ is a regular language. This means you have to describe an NFA (or DFA) from the given M, F and h[8 points]. Show that *every* regular language can be described this way.[12 points]

Question 3[20 points]
Show that the following languages are not regular by using the pumping lemma.

1. $\{a^n b^m a^{n+m} | n, m \geq 0\}$,
2. $\{x | x = x^R, x \in \Sigma^*\}$, where x^R means x reversed; these strings are called *palindromes.* An example is *abba*, a non-example is *baba*.

Question 4[20 points] Show that the following languages are not regular by using the pumping lemma.

1. $\{x \in \{a, b, c\}^* || x| \text{ is a square.}\}$ Here $|x|$ means the length of x.
2. $\{a^{2n} b^n\}$.

Question 5[20 points] We are using the alphabet $\{0, 1\}$. We have a DFA with 5 states, $S = \{s_0, s_1, s_2, s_3, s_4\}$. The start state is s_0 and the only accepting state is also s_0. The transitions are given by the formula

$$\delta(s_i, a) = s_j \text{ where } j = i^2 + a \mod 5.$$

Draw the table showing which pairs of states are inequivalent and then construct the minimal automaton. Remember to remove useless states right from the start, before you draw the table. I am happy with a drawing of the automaton.

Alternate Question 5[20 points] We define the Hamming distance between two strings w, x *of the same length* to be the number of places where they differ. If the strings have different lengths we say that their Hamming distance is infinite. We write is as $H(x, y)$. For example, $H(000, 010) = 1$ and $H(0000, 1001) = 2$. Given a set of words A and a positive integer k we define the new set $N_k(A)$ as follows

$$N_k(A) = \{x | \exists y \in A \text{ such that } H(x, y) \leq k\}.$$

For example $N_1(\{000, 111\}) = \{000, 001, 010, 001, 111, 110, 101, 011\}$ and $N_2(\{000\}) = \{000, 001, 010, 100, 110, 101, 011\}$. Of course, these are only examples and my definition of $N_k(A)$ is perfectly valid when A is an infinite set. **Prove** that if A is regular then $N_2(A)$ is regular. What happens to $N_k(A)$? [You don't have to answer this last question.]

Question 6[0 points] In alternate question 2, we showed how one could have defined regular languages in terms of monoids and homomorphisms instead of in terms of DFA. Given a regular language L, we can define an equivalence relation on words in Σ^* as follows:

$$x \equiv_L y = \forall u, v \in \Sigma^*, uxv \in L \iff uyv \in L.$$

It is easy to see that this is a congrence relation (with respect to concatenation). If we quotient by this equivalence relation we get a monoid called the syntactic monoid of the language L. The syntactic monoid is finite iff L is regular. Now what can you say about the language if the monoid happens to be a group? What if it is not only not a group but contains no subgroup? Yes, a monoid that is not a group could have a submonoid which is a group.

Question 7[0 points] Design an **NFA** K with n states, over a one-letter alphabet, such that K rejects some strings, but the *shortest* string that it rejects has length *strictly* greater than n.