McGill

December 2011
Final Examination

FACULTY OF SCIENCE
FINAL EXAMINATION

COMPUTER SCIENCE COMP 330

Theoretical Aspects of Computer Science

Examiner: Prof. Prakash Panangaden — 15th December 2011
Associate Examiner: Prof. Laurie Hendren — 2 pm to 5 pm

Instructions:

This exam has 5 questions. Please answer all questions. The maximum score for this exam is 50. The marks for each question are indicated just after each question number. This is an **open book exam**: you may use any books or notes that you have, including dictionaries. You have three hours in all. You may **not** use calculators, computers, cell phones or electronic aids of any kind. Please answer all questions **in the official answer book**. You may keep the questions. The questions appear on pages 1 and 2; this title page is not numbered. There are a total of three pages including this title page.

Question 1[10 points]
Is the following language regular?

$$L = \{x \in \{a,b\}^* | \text{ where in } \textbf{every} \text{ prefix of } x, 0 \leq \#_a(x) - \#_b(x) \leq 2\}.$$

If so, give a DFA for it with as few states as possible. If you claim that it is not regular give a pumping lemma proof that it is not regular.

Question 2[13 points]
Consider the language $L = a^n b^m a^n a^m$ with $n, m \geq 0$. Classify this as *one* of the three following:

1. regular,
2. context-free but not regular,
3. recursive but not context-free.

You have to prove each asssertion. For example, if you say that it is regular, you must give an NFA to recognize it; of course, in this case it is obvious that it does not belong to the other two classes. Similarly, if you claim that it is context-free, but not regular, you have to give a context-free grammar **and a proof that it is not regular**, but, of course you will not have to prove that it is recursive. If you claim that it belongs to the last class, you must show that it is not context-free (it is then immediate that it is not regular) *and* give an algorithm to recognize it. **Please read this question with extra care.**

Question 3[10 points]
One of the following questions is decidable and the other is undecidable. For the undecidable question prove undecidability and for the decidable one give an algorithm. Your algorithm can use known algorithms as basic steps. You should not describe well-known graph algorithms or algorithms covered in class in detail. Here L is a context-free language with a given grammar and R is a regular language with a given DFA; both are over the same alphabet Σ.

1. $L \cap R = \emptyset$
2. $L \cup R = \Sigma^*$

Question 4[12 points]
The set $\overline{K} = \{\langle M\rangle | M(\langle M\rangle) \uparrow\}$, where $\langle M\rangle$ is the encoding of M. Prove by an appropriate reduction that this set is not RE.

Question 5[5 points]
Are the following statements true? No explanations are needed.

1. The intersection of two context-free languages can fail to be recursive.
2. The intersection of a context-free language and a recursive language is context-free.
3. Given G_1 and G_2 context-free grammars, it is undecidable whether a word w belongs to $L(G_1) \cap L(G_2)$.
4. It is decidable whether a context-free grammar is ambiguous.
5. The internet is able to compute functions that a Turing machine cannot.