Study guide: Scientific software engineering with a simple ODE model as example

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Mathematical model problem

$$u'(t) = -au(t), \quad t \in (0, T]$$

$$u(0) = I$$

Solution by θ -scheme:

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n$$

 $\theta=0$: Forward Euler, $\theta=1$: Backward Euler, $\theta=1/2$: Crank-Nicolson (midpoint method)

```
from numpy import *
from matplotlib.pyplot import *

A = 1
a = 2
T = 4
dt = 0.2
N = int(round(T/dt))
y = zeros(N+1)
t = linspace(0, T, N+1)
theta = 1
y[0] = A
for n in range(0, N):
    y[n+1] = (1 - (1-theta)*a*dt)/(1 + theta*dt*a)*y[n]

y_e = A*exp(-a*t) - y
error = y_e - y
E = sqrt(dt*sum(error**2))
print 'Norm of the error: %.3E' % E
plot(t, y, 'r-o')
t = linspace(0, T, 1001)
y_e = A*exp(-a*t) - y
error = y_e - y
legend('numerical, theta-%g' % theta, 'exact'])
xlabel('t')
ylabel('t')
ylabel('t')
show()
```

There are major issues with this solution

- The notation in the program does not correspond exactly to the notation in the mathematical problem: the solution is called y and corresponds to u in the mathematical description, the variable A corresponds to the mathematical parameter I, N in the program is called N_t in the mathematics.
- $\ensuremath{ \bullet}$ There are no comments in the program.

```
from numpy import *
from matplotlib.pyplot import *

I = 1
a = 2
T = 4
dt = 0.2
Nt = int(round(T/dt))  # no of time intervals
u = zeros(Nt+1)  # array of u(n) values
t = linspace(0, T, Nt+1)  # time mesh
theta = 1  # Backward Euler method

u[0] = I  # assign initial condition
for n in range(0, Nt):  # n=0,1,..., Nt+1
u[n+i] = (1 - (1-theta)*a*dt)/(1 + theta*dt*a)*u[n]

# Compute norm of the error
u.e = I**xpr(-a*t) - u  # exact u at the mesh points
error = u.e - u
error = u
```

But: Further development of such flat programs require many scattered edits - easy to make mistakes!

The solution formula for u^{n+1} is completely general and should be available as a Python function with all input data as function arguments and all output data returned to the calling code

```
def solver(I, a, T, dt, theta):
         """Solve u'=-a*u, u(0)=I, for t in (0,T] with steps of dt. """

dt = float(dt)  # avoid integer division
                                                                            # avoid integer division
# no of time intervals
          Nt = int(round(T/dt))
        T = Nt+dt # adjust T to fit time step dt
u = np.zeros(Nt+1) # array of u[n] values
t = np.linspace(0, T, Nt+1) # time mesh
          \begin{array}{lll} \mathbf{u}[\,0] &=& \mathbf{I} & \text{\# assign initial condition} \\ \text{for n in range}(\,0\,,\,\, \mathbf{Nt}\,): & \text{\# n=0,1,\dots,Nt-1} \\ & \mathbf{u}[\,\mathbf{n+1}\,] &=& (1\,\,-\,\,\,(1\,-\,\mathbf{theta}\,)\,*\mathbf{a}\!*\mathbf{dt}\,)\,/\,(1\,\,+\,\,\mathbf{theta}\!*\mathbf{dt}\!*\mathbf{a}\,)\,*\mathbf{u}[\,\mathbf{n}] \end{array} 
          return u, t
u, t = solver(I=1, a=2, T=4, dt=0.2, theta=0.5)
```

The DRY principle: Don't repeat yourself!

DRY:

When implementing a particular functionality in a computer program, make sure this functionality and its variations are implemented in just one piece of code. That is, if you need to revise the implementation, there should be one and only one place to edit. It follows that you should never duplicate code (don't repeat yourself!), and code snippets that are similar should be factored into one piece (function) and parameterized (by function arguments).

Make sure any program file is a valid Python module

• Module requires code to be divided into functions :-)

- Why module? Other programs can import the functions

```
from decay import solver
# Solve a decay problem
u, t = solver(\tilde{I} = 1, a=2, T=4, dt=0.2, theta=0.5)
```

or prefix function names by the module name:

```
import decay
# Solve a decay problem
u, t = decay.solver(I=1, a=2, T=4, dt=0.2, theta=0.5)
```

The requirements of a module are so simple

- The filename without .py must be a valid Python variable
- The main program must be executed (through statements or a function call) in the test block.

The test block is normally placed at the end of a module file:

```
if __name__ == '__main__':
    # Statements
```

If the file is imported, the if test fails and no main program is run, otherwise, the file works as a program

```
The module file decay, py for our example
    from numpy import *
from matplotlib.pyplot import *
    def solver(I, a, T, dt, theta):
    def exact_solution(t, I, a):
         return T*exp(-a*t)
    def experiment_compare_numerical_and_exact():
    I = 1;    a = 2;    T = 4;    dt = 0.4;    theta = 1
    u, t = solver(I, a, T, dt, theta)
         t_e = linspace(0, T, 1001)
                                               # very fine mesh for u_e
         u_e = exact_solution(t_e, I, a)
          plot(t, u, 'r--o')
                                                 # dashed red line with circles
          plot(t_e, u_e, 'b-')
                                                 # blue line for u_e
          legend(['numerical, theta=%g' % theta, 'exact'])
         xlabel('t')
         plotfile = 'tmp'
savefig(plotfile + '.png'); savefig(plotfile + '.pdf')
          error = exact_solution(t, I, a) - u
         E = sqrt(dt*sum(error**2))
print 'Error norm:', E
```

```
The module file decay.py for our example w/prefix
     import numpy as np
import matplotlib.pyplot as plt
     def solver(I, a, T, dt, theta):
     def exact_solution(t, I, a):
          return I*np.exp(-a*t)
     def experiment_compare_numerical_and_exact():
    I = 1;    a = 2;    T = 4;    dt = 0.4;    theta = 1
    u, t = solver(I, a, T, dt, theta)
          t_e = np.linspace(0, T, 1001)
                                                           # very fine mesh for u_e
          u_e = exact_solution(t_e, I, a)
           plt.plot(t, u, 'r--o')
                                                        # dashed red line with circles
          plt.plot(t_e, u_e, 'b-') # dashed red line we plt.plot(t_e, u_e, 'b-') # blue line for u_e plt.legend(['numerical, theta=%g' % theta, 'exact']) plt.xlabel('t')
           plt.ylabel('u')
           plt.savefig(plotfile + '.png'); plt.savefig(plotfile + '.pdf')
           error = exact_solution(t, I, a) - u
          E = np.sqrt(dt*np.sum(error**2))
print 'Error norm:', E
```



```
def experiment_compare_schemes():
    """Compare theta=0,1,0.5 in the same plot."""
    I = 1: a = 2: T = 4; dt = 0.4
    leg ends = []
    for theta in [0, 1, 0.5]:
        u, t = solver[I, a, T, dt, theta)
        plt.plot(t, u, '--o')
        legends.append('theta-%g' '% theta)
    t_e = np.limspace(0, T, 1001)  # very fine mesh for u_e
    u_e = exact_solution(t_e, I, a)
    plt.plot(t_e, u_e, 'b-')
    legends.append('aperact')
    plt legend('aperact')
    plt legend('legends, loce')upper right')
    plt legend('legends, loce')upper right')
    plt savefig('plotfile + '.pdf')
```

```
Prefixing imported functions by the module name

MATLAB-style names (linspace, plot):

from numpy import *
from matplotlib.pyplot import *

Python community convention is to prefix with module name (np.linspace, plt.plot):

import numpy as np
import numpy as np
import matplotlib.pyplot as plt
```

```
Output

Outpu
```

```
Example on NumPy-style doc string

def solver(I, a, T, dt, theta):

"""

Solve :math: 'u'--au' with :math: 'u(0)=I' for :math: 't \in (0,T]' with steps of 'dt' and the method implied by 'theta'.

Parameters

Is float

Initial condition.
a: float

Parameter in the differential equation.
I: float

Total simulation time.
theta: float, int

Parameter in the numerical scheme. 0 gives
forward Euler, I Backward Euler, and 0.5
the centered Crank-Bicolson scheme.

Returns

'u': array
Solution array.
't': array

Array with time points corresponding to 'u'.

Examples

Salue ampth: 'u' = 1\text{Meac(11/2} \text{leg u(0)=1.5}'.
```

```
Can use a premade script to use Sphinx to generate an API manual in (e.g.) HTML

Total simulation time.

there, float, int

Parameter in the numerical scheme. O gives Forward Euler, 1 Backward Euler, and 0.5 the confirmed Cross-Nocolous scheme.

Returne:

u. array
Solution array.

t. array

Array with time points corresponding to u.

Examples

Solve u' = - \frac{1}{2}u_1 u(0) = 1.5 with the Crass-Nocolous method:

\[
\begin{array}{c}
\text{c}
\text
```

User interfaces

- Never edit the program to change input!
- Set input data on the command line or in a graphical user interface
- How is explained next

Accessing command-line arguments

- All command-line arguments are available in sys.argv
- sys.argv[0] is the program
- sys.argv[1:] holds the command-line arguments
- Method 1: fixed sequence of parameters on the command line
- Method 2: -option value pairs on the command line (with default values)

Terminal> python myprog.py 1.5 2 0.5 0.8 0.4 Terminal> python myprog.py --I 1.5 --a 2 --dt 0.8 0.4

Reading a sequence of command-line arguments

Required input:

- 1
- a
- T
- name of scheme (FE, BE, CN)
- a list of Δt values

Give these on the command line in correct sequence

Terminal> python decay_cml.py 1.5 0.5 4 CN 0.1 0.2 0.05

Implementation

```
def define_command_line_options():
    import argparse
    parser = argparse.ArgumentParser()
    parser.add_argument(
        '--1', '---initial_condition', type=float,
        default=1.0, help='initial condition, u(0)',
        metavar='?!)
    parser.add_argument(
        '--a', type=float, default=1.0,
        help='coefficient in ODE', metavar='a')
    parser.add_argument(
        '--T', ''--stop.time', type=float,
        default=1.0, help='end time of simulation',
        metavar='?')
    parser.add_argument(
        '--scheme', type=str, default='CN',
        help='FE, BE, or CN')
    parser.add_argument(
        '--dt', ''--time_step_values', type=float,
        default=[1.0], help='time_step_values',
        metavar='dt', nargs='+', dest='dt_values')
    return parser
```

Note:

• sys.argv[i] is always a string

Working with an argument parser

Set option-value pairs on the command line if the default value is not suitable:

Terminal> python decay_argparse.py --I 1.5 --a 2 --dt 0.8 0.4

Code:

```
def read_command_line_argparse():
    parser = define_command_line_options()
    args = parser.parse_args()
    scheme2theta = {"BE": 1, 'CM": 0.5, 'FE': 0}
    data = (args.I, args.a, args.T, scheme2theta[args.scheme],
    args.dt_values)
    return data
```

(metavar is the symbol used in help output)

Normally very much programming required - and much competence on graphical user interfaces.

The Parampool package

- Parampool is a package for handling a large pool of input parameters in simulation programs
- Parampool can automatically create a sophisticated web-based graphical user interface (GUI) to set parameters and view solutions

Remark

The forthcoming material aims at those with particular interest in equipping their programs with a GUI - others can safely skip it.

Making a compute function

- Key concept: a compute function that takes all input data as arguments and returning HTML code for viewing the results (e.g., plots and numbers)
- What we have: decay_plot.py
- ullet main function carries out simulations and plotting for a series of Δt values
- Goal: steer and view these experiments from a web GUI
- What to do:
 - create a compute function
 - call parampool functionality

The compute function must return HTML code

html_text += '
html_text += '
html_text += '\n'
return html_text

Generating the user interface

Make a file decay_GUI_generate.py:

Running decay_GUI_generate.py results in

- decay_GUI_model.py defines HTML widgets to be used to set input data in the web interface,
- templates/decay_GUI_views.py defines the layout of the web page,
- decay_GUI_controller.py runs the web application.

Good news: we only need to run decay_GUI_controller.py and there is no need to look into any of these files!

More advanced use

- The compute function can have arguments of type float, int, string, list, dict, numpy array, filename (file upload)
- Alternative: specify a hierarchy of input parameters with name, default value, data type, widget type, unit (m, kg, s), validity check
- The generated web GUI can have user accounts with login and storage of results in a database

Doctests

Doc strings can be equipped with interactive Python sessions for demonstrating usage and *automatic testing* of functions.

```
def solver(I, a, T, dt, theta):
    """
    Solve u^2-a*u, u(0)-I, for t in (0,T] with steps of dt.

>>> u, t = solver(I=0.8, a=1.2, I=2, dt=0.5, theta=0.5)
>>> for t.n, u.n in zip(t, u):
... print ^1 t^2 x^2. If, u=2. If t^2 x^2 x^2
```

Running doctests

Automatic check that the code reproduces the doctest output:

Terminal> python -m doctest decay.py

Floats are difficult to compare

Limit the number of digits in the output in doctests! Otherwise, round-off errors on a different machine may ruin the test.

Unit testing with nose

- Nose and pytest are a very user-friendly testing frameworks
- Based on unit testing
- Identify (small) units of code and test each unit
- Nose automates running all tests
- Good habit: run all tests after (small) edits of a code
- Even better habit: write tests before the code (!)
- Remark: unit testing in scientific computing is not yet well established

Basic use of nose and pytest

- Implement tests in test functions with names starting with test_.
- Test functions cannot have arguments.
- Test functions perform assertions on computed results using assert functions from the nose.tools module.
- Test functions can be in the source code files or be collected in separate files test*.py.

Example on a test function in the source code

```
Very simple module mymod (in file mymod.py):
```

```
def double(n):
return 2*n
```

Write test function in mymod.py

```
def double(n):
    return 2*n

def test_double():
    n = 4
    expected = 2*4
    computed = double(n)
    assert expected == computed
```

Running one of

```
Terminal> nosetests -s -v mymod
Terminal> py.test -s -v mymod
```

makes the framework run all test_*() functions in mymod.py.

Example on test functions in a separate file

```
Write the test in a separate file, say test\_mymod.py:
```

```
import mymod

def test_double():
    n = 4
    expected = 2*4
    computed = double(n)
    assert expected == computed
```

Running one of

```
Terminal> nosetests -s -v
Terminal> py.test -s -v
```

makes the frameworks run all test_*() functions in all files test*.py in the current directory and in all subdirectories (pytest) or just those with names tests or *_tests (nose)

Tip

Start with test functions in the source code file. When the file contains many tests, or when you have many source code files, move tests to separate files.

Test function for solver

Use exact discrete solution of the heta scheme as test:

$$u^n = I\left(rac{1-(1- heta)a\Delta t}{1+ heta a\Delta t}
ight)^n$$

Can test that potential integer division is avoided too

Warning

If a, Δt , and θ are integers, the formula for u^{n+1} in the solver function may lead to 0 because of unintended integer division.

Packaging the software for other users

Installation of a single module file decay.py:

```
from distutils.core import setup
setup(name='decay',
   version='0.1',
   py_modules=['decay'],
   scripts=['decay.py'],
   )
```

Installation:

Terminal> sudo python setup.py install

(Many variants!)

split.py for several modules in a package

- Python package = several modules
- Modules be in a directory with a __init__.py file
- Name of package = name of directory

setup.py:

The __init__.py file can be empty

```
Empty __init__.py:
```

import decay
u, t = decay.decay.solver(...)

Do this in __init__.py to avoid decay.decay.solver:

from decay import *

Can now write

import decay
u, t = decay.solver(...)
or
from decay import solver

from decay import solve u, t = solver(...)

Always develop software and write reports with Git

- Git keeps track of different versions of files
- Can roll back to previous versions
- Can see who did what when
- Can merge simultaneous edits by different users
- Professionals rely on Git!

The Git work cycle:

```
git pull # before starting a new session # edit files git add mynewfile # remember to add new files! git commit -am 'Short description of what I did' git push origin master # before end of day or a break
```

More pro use with Git See what others have done in the project: git fetch origin # instead of git pull git diff origin/master # what are the changes? git merge origin/master # update my files Develop new features in a separate branch: git branch newstuff git checkout newstuff # edit files git commit - am 'Changed ...' git push origin newstuff When newstuff is tested and matured, merge back in master: git checkout master git merge newstuff

```
from numpy import exp

class Problem(object):
    def __imit__(self, I=i, a=i, T=i0):
        self.T, self.I, self.a = I, float(a), T

def u_exact(self, t):
    I, a = self.I, self.a
    return I*exp(-a*t)
```

```
class Solver(object):
    def __init__(self, problem, dt=0.1, theta=0.5):
        self_problem = problem
        self.dt, self.theta = float(dt), theta

def solve(self):
        self_problem.I, self_problem.a, self_problem.T,
        self_st_dt, self.theta)

def error(self):
    """Return norm of error at the mesh points."""
        u_e = self_problem u_exact(self.t)
        e = u_e - self_u
        E = np.sqrt(self.dt*np.sum(e**2))
        return E
```

```
class Problem(object):
    def __init__(self, I=1, a=1, T=10):
        self.T, self.I = I, float(a), T

    def define_command_line_options(self, parser=None):
        """Return updated (parser) or new ArgumentParser object."""
    if parser is None:
        import argparse
        parser argparse.ArgumentParser()

parser.add_argument(
        '-I', '--initial_condition', type=float,
        defaul=1.0, help='initial_condition, u(0)',
        metavar='I')

parser.add_argument(
        '--a', type=float, default=1.0,
        help='coefficient in ODE', metavar='a')

parser.add_argument(
        '-'-I', ''--stop_time', type=float,
        default=1.0, help='end time of simulation',
        metavar='I')

return parser

def init_from_command_line(self, args):
    """Load attributes from ArgumentParser into instance."""
    self.I, self.a, self.T = args.I, args.a, args.T
```

```
class Solver(object):
    def __init__(self, problem, dt=0.1, theta=0.5):
        self problem = problem
        self. dt, self.theta = float(dt), theta

def define_command_line_options(self, parser):
    """Return updated (parser) or new drgumentParser object."""
    parser.add_argument(
    '--scheme', type=str, default='CN',
        help='FF, BE, or CN')
    parser.add_argument(
    '--dt', '-time_step_values', type=float,
        default=[1:0], help='time_step_values')
        return parser

def init_from_command_line(self, args):
    """Load_attributes_from_lrgumentParser_into_instance."""
        self.dt, self.theta = args_dt, args.theta
```

```
def experiment_classes():
    problem = Problem()
    solver = Solver(problem)

# Read input from the command line options()
    parser = problem define.command_line_options()
    parser = solver. define.command_line_options(parser)
    args = parser.parse.args()
    problem init.from.command.line(args)

# Solve and plot
    solver. solve()
    import matplotlib.pyplot as plt
    t.e = np.linspace(0, T, 1001)  # very fine mesh for u_e
    u_e = problem.u_exact(t.e)

plt.plot(t, u, 'r-o')  # dashed red line with circles
    plt.plot(t, e, u_e, 'b-')  # blue line for u_e
    plt legend(['numerical, theta=/g' % theta, 'exact'])
    plt xlabel('t')
    plt xlabel('u')
    plt show()
```

Performning scientific experiments

Goals:

- Explore the behavior of a numerical method for an ODE
- Show how a program can set up, execute, and report scientific investigations
- Oemonstrate how to write a scientific report
- Demonstrate various technologies for reports: HTML w/MathJax, PTEX, Sphinx, IPython notebooks, ...

Model problem and numerical solution method

Problem:

$$u'(t) = -au(t), \quad u(0) = l, \ 0 < t \le T,$$
 (1)

Solution method (θ -rule):

$$u^{n+1} = \frac{1 - (1 - \theta)a\Delta t}{1 + \theta a\Delta t}u^n, \quad u^0 = I.$$

Plan for the experiments

For fixed I, a, and T, we run the three schemes for various values of Δt , and present in a report the following results:

- visual comparison of the numerical and exact solution in a plot for each Δt and $\theta=0,1,\frac{1}{2},$
- ② a table and a plot of the norm of the numerical error versus Δt for $\theta=0,1,\frac{1}{2}.$

Available software

model.py:

```
Terminal> python model.py --I 1.5 --a 0.25 --T 6 --dt 1.25 0.75 0.5 0.0 1.25: 5.998E-01 0.0 0.75: 1.926E-01 0.0 0.75: 1.926E-01 0.0 0.50: 1.123E-01 0.0 0.10: 1.558E-02 0.5 0.50: 0.75: 1.543E-02 0.5 0.75: 1.543E-02 0.5 0.75: 1.543E-02 0.5 0.50: 7.337E-03 0.5 0.50: 7.337E-03 0.5 0.50: 7.337E-03 0.5 0.50: 8.859E-04 1.0 1.25: 1.766E-01 1.0 0.75: 8.579E-02 1.0 0.75: 8.579E-02 1.0 0.10: 1.411E-02
```

+ a set of plot files of numerial vs exact solution

Required new results

• Put plots together in table of plots

- ullet Table of numerical error vs Δt and heta
- \bullet Log-log convergence plot of numerical error vs Δt for $\theta=0,1,0.5$

Must write a script exper1.py to automate running model.py and generating these results

Terminal> python exper1.py 0.5 0.25 0.1 0.05

 $(\Delta t \text{ values on the comand line})$

Reproducible science is key!

Let your scientific investigations be automated by scripts!

- Excellent documentation
- Trivial to re-run experiments
- Easy to extend investigations

What actions are needed in the script?

- Run model.py program with appropriate input
- Interpret the output and make table and plot of numerical errors
- Combine plot files to new figures

Complete script: exper1.py

Command to be run: python model.py --I 1.2 --a 0.2 --T 8 -dt 1.25 0.75 0.5 0.1 Constructed in Python: # Given I, a, T, and a list dt, values cmd = 'python model.py --I %e --a %e --T %g' % (I, a, T) dt.values.str = ' '.join([str(v) for v in dt_values]) cmd += ' --dt %s' % dt_values_str Run under the operating system: from subprocess import Popen, PIPE, STDOUT p = Popen(cmd, shell=True, stdout=PIPE, stderr=STDOUT) output, dummy = p.communicate() failure = p.returncode if failure: print 'Command failed:', cmd; sys.exit(1)

Interpreting the output from an operating system command

The output if the previous command run by subprocess is in a string output:

Combining plot files: PNG and PDF solutions

PNG:

Terminal> montage -background white -geometry 100% -tile 2x \fi.png f2.png f3.png f4.png f.png
Terminal> convert -trim f.png f.png
Terminal> convert f.png -transparent white f.png

PDF

Terminal> pdftk f1.pdf f2.pdf f3.pdf f4.pdf output tmp.pdf
Terminal> pdfnup --nup 2x2 --outfile tmp.pdf tmp.pdf
Terminal> pdfcrop tmp.pdf f.pdf
Terminal> rm -f tmp.pdf

Easy to build these commands in Python and execute them with subprocess or os.system: os.system(cmd)

Making a report

- Scientific investigations are best documented in a report!
- A sample report
- How can we write such a report?
- First problem: what format should I write in?
- Plain HTML
- HTML with MathJax
- LaTeX PDF, based on LaTeX source
- Sphinx HTML, based on reStructuredText
- IPython notebook, Markdown, MediaWiki, ...
- DocOnce can generate LaTeX, HTML w/MathJax, Sphinx, IPython notebook, Markdown, MediaWiki, ... (DocOnce source for the examples above)
- Examples on different report formats

Publishing a complete project

- Make folder (directory) tree
- Keep track of all files via a version control system (Git!)
- Publish as private or public repository
- Utilize Bitbucket or GitHub
- See the intro to project hosting sites with version control