3.1

$$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & 2\end{bmatrix},\quad B=\begin{bmatrix}4 & 3 & 0\\ 2 & -1 & 1\end{bmatrix},\quad C=\begin{bmatrix}-1 & 2 & -1\\ 0 & -5 & 1\end{bmatrix}$$

$$A-2B+3C=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & 2\end{bmatrix}+\begin{bmatrix}-8 & -6 & 0\\ -4 & 2 & -2\end{bmatrix}+\begin{bmatrix}-3 & 6 & -3\\ 0 & -15 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -1 & -1\\ -1 & -13 & 3\end{bmatrix}$$

$$3A-2B=\begin{bmatrix}3 & -3 & 6\\ 9 & 0 & 6\end{bmatrix}+\begin{bmatrix}-8 & -6 & 0\\ -4 & 2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}-5 & -9 & 6\\ 5 & 2 & 4\end{bmatrix}$$

$$AB^T=\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & 2\end{bmatrix}\begin{bmatrix}4 & 2\\ 3 & -1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 5\\ 12 & 8\end{bmatrix}$$

$$B^T C = \begin{bmatrix} 4 & 2 \\ 3 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & -1 \\ 0 & -5 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -2 & -2 \\ -3 & 11 & -4 \\ 0 & -5 & 1 \end{bmatrix}$$

$$2 \cdot \begin{bmatrix} 3 & -6 & 2 & 0 \\ 1 & 5 & -1 & 8 \\ 4 & 3 & 1 & 7 \end{bmatrix} + 2X = \begin{bmatrix} 5 & 4 & -4 & 2 \\ -7 & 1 & 9 & 4 \\ 6 & -1 & 3 & 9 \end{bmatrix}$$

$$2X = \begin{bmatrix} 5 & 4 & -4 & 2 \\ -7 & 1 & 9 & 4 \\ 6 & -1 & 3 & 9 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 2 & 0 \\ 1 & 5 & -1 & 8 \\ 4 & 3 & 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 10 & -6 & 2 \\ -8 & -4 & 10 & -4 \\ 2 & -4 & 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 5 & -3 & 1 \\ -4 & -2 & 5 & -2 \\ 1 & -2 & 1 & 1 \end{bmatrix}$$

3. $A=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $B=[1,-2,1]$, $C=\begin{bmatrix}-1&0&0\\1&-1&0\\0&1&-1\end{bmatrix}$

$$AB=\begin{bmatrix}1\\2\\3\end{bmatrix}[1,-2,1]=\begin{bmatrix}1&-2&1\\2&-4&2\\3&-6&3\end{bmatrix}.$$

$$BA=[1,-2,1]\begin{bmatrix}1\\2\\3\end{bmatrix}=1-4+3=0$$

$$CA=\begin{bmatrix}-1&0&0\\1&-1&0\\0&1&-1\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}-1\\-1\\-1\end{bmatrix}.$$

$$BCA=[1,-2,1]\begin{bmatrix}-1\\-1\\-1\end{bmatrix}=-1+2-1=0$$

4. $$DA=\begin{bmatrix}\lambda_1&&&\\&\lambda_2&&\\&&\ddots&\\&&&\lambda_n\end{bmatrix}\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&\cdots&\cdots&\vdots\\\vdots&&&\vdots\\a_{n1}&\cdots&\cdots&a_{nn}\end{bmatrix}$$

$$=\begin{bmatrix}\lambda_1a_{11}&\lambda_1a_{12}&\cdots&\lambda_1a_{1n}\\\lambda_2a_{21}&\cdots&\cdots&\lambda_2a_{2n}\\ \\\lambda_na_{n1}&\cdots&\cdots&\lambda_na_{nn}\end{bmatrix}.$$

$$AD = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & & \lambda_n \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 a_{11} & \cdots & \lambda_n a_{1n} \\ \lambda_1 a_{21} & \cdots & \lambda_n a_{2n} \\ \vdots & & \vdots \\ \lambda_1 a_{n1} & \cdots & \lambda_n a_{nn} \end{bmatrix}$$

(2) $\because AD = DA$

$\therefore \lambda_i a_{ij} = \lambda_j a_{ij}$ (i≠j且 $i,j = 1,2 \ldots n$)

$\because \lambda_i \neq \lambda_j$

$\therefore a_{ij} = 0 \quad (i \neq j)$

$\therefore A$ 为对角阵

14 (1) $(A+A^T)^T = A^T + A = A + A^T$

$\therefore$ 对称矩阵

$(A-A^T)^T = A^T - A = -(A-A^T)$

$\therefore$ 反对称矩阵

(2) $B = \frac{1}{2}(A+A^T)$ $C = \frac{1}{2}(A-A^T)$

$B+C = \frac{1}{2}\cdot 2A = A$

$\therefore$ 存在

假设 $A = B_1 + C_1 = B_2 + C_2$，其中 B_1, B_2 对称，

C_1, C_2 反对称。 $B_1 \neq B_2$， $C_1 \neq C_2$

$A^T = B_1^T + C_1^T = B_1 - C_1$

$= B_2^T + C_2^T = B_2 - C_2$

$\therefore \begin{cases} B_1 + C_1 = B_2 + C_2 \\ B_1 - C_1 = B_2 - C_2 \end{cases}$ $\therefore 2C_1 = 2C_2$

$C_1 = C_2$ 与假设矛盾

$\therefore$ 唯一.