# Turing Machines

## Reading: Chapter 8

# Turing Machines are…

- Very powerful (abstract) machines that could simulate any modern day computer (although very, very slowly!)

- Why design such a machine?

  For every input, answer YES or NO

  - If a problem cannot be "solved" even using a TM, then it implies that the problem is *undecidable*

- Computability vs. Decidability

# A Turing Machine (TM)

- $M = (Q, \sum, \Gamma, \delta, q_0, B, F)$

This is like the CPU & program counter

Finite control

Tape is the memory

Tape head

Infinite tape with tape symbols

| … | B | B | B | $X_1$ | $X_2$ | $X_3$ | … | $X_i$ | … | $X_n$ | B | B | … |

Input & output tape symbols

B: blank symbol (special symbol reserved to indicate data boundary)
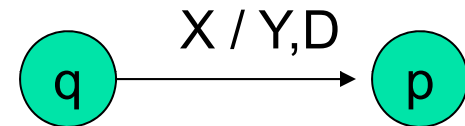
# Transition function

- One move (denoted by |---)
  in a TM does the following:
  - $\delta(q,X) = (p,Y,D)$

    X / Y,D

    q ⟶ p

    - q is the current state
    - X is the current tape symbol pointed by tape head
    - State changes from q to p
    - <u>After the move:</u>
      - X is replaced with symbol Y
      - If D="L", the tape head moves "left" by one position.
        Alternatively, if D="R" the tape head moves "right" by one position.

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# ID of a TM

- **Instantaneous Description or ID :**
  - $X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n$
    means:
    - $q$ is the current state
    - Tape head is pointing to $X_i$
    - $X_1X_2\ldots X_{i-1}X_iX_{i+1}\ldots X_n$ are the current tape symbols

- $\delta(q,X_i) = (p,Y,R)$ is same as:
  $X_1\ldots X_{i-1}qX_i\ldots X_n$ |---- $X_1\ldots X_{i-1}YpX_{i+1}\ldots X_n$
- $\delta(q,X_i) = (p,Y,L)$ is same as:
  $X_1\ldots X_{i-1}qX_i\ldots X_n$ |---- $X_1\ldots pX_{i-1}YX_{i+1}\ldots X_n$
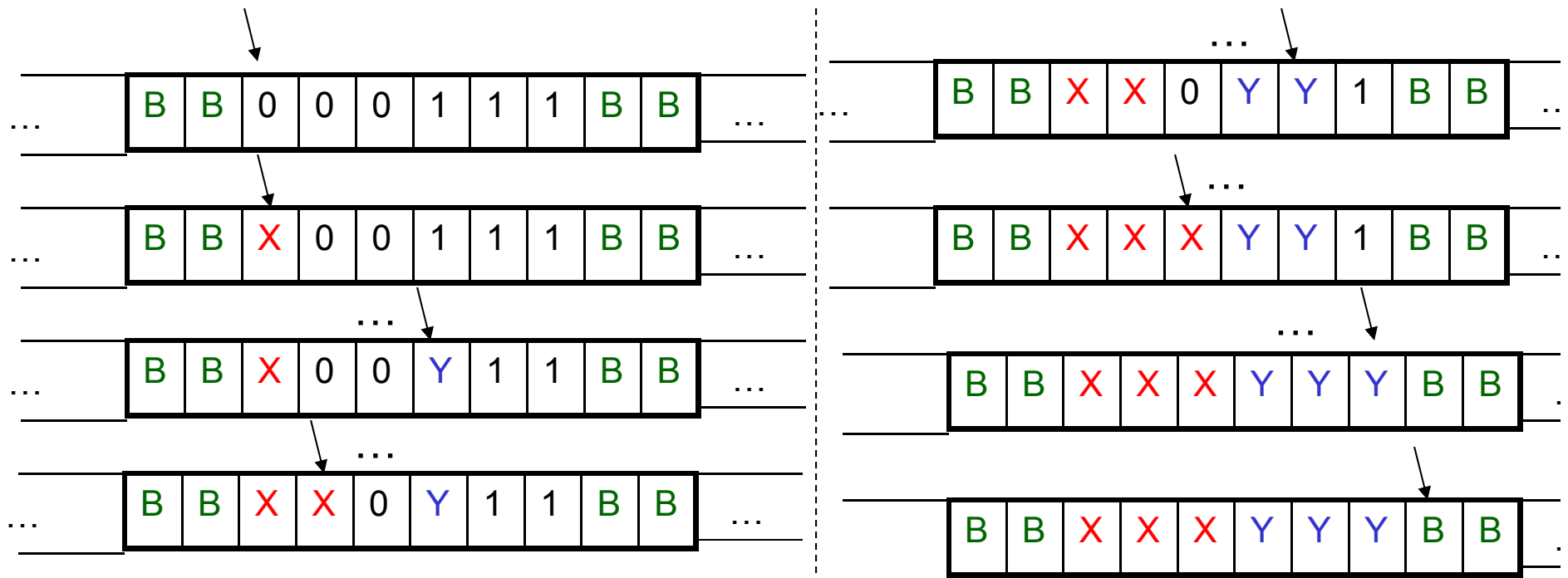
# Way to check for Membership

- **Is a string *w* accepted by a TM?**

- <u>Initial condition:</u>
    - The (whole) input string *w* is present in TM, preceded and followed by infinite blank symbols
- <u>Final acceptance:</u>
    - Accept *w* if TM enters <u>final state</u> and halts
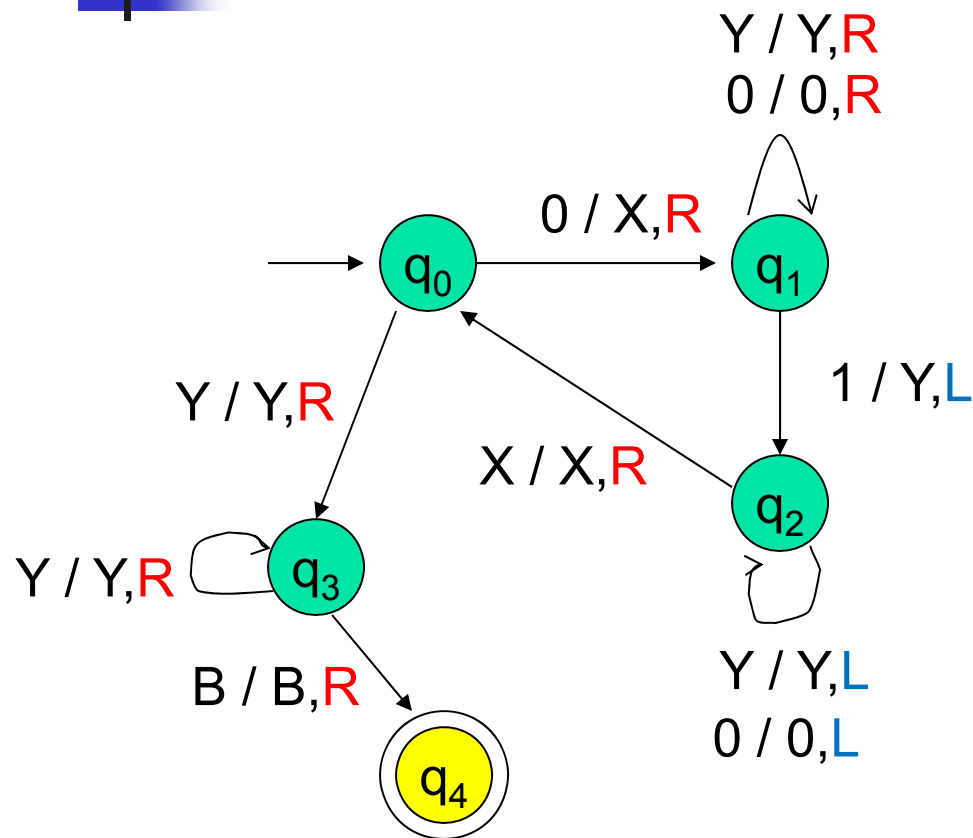    - If TM halts and not final state, then reject

# Example: L = $\{0^n1^n \mid n \geq 1\}$

- Strategy:     w = 000111



Accept

# TM for $\{0^n1^n \mid n \geq 1\}$

$$Y / Y, R$$
$$0 / 0, R$$

$$0 / X, R$$

$q_0 \longrightarrow q_1$

$$1 / Y, L$$

$$Y / Y, R$$

$$X / X, R$$

$q_2$

$$Y / Y, R$$

$q_3$

$$B / B, R$$

$q_4$

$$Y / Y, L$$
$$0 / 0, L$$

1. Mark next unread 0 with X and move right
2. Move to the right all the way to the first unread 1, and mark it with Y
3. Move back (to the left) all the way to the last marked X, and then move one position to the right
4. If the next position is 0, then goto step 1.
   Else move all the way to the right to ensure there are no excess 1s. If not move right to the next blank symbol and stop & accept.

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# TM for $\{0^n 1^n \mid n \geq 1\}$

| Curr. State | Next Tape Symbol | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | X | Y | B |
| $q_0$ | $(q_1,X,R)$ | - | - | $(q_3,Y,R)$ | - |
| $q_1$ | $(q_1,0,R)$ | $(q_2,Y,L)$ | - | $(q_1,Y,R)$ | - |
| $q_2$ | $(q_2,0,L)$ | - | $(q_0,X,R)$ | $(q_2,Y,L)$ | - |
| $q_3$ | - | - | - | $(q_3,Y,R)$ | $(q_4,B,R)$ |
| *$q_4$ | - | -- | - | - | - |

Table representation of the state diagram