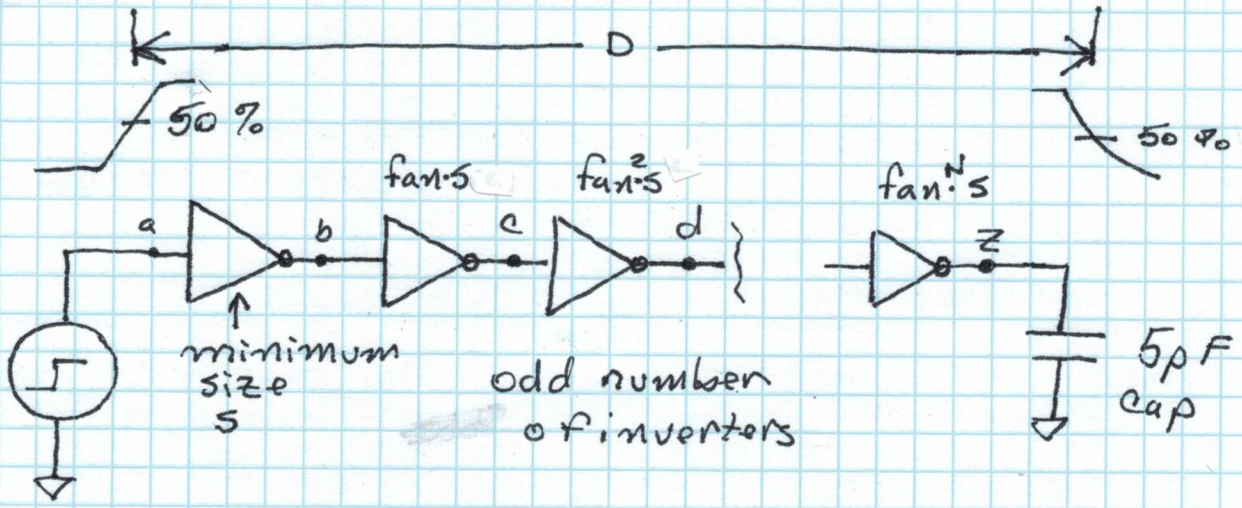

$$\text{Total delay} = D = N \, \text{fan} \, \delta$$

(N: no of stages)

$$R_{\text{atio}} = \frac{C_{in_{next}}}{C_{gate\ c_{in}}^{min}} = \text{fan}^{N} = \left(\frac{C_{load}}{c_{in}}\right)$$

(driver sees this: C_{load}; smallest gate; fan out; N: no of stages)

call it → R

$$\text{fan out} = \frac{I_o}{I_i} \Rightarrow \frac{W_i}{W_{i-1}}$$

now write an equation to minimize total delay as a function of # of stages

$$\frac{\partial}{\partial N}\left(N R^{1/N} \delta\right) = R^{1/N}\delta + N\delta \frac{\partial}{\partial N}\left(R^{1/N}\right)$$

$$= R^{1/N}\delta + \frac{N\delta \ln(R)}{N^2} R^{1/N}$$

minimize

$$0 = R^{1/N}\delta\left(1 - \frac{\ln R}{N}\right)$$

$$\frac{\ln(R)}{N} = 1$$

$R = \frac{C_{in}}{C_{gate}^{min}}$

no of stages $N = \ln(R)$

solve $\rightarrow$ $R = e^N$

Example

$C_{in} = 0.1\,pF$

last stage $C = 5pF$

$fan = R^{1/N}$

$fan = (e^N)^{1/N} = e$

$$\frac{0.1\,pF}{5pF} = \frac{1}{50} = -3.192 \Rightarrow 4 \text{ stages}$$

Each stage is succesively wider

$$\text{fan out} = \frac{I_{out}}{I_{in}} = \frac{W_i}{W_{i-1}}$$

optimum fan out is $fan = e = 2.718$

but we will typically use an integer (standard cell)

this analytical solution is good but treats each inverter as perfect with no capacitance

so now solve this problem in Hspice