

Thermal simulation

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Thermal simulation

- There are a range of types of thermal problems

- 1) steady state or time dependent

- steady state is the solution it would settle to after infinite time

- 2) heat transfer formula or heat equation

- 3) 1, 2 or 3 dimensions

- very often one dimension dominates so you can approximate and make the calculation much shorter

Heat Equation

- The heat equation describes how temperature diffuses through a material over time
- Heat equation has the form
where K is the coefficient of thermal diffusivity
- A system obeys this equation if heat is not added or subtracted at the boundaries, and is neither created or destroyed within the system
- T is temperature and is a function of space and time, in 1 dimension this might be $T(x,t)$
- Under the condition – we can use separation of variables we can separate $T(x,t) = X(x) T(t)$
 - the temperature dependence in x is only described by $X(x)$ and the temperature dependence with time is only described by $T(t)$
 - Now we can solve for $X(x)$ and $T(t)$ independently

$$K \nabla^2 T = \frac{dT}{dt}$$

Separable Solution

- $T(0,0)=T_1, T(0,L)=T_2$ $\kappa \nabla^2 T = \frac{dT}{dt}$

$$\kappa \nabla^2 XT = T \kappa \nabla^2 X = \frac{dXT}{dt} = X \frac{dT}{dt}$$

$$\kappa \frac{X''}{X} = \frac{T'}{T} = -\alpha$$

$$X'' = \frac{-\alpha}{\kappa} X, \quad X(x) = A \sin\left(\sqrt{\frac{\alpha}{\kappa}} x\right) + B \cos\left(\sqrt{\frac{\alpha}{\kappa}} x\right)$$

$$T' = -\alpha T, \quad T(t) = Ce^{-\alpha t} + De^{\alpha t}$$

Separable Solution

- At time $t=0$
- $T(0,0)=T_1$, $X(0,L)=T_2$
- $X(0)=BT_1$, $X(L)=CT_2$
- The time function describes how this
- changes with time

$$X(0) = B \cos \left(\sqrt{\frac{\alpha}{\kappa}} 0 \right) = T_1, \quad B = T_1$$

$$X(L) = A \sin \left(\sqrt{\frac{\alpha}{\kappa}} L \right) + T_1 \cos \left(\sqrt{\frac{\alpha}{\kappa}} L \right) = T_2$$

$$A = \frac{T_2 - T_1 \cos \left(\sqrt{\frac{\alpha}{\kappa}} L \right)}{\sin \left(\sqrt{\frac{\alpha}{\kappa}} L \right)}$$

Heat Transfer Formula

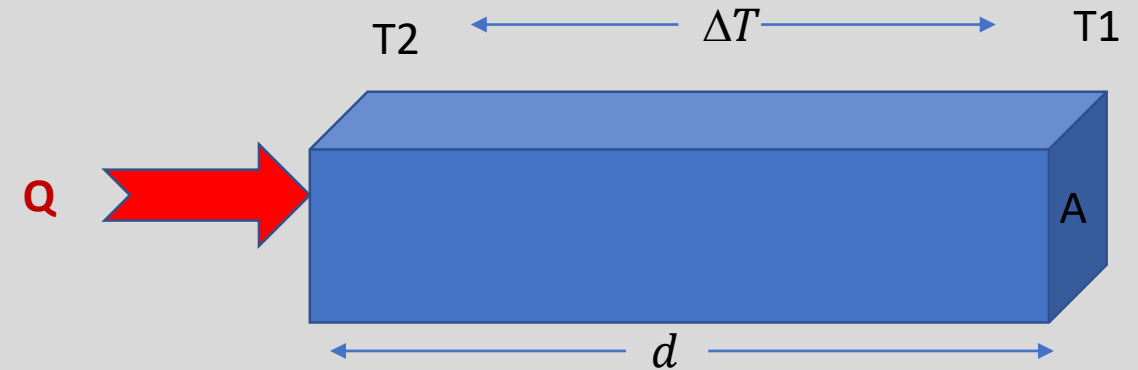
- Those assumptions don't always hold, in some cases we have heat added to the system or dissipated to the environment
- We need an equation that includes terms representing these interactions with the environment

Consider

$$\kappa \nabla^2 T + \frac{\kappa}{\lambda} g_v + \frac{\kappa}{\lambda} \nabla \lambda \nabla T = \frac{dT}{dt}$$

Heat flow

- Imagine a block of homogeneous material of area - A
- κ - thermal conductivity (W/mK)
- Q - rate of heat flow
- d – thickness of the block
- ΔT – temperature difference across the block



The rate that heat is conducted through a material is proportional to the area normal to the heat flow, and the temperature gradient along the heat flow path

$$Q = \kappa A \Delta T / d$$

Thermal Resistance

- R - Thermal resistance

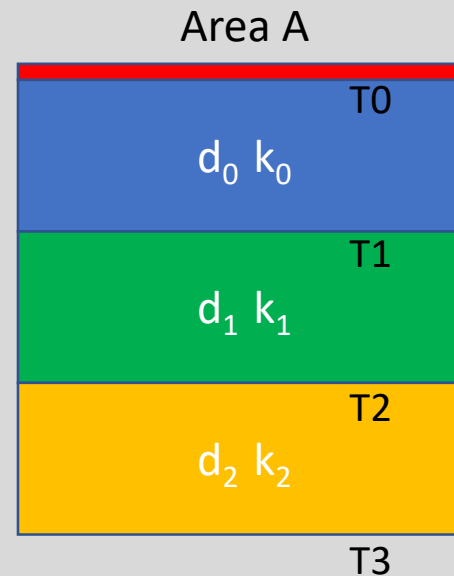
$$R = A \Delta T / Q$$

$$\text{So } R = d/k$$

$$\theta = R_{\text{material}} + R_{\text{contact}}$$

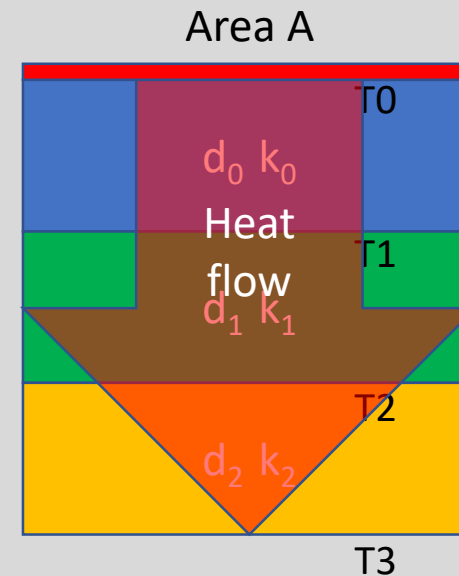
We can solve for the temperature of each interface between these layers

- Consider a stack of three materials of thicknesses d_i and thermal conductivity k_i
- Assume that the heat comes from components on the top surface which dissipate heat
- Assume for simplicity that this component covers area A at the top



We can solve for the temperature of each interface between these layers

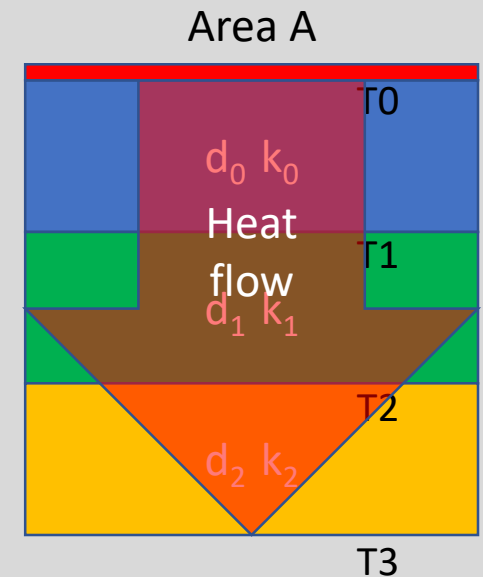
- Consider a stack of three materials of thicknesses d_i and thermal conductivity k_i
- Assume all heat flows uniformly (because it is homogeneous) through the material from top to bottom, no heat flows out the side
- This is a Dirchelet boundary condition at top and bottom and VonNewman on the sides --- check this



We can solve for the temperature of each interface between these layers

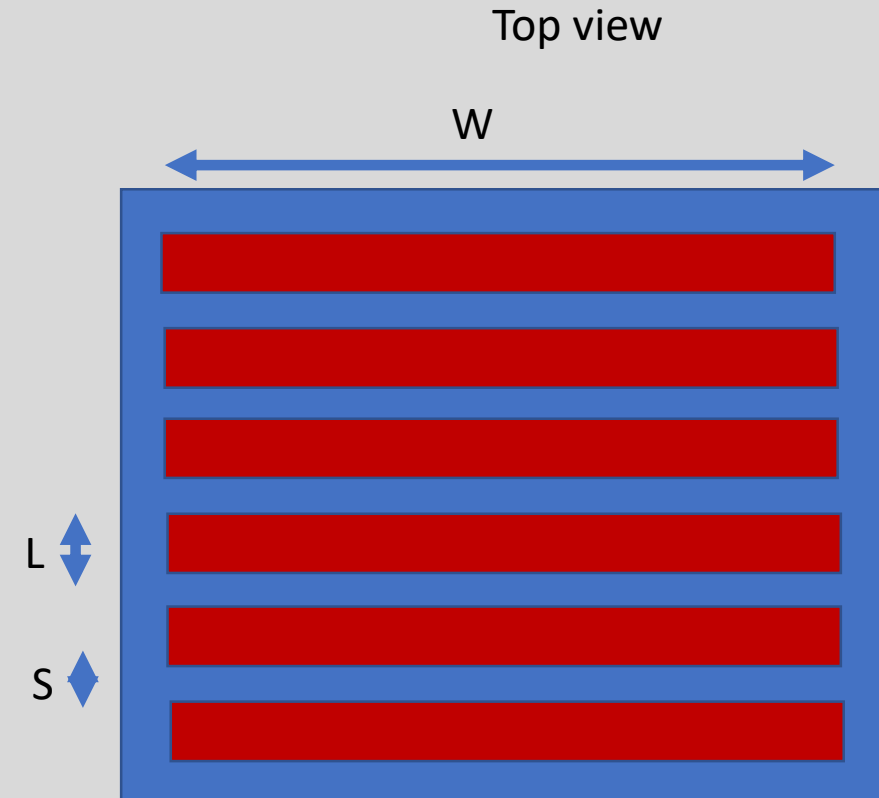
- Consider pseudo code that describes this phenomenon

```
for i=2 to 0 by -1 // go through each layer
     $R_i = d_i / (k_i A)$  // calculate thermal resistivity of each layer i
     $T_i = T_{i+1} + QR_i$  // Calculate the temperature of each layer interface
```



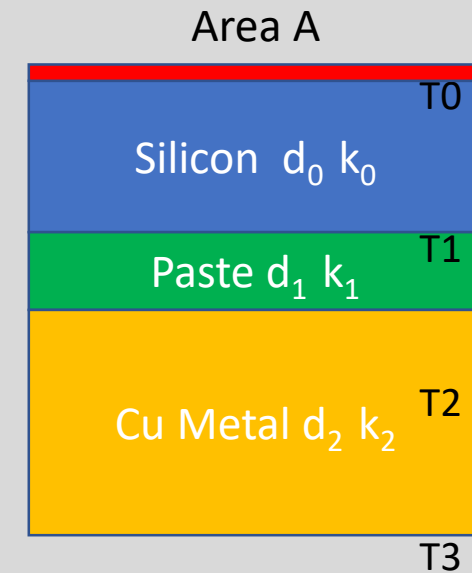
Use this simple model as an approximation

- Consider a MOSFET power transistor driving an external load which requires current
- In EEE 202 we learned that the output impedance of this transistor should be the conjugate of the load impedance for maximum power transfer to the load
 - The consequence is that half the power is dissipated in the part
- Consider a MOSFET made with $N=6$ fingers, $W= 100\mu\text{m}$ wide and each $L = 5\mu\text{m}$
- $V_{dd} = 3$ Volts, and the output impedance of each finger is $R = 300$ ohm
- $P_{diss} = N(V_{dd}^2 / R)$
- $\text{Area} = (W+2S)*((L+S)*N+S)$



Apply this for specific device design

- Layer 0 is Silicon with thickness of 5 microns, and thermal conductivity $k_0 =$
- Layer 1 is a thermal paste 25 microns thick with a thermal conductivity of $k_1 =$
- Layer 2 is the metal flag of thickness 1000 um and thermal conductivity $k_2 =$
- $T_3 = 90 \text{ deg C}$
- We want to keep the device at less than 150 deg C
- This creates a design limitation where
$$T_0 < 150 \text{ deg C}$$



Python code

- Make sure you use consistent units and pay careful attention to dimensional analysis
- Build a table of temperatures as a function of V_{dd}
- We anticipate threshold and transconductance to be heavily dependent on temperature