Simulation in Computer Graphics
Bounding Volume Hierarchies

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Outline

- Introduction
- Bounding volumes BV
- Hierarchies of bounding volumes BVH
- Generation and update of BVs
- Design issues of BVHs
- Performance

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Motivation

- Detection of interpenetrating objects
- Object representations in simulation environments do not consider impenetrability
- Aspects
  - Polygonal, non-polygonal surface
  - Convex, non-convex
  - Rigid, deformable
  - Collision information

Example

- Collision detection is an essential part of physically realistic dynamic simulations
- In each time step
  - Detect collisions
  - Resolve collisions
  - Compute dynamics

[UNC, Univ of Iowa]

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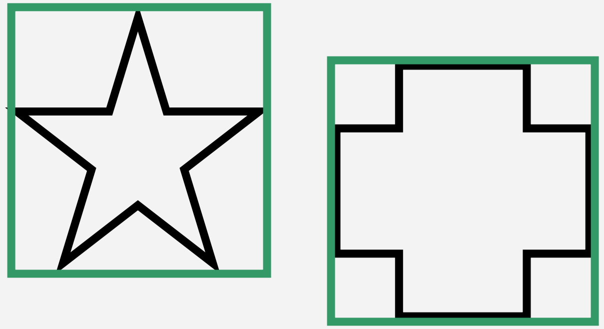
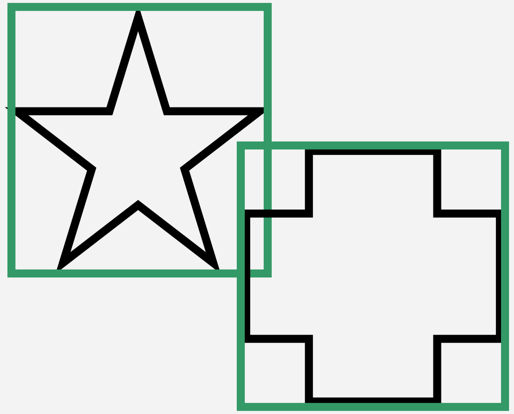
Motivation

- Collision detection for polygonal models is in $O(n^2)$
- Simple bounding volumes – encapsulating geometrically complex objects – can accelerate the detection of collisions



No overlapping bounding volumes
$\rightarrow$ No collision



Overlapping bounding volumes
$\rightarrow$ Objects **could** interfere

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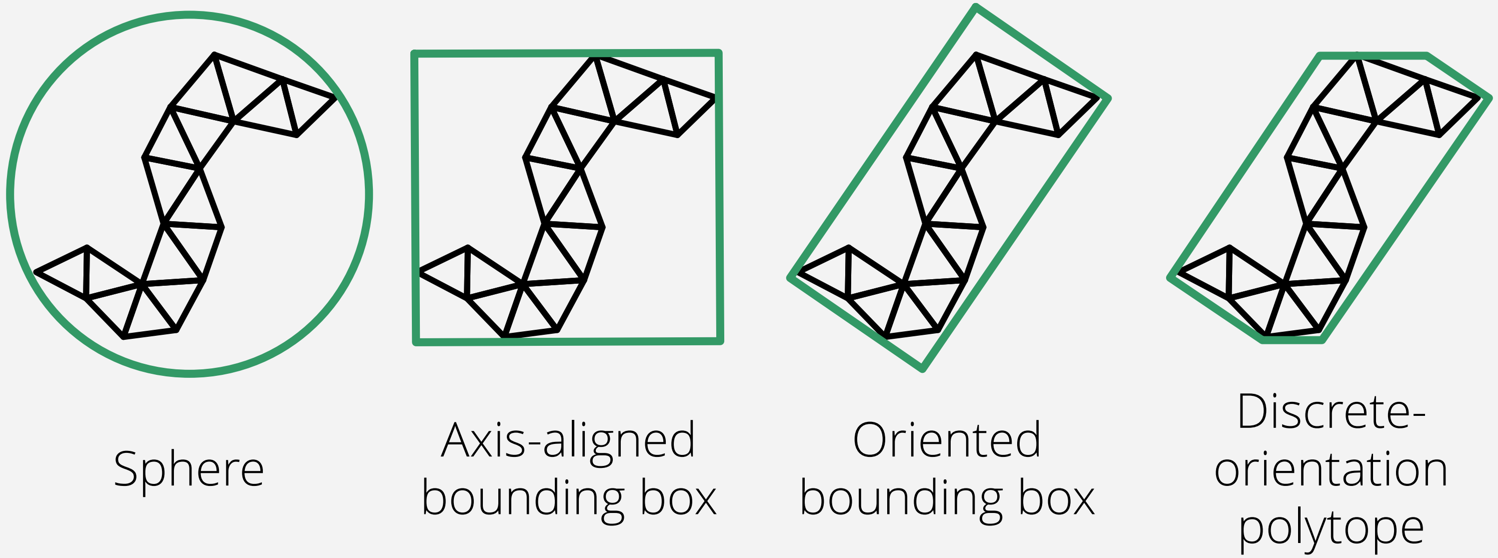
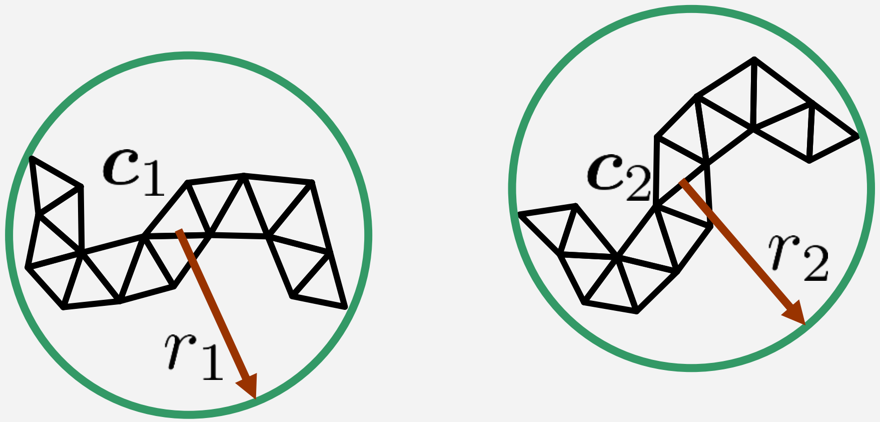
Examples and Characteristics



- Desired characteristics
  - Efficient intersection test, memory efficient
  - Efficient generation and update in case of transformations
  - Tight fitting

Sphere

- Spheres are represented by
  - The center position $\boldsymbol{c}$
  - The radius r
- Two spheres do not overlap if $(\boldsymbol{c}_1 - \boldsymbol{c}_2)(\boldsymbol{c}_1 - \boldsymbol{c}_2) > (r_1 + r_2)^2$



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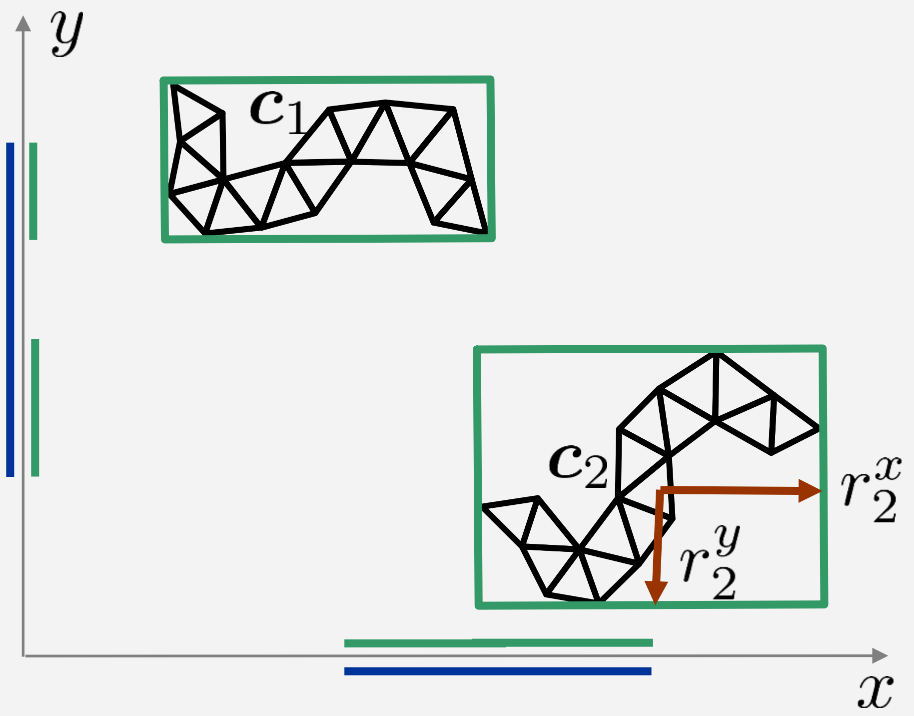
Axis-Aligned Bounding Box AABB

- AABBs are represented by
  - The center positions $\boldsymbol{c}$
  - The radii r^x, r^y
- Two AABBs in 2D do not overlap if

$$\left|(\boldsymbol{c}_1 - \boldsymbol{c}_2)\begin{pmatrix}1\\0\end{pmatrix}\right| > r_1^x + r_2^x$$

or

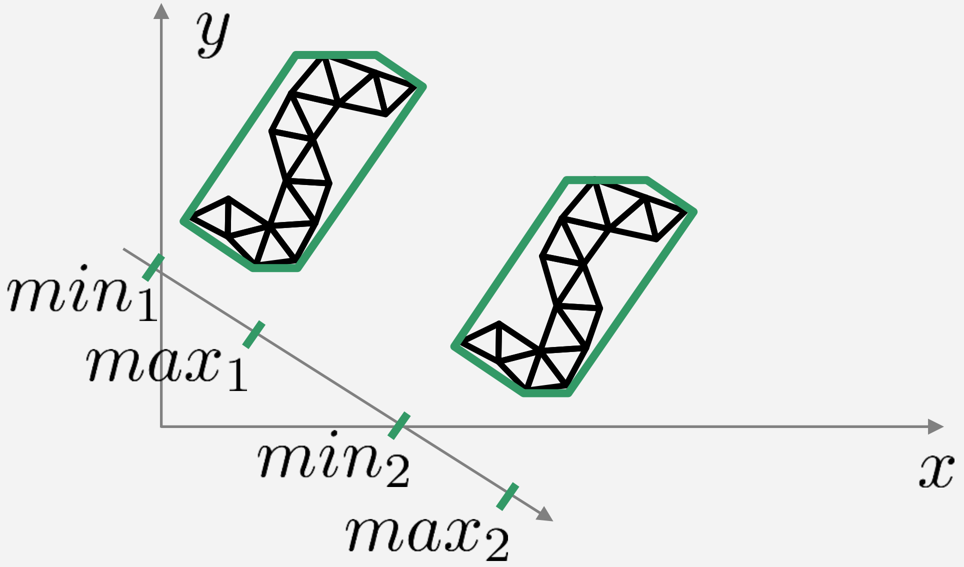
$$\left|(\boldsymbol{c}_1 - \boldsymbol{c}_2)\begin{pmatrix}0\\1\end{pmatrix}\right| > r_1^y + r_2^y$$



Discrete-Orientation Polytope k-DOP

- Convex polytope whose faces are determined by a fixed set of normals
- *k*-DOPs are represented by
  - *k* / 2 normals
  - *k* / 2 min-max intervals
- If any pair of intervals does not overlap, *k*-DOPs do not overlap

$\exists \text{direction} : max_1 < min_2 \vee min_1 < max_2$



Discrete-Orientation Polytope k-DOP

- AABB is a 4-DOP. Are all 4-DOPs AABBs?
- All k-DOPs share the same pre-defined normal set
- Only min-max intervals are stored per *k*-DOP
- Larger *k* improves the approximation quality
- Intersection test is more expensive for larger *k*

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Oriented Bounding Box OBB

- Similar to AABB, but with flexible orientations
- OBBs have not to be aligned with respect to each other or to a coordinate system
- In contrast to AABBs and *k*-DOPs,
  - OBBs can be rotated with an object
  - OBBs are more expensive to check for overlap

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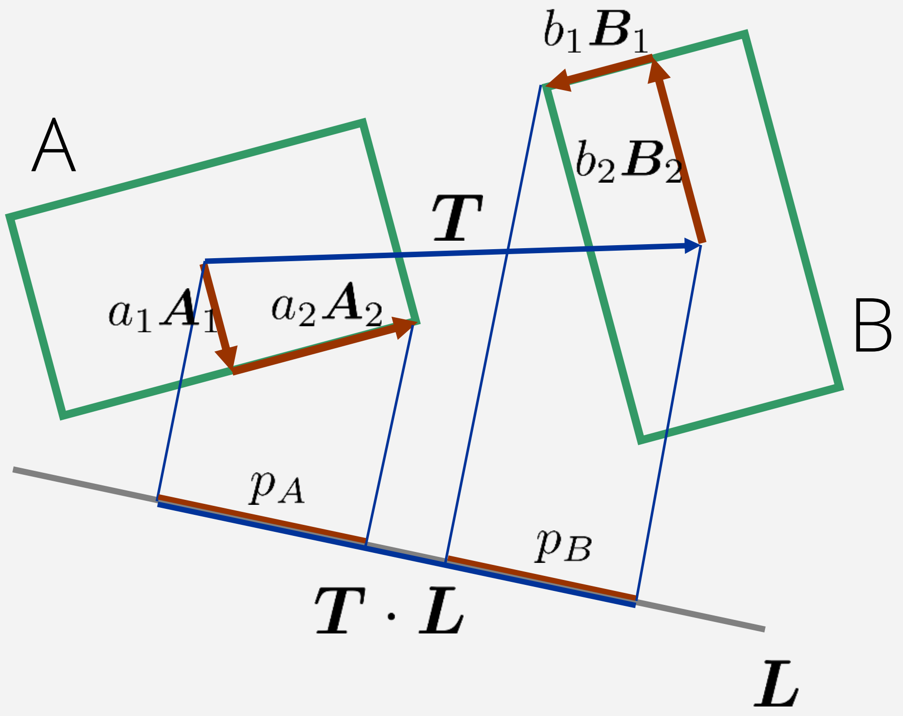
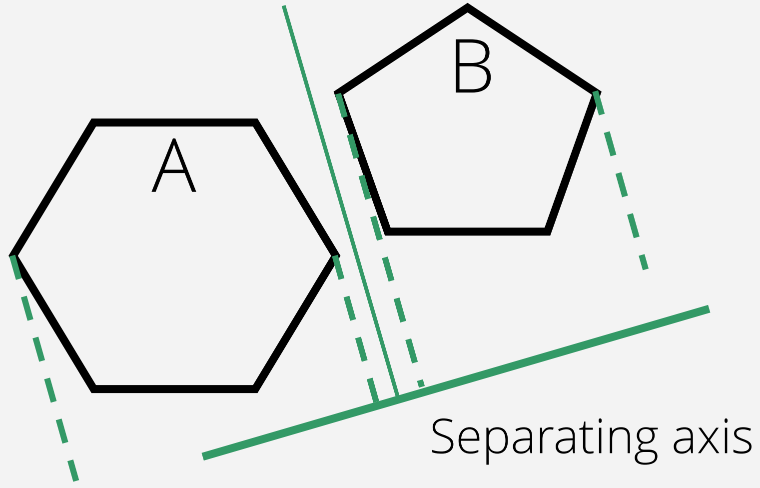
OBB Overlap Test in 2D

- $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{B}_1, \boldsymbol{B}_2$ are normalized axes of A and B
- a_1, a_2, b_1, b_2 are radii of A and B
- $\boldsymbol{L}$ is a normalized direction
- $\boldsymbol{T}$ is the distance of centers of A and B
- $p_A = a_1 \boldsymbol{A}_1 \boldsymbol{L} + a_2 \boldsymbol{A}_2 \boldsymbol{L}$
- $p_B = b_1 \boldsymbol{B}_1 \boldsymbol{L} + b_2 \boldsymbol{B}_2 \boldsymbol{L}$
- A and B do not overlap in 2D if $\exists \boldsymbol{L} : \boldsymbol{T} \cdot \boldsymbol{L} > p_A + p_B$



Separating Axis Test

- Motivation
  - Two objects A and B are disjoint if for some vector $\boldsymbol{v}$ the projections of the objects onto $\boldsymbol{v}$ do not overlap. In this case, $\boldsymbol{v}$ is a separating axis.
  - If A and B are convex, the separating axis exists if and only if A and B do not overlap.



For concave objects, a separating axis does not necessarily exist, if both objects are disjoint.

Separating Axis Test

- For polyhedral objects, only a few axes have to be tested
  - Axes parallel to face normals of A
  - Axes parallel to face normals of B
  - Axes parallel to all cross products of edges of A and B
- In case of 3D OBBs, 3+3+3·3 axes have to be tested
- General overlap test
- Does not provide information on the intersection geometry

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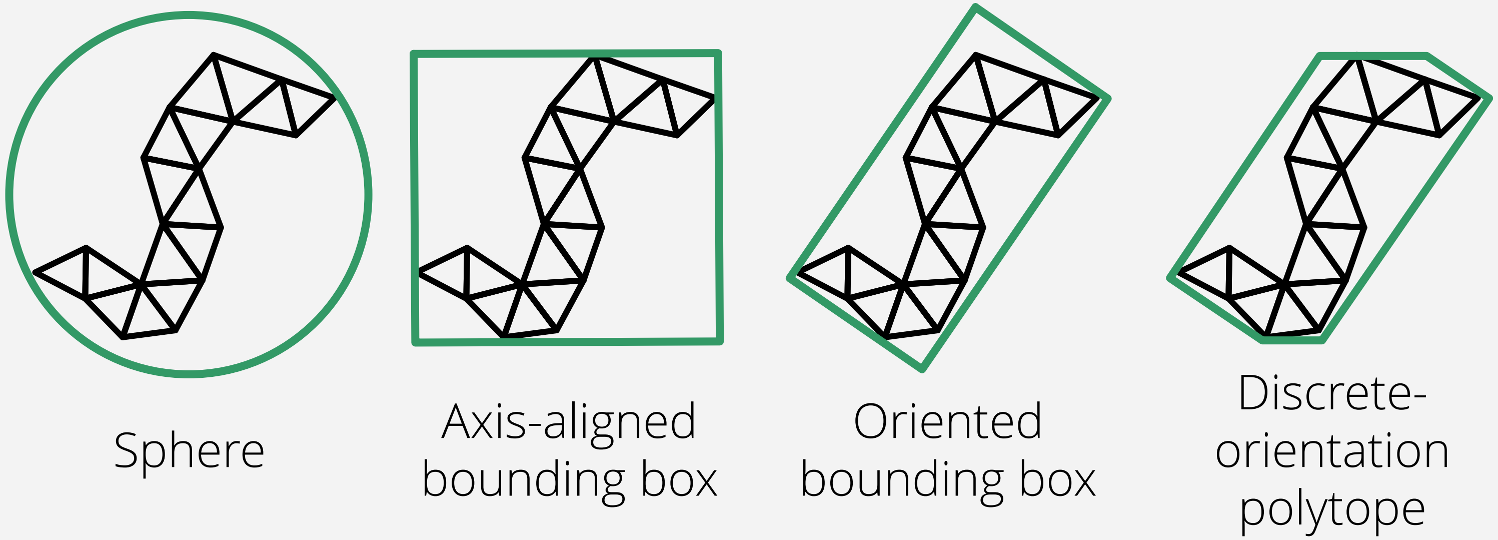
Optimal Bounding Volume

– It depends …

Tight approximation

Efficient overlap test



Summary - Bounding Volumes

- Simple geometries that encapsulate complex objects
- Efficient overlap rejection test
- Tight object approximation,
  memory efficient, fast overlap test
- Spheres, AABBs, OBBs, k-DOPs

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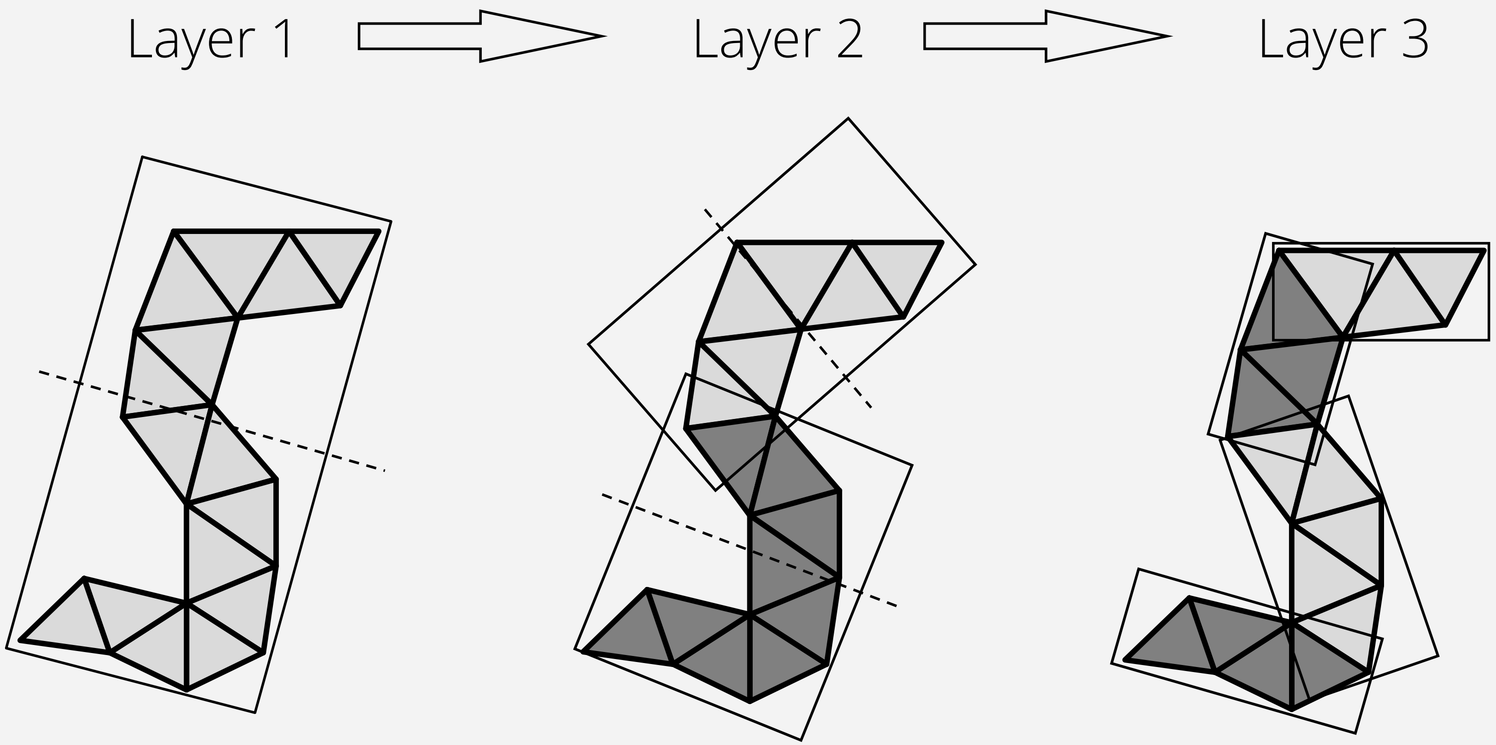
Bounding Volume Hierarchies

- Efficient overlap rejection test for parts of an objects
- E.g., object can be subdivided to build a BVH



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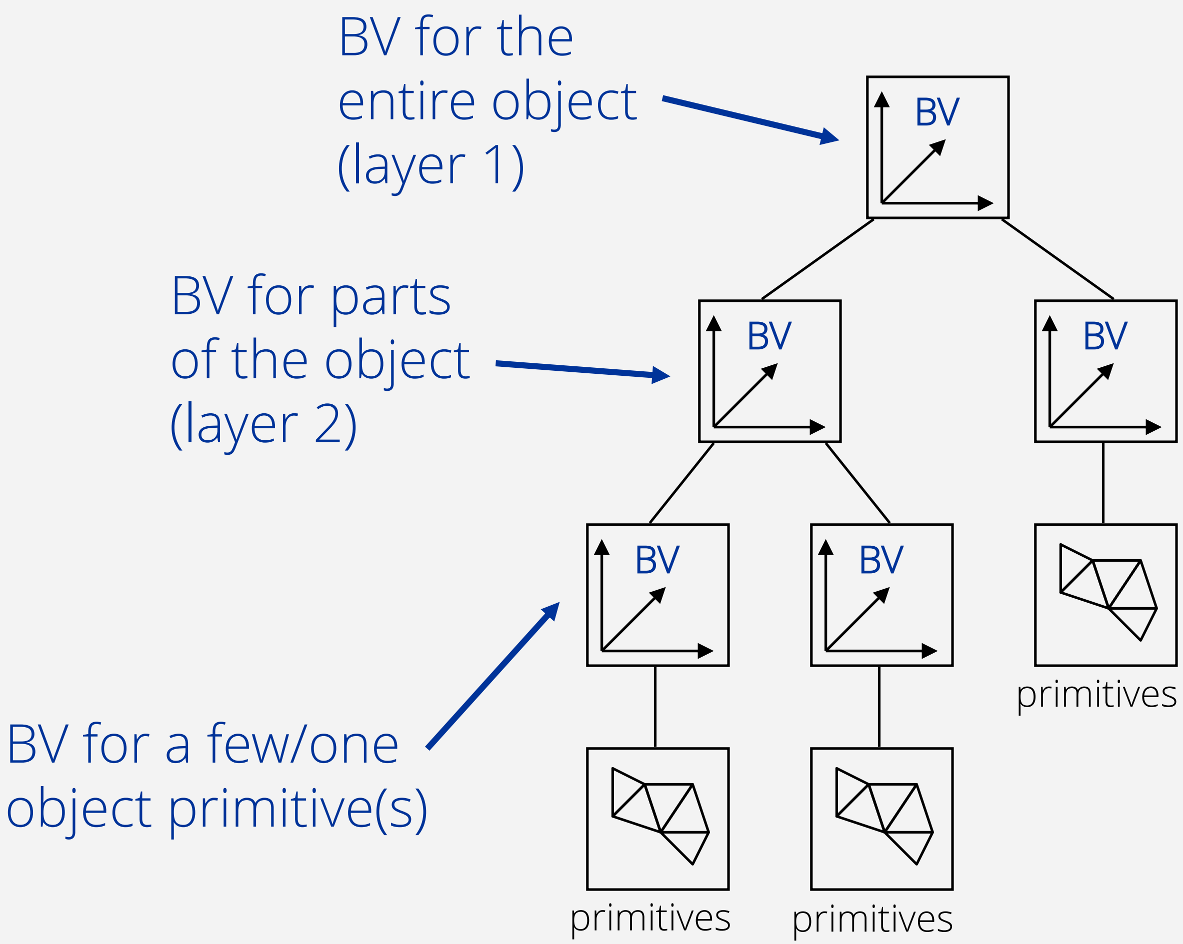
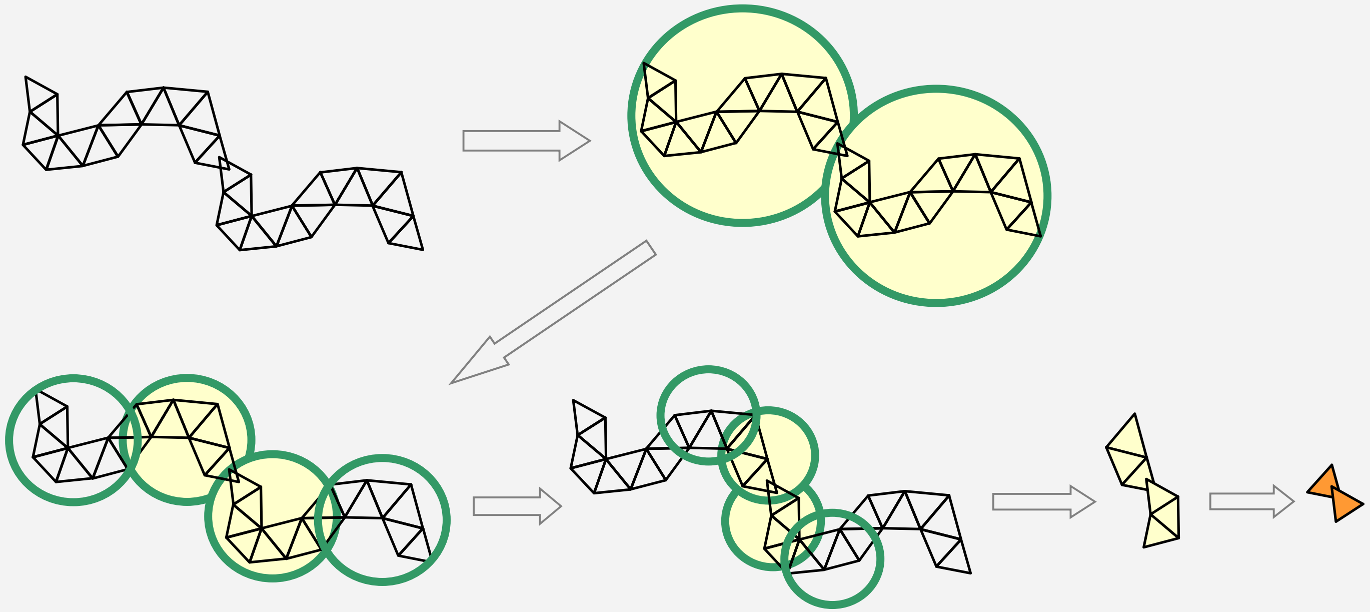
Data Structure

- Tree of bounding volumes
- Nodes contain BV information
- Leaf nodes contain object primitives



Overlap Test for BV Trees



- If BVs in a layer overlap, their children are checked
- At a leaf, primitives are tested with BVs and primitives
- Efficient culling of irrelevant object parts

Pseudo Code

1. Overlap test for two parent nodes (root)
2. If no overlap then "no collision" else
3. All children of one parent node are checked against children of the other parent node
4. If no overlap then "no collision" else
5. If at leaf nodes then "collision" else go to 3.

– Step 3. checks BVs or object primitives for intersection

– Required tests: BV-BV, BV–primitive, primitive-primitive

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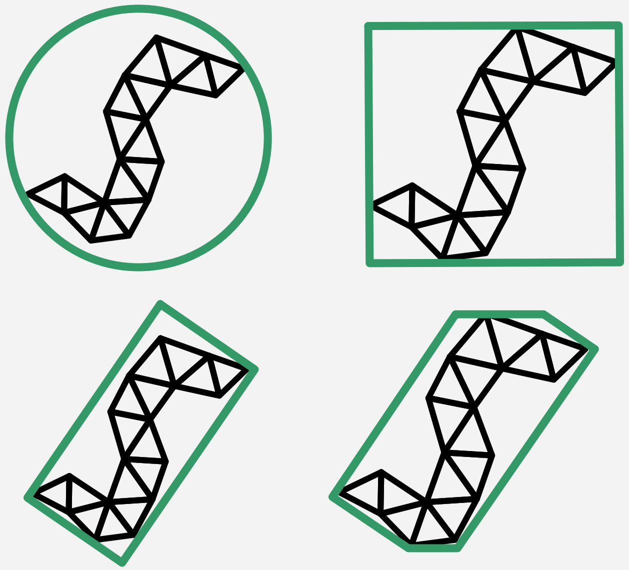
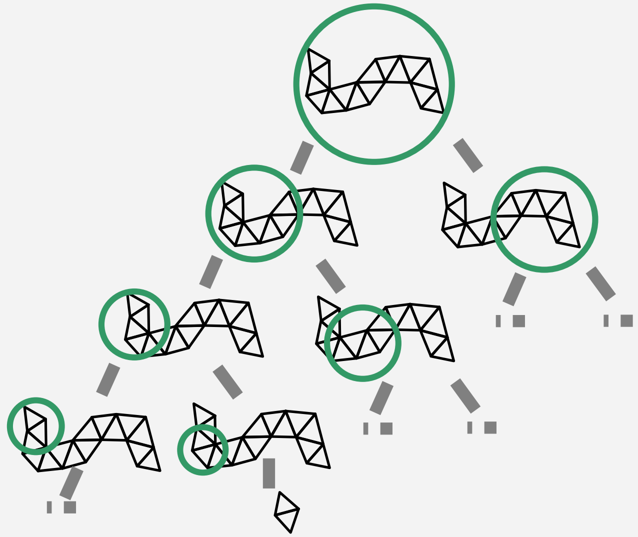
Summary - BVHs

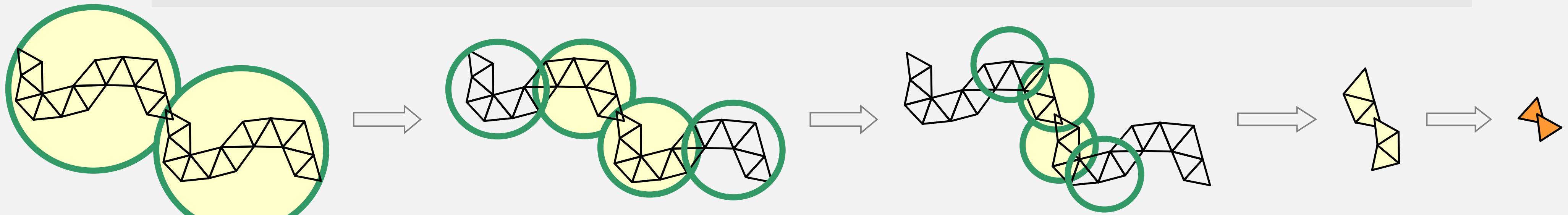
(1) Bounding volumes



(2) Bounding volume tree



(3) Collision detection test



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Sphere

- Start with AABB of all points
  - The center of the sphere is given by the center of the AABB
  - The radius of the sphere is given by the largest distance from the center to a point
- Iteratively improve an initial guess (Ritter 1990)
  - Six extremal points of an AABB are computed
  - Choose pair of points with largest distance to get the center of the sphere and an initial guess of the radius
  - Iteratively enlarge the radius for points outside the sphere
- Minimum bounding sphere (Welzl 1991)
  - Randomized algorithm that runs in expected linear time
- Spheres can be translated and rotated with objects

AABB

- Compute six extremal points for the center and the radii
- AABBs can be translated with an object
- AABBs cannot be rotated with an object (the overlap test does not work for arbitrarily oriented AABBs)
- Rotation issue is addressed by
  - Computing the AABB for the bounding sphere
  - Update the AABB only considering the new positions of the original extremal points
  - Hill climbing on convex objects or pre-computed convex hulls of concave objects (check adjacent points of the original extremal points to update extremal points)

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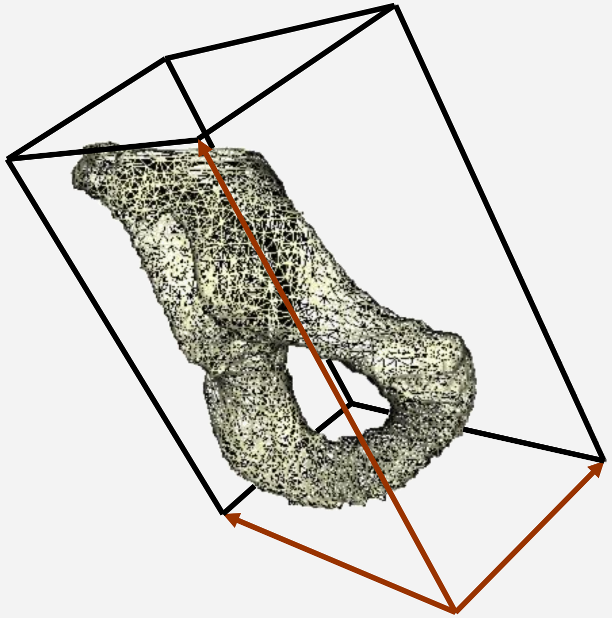
OBB

- Directions given by eigenvectors of the covariance matrix (PCA) (Barequet 1996)

$$\boldsymbol{C}_{jk} = \frac{1}{n}\sum_{i=1}^{n}(\boldsymbol{x}_i - \mu)_j(\boldsymbol{x}_i - \mu)_k \quad \mu = \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{x}_i$$

- Compute an AABB (Barequet 1999)
  - Choose two extremal points with the largest distance opposite to each other to define the first direction for the OBB
  - Choose two orthogonal directions
- Can be translated and rotated with an object



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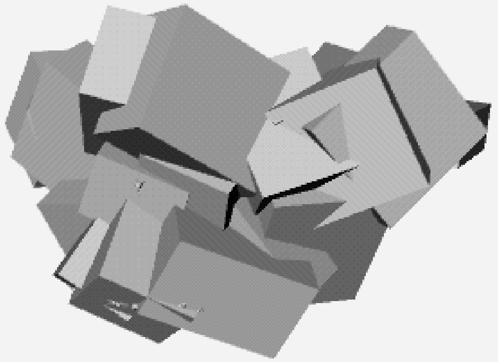
Construction of BVHs

- Goals
  - Balanced tree
  - Tight-fitting bounding volumes
  - Minimal redundancy
    (primitives in more than one BV per level)



- Parameters
  - BV type
  - Top-down / bottom-up
  - What and how to subdivide or merge: primitives or BVs
  - How many primitives per leaf in the BV tree
  - Re-sampling of the object

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Collision Detection Libraries

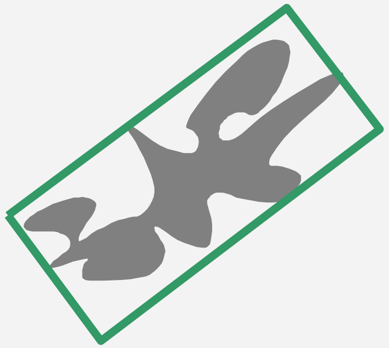
SOLID

Axis-aligned bounding box

van den Bergen
Eindhoven University
1997

RAPID

Oriented bounding box



Gottschalk et al.
University of North Carolina
1995

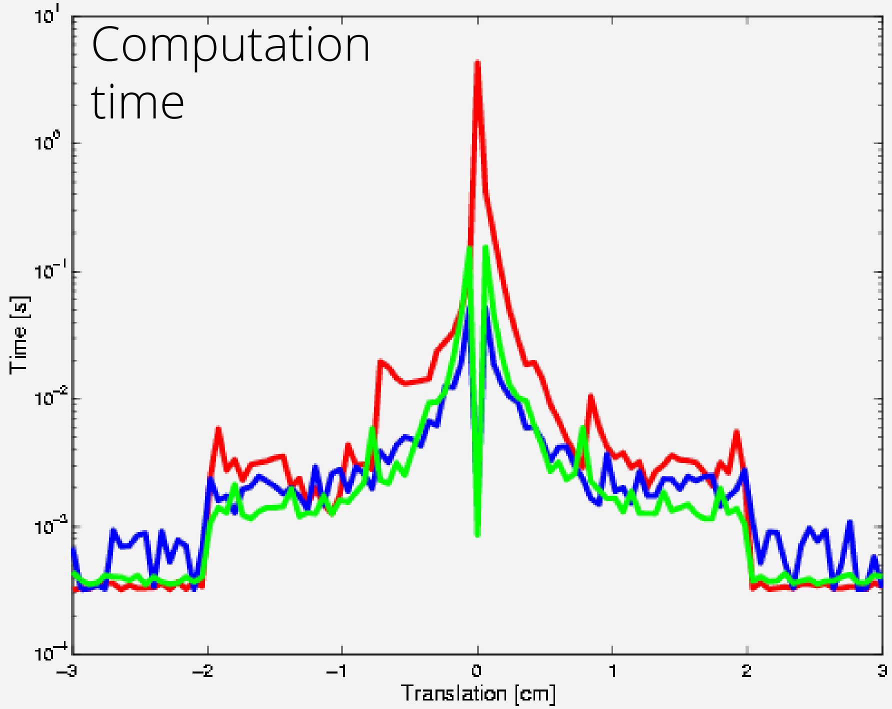
QuickCD

Discrete orientation polytope

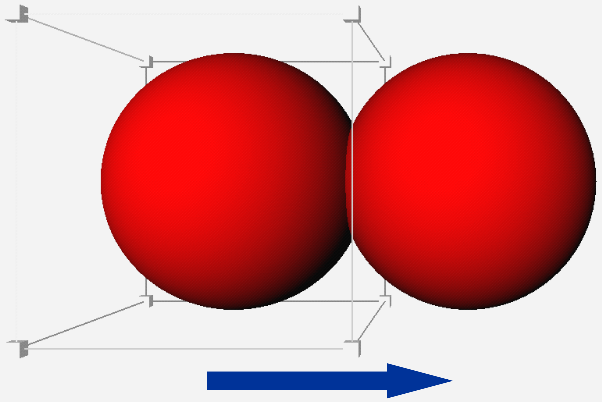
Klosowski et al.
University of New York
1998

Comparison

– Two spheres with radius 1 and 10000 triangles per sphere



Distance between sphere centers



8-DOP

OBB

AABB