

CS5489 - Machine Learning

Lecture 6b - Linear Dimensionality Reduction for Text

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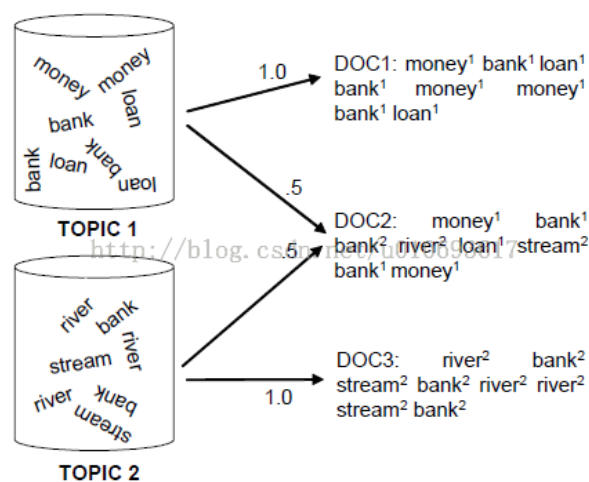
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Outline

1. Linear Dimensionality Reduction for Vectors
2. **Linear Dimensionality Reduction for Text**
3. Non-linear Dimensionality Reduction
4. Manifold Embedding

Dimensionality Reduction

- **Goal:** Transform high-dimensional vectors into low-dimensional vectors.
 - Dimensions in the low-dim data represent co-occurring features in high-dim data.
 - Dimensions in the low-dim data may have semantic meaning.
- **For example:** document analysis
 - high-dim: bag-of-words vectors of documents
 - low-dim: each dimension represents similarity to a topic.



Latent Semantic Analysis (LSA)

- Also called *Latent Semantic Indexing*
- Consider a bag-of-words representation (e.g., TF, TF-IDF)
 - document vector \mathbf{x}_i
 - $x_{i,j}$ is the frequency of word j in document i
- Approximate each document vector as a weighted sum of topic vectors.
 - $\hat{\mathbf{x}} = \sum_{n=1}^p w_p \mathbf{v}_p$
 - Topic vector \mathbf{v}_p contains co-occurring words.

- corresponds to a particular *topic* or *theme*.
- Weight w_p represents similarity of the document to the p-th topic.
- Objective:
 - minimize the squared reconstruction error (Similar to PCA):
 - $\min_{\mathbf{v}, \mathbf{w}} \sum_i ||\mathbf{x}_i - \hat{\mathbf{x}}_i||^2$
 - Solution computed using SVD.
- Represent each document by its topic weights.
 - Apply other machine learning algorithms...
- **Advantage:**
 - Finds relations between terms (synonymy and polysemy).
 - distances/similarities are now comparing topics rather than words.
 - higher-level semantic representation

Example on Spam Email dataset

- use bag-of-words representation with 50 words
- term-frequency (TF) normalization

```
In [2]: # Load spam/ham text data from directories
textdata = datasets.load_files("email", encoding="utf8", decode_error="replace")

# convert to bag-of-words representation
cntvect = feature_extraction.text.CountVectorizer(stop_words='english', max_features=50)
X = cntvect.fit_transform(textdata.data)
Y = textdata.target

# TF representation
tf_trans = feature_extraction.text.TfidfTransformer(norm='l1', use_idf=False)
Xtf = tf_trans.fit_transform(X)

# print the vocabulary
print(cntvect.vocabulary_)

{'online': 39, 'watches': 49, 'brands': 13, 'bags': 11, 'new': 35, 'today': 46, 'h
i': 25, 'peter': 42, 'contact': 17, 'mr': 34, 'email': 21, 'com': 16, 'number': 3
7, 'country': 19, 'nigeria': 36, 'answer': 10, 'status': 44, 'good': 23, 'day': 2
0, 'john': 30, 'united': 47, 'payment': 40, 'bank': 12, 'address': 9, 'informatio
n': 28, 'inform': 27, '10': 2, 'office': 38, 'kamara': 32, '000': 1, 'codeine': 1
5, '30mg': 6, '30': 5, '00': 0, '60': 8, 'pills': 43, '120': 3, 'just': 31, 'cos
t': 18, 'buy': 14, 'mg': 33, 'percocet': 41, '50': 7, '15mg': 4, 'visa': 48, 'isid
oro': 29, 'store': 45, 'http': 26, 'google': 24, 'files': 22}
```

LSA on Spam data

- Apply LSA with 5 topics
 - implemented as `TruncatedSVD`

```
In [3]: lsa = decomposition.TruncatedSVD(n_components=5, random_state=4487)
wlsa = lsa.fit_transform(Xtf)

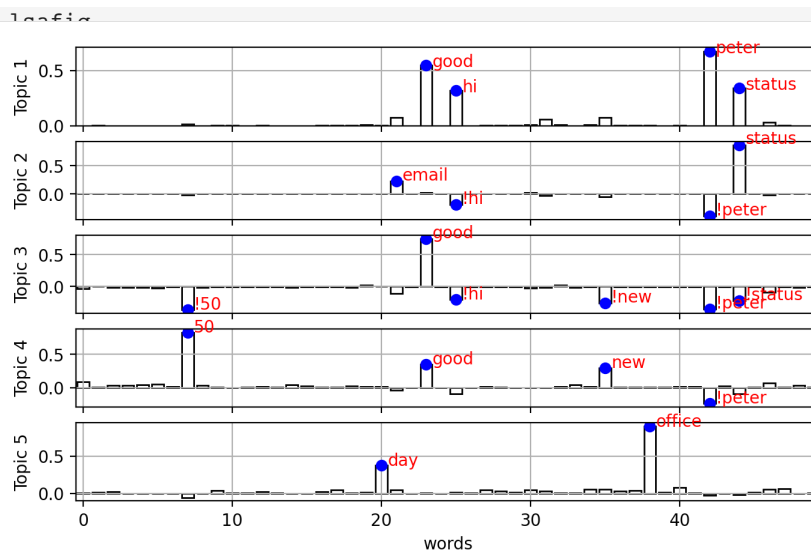
# components
V = lsa.components_
```

Topic vectors

- topic vectors contain frequent co-occurring words

```
In [5]: vocab = asarray(cntvect.get_feature_names_out())
lsafig = plot_topics(lsa, vocab, figsize=(8,5))
```

Out [5]:

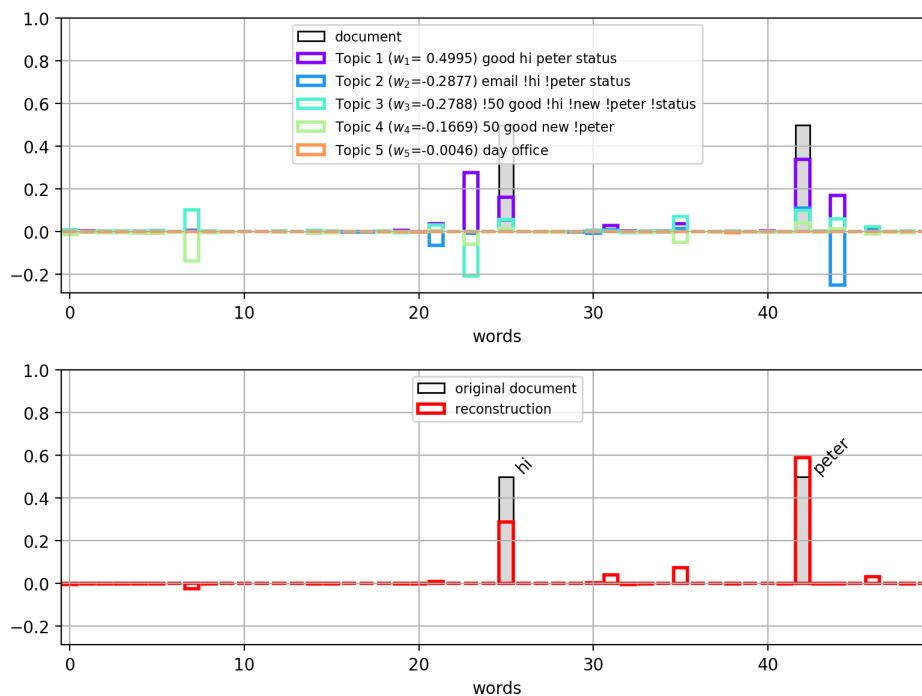


Document representation

- Documents are a combination of topics

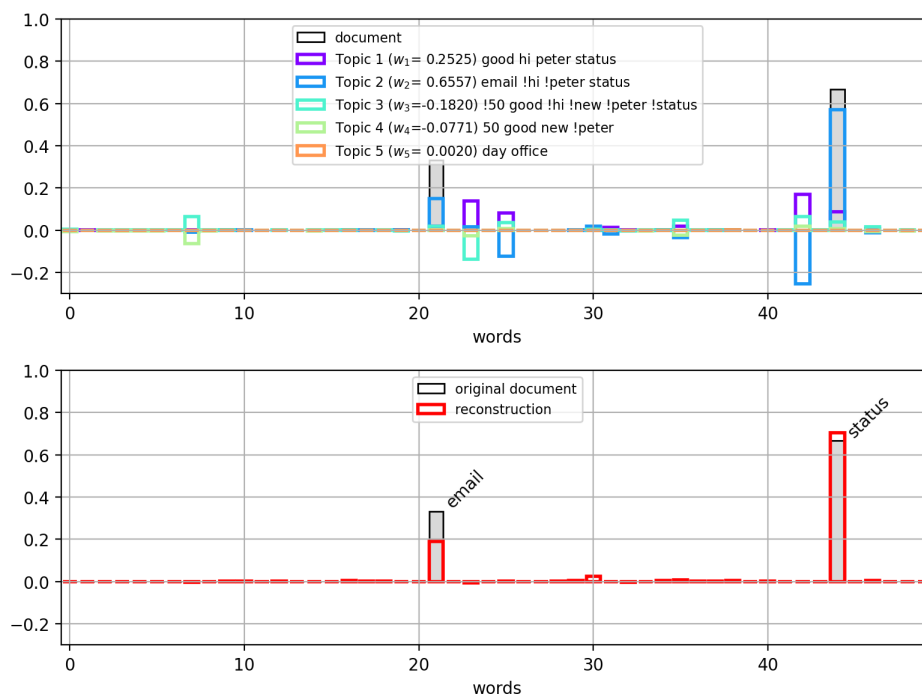
In [7]: `plot_doc_topic(Xtf[2,:], Wlsa[2,:], lsa, vocab)`

Out [7]:



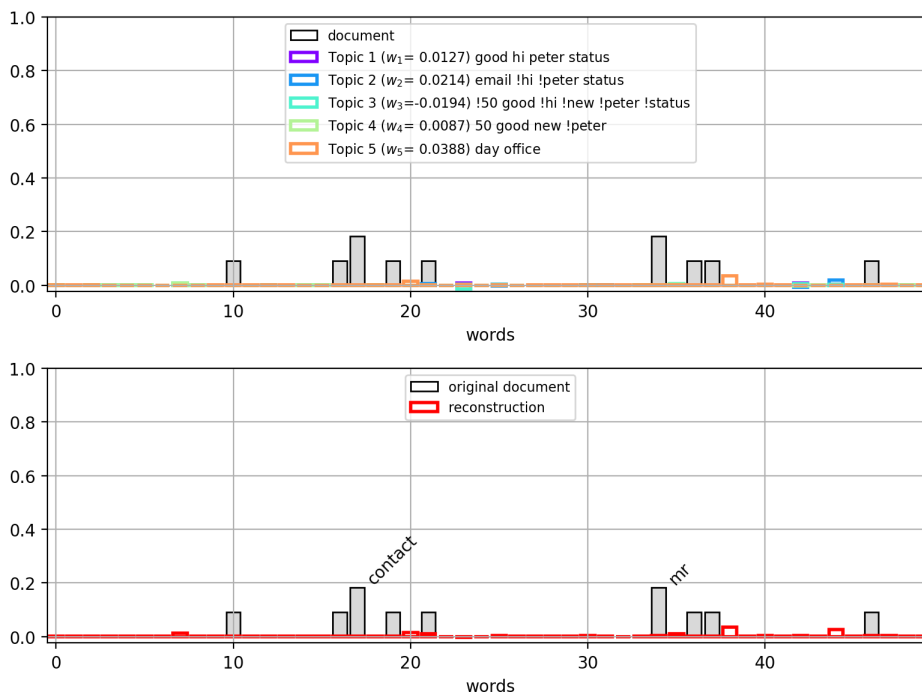
In [8]: `plot_doc_topic(Xtf[4,:], Wlsa[4,:], lsa, vocab)`

Out [8]:



In [9]: `plot_doc_topic(Xtf[3,:], Wlsa[3,:], lsa, vocab)`

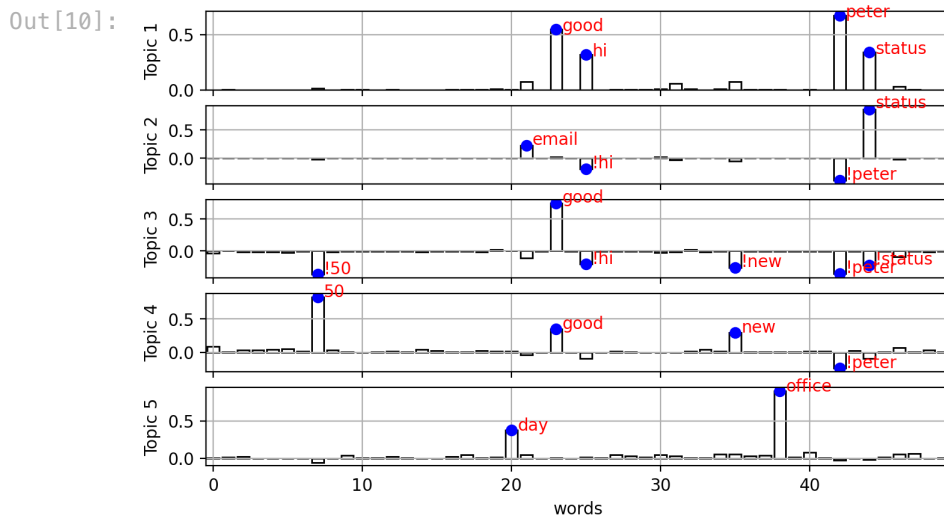
Out [9]:



Problem with LSA

- In the topic vector, the "frequency" of a word can be negative!
 - Doesn't really make sense for document bag-of-words model.

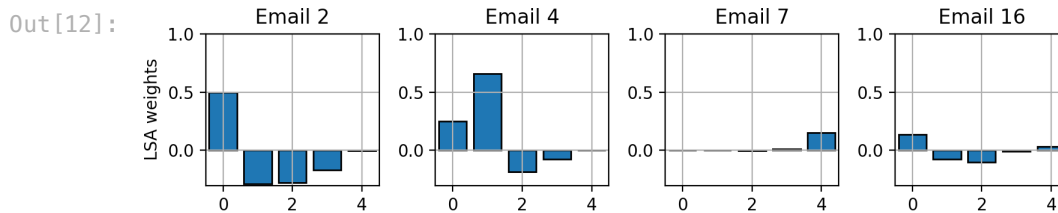
In [10]: `lsafig`



Problems with LSA

- The weights for each topic can be negative!
 - Topics should only be "additive"
 - Topics should increase probability of some topic-related words, but not decrease probability of other words.
 - It doesn't make sense to "remove" a topic using a negative topic weight.

In [12]: pfig



Non-negative Matrix Factorization (NMF)

- **Solution:** constrain the topic vector and weights to be non-negative.
- Similar to LSA
 - Approximate each document vector as a weighted sum of topic vectors.
 - $\hat{\mathbf{x}}_j = \sum_{n=1}^p w_p \mathbf{v}_p$
 - But now, each entry of topic vector $\mathbf{v}_p \geq 0$ and topic weight $w_p \geq 0$
 - Objective: minimize the squared reconstruction error

$$\min_{\mathbf{v} \geq 0, \mathbf{w} \geq 0} \sum_j \|\mathbf{x}_j - \hat{\mathbf{x}}_j\|^2$$

- subject to the non-negative constraints.
- no closed-form solution, need to use an optimizer.

In [13]:

```
# Run NMF
nmf = decomposition.NMF(n_components=5)
Wnmf = nmf.fit_transform(Xtf)

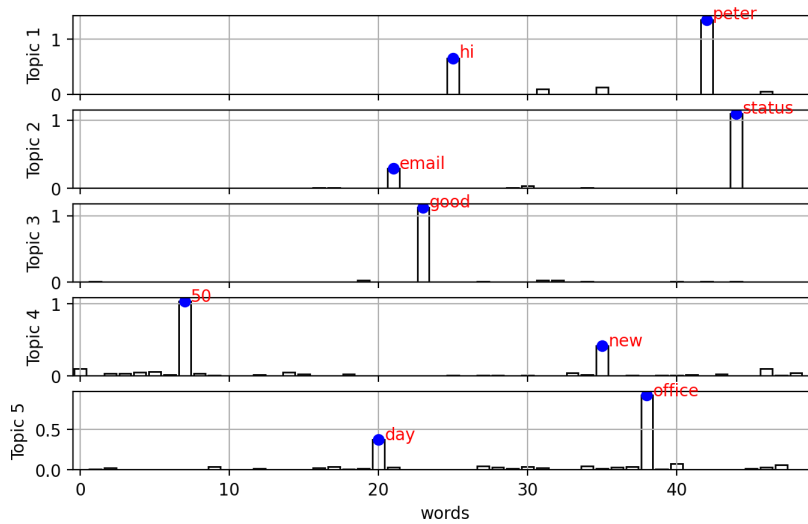
# components
V = nmf.components_
```

Topic vector

- all non-negative entries
- looks much cleaner (less small entries)

```
In [14]: plot_topics(nmf, vocab, figsize=(8,5))
```

Out[14]:

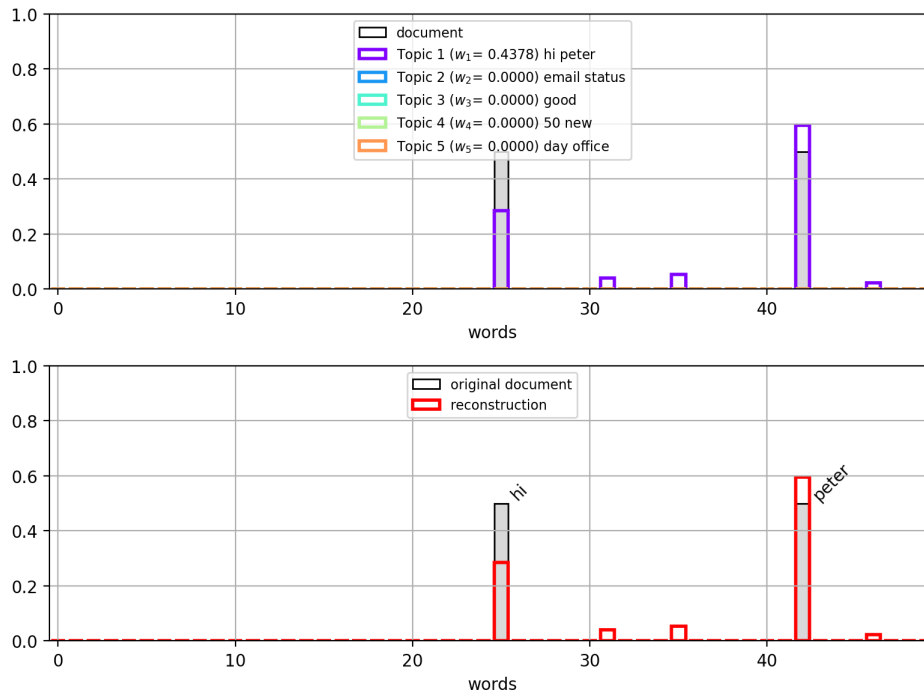


Document vector

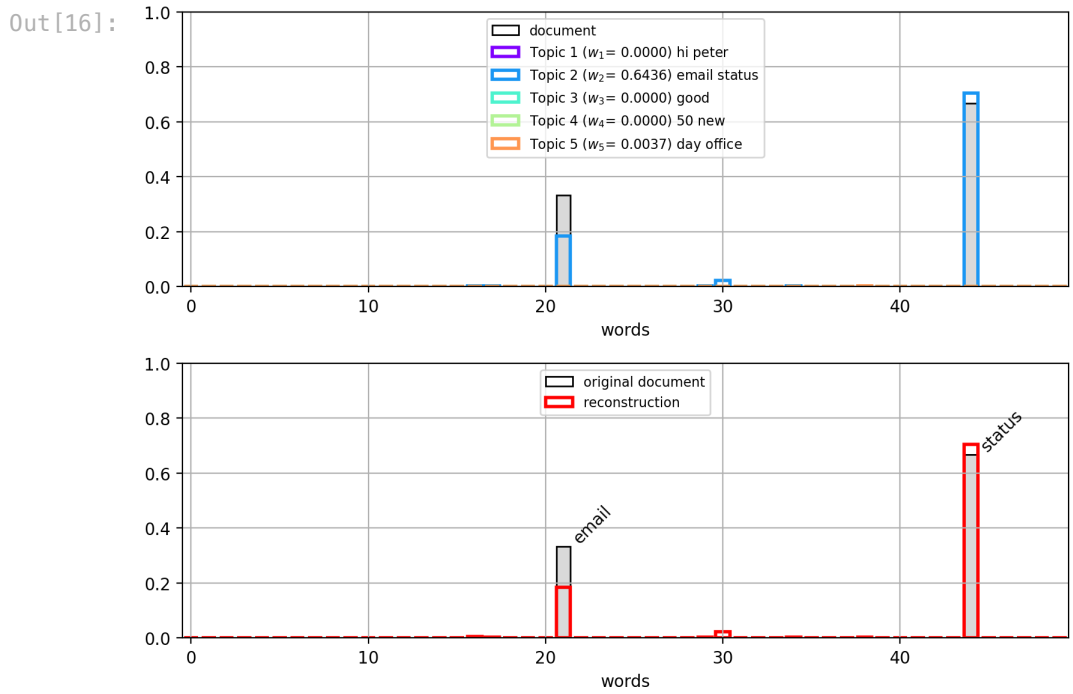
- additive combination of topics

```
In [15]: plot_doc_topic(Xtf[2,:], Wnmf[2,:], nmf, vocab)
```

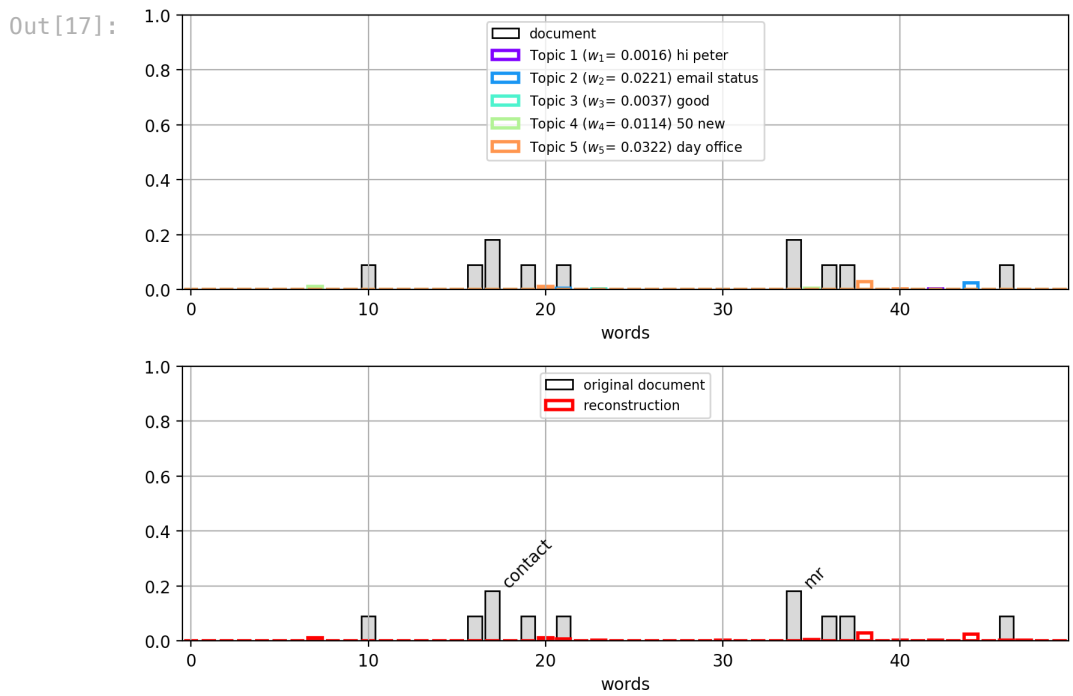
Out[15]:



```
In [16]: plot_doc_topic(Xtf[4,:], Wnmf[4,:], nmf, vocab)
```



In [17]: `plot_doc_topic(Xtf[3,:], Wnmf[3,:], nmf, vocab)`

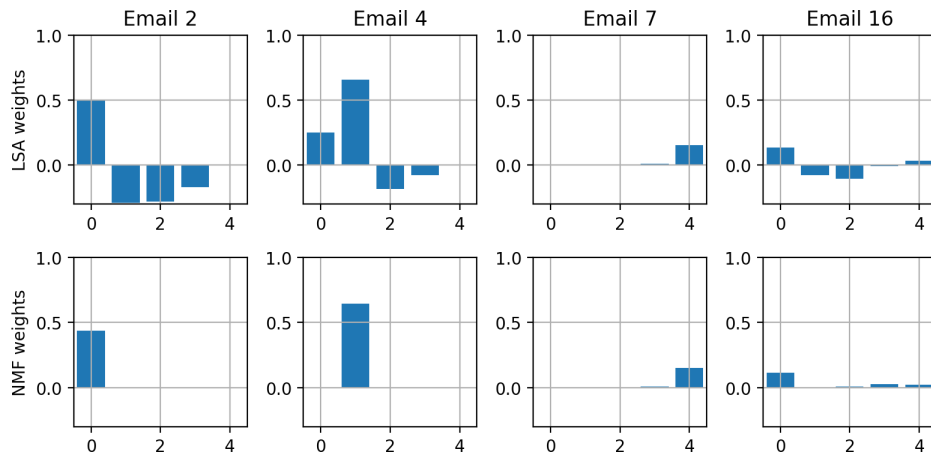


Sparseness

- For NMF representation, most topic weights for a document are zero.
 - this is called a *sparse* representation.
 - each document is only composed of a few topics.

In [19]: `spfig`

Out [19]:

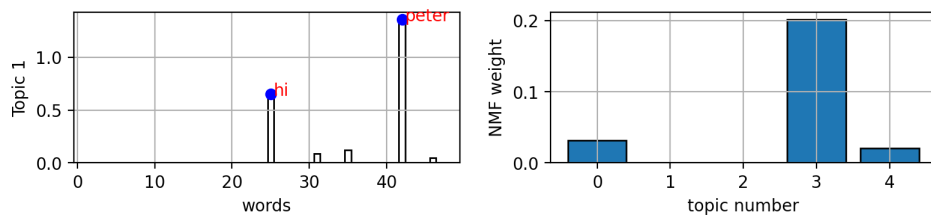


Problem with NMF

- While the weights and component vectors are non-negative, NMF does not enforce them to be probabilities.

In [21]: `nfig`

Out [21]:



- TF/TFIDF is a probabilistic model of words in a document
 - the vector of probabilities sums to 1
 - probabilities are between 0 and 1
- The NMF components and weights are difficult to interpret.
- We will solve this problem later using probabilistic graphical models.

Linear Dimensionality Reduction - Summary

- Goal:** given set of input vectors $\{\mathbf{x}_i\}_{i=1}^n$, with $\mathbf{x}_i \in \mathbb{R}^d$, represent each input vector as lower-dimensional vector $\mathbf{w}_i \in \mathbb{R}^p$.
 - Approximate \mathbf{x} as a weighted sum of basis vectors $\mathbf{v}_j \in \mathbb{R}^d$
 - $\hat{\mathbf{x}} = \sum_{j=1}^p w_j \mathbf{v}_j$
 - minimize the reconstruction error of $\hat{\mathbf{x}}$.
 - enables faster processing, or reduces noise.

| Name | Objective | Advantages | Disadvantages |
|---|--|--|--|
| Principal component analysis (PCA) | minimize reconstruction error; preserve the most variance of data | - captures correlated dimensions, removes redundant dimensions, removes noise. - closed-form solution | - does not consider end goal (e.g., classification) |
| Random Projections | sample random basis vectors. | - fast. - preserves pairwise distances between points (up to accuracy factor). | - adds noise to the pairwise distances. |
| Fisher's Linear Discriminant (FLD) | maximize class separation | - preserves class separation | - requires class information |
| Latent Semantic Analysis (LSA) | minimize reconstruction error | - topic vectors have semantic meaning (co-occurring words) - closed-form solution | - topic weights and topic vectors can be negative - does not consider end goal (e.g., classification) |
| Non-negative Matrix Factorization (NMF) | minimize reconstruction error; non-negative weights and basis vectors. | - "additive" topic/parts model for text or images - sparse topic weights. | - solution requires iterative algorithm. - does not consider end goal (e.g., classification) |

Other things

- *Feature Normalization*

- PCA and LDA are based on the covariance between input dimensions.
- applying *per-feature* normalization will yield a different PCA result!
 - normalizing each input dimension changes the relative covariances.

In [24]: `nfig`

Out [24]:

