CSC2549 Physics-Based Animation

Paddington | Frame Store

Reminders

Assignment #1 Marks Released (Avg: 96%)

Assignment #3 is live and is due on October 18th

https://github.com/dilevin/CSC2549-a3-finite-elements-3d

Assignment #4 (Cloth Simulation) coming soon.

Graphics Reading Group

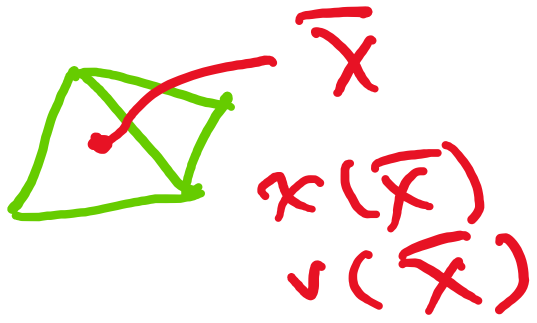
Seminar Room in BA5166 (Dynamic Graphics Project)

Wednesdays 11am

$$N(\bar X) = \left(\lambda_0(\bar X) I,\ \lambda_1(\bar X) I,\ \lambda_2 I,\ \lambda_3(X) I\right)$$

$I \in \mathbb{R}^{3\times 3}$, identity

$$N \in \mathbb{R}^{3\times 12}$$



$$x(\bar X) = N(\bar X)\, q, \quad q = \begin{pmatrix} x_0 \\ \vdots \\ x_3 \end{pmatrix}$$

$$v(\bar X) = \frac{d}{dt} x = N(\bar X)\, \dot q$$

Potential Energy

Neohookean material model measures ”strain energy density” as a function of the deformation gradient

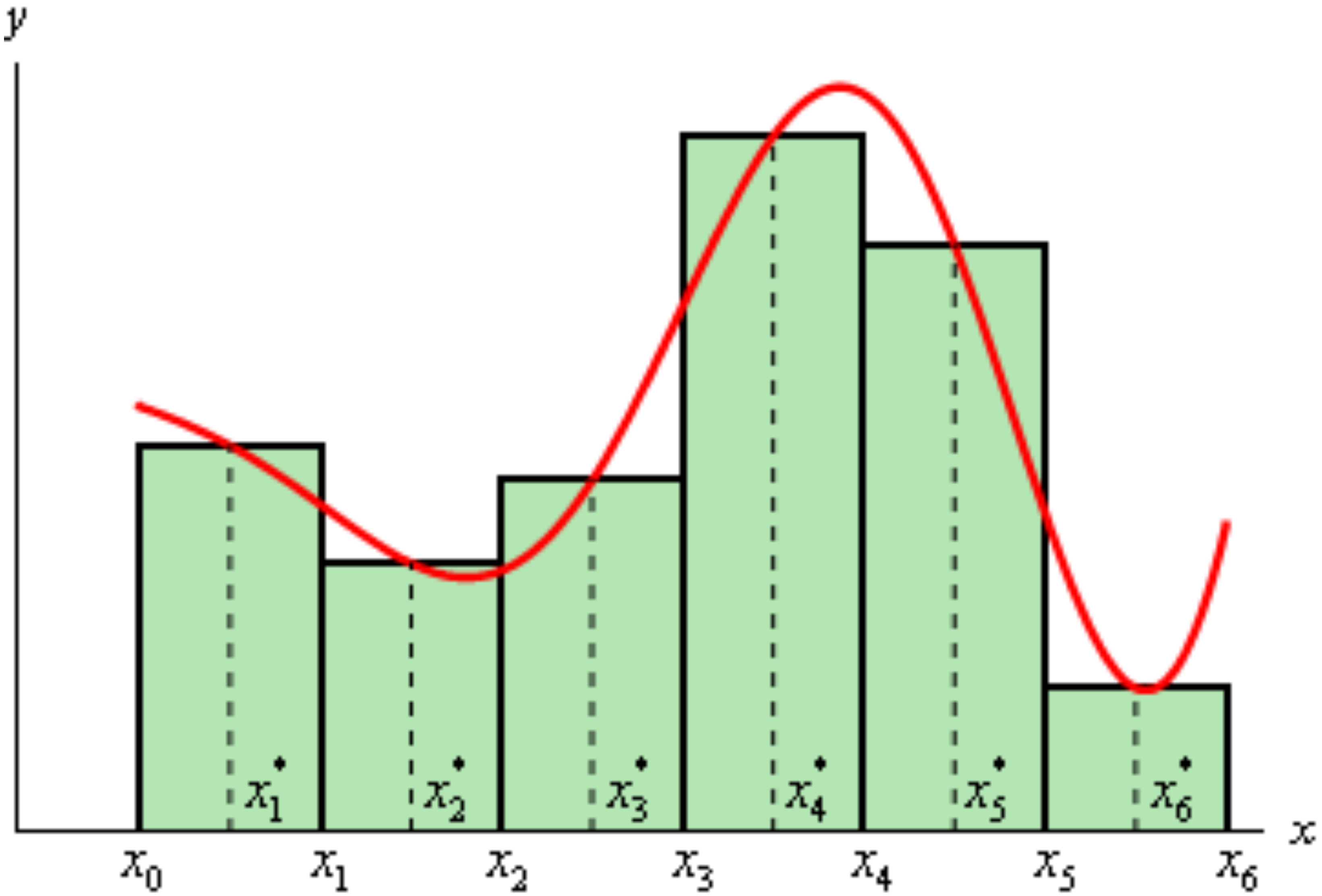
Strain Energy Density is the strain energy *per unit volume*

We need to integrate to to get the actual potential energy

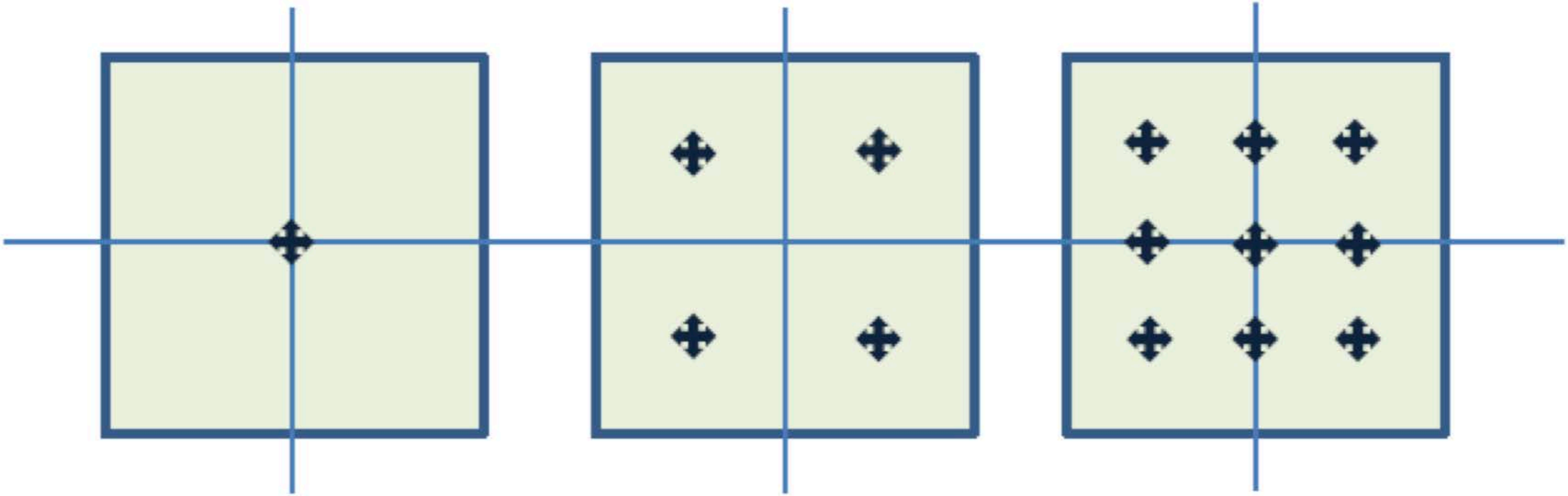
$$NH(F(\bar{x})) \rightarrow E \in \mathbb{R}$$

$$\int_{\text{Tetrahedron}} \Psi(F(\bar{x}))\, d\bar{x} = U = \text{potential energy}$$

Quadrature



Quadrature



Quadrature

$$\int_{\Omega} F(\bar{x})\, d\bar{x} = \sum_{i=0}^{n} F(\bar{x}_i)\, \omega_i$$

weight

quadrature point

OR $F(\bar{x}_{\text{ANYWHERE}})$ volume (tetrahedra)

Potential Energy

$$V_{Tet} = \Psi(F(\bar{x}_{ANY}))\,\mathrm{vol}(\mathrm{tetrahedra})$$

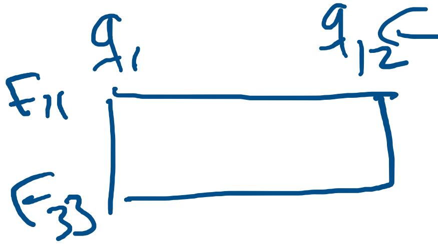
Forces

$$\text{force} = -\frac{\partial U}{\partial q} = -\frac{\partial \psi(F(q))}{\partial q}\,\mathrm{vol}(tet)$$

$$\text{forces} = \frac{\partial F}{\partial q}^T \frac{\partial \psi}{\partial F}\,\mathrm{vol}(tet)$$

J

B^T



$$F \in \mathbb{R}^{3\times 3}$$

$$F = [F_{11}\ F_{12}\ F_{13} \cdots F_{31}\ F_{32}\ F_{33}]$$

The Stiffness Matrix

$$K = -B^T \frac{\partial^2 V}{\partial F^2} B \,\mathrm{vol}(tet)$$

Assemble f, K and do things

Equations of Motion for 3D Finite Elements

$$M\ddot{q} = -\frac{\partial V}{\partial q} \quad , \quad V \in \mathbb{R}, \; q \in \mathbb{R}^n$$

Implicit Euler

Linearly Implicit Euler

$$(M - \Delta t^2 K)\,\dot{q}^{t+1} = M\dot{q}^t + \Delta t F(q^t)$$

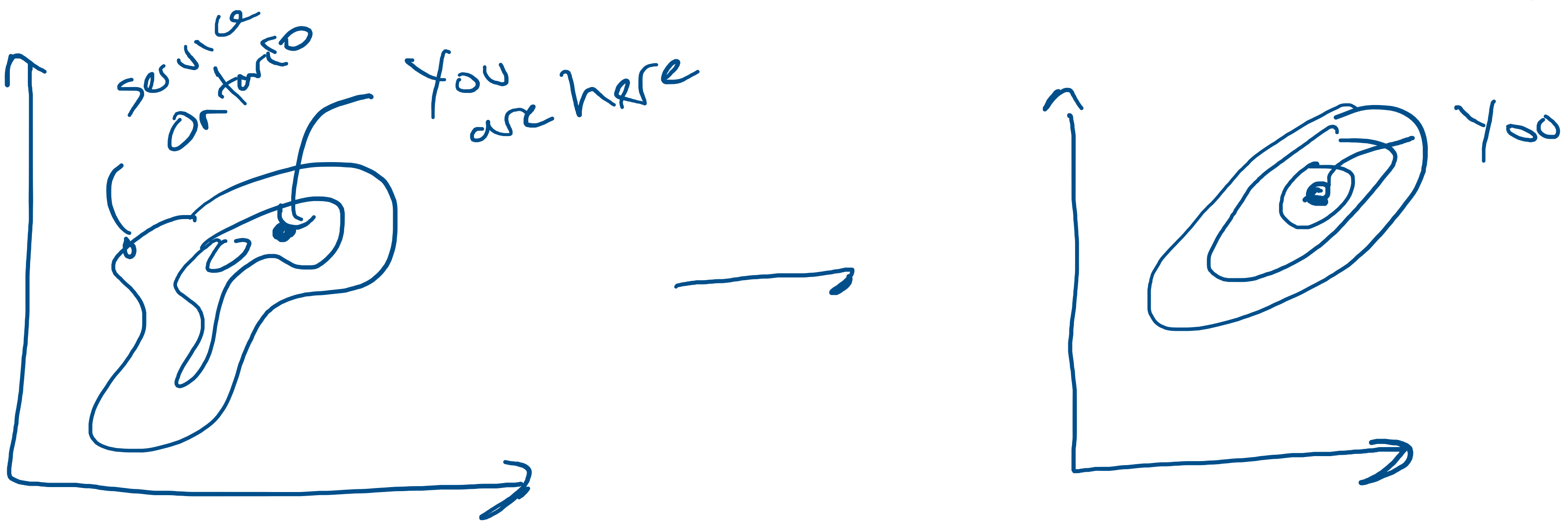
Fully Implicit Euler

$$M\ddot{q} = F(q^{t+1})$$

$$M\dot{q}^{t+1} - M\dot{q}^t + \Delta t\, F(q^t + \Delta t\, \dot{q}^{t+1}) = 0$$

Backward Euler using Optimization

$$\dot{q}^{t+1} = \underset{v}{\arg\min}\ \underbrace{\frac{1}{2}\left(v-\dot{q}^{t}\right)^{T} M \left(v-\dot{q}^{t}\right) + V\left(q^{t}+\Delta t\, v\right)}_{E(\dot{q}^{t+1})}$$



$$\dot{q}^{t+1} = \dot{q}^{t} + \Delta v$$

$$\Delta v^{*} = \underset{\Delta v}{\operatorname{argmin}} \; \frac{1}{2} \Delta v^{T} \underline{H} \Delta v + \Delta v^{T} \underline{g}$$

hessian gradient

$$\Delta v^{*} = -H^{-1} g \quad (1)$$

$$\dot{q}^{t+1}_{(k)} = \dot{q}^{t+1}_{(k-1)} + \Delta v \quad (2)$$

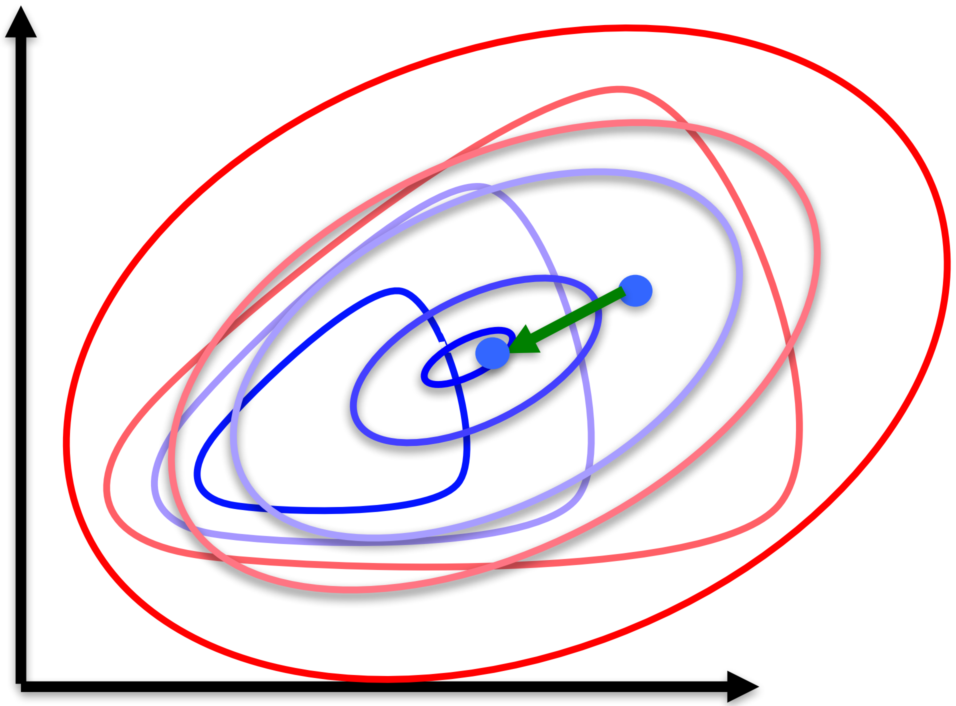
$$q^{t+1}_{(k)} = q^{t} + \Delta t \, \dot{q}^{t+1}_{(k)} \quad (3)$$

$$g = M\dot{q}^{t+1}_{(k)} - M\dot{q}^{t} + \Delta t \frac{\partial V}{\partial q}\Big|_{q^{t+1}_{(k)}}$$

$$H = \frac{M + \Delta t^2 \frac{\partial^2 V}{\partial q^2}}{L\left(M - \Delta t^2 K\right)}\Bigg|_{q^{t+1}_{(k)}}$$

$$\frac{\partial V}{\partial \dot{q}^{t+1}}\left(q^t + \Delta t\left(\dot{q}^t + \Delta v^{(k)}\right)\right) = \Delta t \frac{\partial V}{\partial q}\left(q^t + \Delta t\,\underbrace{\left(\dot{q}^t + \Delta v^{(k)}\right)}_{\dot{q}^{t+1}_{(k)}}\right)$$

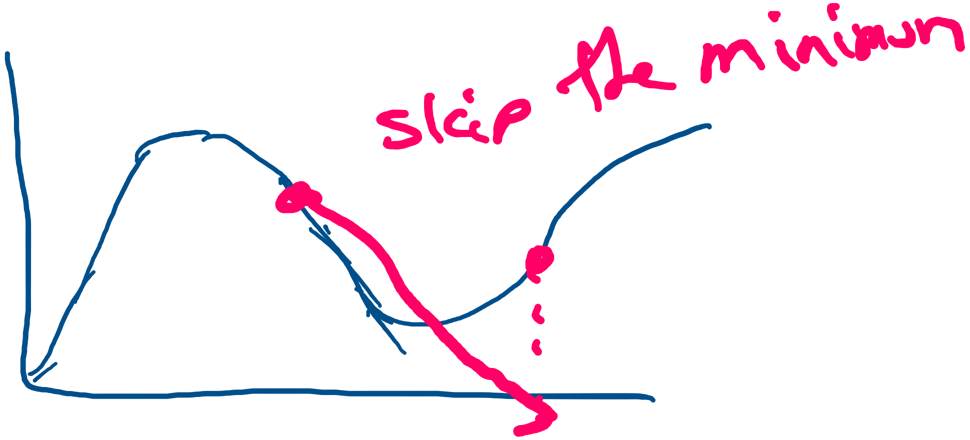
Newton's Method



Newton's Method

$$\dot{q}^{t+1} = \dot{q}^{t+1}_{c(k)} + \Delta V$$

Landscape

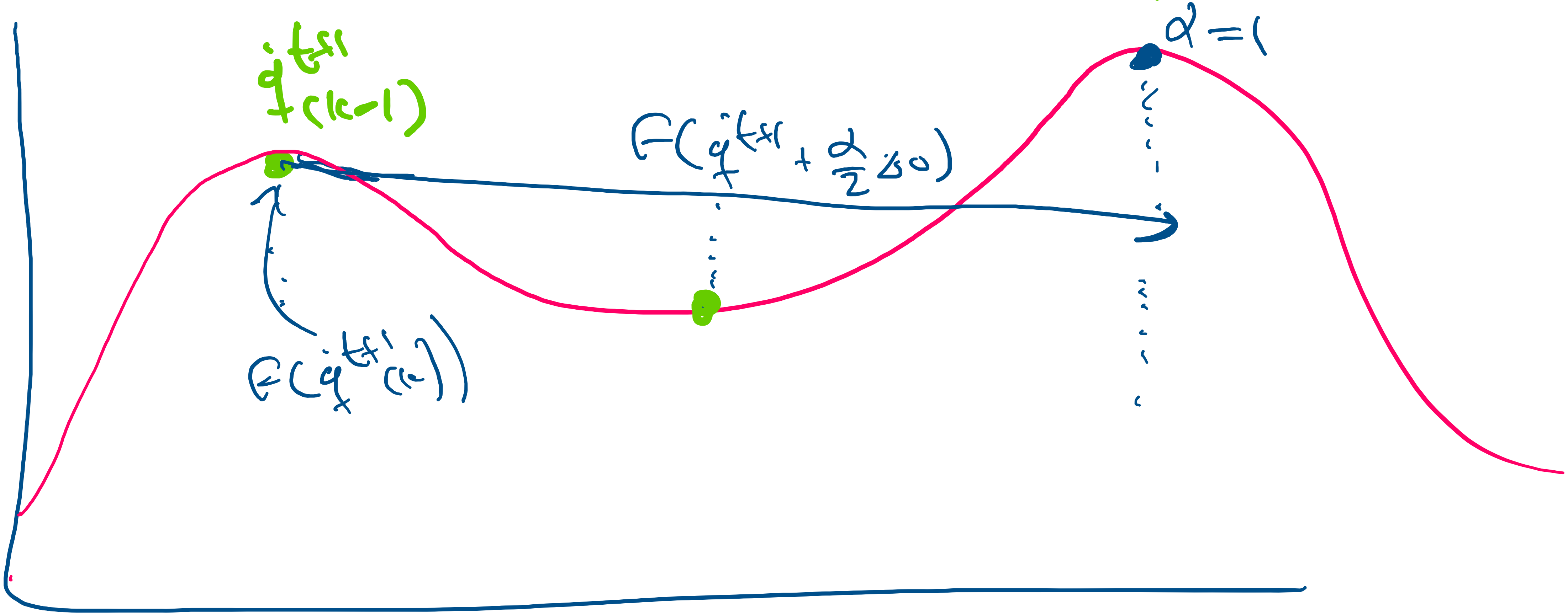


$$\dot{q}^{t+1} = \dot{q}^{t+1}_{c(k)} + \underbrace{\alpha}_{\text{step size}} \Delta V$$

LINE SEARCH

Backtracking Line Search

1.) $F(\dot{q}^{t+1}_{(k-1)} + \alpha \Delta v) < F(\dot{q}^{t+1}_{(k-1)})$



Sufficient decrease: $F(\dot{q}^{t+1} + \alpha \Delta v) < F(\dot{q}^{t+1}_{(k)}) + \underbrace{\beta}_{1e-8} g^T \Delta v$

Newton's Method /w Line Search

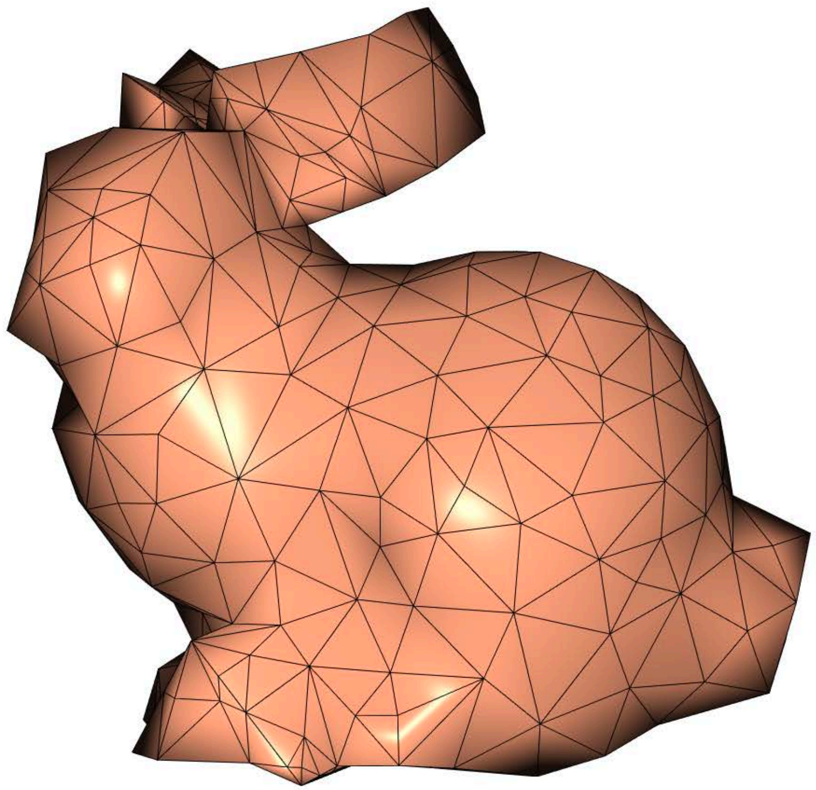
$$\Delta v = -H^{-1} g$$

α using linesearch

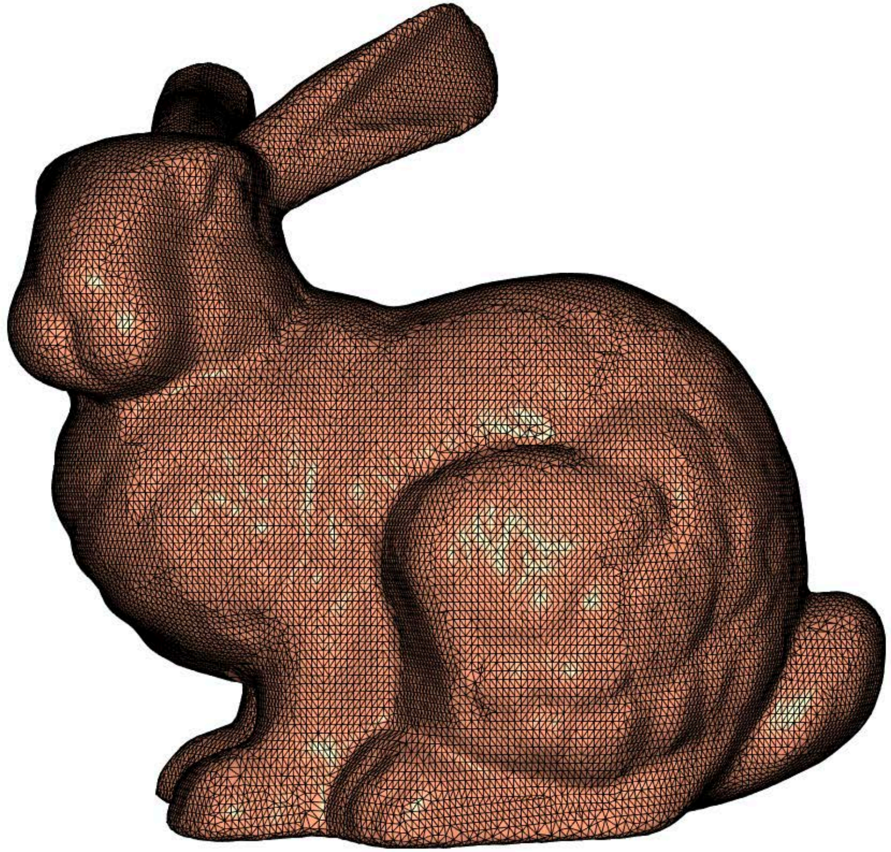
$$\dot{q}^{t+1}_{(k)} = \dot{q}^{t+1}_{(k-1)} + \alpha \Delta v$$

$$q^{t+1}_{(k)} = q^{t} + \Delta t \, \dot{q}^{t+1}_{(k)}$$

Skinning Simulations



Simulation Tetrahedral Mesh



Visualization Triangle Mesh

Skinning Simulations

#q 1

|Nvert|

$x_1 \; x_2 \, x_3 \ldots$

q