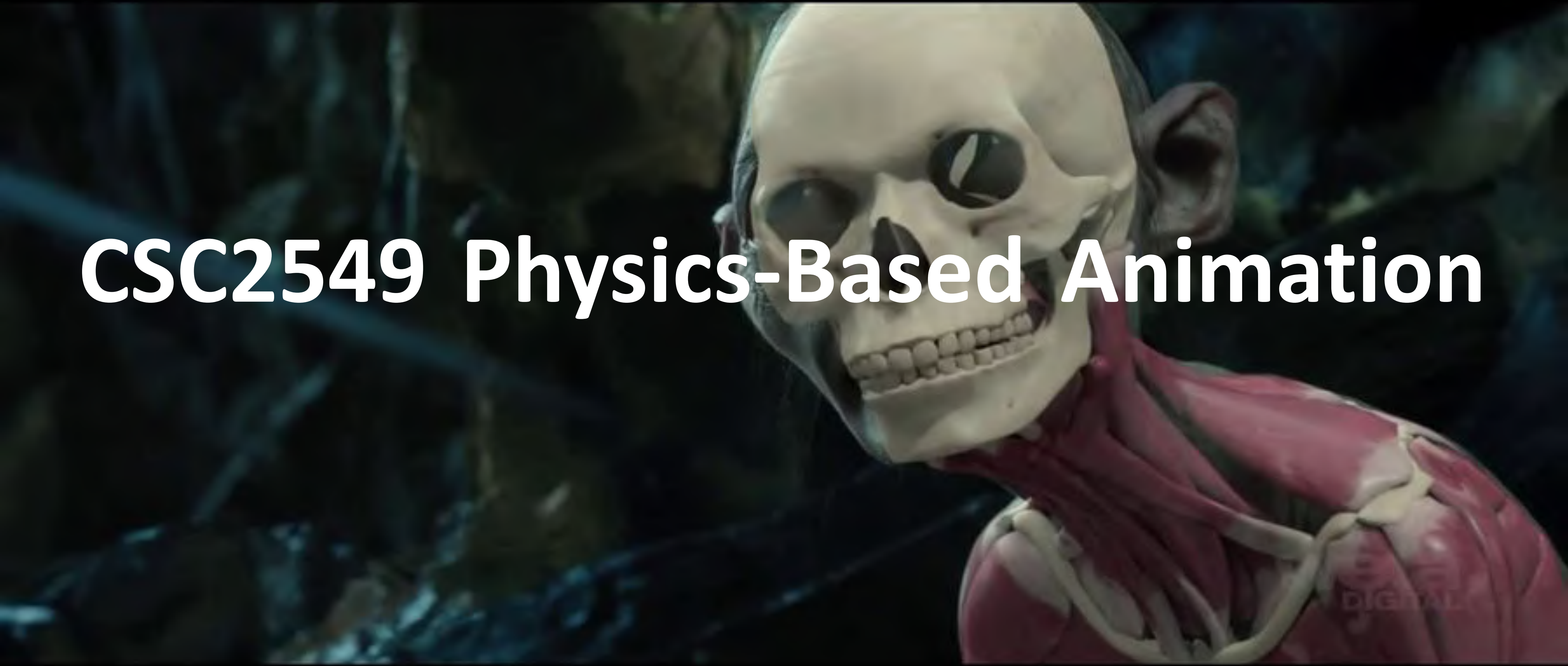
CSC2549 Physics-Based Animation



Lord of the Rings | Weta Digital

Today: Introduction and Springs

CSC2549 Physics-Based Animation

Course web site (includes course information sheet):

https://github.com/dilevin/CSC2549-physics-based-animation

Instructor:

Prof. David I.W. Levin diwlevin@cs.toronto.edu

TA:

Derek Liu

Office Hours:

Dave – Wednesday 5-6pm BA5268

CSC2549 Physics-Based Animation

Discussion Board is COMING SOON

Assignments will be submitted via MarkUs, also COMING SOON ☺

Week	Topic / Event
1	Introduction, the 1D mass-spring system, Assignment 1 (1D mass-springs) due 27/09
2	Explicit and implicit time integration
3	Mass-spring systems in three dimensions, Assignment 2 (3D mass-springs) due 04/10
4	Finite Elements for simulating nonlinear elastodynamics of solids, Assignment 3 (3D FEM) due 11/10
5	Finite Elements for simulating cloth and shells, Assignment 4 (Cloth simulation) due 18/10
6	Fluid simulation using Finite Volume Methods
7	Rigid body mechanics, Assignment 5 (Rigid body simulation) due 01/11
October 28	Drop date (consider if grade so far is <50%)
8	Jointed Rigid Body Systems
9	Collision detection and contact resolution, Assignment 6 (Rigid body collision resolution) due 08/11
10	Fast algorithms for physics-based animation
11	Special Lecture
12	Final Project Presentations

Academic Honesty Policy

It’s on the webpage and is mandatory reading!

Administrivia

Grading:

%	Item
60%	Assignments
30%	Final Project
10%	Class Participation

Today

1. Introduction to Physics-Based Animation
2. Variational Mechanics
3. Mass-Spring System in 1D
4. Preview Assignment 1

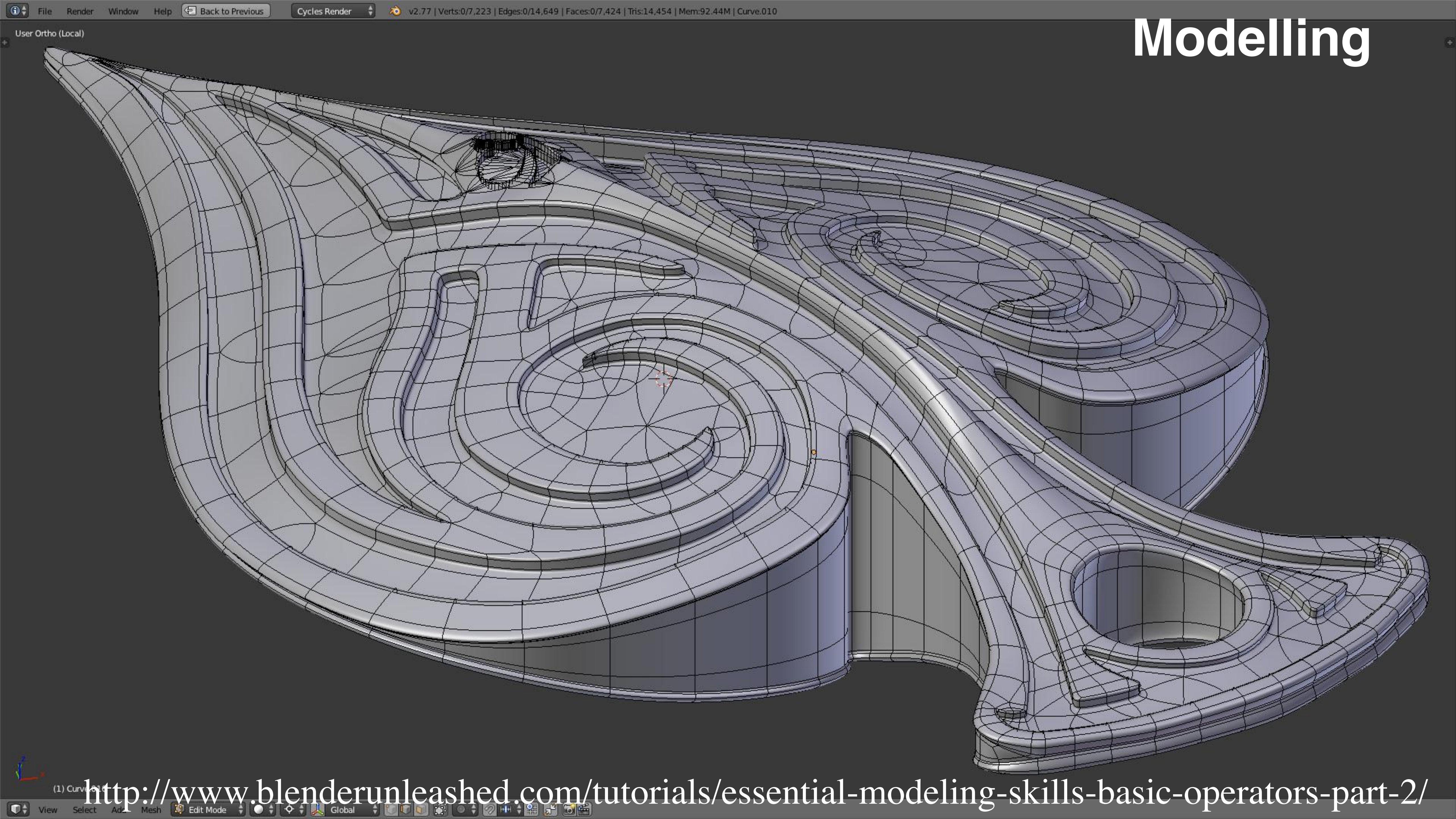
“Core” Areas of Computer Graphics

Modeling

Rendering

Animation

Modelling

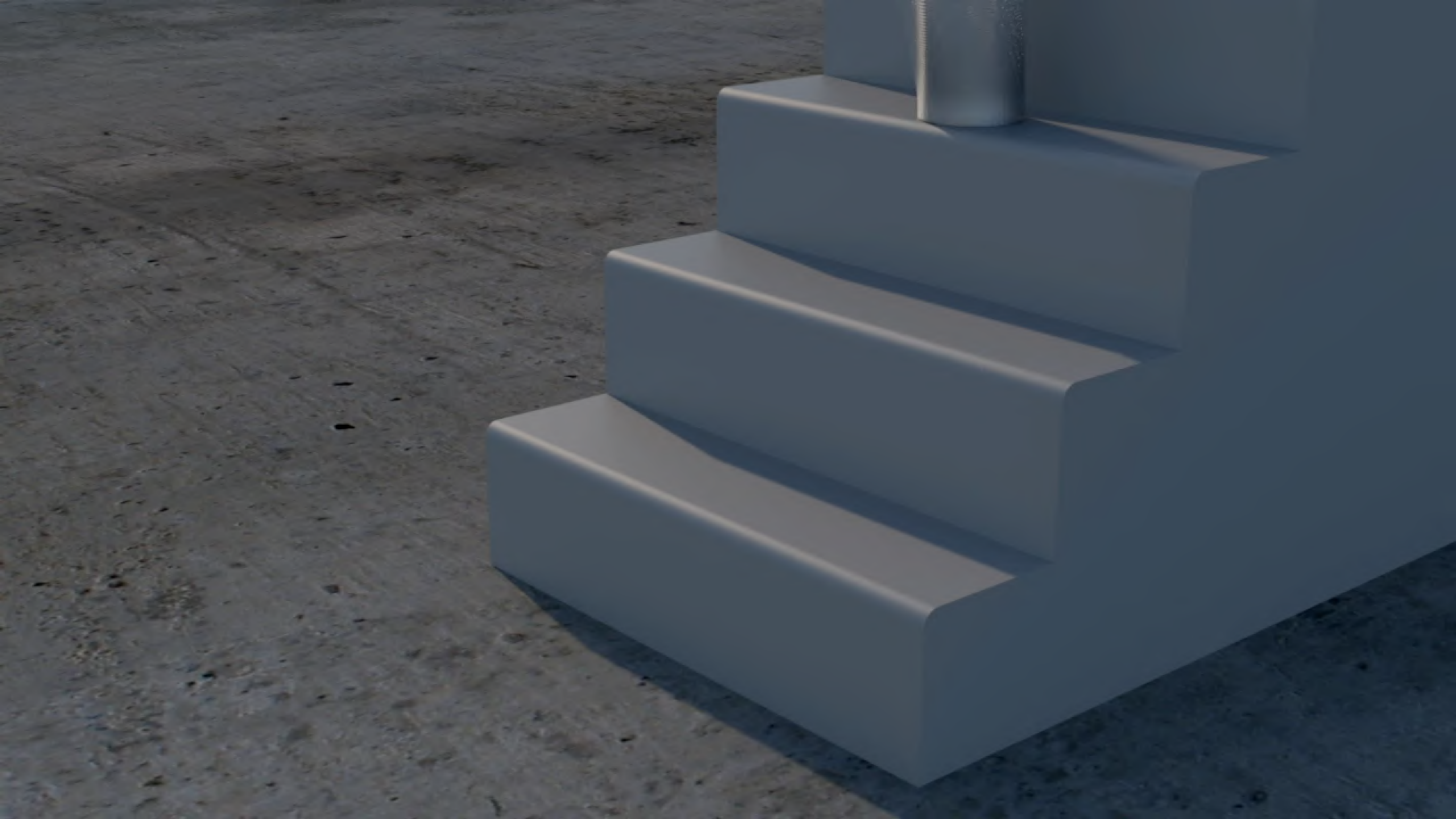


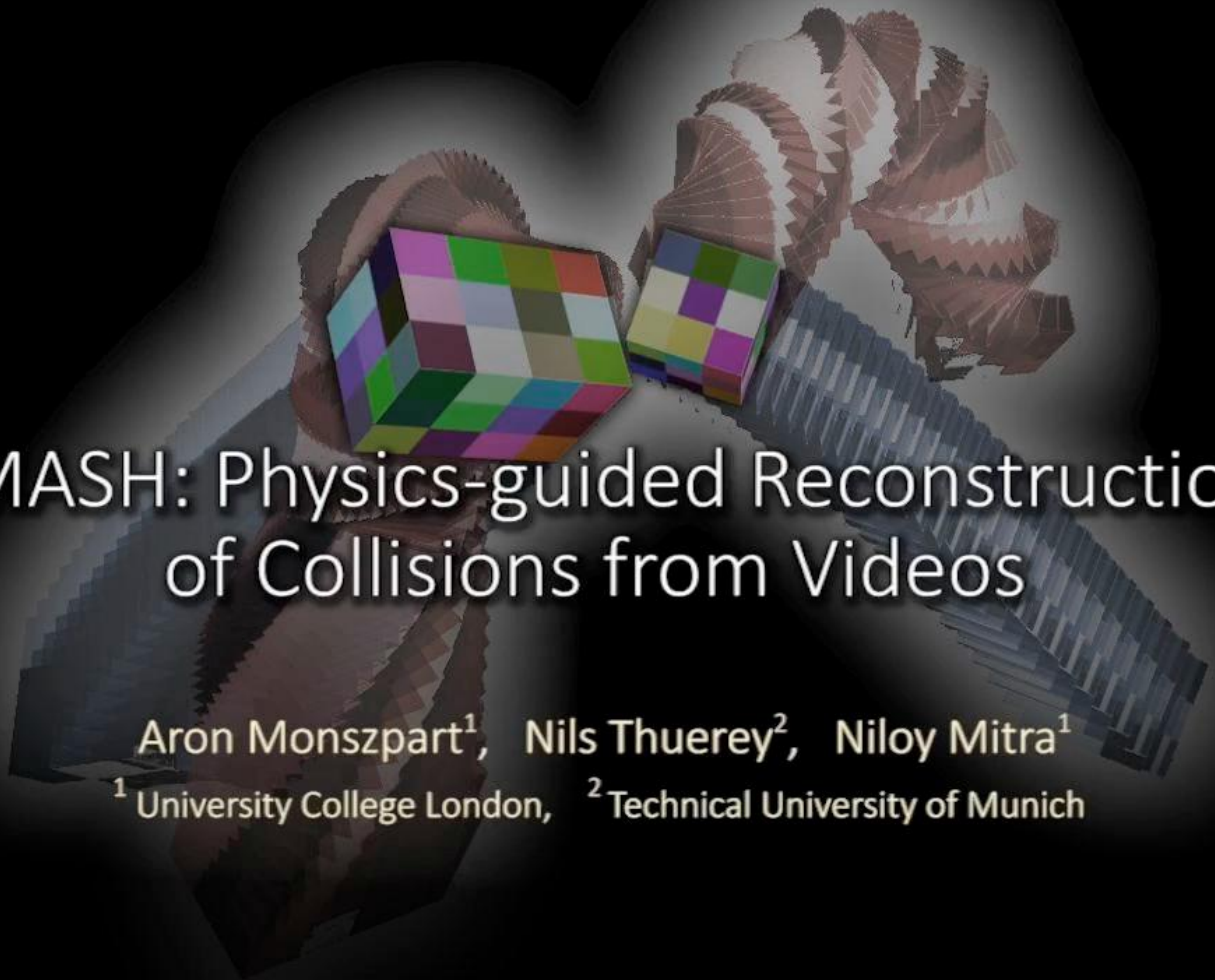
http://www.blenderunleashed.com/tutorials/essential-modeling-skills-basic-operators-part-2/


Rendering

Animation


skeleton
Image courtesy of Weta Digital





SMASH: Physics-guided Reconstruction of Collisions from Videos

Aron Monszpart[1], Nils Thuerey[2], Niloy Mitra[1]

[1] University College London, [2] Technical University of Munich

Reasons you might be taking this course

1. Just curious

2. You might want to use physics simulation in your research

3. You want to do research in physics simulation

Newton’s Laws

1. Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force

2. The force acting on an object is equal to the time rate-of-change of the momentum

3. For every action there is an equal and opposite reaction

Newton's Laws

Momentum $= m\vec{v}$

$$\frac{d}{dt}(m\vec{v}) = m\dot{\vec{v}} = \underline{\underline{m\vec{a} = \vec{F}}}$$

Vectorial Mechanics

Variational Mechanics

Also called ”Analytical Mechanics”

Based on two fundamental energies rather than two vectorial quantities

Kinetic and Potential Energy

Kinetic Energy: Energy due to motion

Potential Energy: Energy “held within” an object due to its position, internal stresses, electrical charge etc …

Potential energy has the potential to become kinetic energy

Variational Mechanics

Also called "Analytical Mechanics"

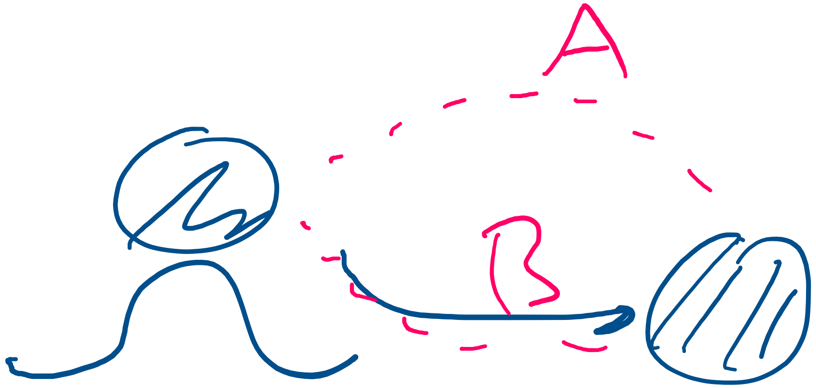
Based on two fundamental energies rather than two vectorial quantities

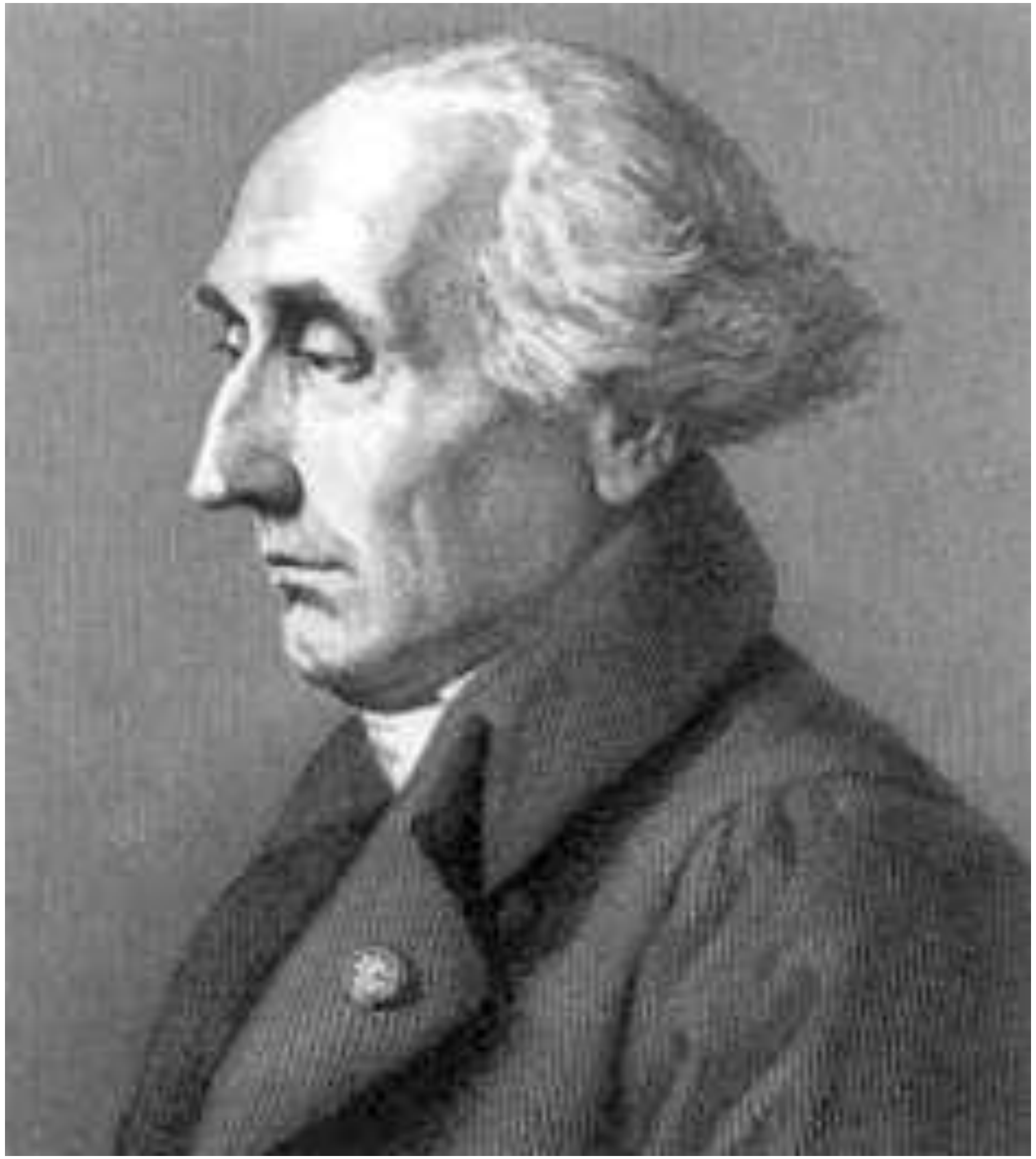
Motion chosen via finding a stationary point of a variational principle

$E(f(t)) \to \mathbb{R}$

The Lagrangian

$$L = \underbrace{T}_{\text{kinetic}} - \overbrace{V}^{\text{potential}}$$

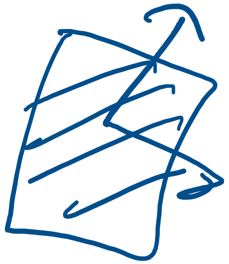




Generalized Coordinates

generalized coordinates

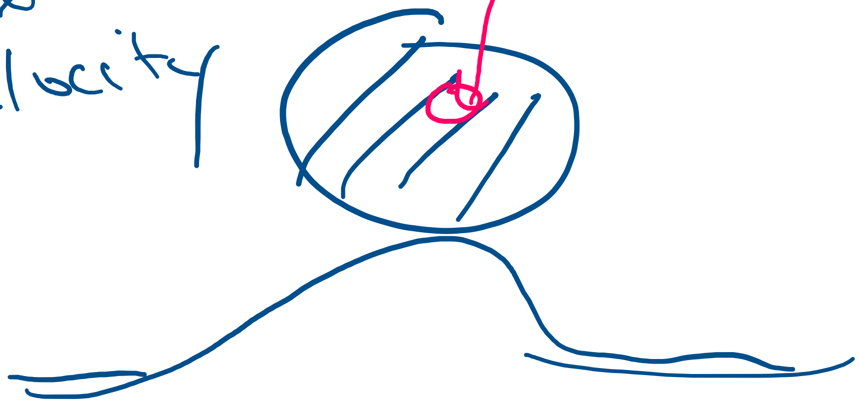
$$f(q) \longrightarrow x(t)$$



R - rotation

P - position

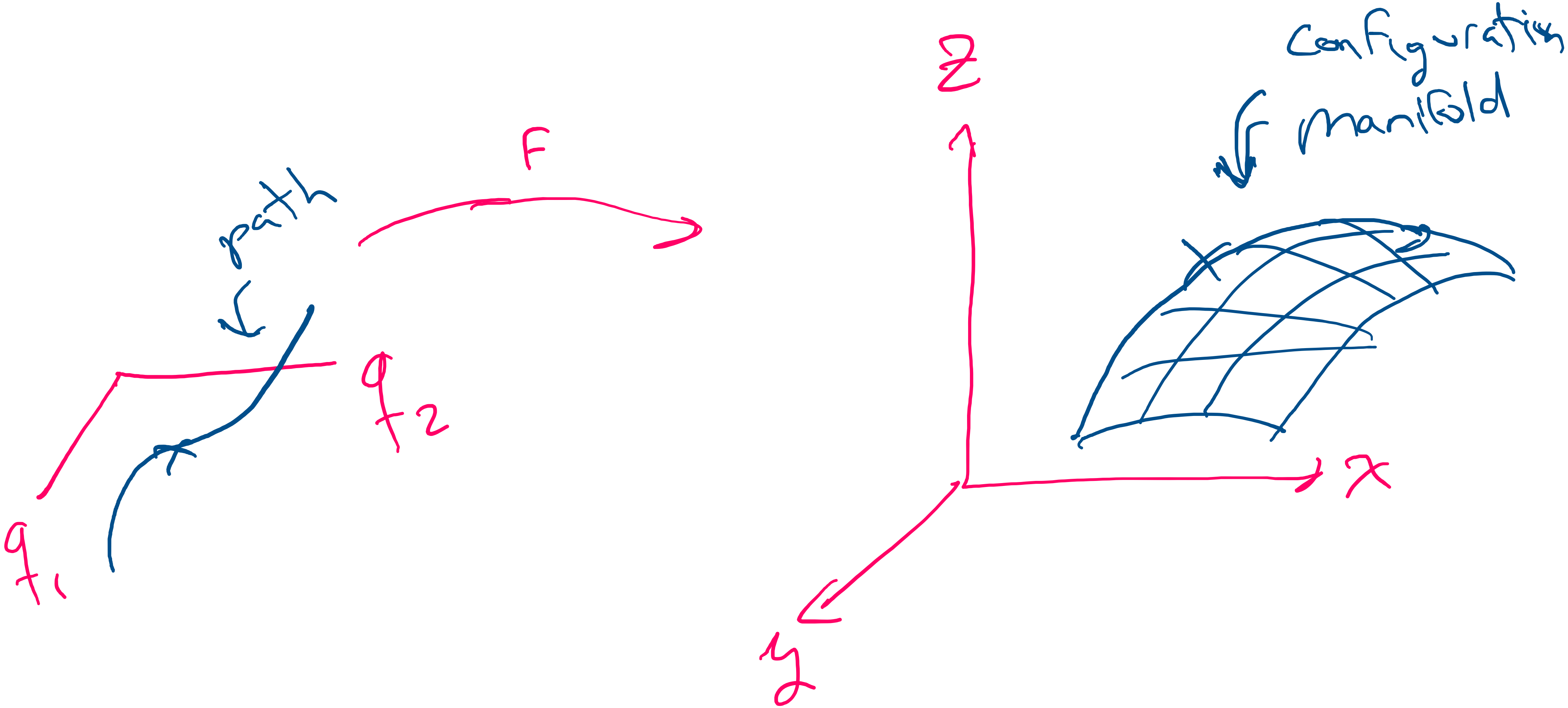
x(t)



generalized velocity

$$\vec{v} = \frac{d}{dt}x(t) = \frac{\partial f}{\partial q}\dot{q}$$

Jacobian (J)


path
q_1
q_2
F
z
x
y
Configuration manifold

The Principle of Least Action

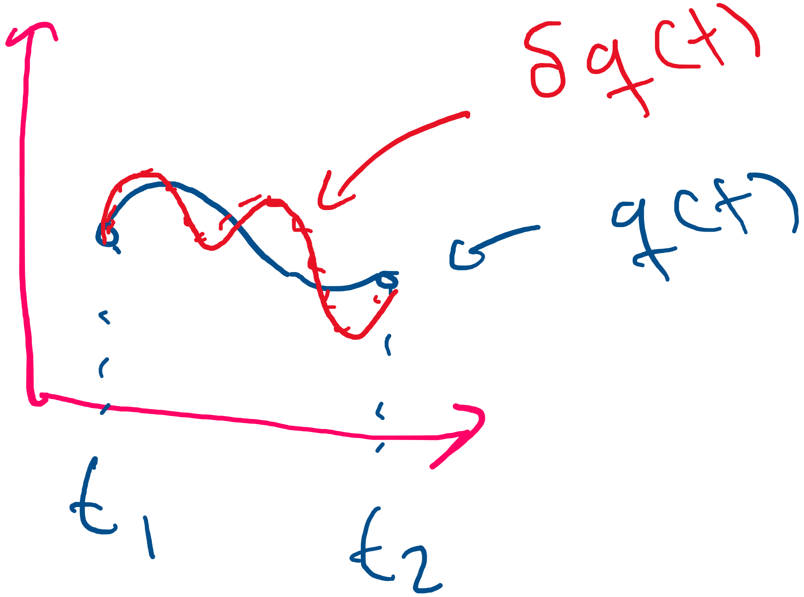
$$S = \int_{t_0}^{t_1} T(q, \dot{q}) - V(q, \dot{q})\, dt$$

Action

Given $q(t_0)$, $q(t_1)$ $\Rightarrow$

First variation $(\delta S = 0)$

Calculus of Variations



Finding a Stationary Point

$$\delta S = 0$$

$$S(q+\delta q, \dot{q}+\delta\dot{q}) = \int_{t_0}^{t_1} L(q+\delta q, \dot{q}+\delta\dot{q})\, dt$$

$$= \int_{t_0}^{t_1} \underbrace{L(q,\dot{q})}_{S} + \underbrace{\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta\dot{q}}_{\delta S}\, dt$$

$$= S + \delta S$$

$$\delta S = \int_{t_0}^{t_1} \frac{\partial L}{\partial q}\delta q + \underline{\frac{\partial L}{\partial \dot q}\delta \dot q}\, dt = 0$$

$$\underbrace{\int \left(\frac{\partial L}{\partial \dot q}\delta q\right)}_{\cancel{\left.\frac{\partial L}{\partial \dot q}\delta q\right|_{t_0}^{t_1}}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\delta q = \underline{\underline{\frac{\partial L}{\partial \dot q}\delta \dot q \, dt}}$$

$\delta q(t_0),\ \delta q(t_1) = 0$

$$\int_{t_0}^{t_1} \frac{\partial L}{\partial q}\,\underline{\delta q} - \frac{d}{dt}\frac{\partial L}{\partial \dot q}\,\underline{\delta q} = 0$$

Good-bye

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q} = \frac{\partial L}{\partial q}$$

Euler-Lagrange Equation

Euler-Lagrange Equations

Why do we care ?

Unifying principle!

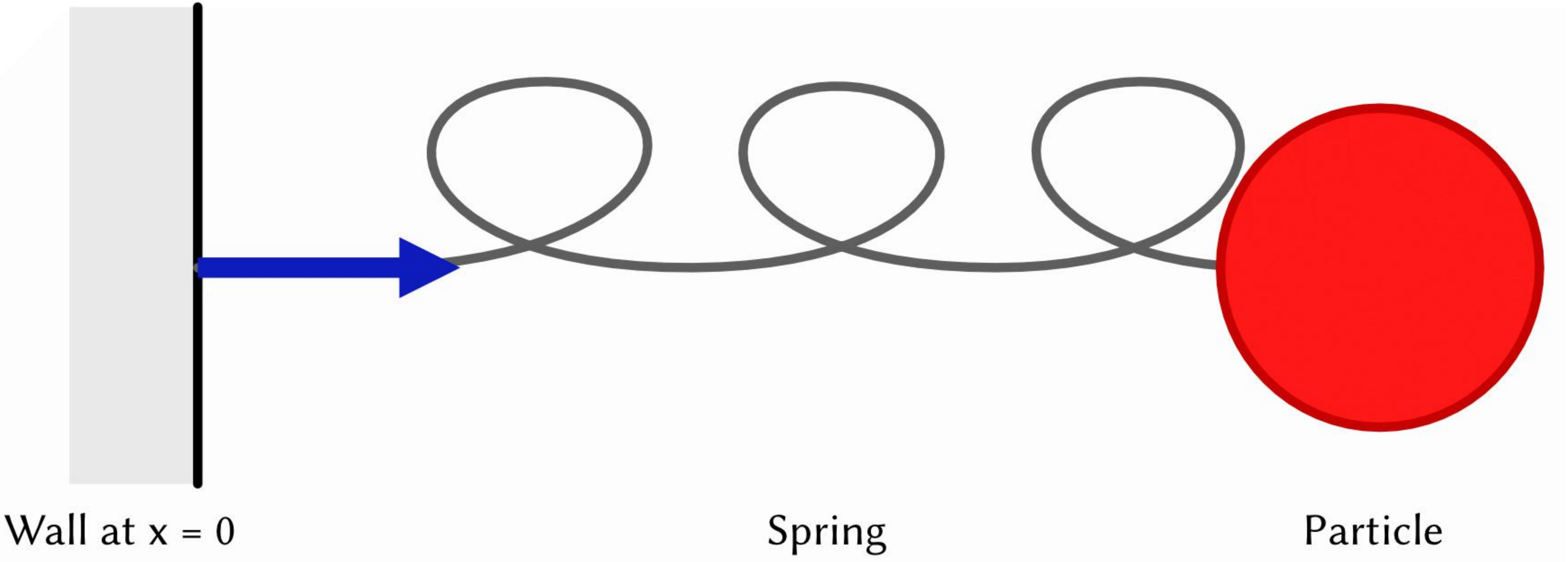
Can derive equations of motion for *more* than just particles

Deformable Objects

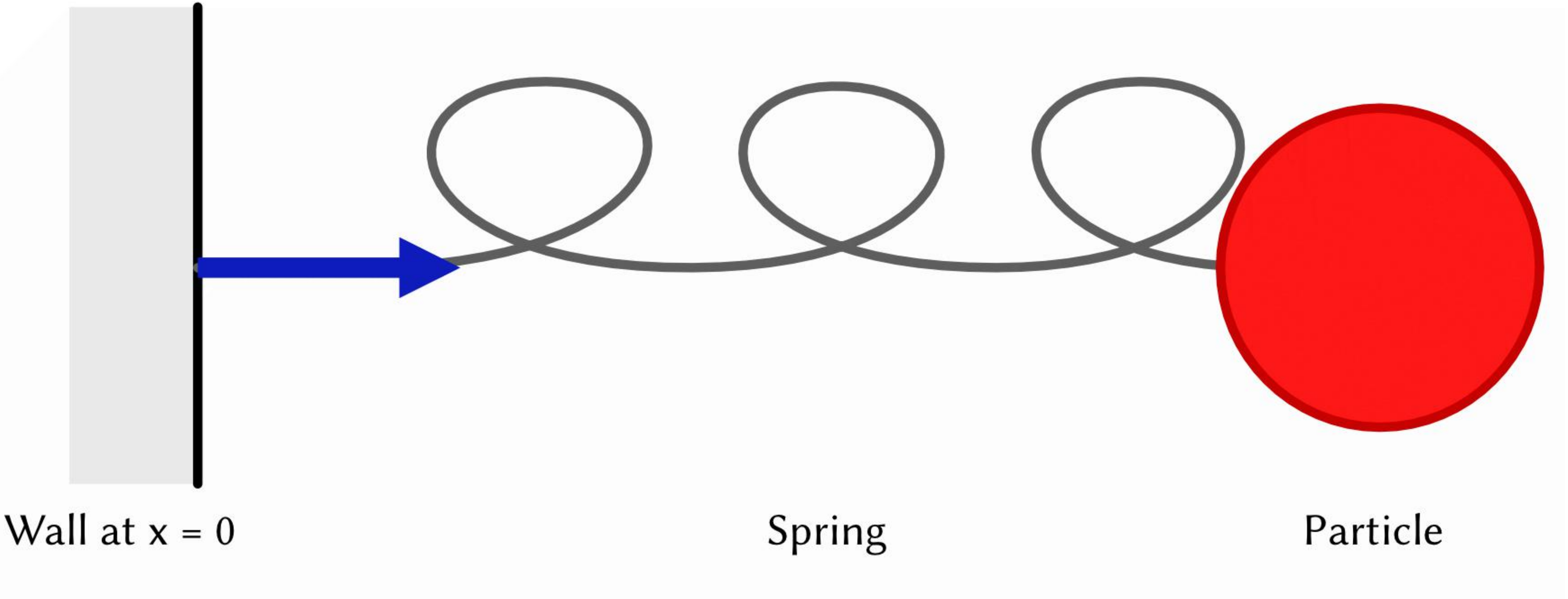
Fluids

Rigid Bodies and More !

Mass-Spring Systems in 1D



Choosing Generalized Coordinates



Generalized Coordinates for Mass Spring System

$$q = x$$

$$\dot{q} = \dot{x} = v$$

Kinetic Energy for Mass-Spring System

$$T = \frac{1}{2} m v^2$$

Yay!!

Potential Energy for Mass-Spring System

Potential Energy is the negative Work done on the system

Work is defined as the product of force and displacement

Potential Energy from a Spring

$$V = \frac{1}{2} k x^2$$

Hooke's Law $\quad F = -kx$

$$W = -\int_{t_0}^{t_1} k x v \, dt \implies V = -W = \frac{1}{2} k x^2$$

Stuff Everything into the Euler Lagrange Equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$$

$$L = T - V$$

$$T = \frac{1}{2}mv^2$$

$$V = \frac{1}{2}kx^2$$

$$q = x, \quad \dot{q} = \dot{x} = v$$

$$\frac{d}{dt}(mv) = \underbrace{-kx}_{\text{Force}}$$

$$ma = -kx$$

generalized Force $= -\dfrac{\partial V}{\partial q}$

Final Equations of Motion

$$ma = -kx$$

$$m\ddot{x} = -kx$$

2nd order - ODE

Next Week: Time Integration

Demo Assignment 1

Finishing Up

Office hours are now!

Variational Stokes:
A Unified Pressure-Viscosity Solver for Accurate Viscous Liquids

Egor Larionov* — UNIVERSITY OF WATERLOO

Christopher Batty* — UNIVERSITY OF WATERLOO

Robert Bridson — AUTODESK

* joint first authors