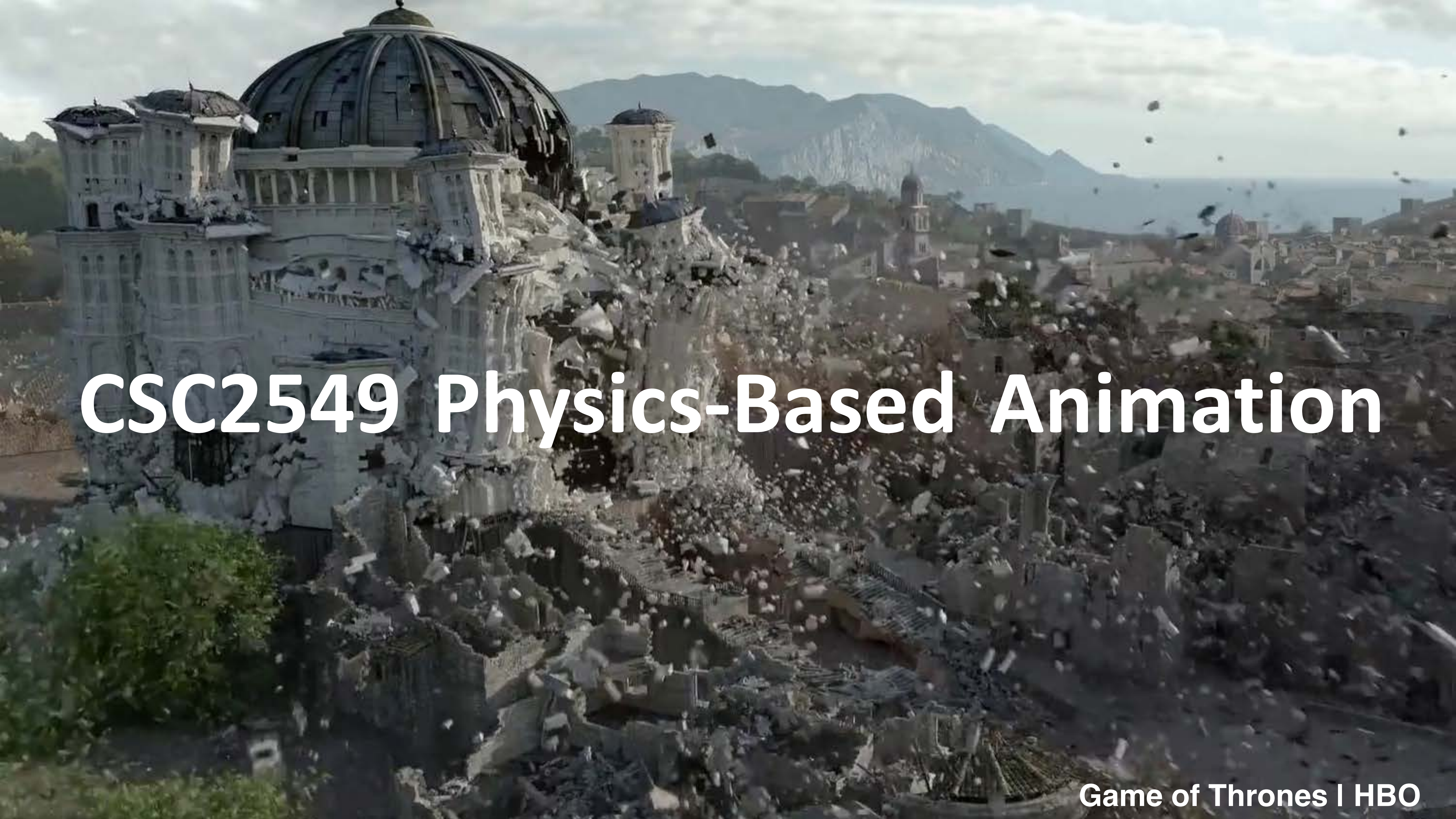
CSC2549 Physics-Based Animation



Game of Thrones | HBO

Reminders

Assignment #4 is due Friday

https://github.com/dilevin/CSC2549-a4-cloth-simulation

Assignment #5 will be live tomorrow, due November 1st

Graphics Reading Group

Seminar Room in BA5166 (Dynamic Graphics Project)

Wednesdays 11am

Let’s Talk about Final Projects

30% of your final grade

Two components

- Presentation – 15% of the mark
- Due date: last class

- Write up in SIGGRAPH style - 15% of the mark
- Due date: 1 week before the end of the exam period

Final Presentation Guide

Duration: 5-6 minutes with 2-3 minutes for question

Content:

Problem Statement (1 Slide)

Related Work (1-2 Slides)

Methodology (as many slides as you need)

Results or Anticipated Results (at least one slide)

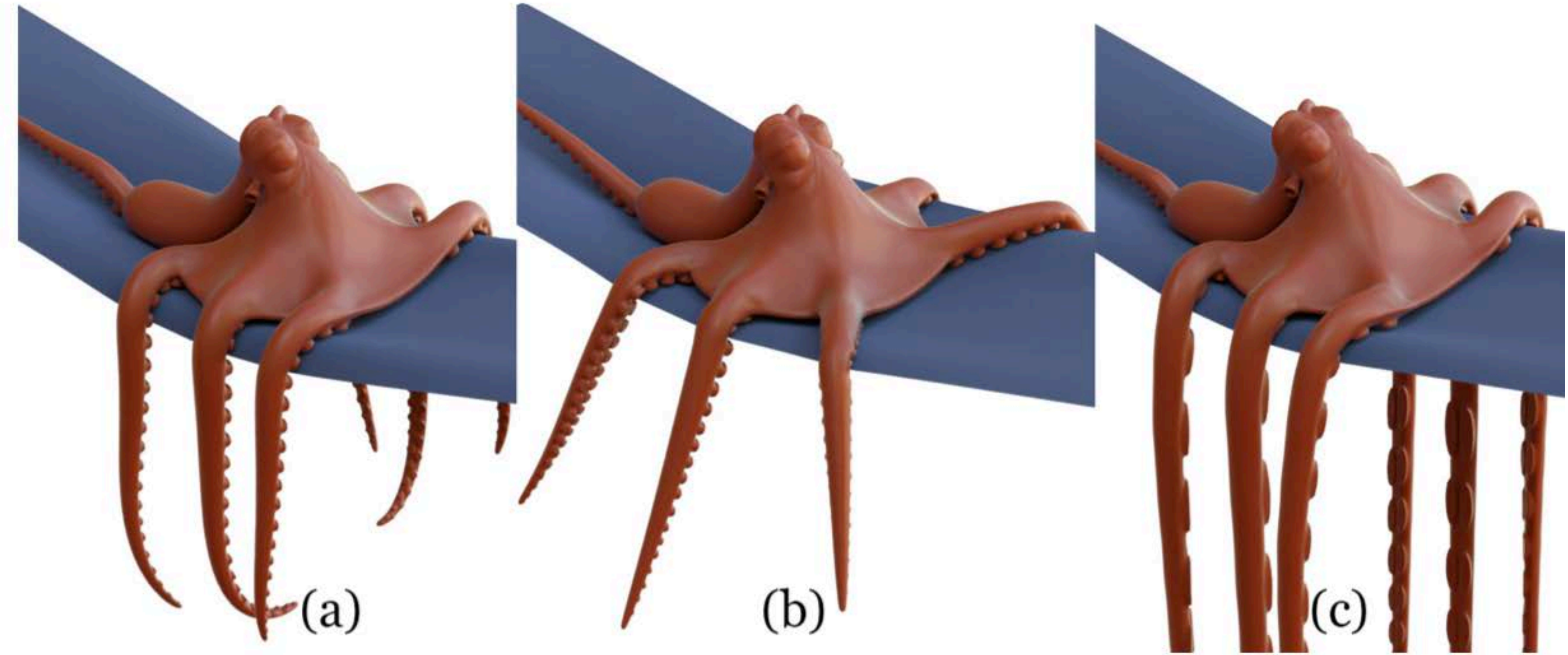
Let’s Talk about Final Projects

If you don’t have a project idea, or haven’t checked with me about your current plan, please do so.

How does everyone feel about the assignment pace …

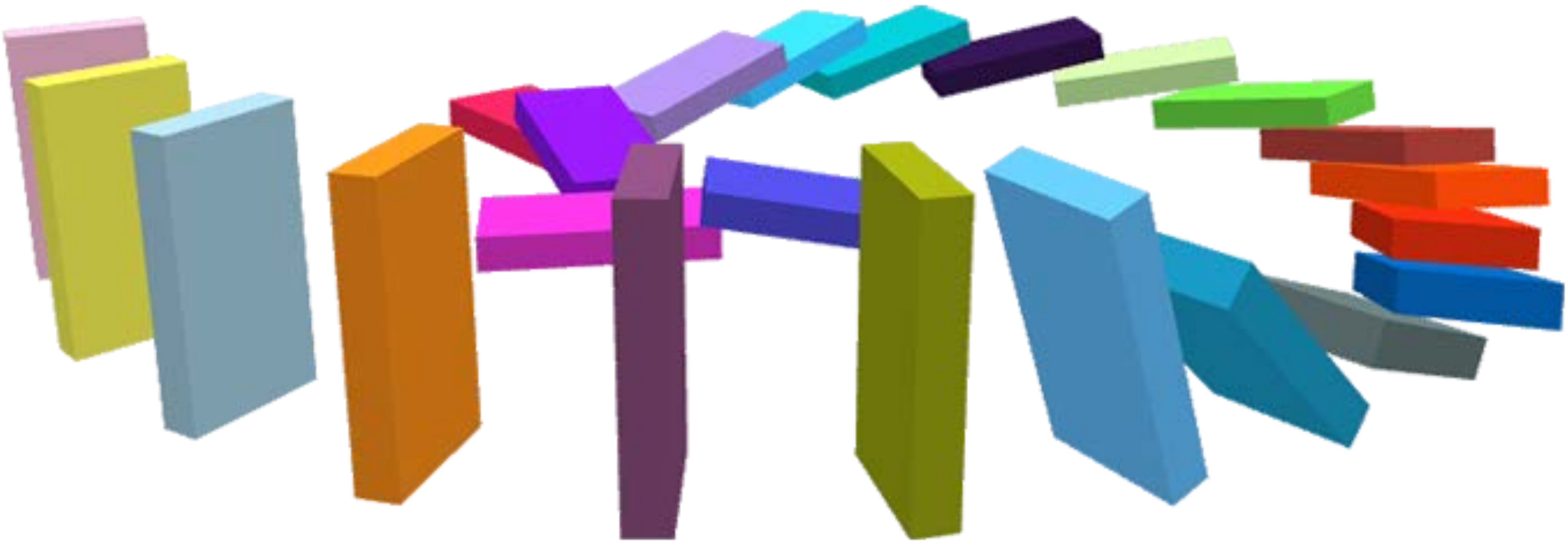
… for the remainder of the class ?

Questions

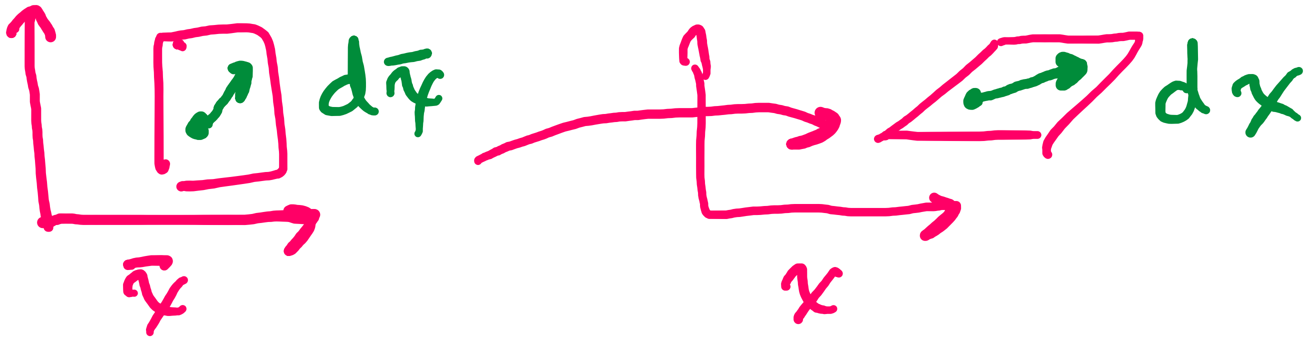


Kim et al. | Anisotropic Elasticity for Inversion-Safety and Element Rehabilitation

Generalized Coordinates for Rigid Bodies



What makes an object rigid ?



$$|dx|^2 = d\bar{x}^T F^T F d\bar{x}$$

Rigid if: $|dx|^2 = |d\bar{x}|^2$

$$dx = F d\bar{x}$$

What does this imply?

$F^T F = I \rightarrow$ want

$F \in$ orthogonal
$\det(F) = +1$ $\Big\}$ $F \in$ Special orthogonal

$$F = \underbrace{R}_{\text{rotation}}$$

$$F = \frac{\partial x}{\partial \bar{x}} = R \rightarrow \overbrace{x = \underbrace{R}_{\text{rotation}}\bar{x} + \underbrace{p}_{\text{translation}}}^{\text{rigid body transformation}}$$

$$q = \{R, p\}$$

$$R \in \mathbb{R}^{3\times 3}, \quad p \in \mathbb{R}^3$$

Generalized coordinates

Code: 12×1 VectorXd, $[\overset{9}{R} \mid \overset{3}{p}]$

Generalized Velocities

$$x = R\bar{x} + p$$

$$\frac{dx}{dt} = \underbrace{\dot{R}}_{\text{Not Nice}}\bar{x} + \underbrace{\dot{p}}_{\text{NICE}}$$

$$R = \underbrace{\text{expm}}_{\text{matrix exponential}}([r])$$

$r \in \mathbb{R}^3$

$$[\,] \rightarrow \begin{bmatrix} 0 & r_z & -r_y \\ -r_z & 0 & r_x \\ r_y & -r_x & 0 \end{bmatrix}$$

$$\dot{y} = A y$$

$$y(t) = \exp(At)\, y_0$$

$$\dot{x} = \omega \times x$$

$$\dot{x} = [\omega]\, x$$

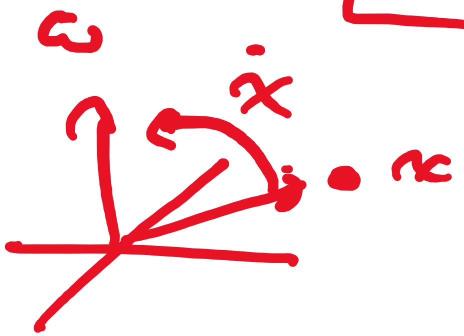
$$x = \underbrace{\text{expm}([\omega]\, t)}_{\text{Rotation}}\, x_0$$

Code:

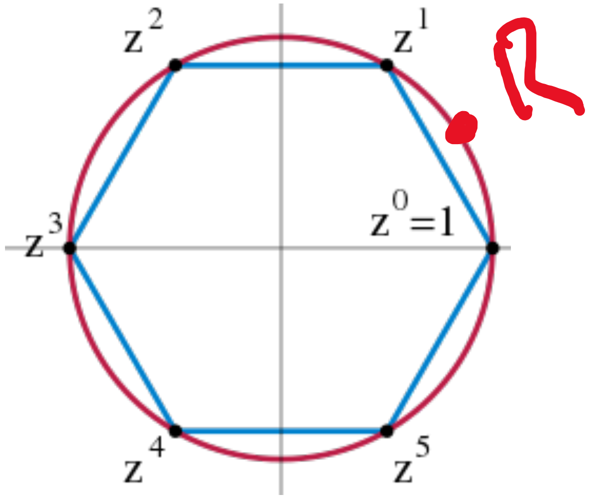
expm(A), NEVER!!!

$$A = V \lambda V^T$$

$$\rightarrow V \exp(\lambda) V^T$$



The Time Derivative of a Rotation Matrix



Parameterizing the group of rotations

$$\dot{R} = \lim_{t \to 0} \underbrace{\frac{d}{dt} \operatorname{expm}([r] + [\omega] t)}_{\text{Differential}}$$

$$\operatorname{expm}([r] + [\omega]t) = \ldots$$

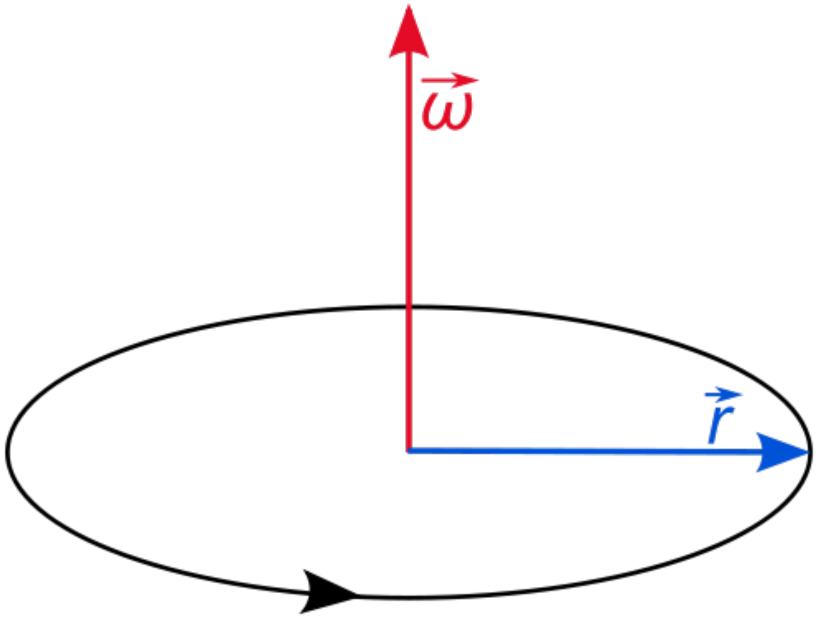
$$\lim_{t \to 0} \frac{d}{dt}\left(I + [r] + [\omega]t + \frac{1}{2}([r] + [\omega]t)^2 \ldots\right)$$

$$= \underbrace{\left(I + [r] + \frac{1}{2}[r][r] + \ldots\right)}_{R}[\omega]$$

$$\dot{R} = R[\omega]$$

(useful)

The Time Derivative of a Rotation Matrix



Angular Velocity Vector

$$\dot{x} = \underbrace{R}_{\text{world space}}\underbrace{[\overbrace{\bar{\omega}}^{\text{undeformed space}}]\bar{x}}_{\text{vel. rotating the space}} + \underbrace{\dot{p}}_{\text{vel. translation}}$$

$$= R[\bar{x}]^T\bar{\omega} + \dot{p}$$

Generalized Velocities

ω - angular velocity WORLD

$\dot{p}$ - linear velocity WORLD

$$\dot{x} = \underbrace{\left(R[\bar{x}]^T R^T \quad I\right)}_{N(\bar{x})}\begin{pmatrix}\omega \\ \dot{p}\end{pmatrix}$$

Kinetic Energy

$$T = \frac{1}{2}\int_{\Omega} \rho \dot{x}^T \dot{x} \, dV$$

$$T = \frac{1}{2}\begin{bmatrix}\omega^T & \dot{p}^T\end{bmatrix} \underbrace{\int_{\Omega} \rho \, N(\bar{x})^T N(\bar{x}) \, dV}_{M} \begin{bmatrix}\omega \\ \dot{p}\end{bmatrix}$$

$$M = \int_{\Omega} \rho \begin{bmatrix} R[\bar{x}]R^T \\ I \end{bmatrix} \begin{bmatrix} R[\bar{x}]R^T & I \end{bmatrix} dV$$

$$\underbrace{\begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix} \underbrace{\int_\Omega \rho \left[\begin{array}{c|c} \overset{A}{[\bar{x}][\bar{x}]^T} & \overset{B}{[\bar{x}]} \\ \hline \underset{B^T}{[\bar{x}]^T} & \underset{C}{I} \end{array}\right] dV}_{M_0} \begin{bmatrix} R^T & 0 \\ 0 & I \end{bmatrix}}_{M}$$

$$C = \int_\Omega \rho I \, dV = m I$$

$$B = \int_\Omega \rho\,[\bar{x}]\,dV, \quad \bar{r} = \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}$$

$$[\bar{x}] = \begin{bmatrix} 0 & \bar{z} & -\bar{y} \\ -\bar{z} & 0 & \bar{x} \\ \bar{y} & -\bar{x} & 0 \end{bmatrix}$$

$$\underbrace{\int_\Omega \rho\,\bar{x}\,dV}_{\text{center-of-mass}\,\cdot\,(\text{origin})}$$

doing this

$$= 0$$

$$M_o = \begin{bmatrix} \int_\Omega \rho[\bar{x}][\bar{x}]^T d\bar{x} & 0 \\ \underbrace{0}_{\text{com origin}} & mI \end{bmatrix}$$

$$\int_\Omega F(\bar{x})\, d\bar{x} = \int_\Omega \nabla \cdot g(\bar{x})\, d\bar{x} \quad \int_{\underbrace{\partial\Omega}_{\text{surface}}} g(\bar{x})^T n \, d\bar{x}$$

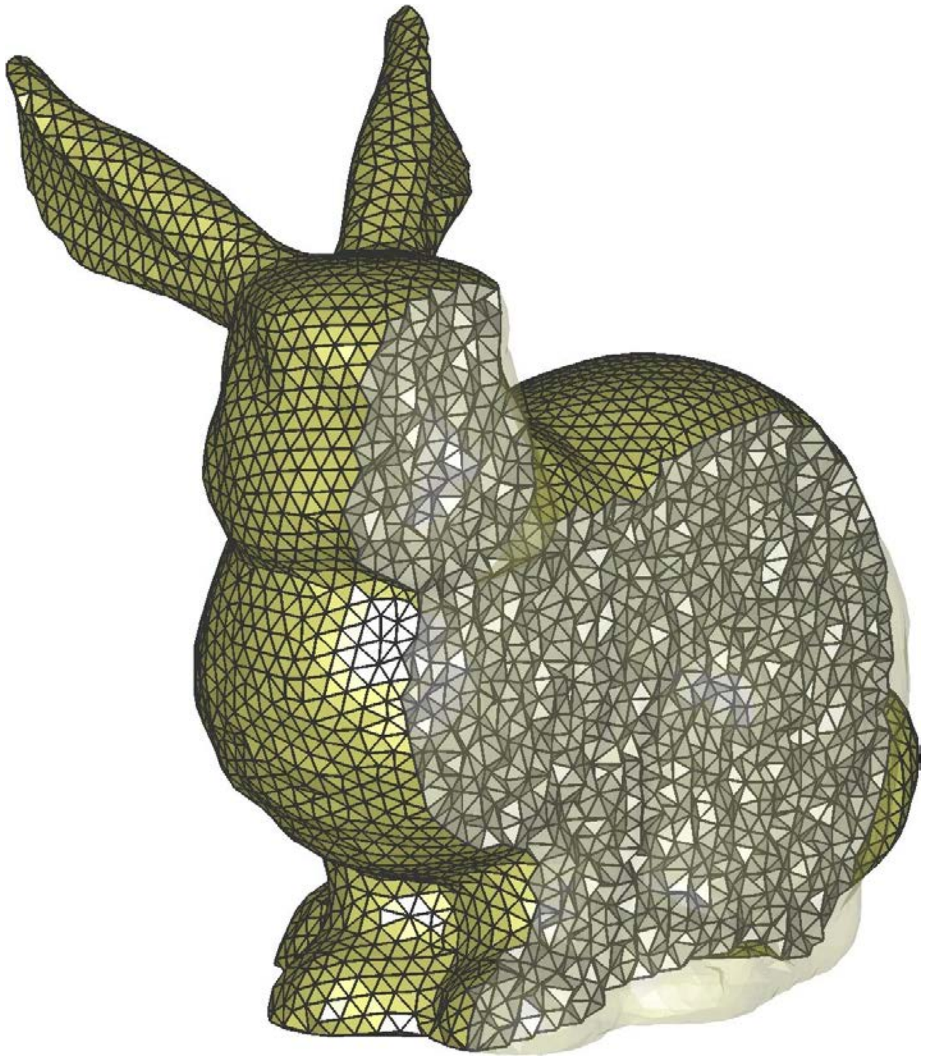
$$\nabla \cdot g = F$$

$$m = \int_{\Omega} \rho \, 1 \, d\bar{x} = \int_{\Omega} \rho \, \nabla \cdot [\bar{x}' \; 0 \; 0] \, d\bar{x}$$

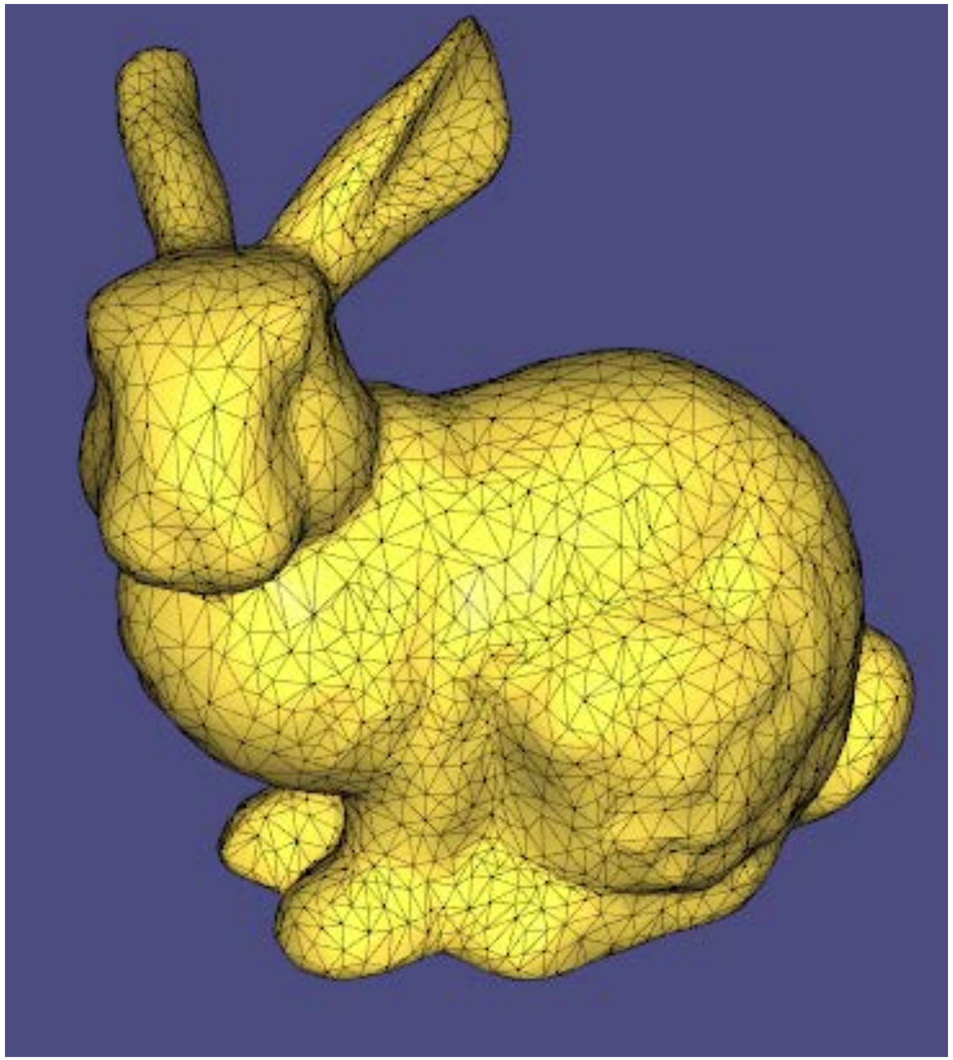
$$= \int_{\underbrace{\partial\Omega}_{\text{surface}}} \rho \, \bar{x} \cdot n_x \, d\Omega = m$$

$$= \sum_{i=1}^{|\text{triangles}|} \int_{\Delta_i} \rho \, \bar{x} \, d\Omega \cdot \underbrace{n_x^i}_{\Delta \text{ normals are constant}}$$

Surface-Only Integration



vs



The Newton-Euler Equations

$$L = \frac{1}{2}\begin{bmatrix}\omega^T & \dot{p}\end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix} M_0 \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} \omega \\ \dot{p} \end{bmatrix}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$$

$\Rightarrow$ need new calculus of variations

$$m\ddot{p} = f_{ext}$$

$$I = \int \rho [x][x]^T dV$$

$$R M_0 R^T \dot{\omega} = \underbrace{\omega \times \left(R I R^T \omega\right)}_{\text{quadratic velocity vector}} + \tau_{ext}$$

Exponential Explicit Euler

Input: ω^t, τ_{EXT}

Output: ω^{t+1}

1. Velocity Level

$$R I R^T \omega^{t+1} = R I R^T \omega^t + \Delta t \left(q_{vv}(\omega^t) + \tau_{EXT} \right)$$

2. Position Level

$$x(t) = \underbrace{\mathrm{expm}([\omega]\Delta t) R^t}_{R^{t+1}} \bar{x}$$

The Result