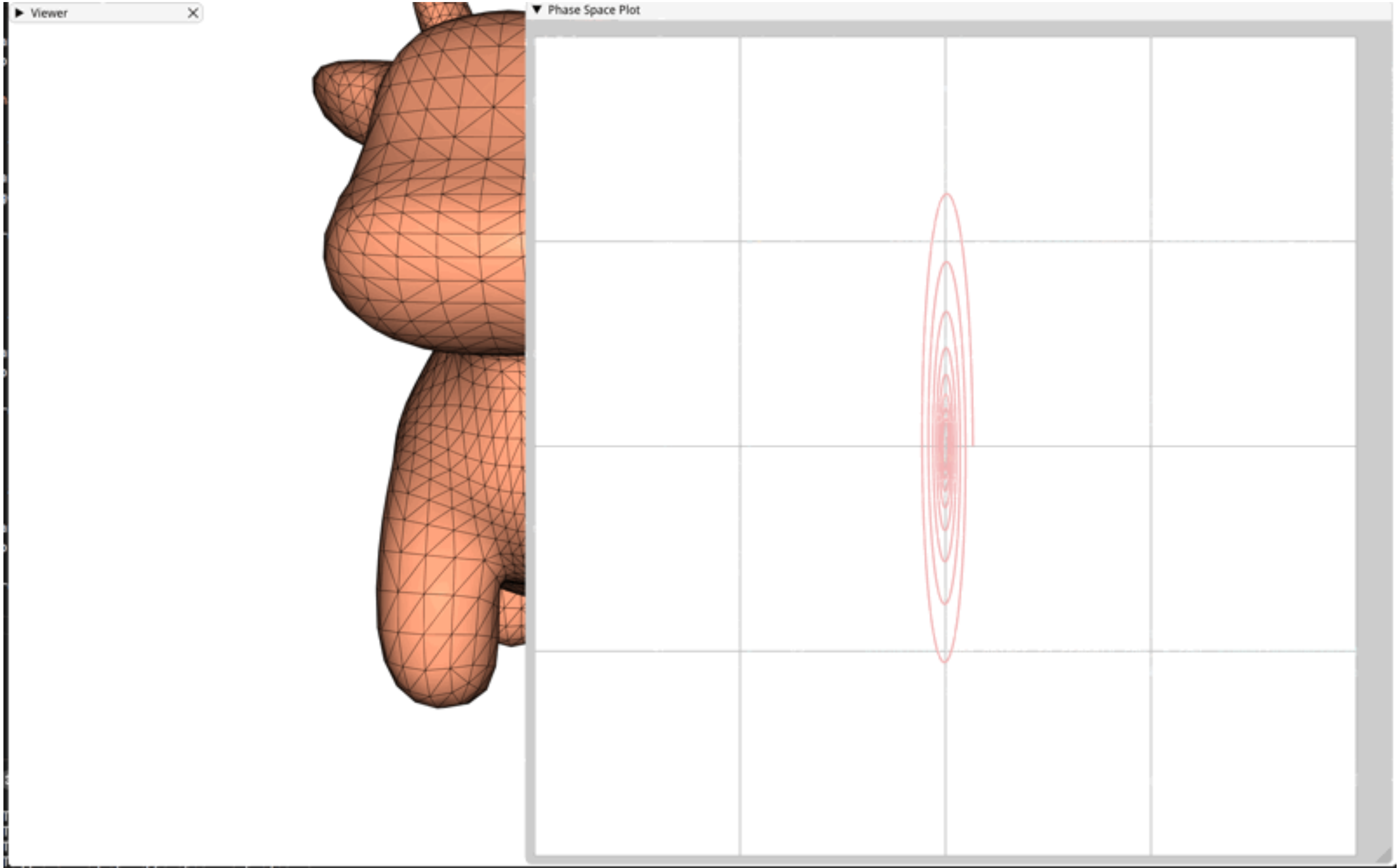
CSC2549 Physics-Based Animation

Scooby Doo 2 | Frantic Films

Last Week: Time Integration



Today: Enter the Third Dimension !

More Reminders

Assignment #1 is due Friday

https://github.com/dilevin/CSC2549-a1-mass-spring-1d

Assignment #2 is live and is due on October 4th

https://github.com/dilevin/CSC2549-a2-mass-spring-3d

Graphics Reading Group

Seminar Room in BA5166 (Dynamic Graphics Project)

Wednesdays 11am

Final Project Information

Projects can be done in teams of up to two (2)

Basic Project Types:

1. Implement a physics simulation
2. Implement something that uses physics simulation

Check with me that your project and the goals are appropriate (i.e not too difficult, will get you a good grade)

Final Project Information

Basic Project Types:

1. Implement a physics simulation
    1. Implement a SIGGRAPH (or equivalent) paper
    2. Research Project
2. Implement something that uses physics simulation
    1. Implement (for instance) a NeurIPS paper that uses physics
    2. Research Project

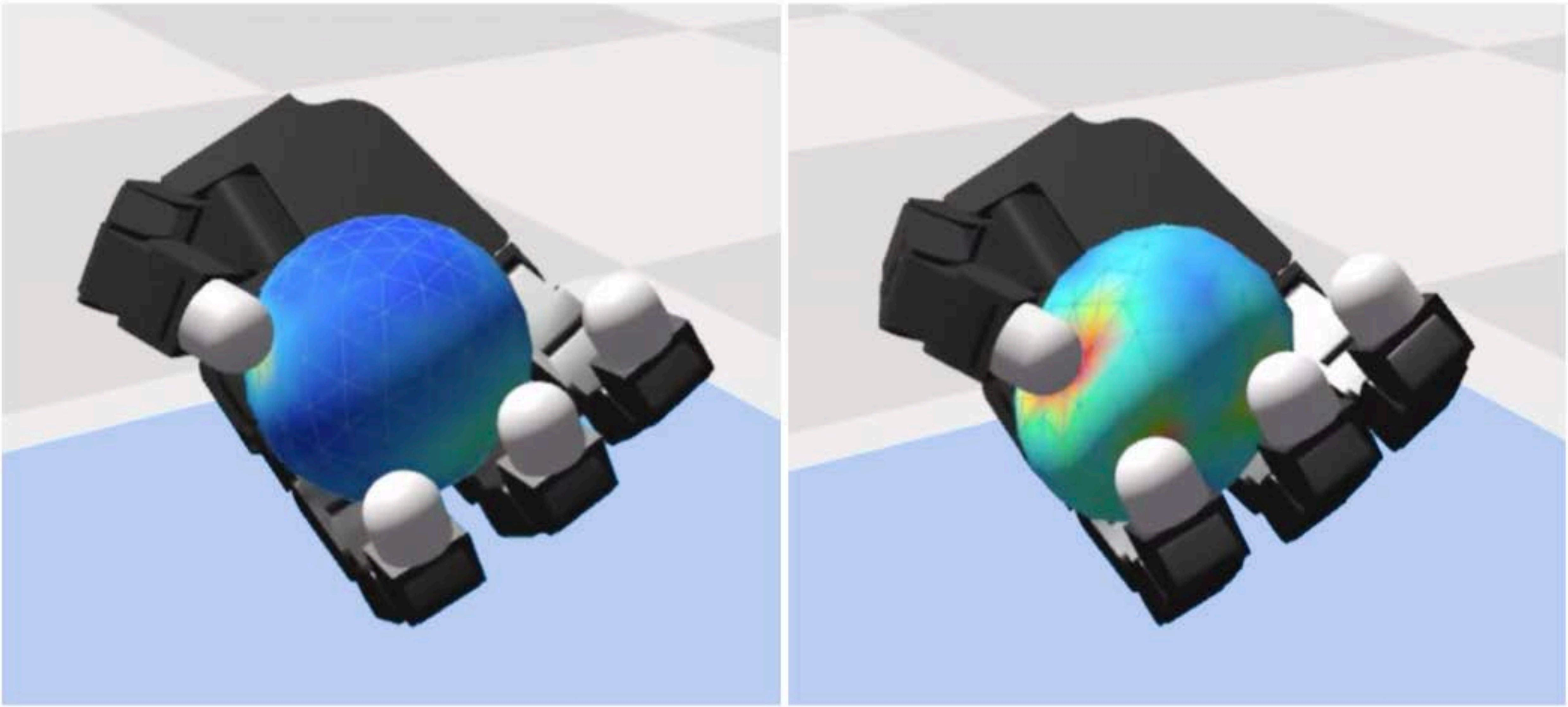
Some Project Ideas

1. Fluid simulation inside of a kidney
2. Simulation of rods (e.g. hair) using splines or subdivided curves
3. Recover the material properties of an object from video
4. Neural Rendering for Simulations
5. Non-Smooth Newton’s Method for Contact and Friction
6. Fast Physics Simulation using new Optimization Algorithms
7. Measuring Material Properties using Hand Tracking
8. Fast Simulation of Rigid Body Mechanisms

Today

1. Questions about the last lecture
2. Potential Energy of a 3D spring
3. Assembly
4. The Linearly-Implicit Time Integrator

Any Questions about the Last Lecture ?

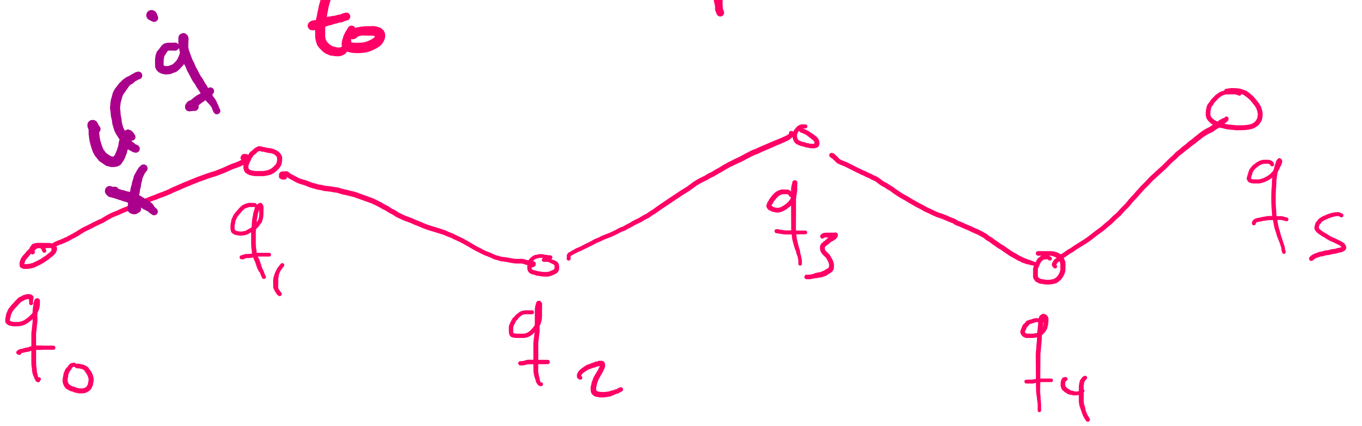


Non-Smooth Newton Methods for Deformable Multi-Body Dynamics | Macklin et al.

Discrete Variational Principles

Discrete Euler-Lagrange Equations

$$S = \int_{t_0}^{t_1} L(q,\dot{q})\,dt$$



$$\dot{q} = \frac{q^{t+1} - q^{t}}{\Delta t}$$

$$L(q,\dot{q}) = L\left(q^{t}, \frac{q^{t+1} - q^{t}}{\Delta t}\right)$$

$$\Rightarrow \quad L(q^{t}, q^{t+1})$$

$$S_D = \sum_{t=0}^{n-1} L(q^t, q^{t+1})\,\Delta t$$

pertubation

$$DS_D = \sum_{t=0}^{n-1} \underbrace{D_1}_{q^t} L\,\Delta q^t + \underbrace{D_2}_{q^{t+1}} L\,\Delta q^{t+1} = 0$$

Discrete "IVP"

$$DS_D = D_1 L\,\Delta q^0 + \sum_{t=1}^{n-1} \left[D_2 L(q^{t-1}, q^t) + D_1 L(q^t, q^{t+1}) \right)$$

$$DS_n = \cancel{D_1 \bar{L} \Delta q^0} + \sum_{t=1}^{n-1} \underbrace{\left[D_2 L(q^{t-1}, q^t) + D_1 L(q^t, q^{t+1}) \right]}_{0} \Delta q^t$$

$$D_2 L(q^{t-1}, q^t) + D_1 L(q^t, q^{t+1}) = 0$$

Fun Math Question

Show that Symplectic Euler results from extremizing the Discrete Principle of Least Action with the assumptions that:

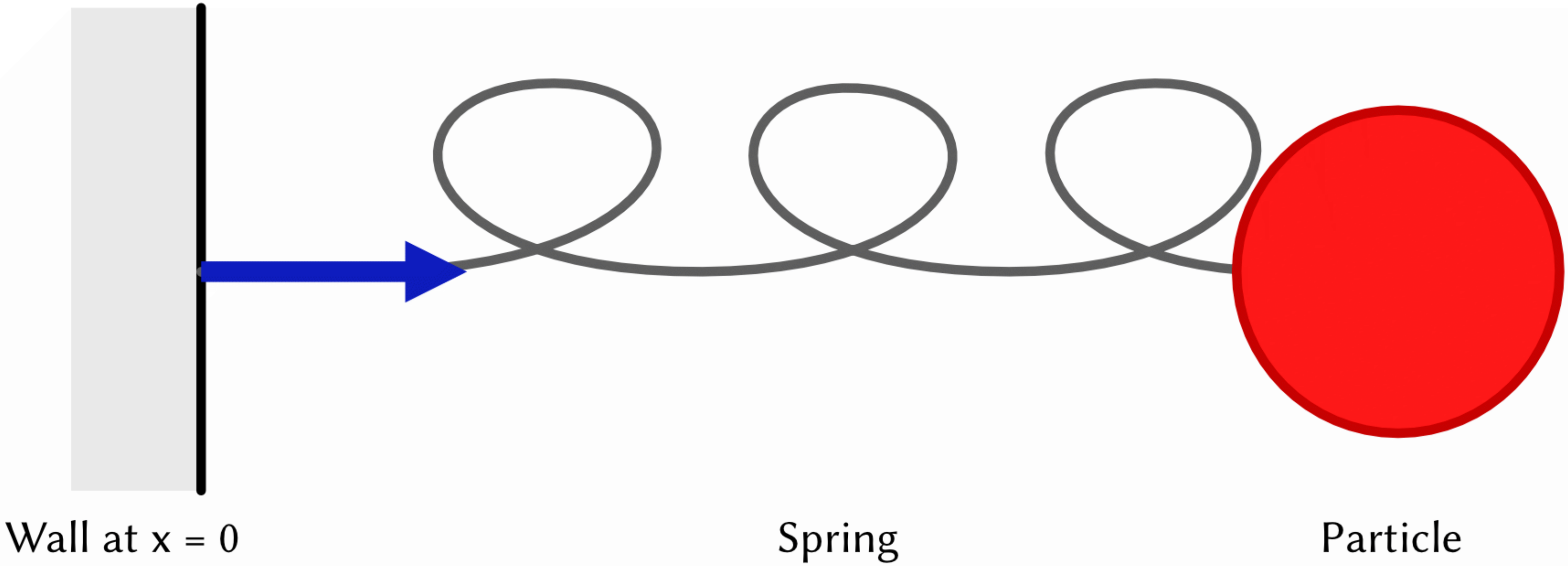
$$\dot{\mathbf{q}} = \frac{\mathbf{q}^{t+1} - \mathbf{q}^{t}}{\Delta t}$$

$$V\left(\mathbf{q}\right) = V\left(\mathbf{q}^{t}\right)$$

A Single Spring in 3D

Generalized Coordinates

$$q = \begin{bmatrix} \vec{x}_0 \\ \vec{x}_1 \end{bmatrix} \in \mathbb{R}^6$$



Wall at x = 0
Spring
Particle

3D Spring Potential Energy

$$\text{Strain} = l - l_0$$

$$V(q) = k(l - l_0)^2$$

k ← stiffness (material property)

$$l = \underbrace{|x_0 - x_1|}_{dx}$$

$$dx = x_0 - x_1$$

$$= 3\overset{B,\ 6}{\begin{bmatrix} I & -I \end{bmatrix}} \overset{q}{\begin{bmatrix} \vec{x}_0 \\ \vec{x}_1 \end{bmatrix}}$$

$$V(q) = \left(\underbrace{\sqrt{q^T B^T B q}}_{l} - l_0\right)^2 k$$

$$\sqrt{dx^T dx} = l$$

3D Kinetic Energy

1D: $T(\dot{q}) = \frac{1}{2}m\dot{q}^2 \rightarrow |\dot{q}|^2$

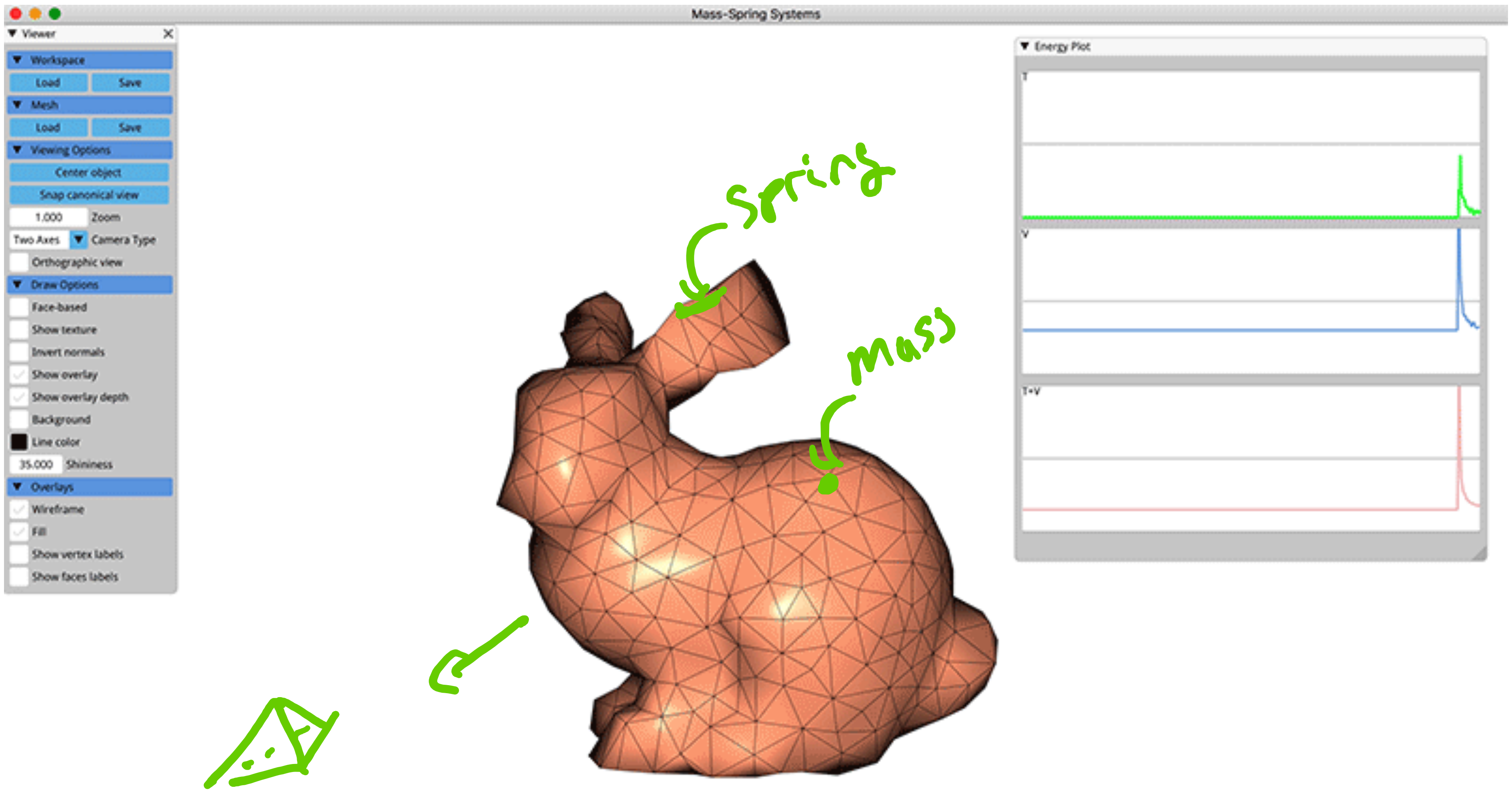
3D: $T(\dot{q}) = \frac{1}{2}m\dot{q}^T\dot{q}$

Large Mass-Spring Systems in 3D

$V \in n \times 3$

$E \in m \times 2$

edge



Generalized Coordinates

$$q = \begin{bmatrix} \vec{x}_0 \\ \vdots \\ \vec{x}_n \end{bmatrix} \in \mathbb{R}^{3 \cdot n}$$

\# vertices

$$\dot{q} = \begin{bmatrix} \dot{\vec{x}}_0 \\ \vdots \\ \dot{\vec{x}}_n \end{bmatrix}$$

Potential Energy

$$\begin{bmatrix} q_0 \\ q_1 \end{bmatrix} \in \mathbb{R}^6$$

$$V = \sum_{i=0}^{m-1} V_i(\underbrace{q_0, q_1}_{\text{endpoints}})$$

$$\begin{array}{cc} & \begin{array}{c} n \end{array} \\ 6 & \underbrace{\left[\begin{array}{c:c:c:c:c} I & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & I & 0 \end{array}\right]}_{\text{Selection } S_i} \end{array} \begin{bmatrix} q_0 \\ q_1 \\ \vdots \\ q_n \end{bmatrix}$$

3x3

q

E: $\boxed{6 \mid 3}$

$$V(q) = \sum_{i=0}^{m-1} V_i(S_i q)$$

Kinetic Energy

$$T(\dot{q}) = \sum_{i=0}^{n-1} \frac{1}{2} m_i \, \dot{q}_i^T \dot{q}_i$$

$$\underbrace{T = \frac{1}{2} \dot{q}^t M \dot{q}}$$

$$M \in \text{diagonal}, \mathbb{R}^{n \times n}$$

mass matrix

inertia tensor

Equations of Motion

$$L = T - V = \frac{1}{2}\dot{q}^T M \dot{q} - V(q)$$

$$M\ddot{q} = -\frac{\partial V}{\partial q} \leftarrow \text{generalized forces}$$

Generalized Forces

$$\frac{\partial V}{\partial q} = \frac{\partial}{\partial q} \sum_{i=0}^{m-1} V_i(S_i q)$$

$$= \sum_{i=0}^{m-1} \frac{\partial}{\partial q} V_i(S_i q)$$

$$= \sum_{i=0}^{m-1} \overbrace{{}^6 S_i^T \frac{\partial V_i}{\partial (q_0, q_\phi)}}^{\text{assembly}}$$

$$[\;] \in \mathbb{R}^6$$

$$S_i \quad \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \qquad \vec{q} \Rightarrow \vec{q}_i \in \mathbb{R}^6$$

$$S_i^T \quad \begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \vec{f}_1 \\ \vec{f}_2 \end{bmatrix} \leftarrow \mathbb{R}^6$$

Assembly

Linearly-Implicit Time Integration

$$M\dot{q}^{t+1} = M\dot{q}^{t} + \Delta t\, f(q^{t+1})$$

$$q^{t+1} = q^{t} + \Delta t\, \dot{q}^{t+1}$$

sub

Taylor expand

$$M\dot{q}^{t+1} = M\dot{q}^{t} + \Delta t\, f(q^{t}) + \Delta t \left.\frac{\partial f}{\partial q}\right|_{q^t} (\Delta t\, \dot{q}^{t+1})$$

K stiffness matrix

$$\left(M - \Delta t^2 \left.\frac{\partial f}{\partial q}\right|_{q^t}\right) \dot{q}^{t+1} = M\dot{q}^{t} + \Delta t\, f(q^{t})$$

Sparse Matrices

$$A\dot{q} = b \longrightarrow M\dot{q}^t + f(q)$$

$$(M - \Delta t^2 K)$$

Eigen::SparseMatrixd

SimplicialLDLT

use this!

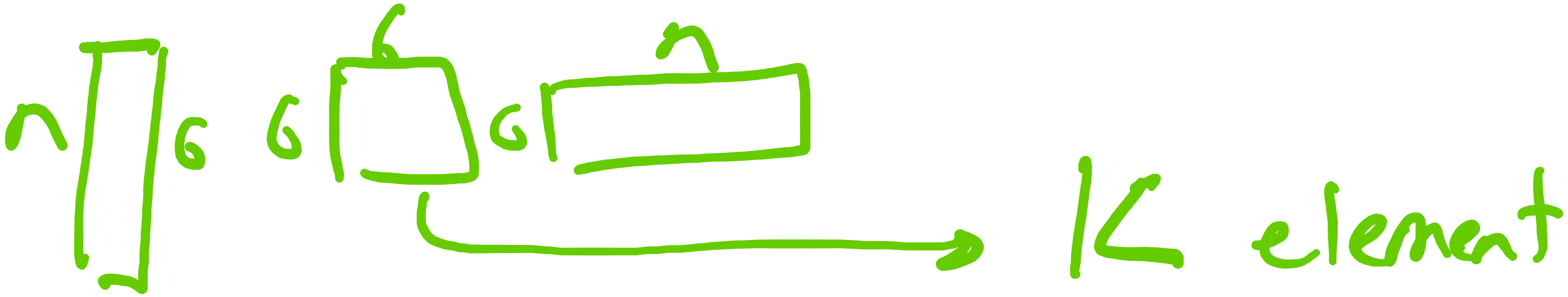
Eigen::Triplet (row, col, val)

SparseMatrixd A

A. setFromTriplets (list)

$$K = -\frac{\partial^2 V}{\partial q^2} = -\frac{\partial}{\partial q} \sum S_i^T \frac{\partial V(S_i q)}{\partial (q_0, q_1)}$$

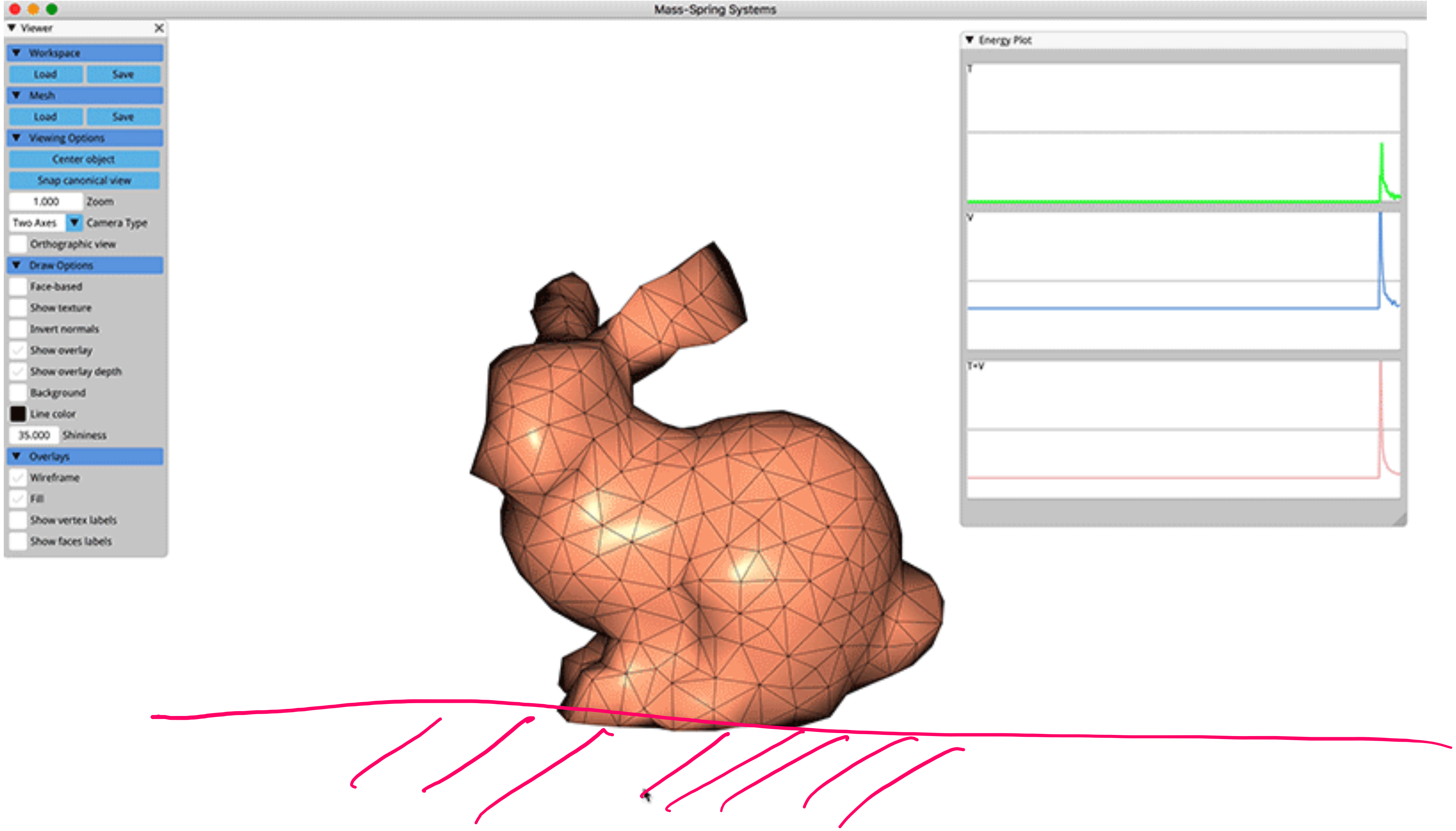
$$K = -\sum S_i^T \frac{\partial^2 V}{\partial q^2} S_i$$



Sparse Matrices in Eigen

Solving Linear Systems in Eigen

Fixed Boundary Conditions



$\bar{i}$ - index of a fixed particle

$$q(i) = \vec{C}\text{onstant}$$

$$q(j \neq i) = ?$$

$$\vec{b} = \begin{array}{cl} \begin{bmatrix} 0 \\ \vec{c} \end{bmatrix} & \begin{array}{l} j \neq i \\ j = i \end{array} \end{array}, \quad \hat{q} \in \mathbb{R}^{3(n-|i|)}$$

$$\vec{q} = P^T \underbrace{\hat{q}}_{\text{moving}} + \underbrace{\vec{b}}_{\text{fixed}}$$

$$\hat{q} = P_{\bar{f}} q$$

Selection → moving (non-fixed) vertices

Assignment 2 Demo

Next Week:

Finite Element Methods for 3D elastica