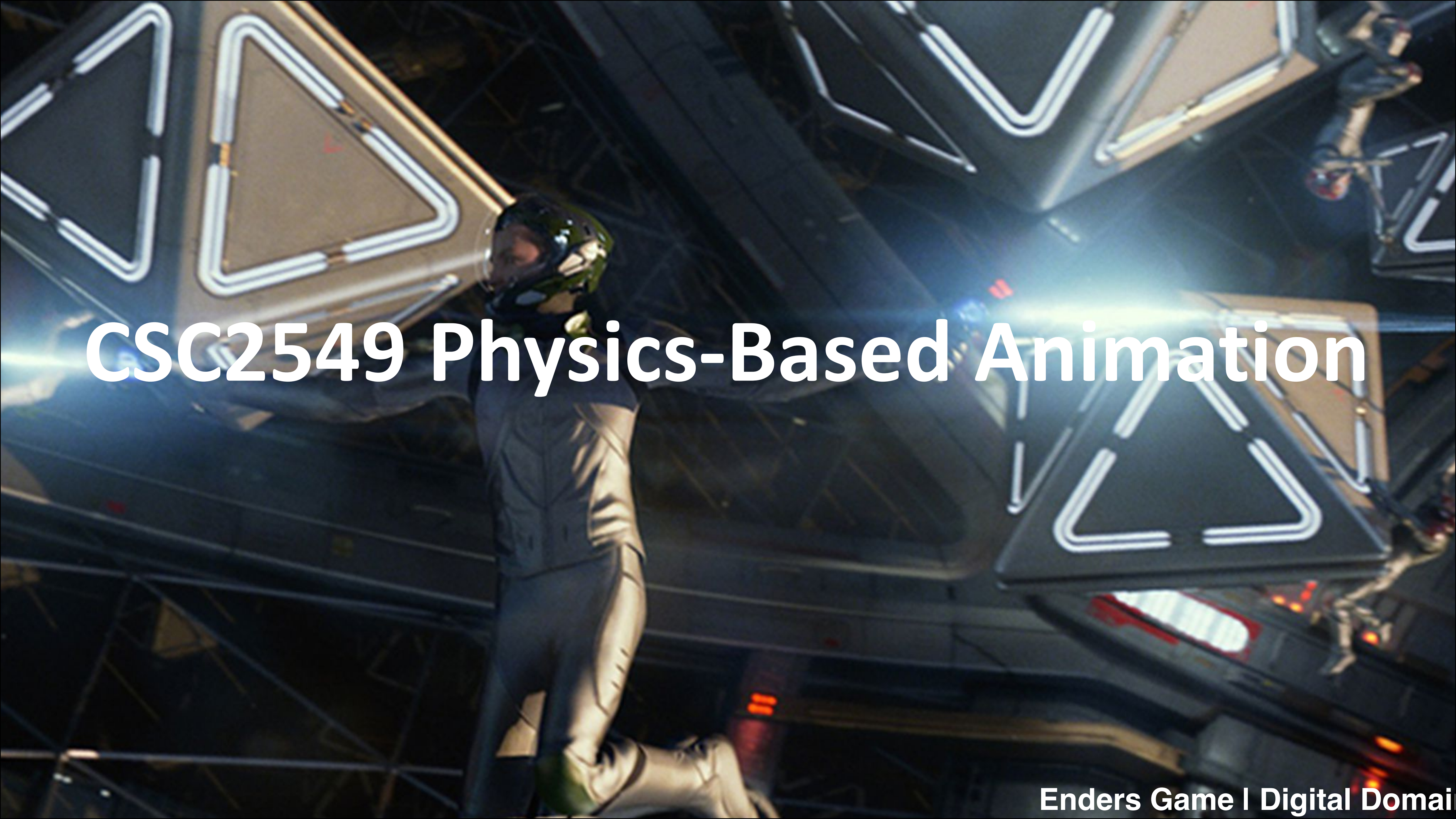
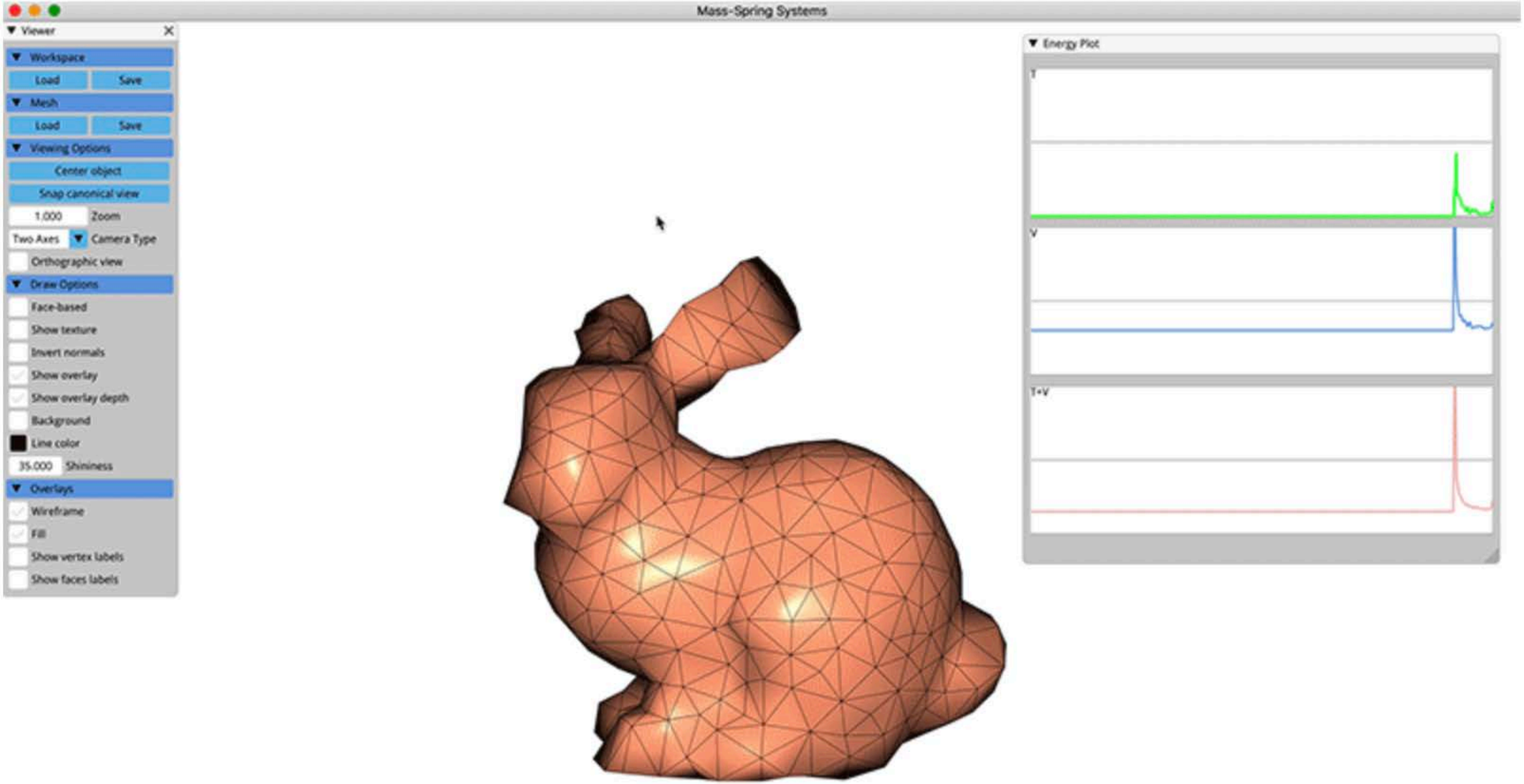
CSC2549 Physics-Based Animation



Enders Game | Digital Domai

Last Week: Mass-Spring Systems



Today: Finite Elements

More Reminders

Assignment #2 is due Friday

https://github.com/dilevin/CSC2549-a2-mass-spring-3d

Assignment #3 is live and is due on October 11th

https://github.com/dilevin/CSC2549-a3-finite-elements-3d

Graphics Reading Group

Seminar Room in BA5166 (Dynamic Graphics Project)

Wednesdays 11am

Change in Late Policy

Everyone gets 21 grace credits (late days)

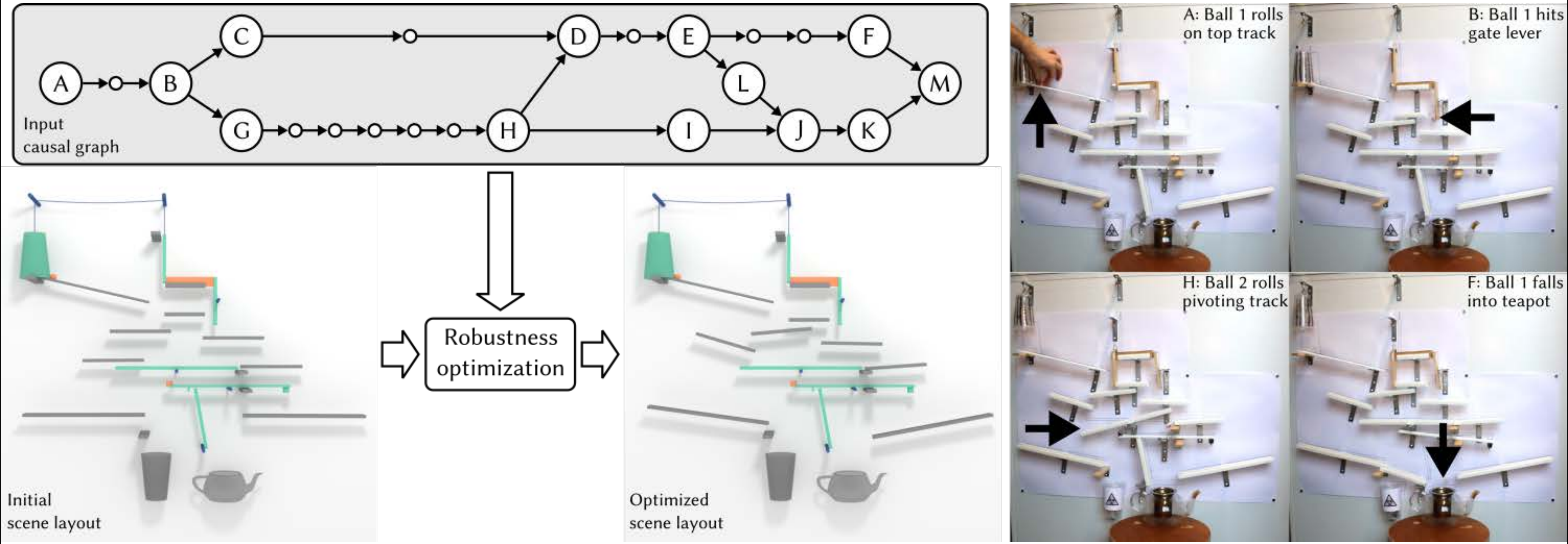
Can use them on any assignment.

Try to hand things in on time, but this allows you to load balance your schedule to avoid conflicts with other assignments or exams.

Today

1. Questions about the last lecture
2. Finite Elements for Elasticity
3. Backward Euler for Finite Elements
4. Skinning a Simulation

Any Questions about the Last Lecture ?



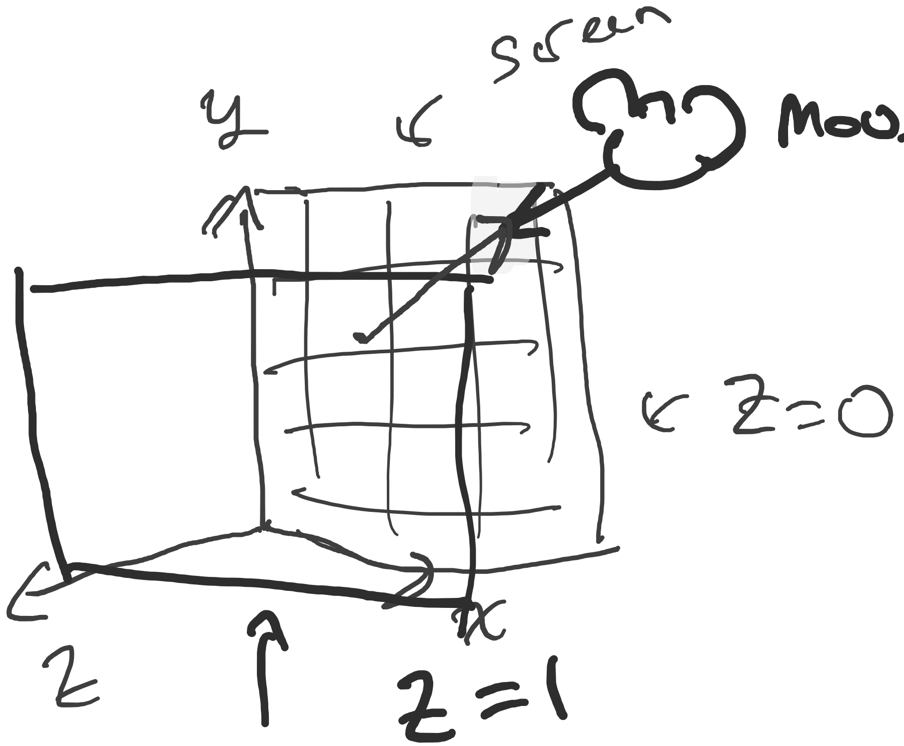
Designing Chain Reaction Contraptions from Causal Graphs | **Roussel et al.**

Picking Points in 3D

Barycentric coordinates

$\lambda_1, \lambda_2, \lambda_3 = 1 - \lambda_1 - \lambda_2$

choose the nearest vertex



camera plane

mouse window $\rightarrow (x, y, 0)$

camera $\rightarrow (x, y, 1)$

igl::unproject

In: a point in canonical space

Out: " " " world space

Indicial Notation for Matrix Derivatives

$$\vec{x} = A\vec{y}$$

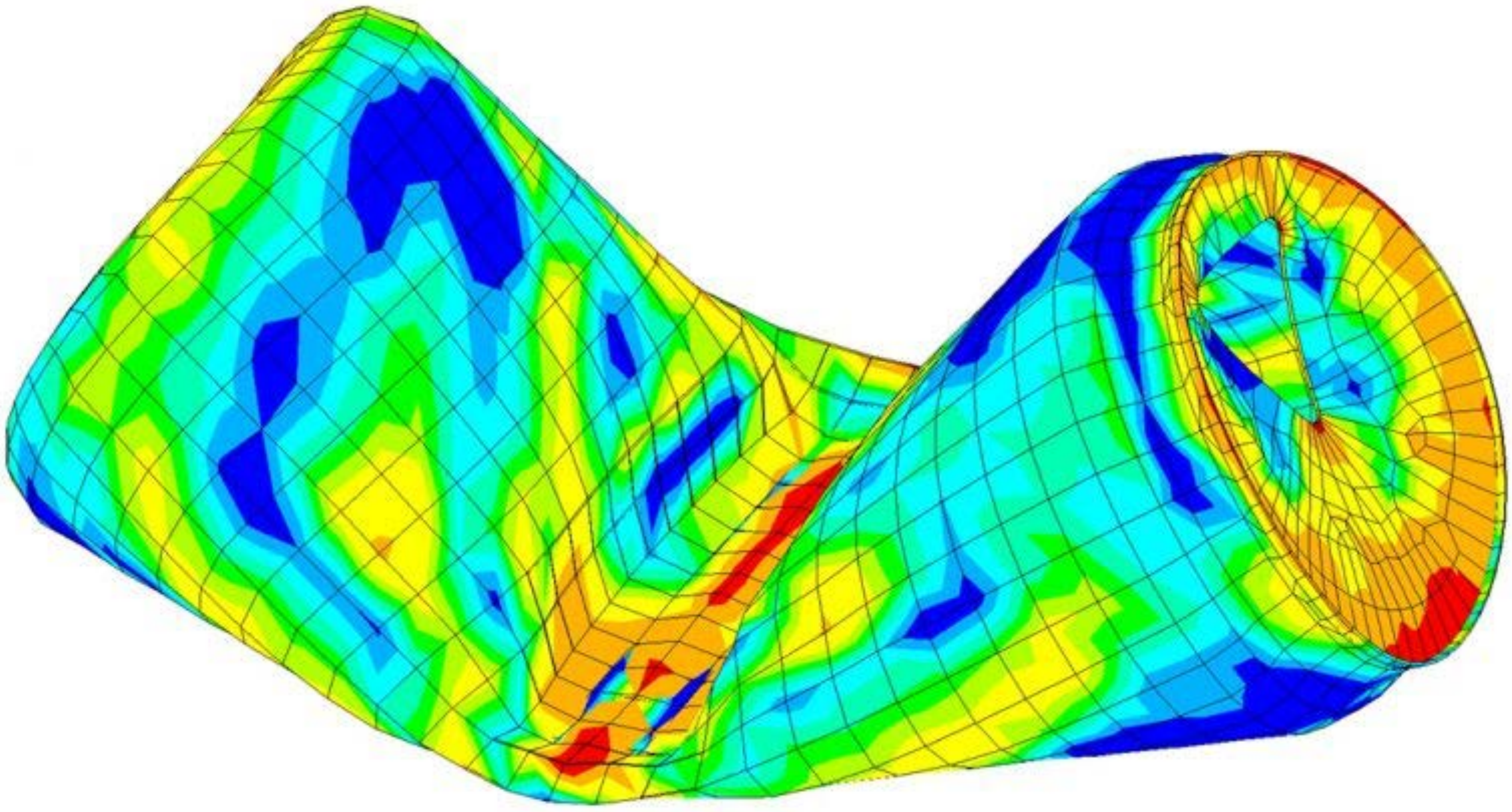
$$A_{ik} = A$$

$$\frac{d\vec{x}}{d\vec{y}} = \sum_j A_{ij} y_j$$

$$\frac{dx_i}{dy_k} = \frac{d}{dy_k} A_{ij} y_j \quad (ESC)$$

$$= A_{ij} \left[\frac{d}{dy_k} y_j\right] \rightarrow \delta_{kj} \begin{cases} 1 & \text{if } k=j \\ 0 & \text{else} \end{cases}$$

Finite Elements



Finite Elements

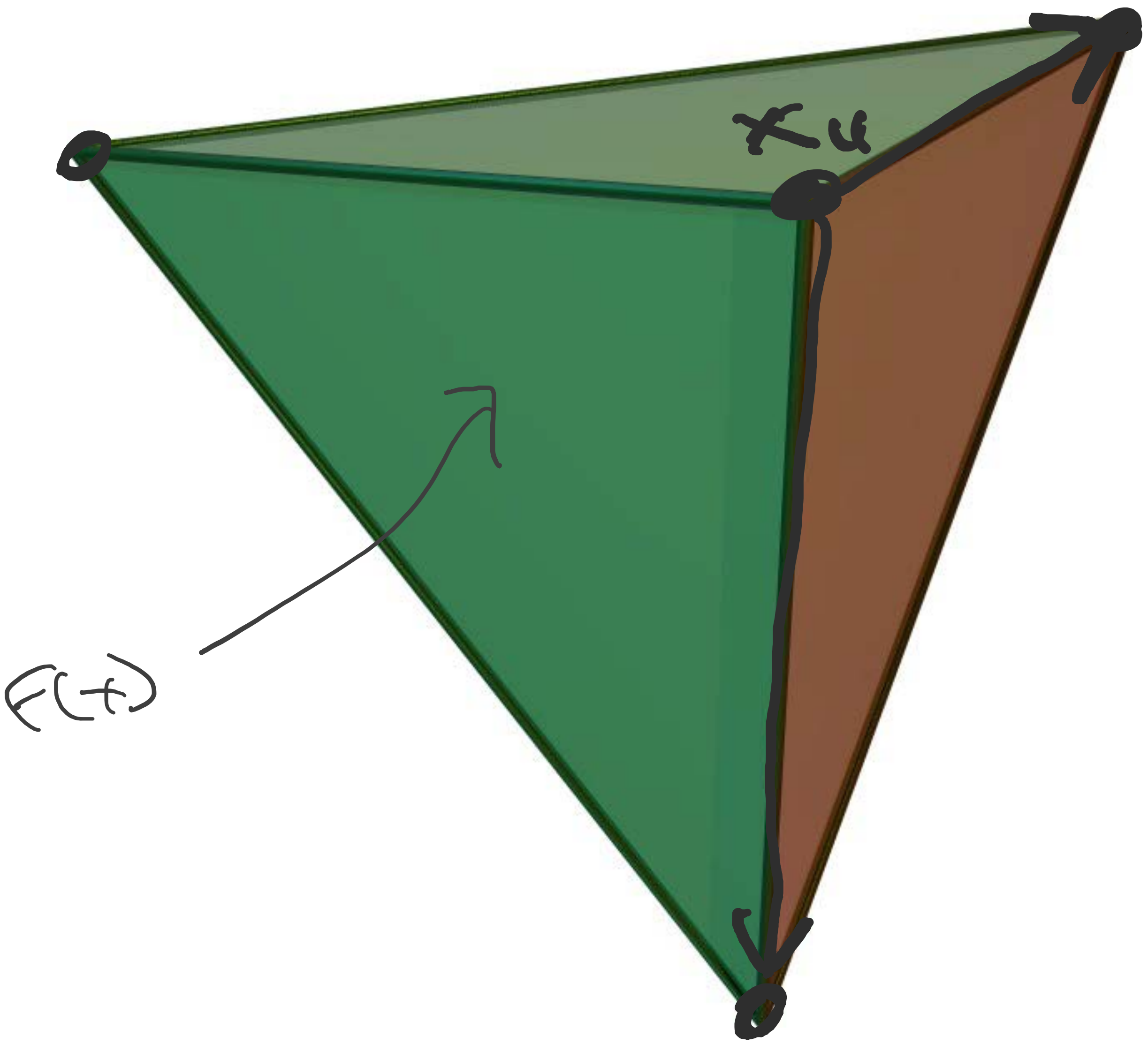
Defining basis or shape functions

$$f(x) = \sum \underbrace{\emptyset_i(x)}_{\text{basis}} w_i \leftarrow \text{solve for}$$

peanut

Better approximation of the real world

Tetrahedral Finite Elements



Barycentric Coordinates as Shape Functions

$$\vec{x} = \lambda_1 \vec{x}_1 + \lambda_2 \vec{x}_2 + \lambda_3 \vec{x}_3 + \lambda_4 \vec{x}_4$$

$$\cancel{y = \lambda_1 y_1 + \lambda_2 y_2 + \lambda_3 y_3}$$

$$\lambda_4 = 1 - \lambda_1 - \lambda_2 - \lambda_3$$

$$\vec{x} = \lambda_1 \vec{x}_1 + \lambda_2 \vec{x}_2 + \lambda_3 \vec{x}_3 + (1 - \lambda_1 - \lambda_2 - \lambda_3)\vec{x}_4$$

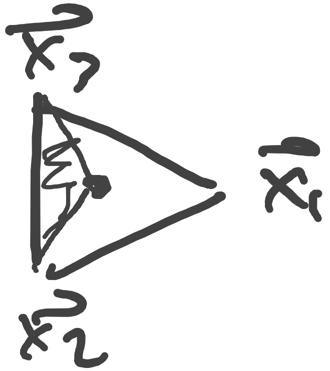
$$\underbrace{\left[\vec{x}_1-\vec{x}_4 \;\middle|\; \vec{x}_2-\vec{x}_4 \;\middle|\; \vec{x}_3-\vec{x}_4\right]}_{T}\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix} = (\vec{x}-\vec{x}_4)$$

$$\lambda_4 = 1-\lambda_1-\lambda_2-\lambda_3$$

T

$$\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix} = T^{-1}(\vec{x}-\vec{x}_4)$$

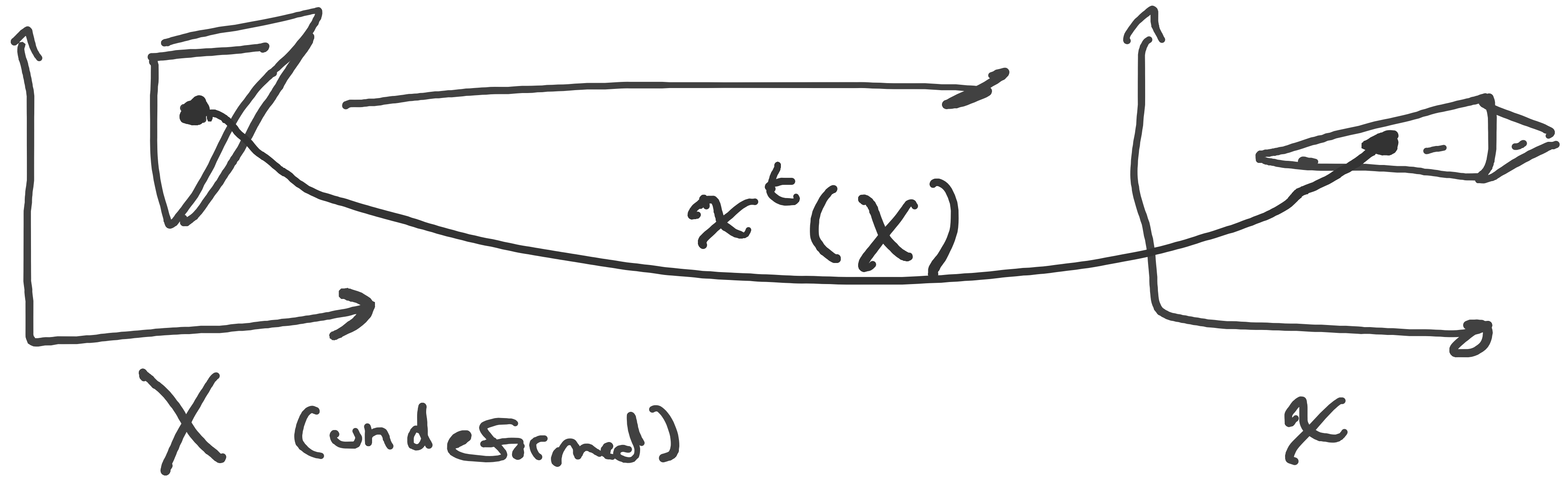
$\lambda_1(\vec{x})$



Tetrahedron FEM

$$F(\vec{x}) = f_1\lambda_1 + f_2\lambda_2 + f_3\lambda_3 + f_4\lambda_4$$

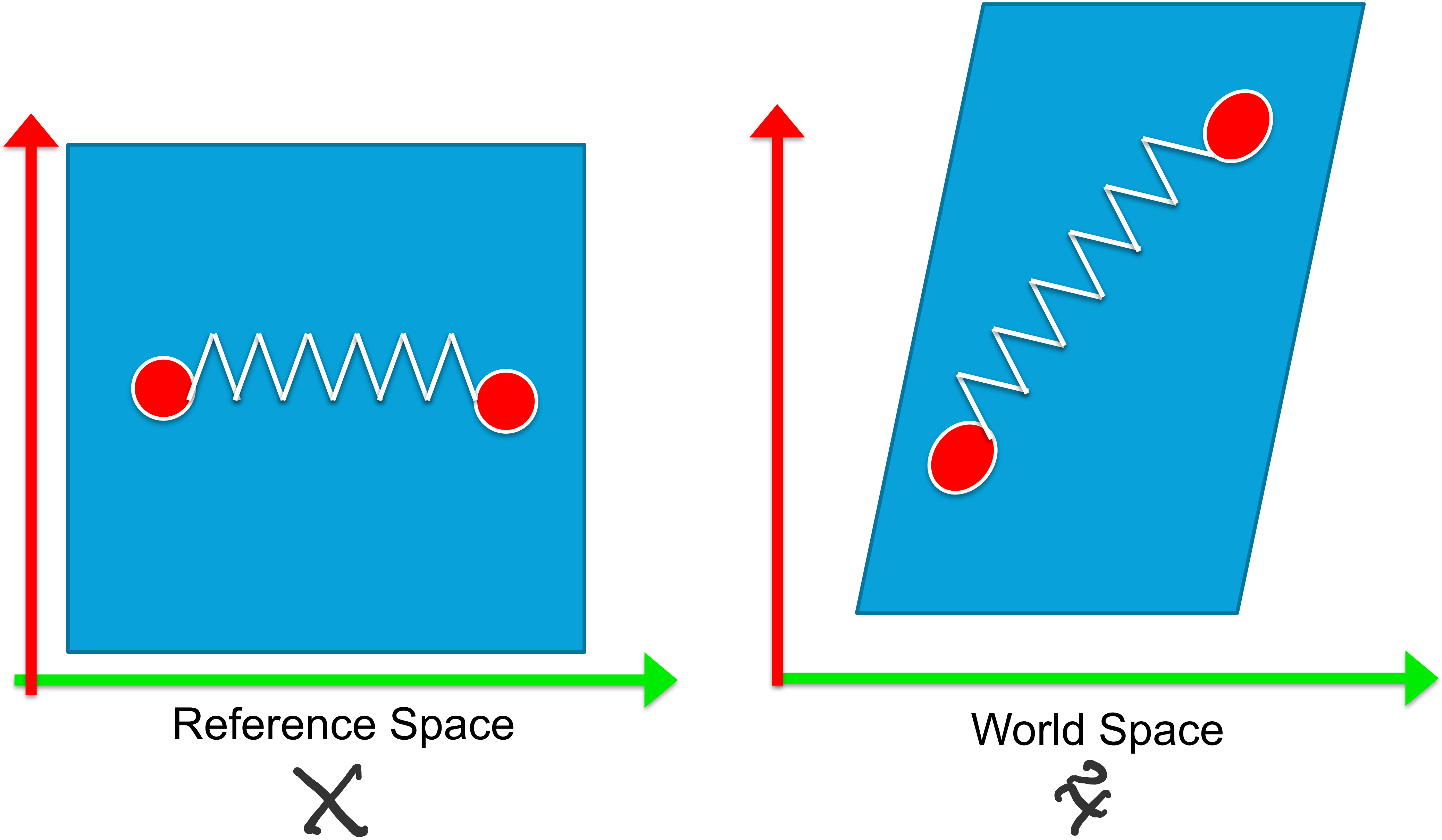
Generalized Coordinates



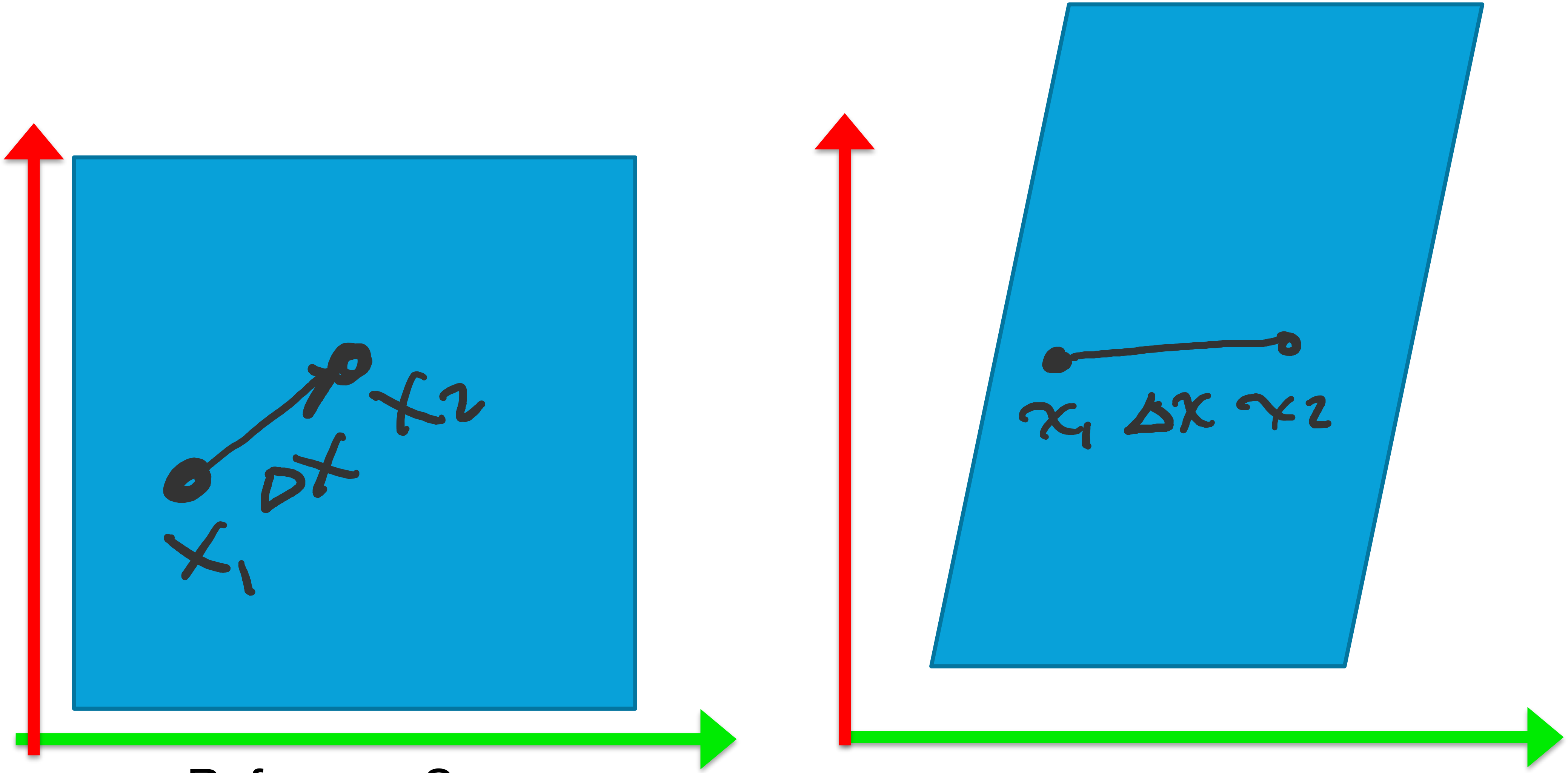
$$x^t(X) = \sum_{i=1}^{4} x_i^t \lambda_i(X)$$

$$q = [\vec{x}_1^t \ \vec{x}_2^t \ \vec{x}_3^t \ \vec{x}_4^t]^T \in \mathbb{R}^{12}$$

Deformation



Deformation



Reference Space

World Space

$$\Delta\vec{x} = \vec{x}_2(\underbrace{X_2}_{X_1 + \Delta X}) - \vec{x}_1(X_1)$$

$$\Delta\vec{x} \approx \cancel{\vec{x}_1} + \frac{\partial \vec{x}}{\partial X}\Delta X - \cancel{\vec{x}_1}$$

$$\Delta\vec{x} \approx \overbrace{\underbrace{\frac{\partial \vec{x}}{\partial X}}_{\frac{d\ \text{world space}}{d\ \text{undeformed space}} \rightarrow \text{deformation gradient}}}^{F} \Delta X$$

1D: $(l - l_0)^2$

$\downarrow$

$l(q)$

3D: $l^2(\mathbf{q}) = \Delta x^T \Delta x = \Delta X^T \underbrace{F^T F}_{\text{right Cauchy-Green strain}} \Delta X$

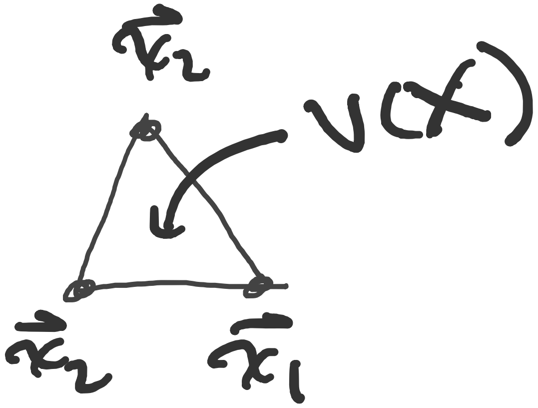
$F = \underbrace{R}_{\text{rotation}} \overbrace{S}^{\text{symmetric component}}$, $F^T F = S^T \cancel{R^T R} S$

$$\frac{d\vec{x}^t}{dX} = \frac{d}{dX}\sum_{i=1}^{4} \vec{x}_i^t \lambda_i(X)$$

$$= \sum_{i=1}^{4} \vec{x}_i^t \frac{\partial \lambda_i}{\partial X}(X) \in \mathbb{R}^{3\times 3}$$

Kinetic Energy and the Mass Matrix

$$T = \frac{1}{2} m |\vec{v}|^2 = \frac{1}{2} m \dot{q}^T \dot{q}$$



$$\frac{1}{2} \int_{\Delta} \rho |v|^2 \, dV$$ ← little volume element

$$\frac{d\vec{x}}{dt} = \frac{d}{dt} \sum \vec{x}_i \lambda_i(X) = \sum_i \lambda_i(X) \vec{v}_i$$

$$v(x) = \sum_i \vec{v}_i \lambda_i(x) = \underbrace{\left[\lambda_1 I \,\vdots\, \lambda_2 I \,\vdots\, \lambda_3 I \,\vdots\, \lambda_4 I\right]}_{N(x) \in \mathbb{R}^{3\times 12}} q$$

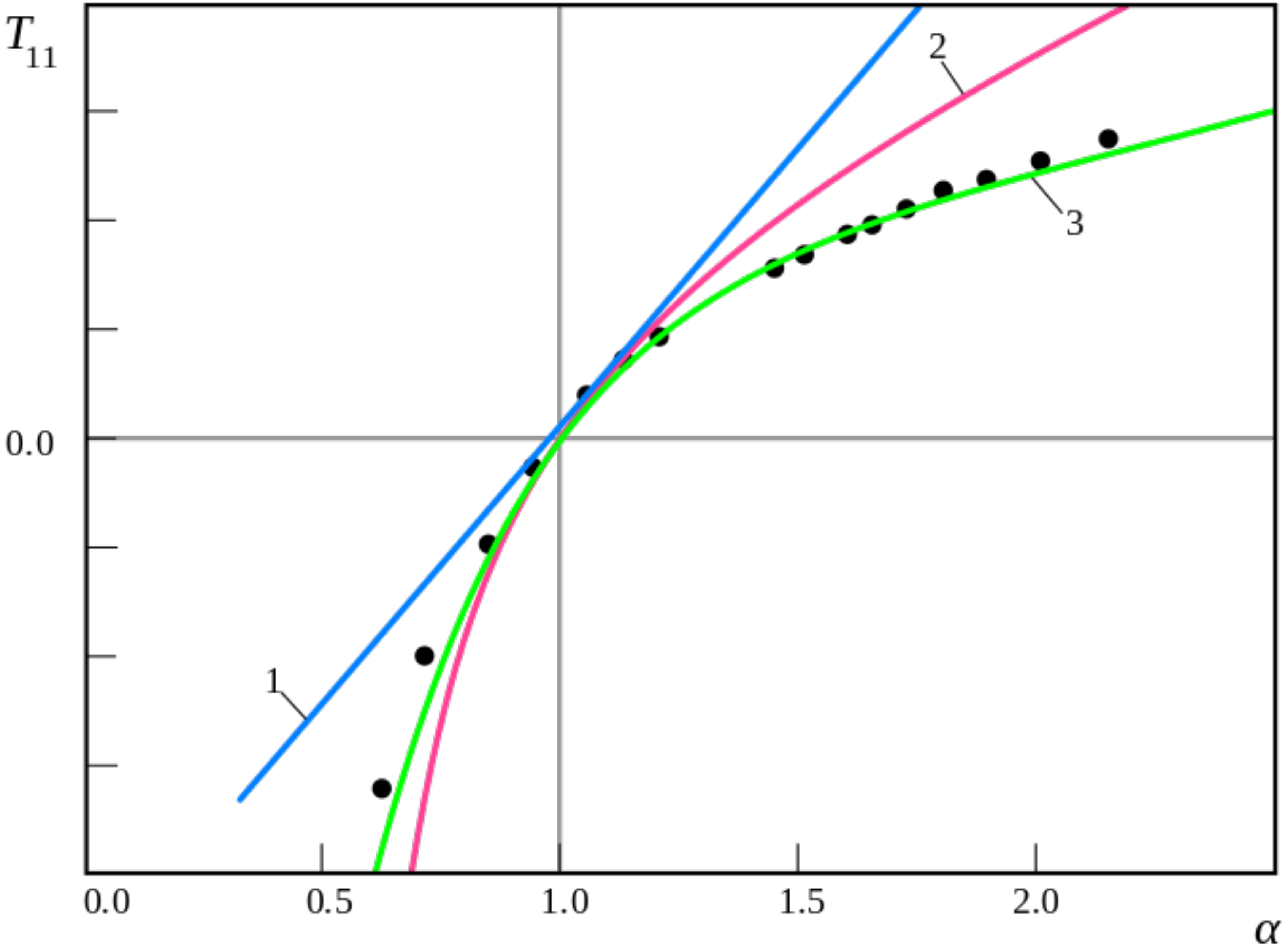
$$q = \begin{bmatrix} \vec{v}_1 \\ \vdots \\ \vec{v}_{4} \end{bmatrix}$$

$$v(x) = Nq$$

$$T = \frac{1}{2} \int_{\Delta} \rho \, \dot{q}^T N(x)^T N(x) \, \dot{q} \, dV$$

$$T = \frac{1}{2}\dot{q}^T \underbrace{\int_{\Delta} \rho N(x)^T N(x)\, dV}_{M_e} \dot{q}$$

Potential Energy using Neo Hookean Material Model



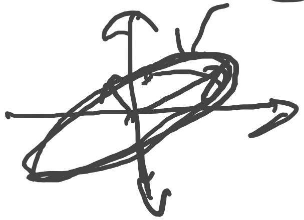
$$NH: C(\bar{I}_1 - 3) + D(J-1)^2$$

$$\bar{I}_1 = \underline{\underline{tr(F^T F)}} \cdot \frac{1}{J^{2/3}} \approx e_1^2 + e_2^2 + e_3^2$$

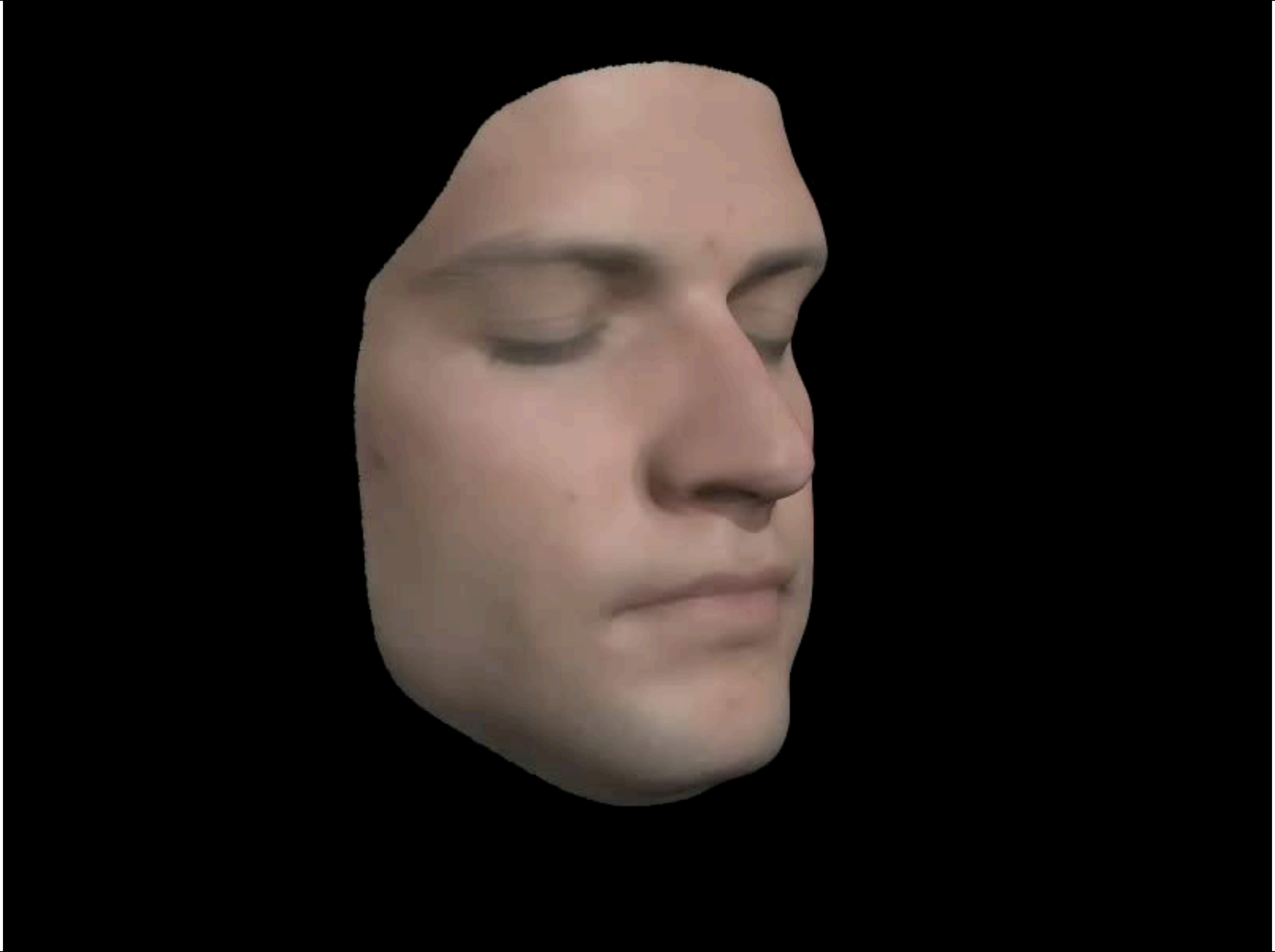
right!

$$J = \det(F)$$

$$F^T F = S$$



eigenvalues



Quadrature

$$V = NH(F(q))$$

$$\frac{\partial V}{\partial q} = \underbrace{\frac{\partial F}{\partial q}^T}_{B} \overbrace{\frac{\partial NH}{\partial F}}^{\in \mathbb{R}^{9\times 1}} \to g$$

$$B \in \mathbb{R}^{9\times 12}, \quad B^T g \in \mathbb{R}^{12\times 1}$$

In practice

$$F = \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ \vdots \\ \vdots \end{bmatrix} \in \mathbb{R}^{9\times 1}$$

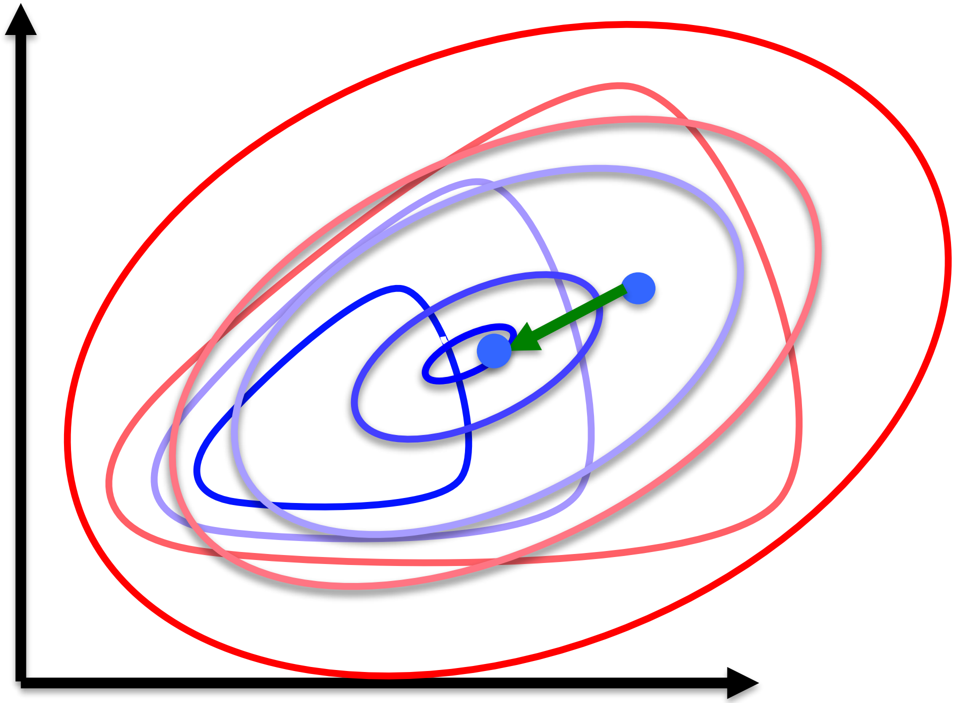
$$\underbrace{\frac{\partial}{\partial q}\frac{\partial NH}{\partial q}}_{\text{Hessian}} = B^T \underbrace{\frac{\partial^2 NH}{\partial F^2}}_{\mathbb{R}^{9\times 9}} \overset{\mathbb{R}^{9\times 12}}{B}$$

Forces

The Stiffness Matrix

Backward Euler using Optimization

Newton’s Method



Newton’s Method

Newton’s Method

Backtracking Line Search

Skinning Simulations

Next Week:

Finite Element Methods for Cloth Simulation !