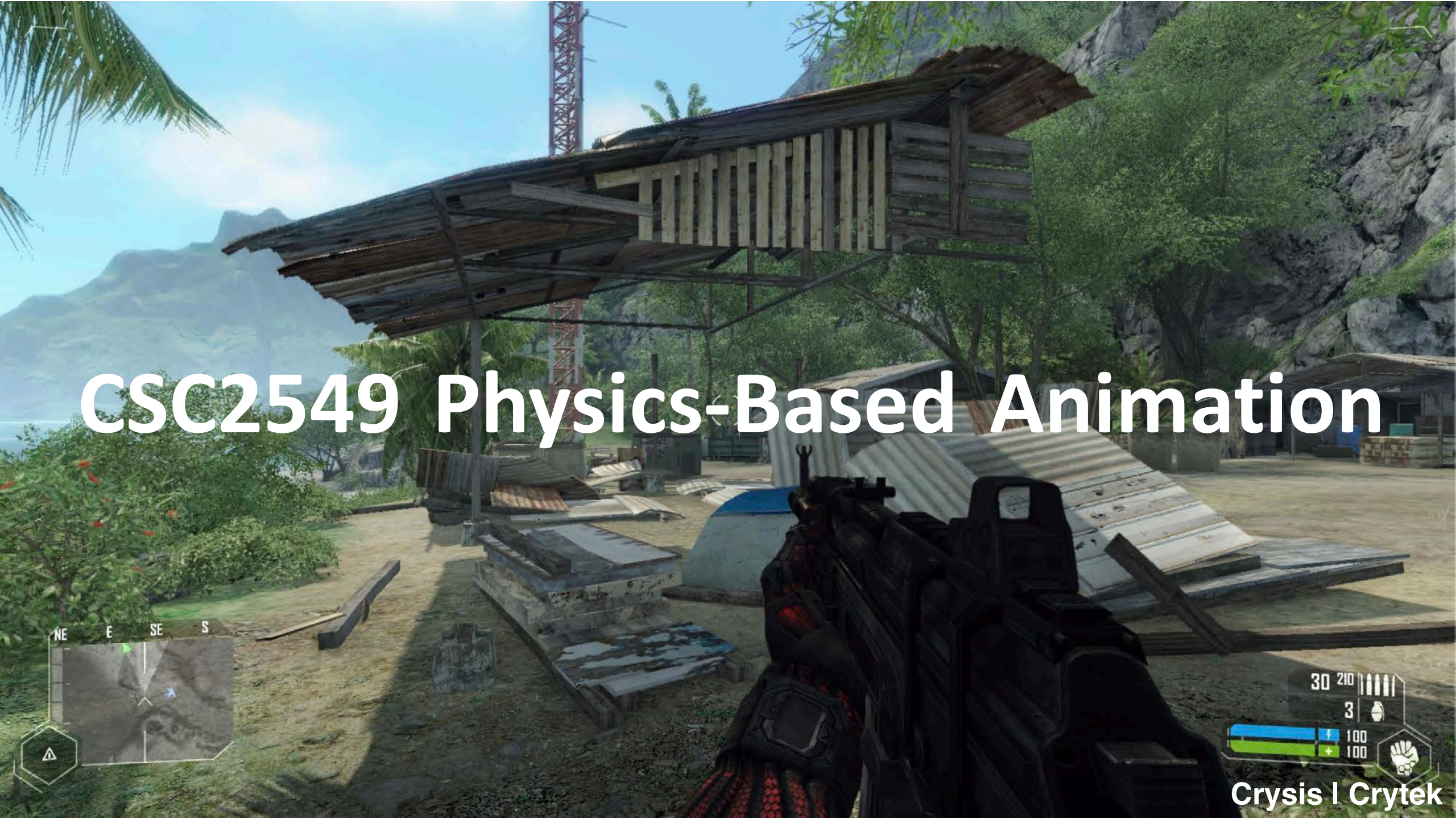


CSC2549 Physics-Based Animation



Reminders

Assignment #6 is due Friday

<https://github.com/dilevin/CSC2549-a6-rigid-body-contact>

Graphics Reading Group

Seminar Room in BA5166 (Dynamic Graphics Project)

Wednesdays 11am

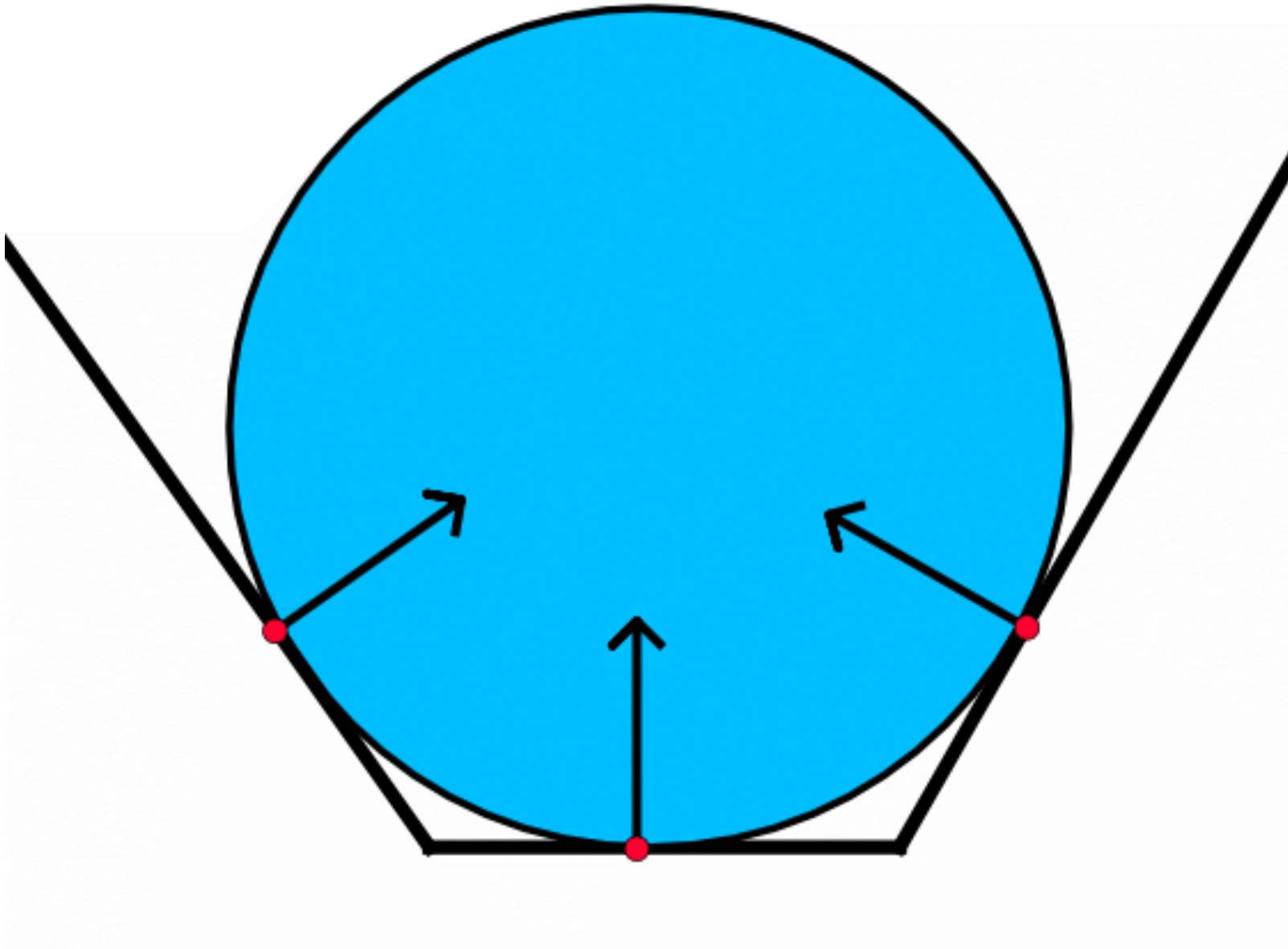
Next Week

I'm Away

Hopefully Ryan Goldade from University of Waterloo and SideFX software will come and talk about fluid simulation

If not ... no class (watch your email).

Rigid Body Contact

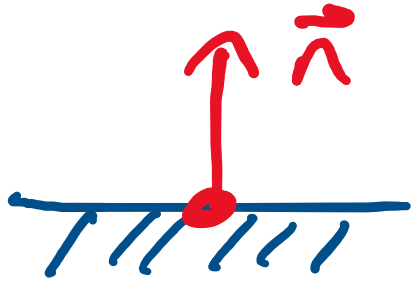


Four Rules of Contact Mechanics

For a single contact point:

1. No interpenetration @ contact point
2. Contact force in direction of ^{+ve} normal
3. when there's contact

Signorini Conditions



$$F_{\text{contact}} = \vec{n} \alpha, \quad \alpha \in \mathbb{R}$$

$$\alpha \geq 0 \quad (\text{push away})$$

$$d \geq 0$$

Sign distance
between
objects

$$d \perp \alpha \quad (\alpha \text{ only non-zero when } d = 0)$$
$$(d \cdot d = 0)$$

Equations of Motion with Signorini Conditions

$$M \ddot{q}^{t_k} = M \dot{q}^k + \sigma F + \sigma \vec{N} \alpha$$

$$d \perp \alpha$$

$$d \geq 0$$

$$d \geq 0$$

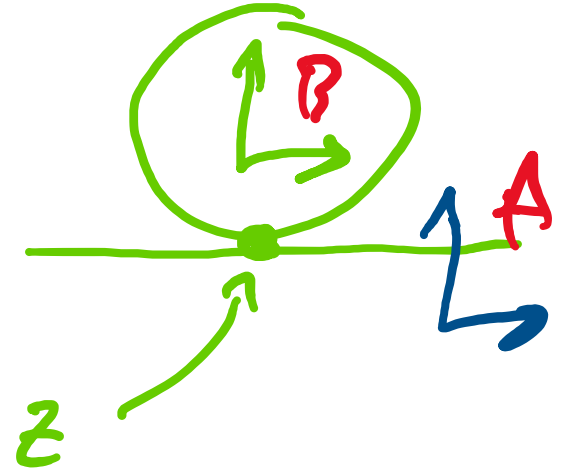
Velocity Level Equations

$$M_{\dot{q}}^{t+1} = M_{\dot{q}}^t + \Delta t F + \Delta t \vec{n} \alpha$$

$$\alpha \geq 0$$

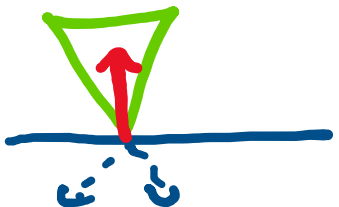
$$\vec{n}^T (\dot{z}^B - \dot{z}^A) \geq 0$$

$$\alpha \perp \vec{n}^T (\dot{z}^B - \dot{z}^A)$$



$\dot{z}^A \rightarrow$ world space of A

$\dot{z}^B \rightarrow$ world of B



Solving the LCP

$$\dot{q}^{t+1} = \dot{q}^{t,unc} + \Delta t M^{-1} G^T \vec{n} \Delta$$

$$\frac{d\chi(q, \bar{x})}{d\bar{x}} = \frac{\partial \chi}{\partial q}(q) \dot{q}$$

$$\dot{z}^A = \underbrace{\frac{\partial \chi^A}{\partial q}}_{G^A}(\bar{z}^A) \dot{q}^{t+1}$$

$$\eta^T (\dot{z}^B - \dot{z}^A) \quad \gamma$$

$$= \left(\eta^T G^B q_{unc}^B - \eta^T G^A q_{unc}^A \right) + \dots$$

$$\Delta t \left(\eta^T G^B M^{-1} G_n^{BT} - \eta^T G^A M^{-1} G_n^{AT} \right) \alpha = 0$$

$$\Delta t \left(\eta^T G^B M^{-1} G_n^{BT} - \eta^T G^A M^{-1} G_n^{AT} \right) \alpha$$

$$\alpha = \frac{-\gamma}{\delta}$$

$$\alpha = \max(0, -\frac{\gamma}{\delta})$$

Unified Particle Physics for Real-Time Applications

Miles Macklin Matthias Müller Nuttapong Chentanez Tae-Yong Kim

NVIDIA

Position-Based Dynamics

“In this paper we present a unified approach that makes some compromises in realism to meet our goal of real-time performance, but enables a wider range of effects than was previously possible “

No Energies, Just Constraints !

o \vec{x}_1

$$C_i(\vec{x}^t + \Delta x) = 0$$

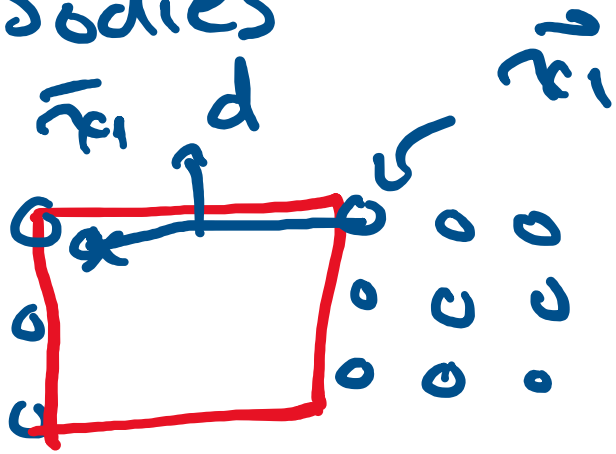
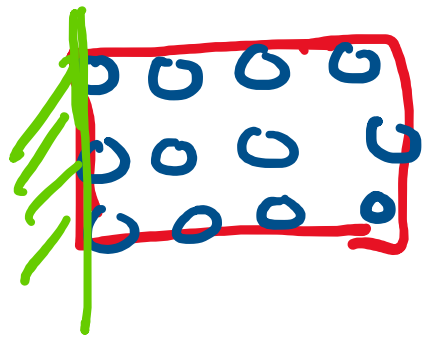
o \vec{x}_2

$$C_j(\vec{x}^t + \Delta x) \geq 0$$

$$M \frac{\Delta x^{t+1}}{\Delta t} = M \frac{\Delta x^t}{\Delta t}$$

$$\Delta x^{t+1} = \Delta x^t + M^{-1} \frac{2C_i^T}{2\vec{x}} \alpha$$

Deformable Bodies



$$C = \underbrace{x_i^b - \bar{x}_i}_{\text{vector force}} = 0$$

k (vector force)

$$k \in (0, 1)$$

Why is it Fast?

1. Iterative method – just stop when you run out of time
2. Parallel implementation, evaluate constraints in parallel
3. No matrix assembly, no large matrix inversion,
4. Constraints are formulated to be robust, don't need to spend time fixing problems

Fast Physics via Alternating Algorithms

Build a solver that has similar properties to the PBD approach but works for “real” physics energies

Based on an optimization technique called alternating projections.

ADMM \supseteq Projective Dynamics: Fast Simulation of General Constitutive Models

Rahul Narain Matthew Overby George E. Brown

University of Minnesota

Alternating Direction Method of Multipliers (ADMM)