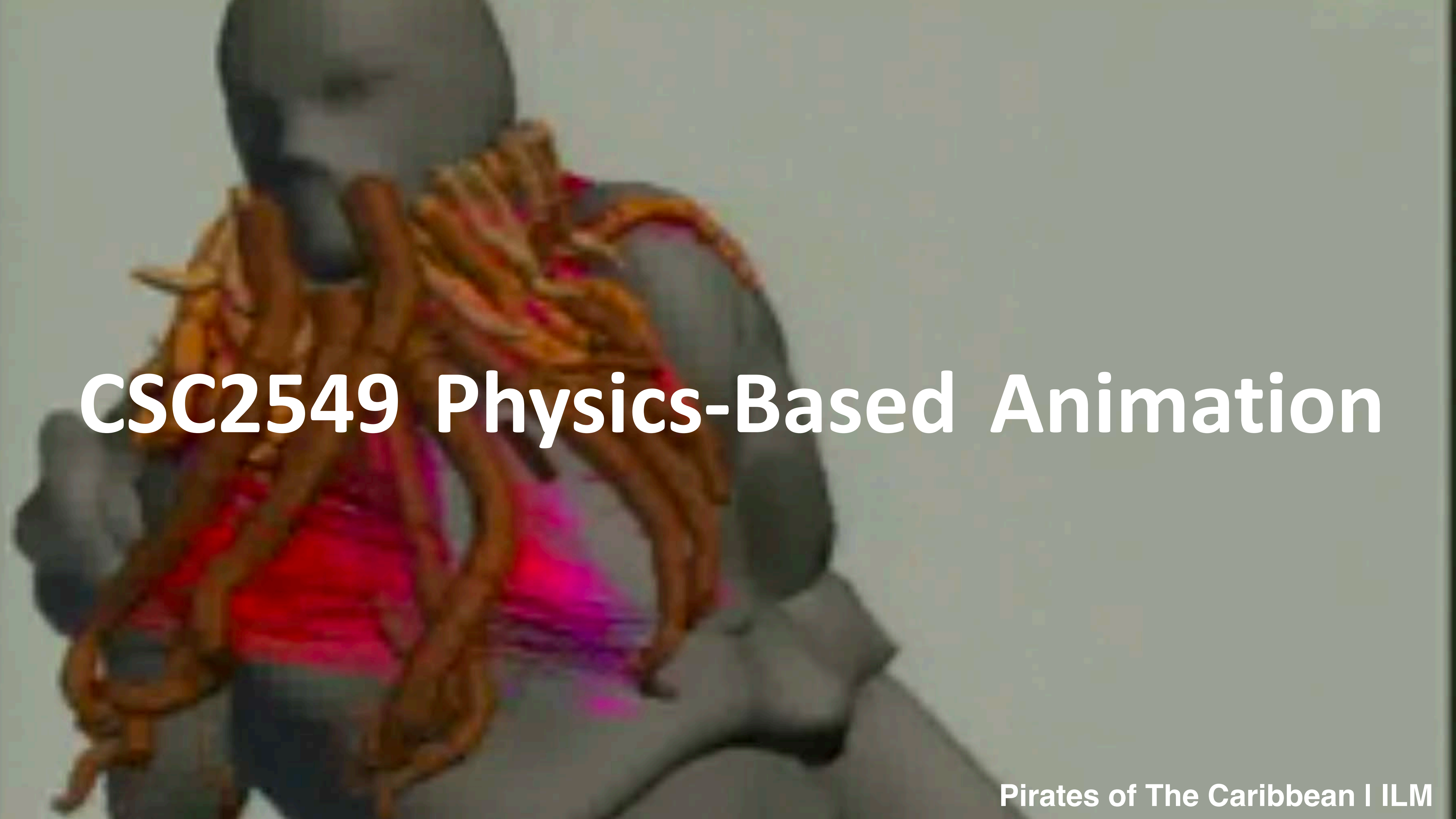
CSC2549 Physics-Based Animation



Pirates of The Caribbean | ILM

Reminders

Assignment #4 is due Friday

https://github.com/dilevin/CSC2549-a4-cloth-simulation

Assignment #5 is live and is due on 8/11

https://github.com/dilevin/CSC2549-a5-rigid-bodies

Graphics Reading Group

Seminar Room in BA5166 (Dynamic Graphics Project)

Wednesdays 11am

Let’s Talk about Final Projects

30% of your final grade

Two components

- Presentation – 15% of the mark
- Due date: November 27th

- Write up in SIGGRAPH style - 15% of the mark
- Due date: December 16th

Final Presentation Guide

Duration: 5-6 minutes with 2-3 minutes for question

Content:

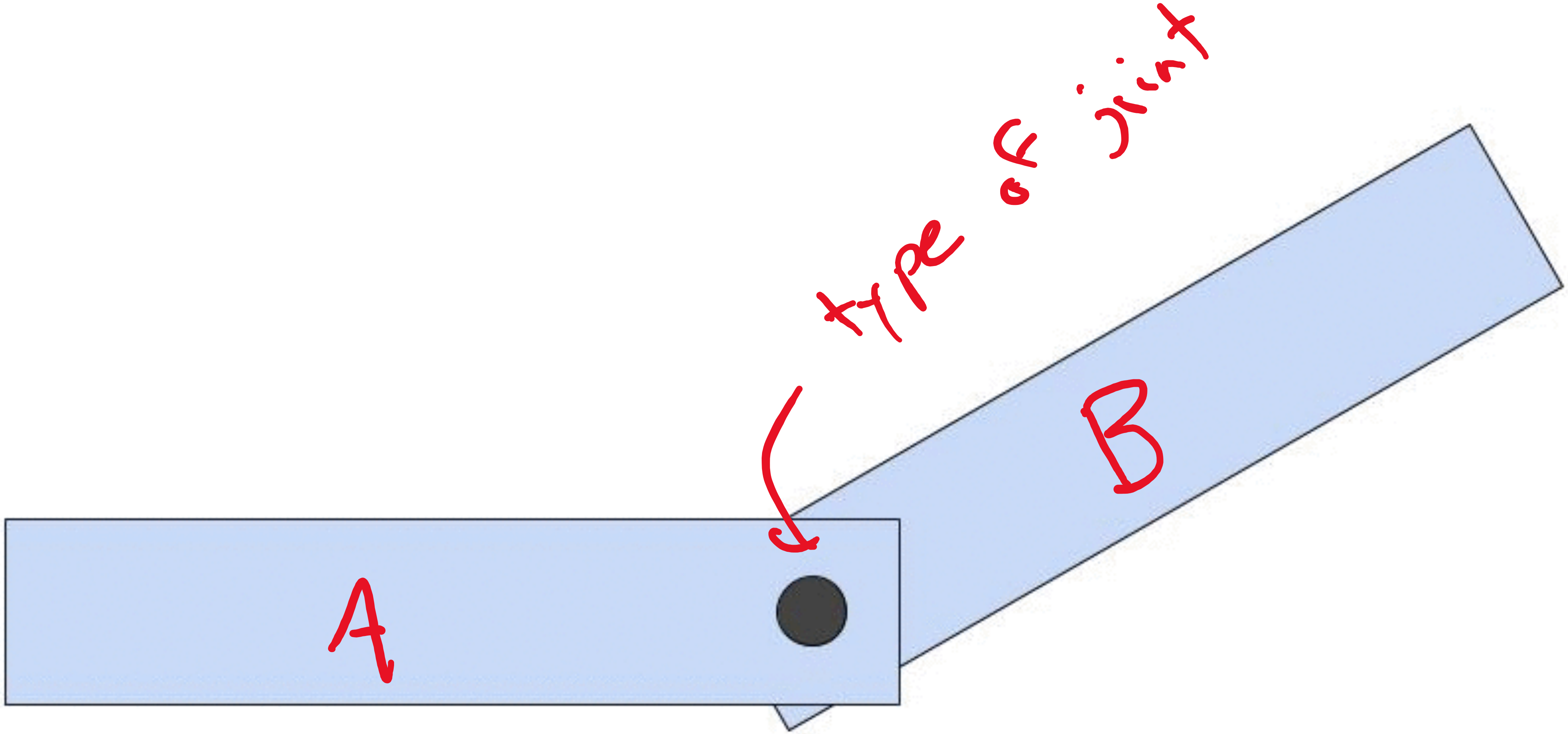
Problem Statement (1 Slide)

Related Work (1-2 Slides)

Methodology (as many slides as you need)

Results or Anticipated Results (at least one slide)

Example



The Two Approaches

1. "Maximal" Coordinates

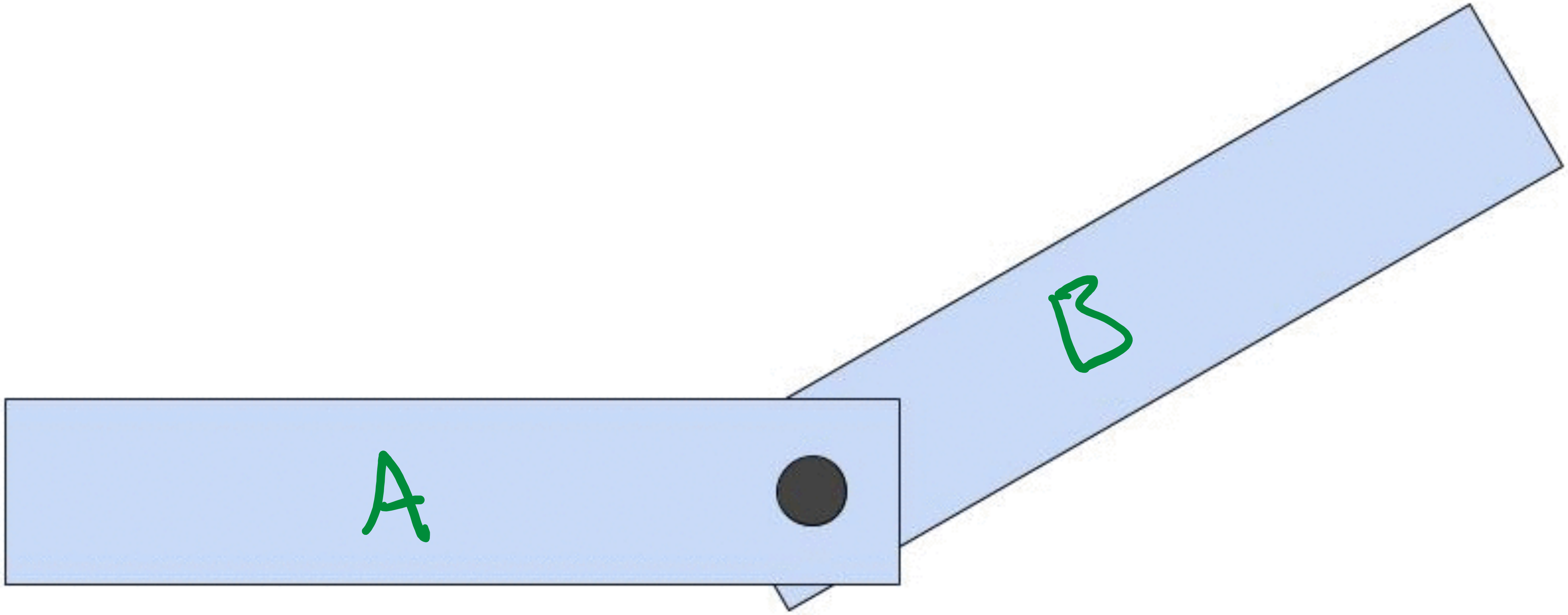
Rigid Body has R, p

Constraints

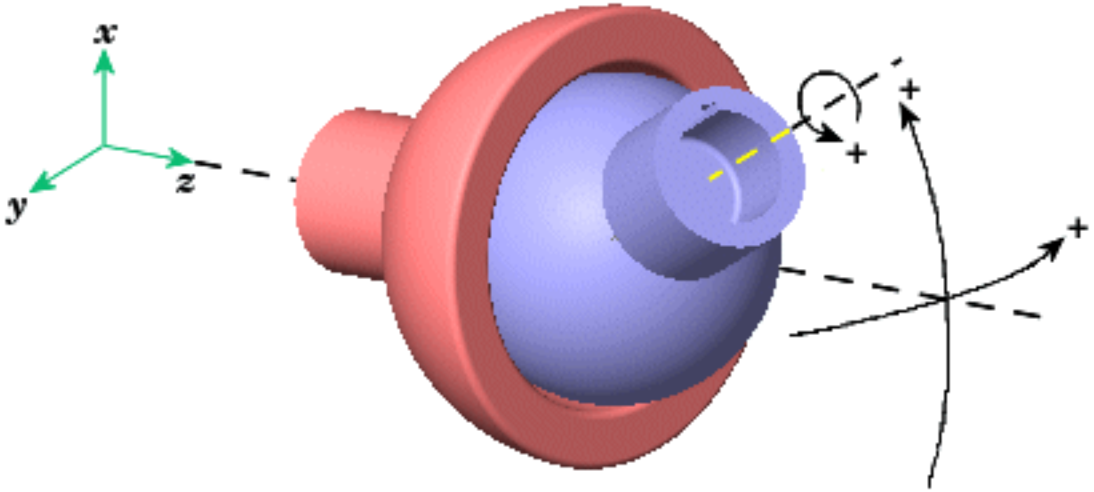
2. "Reduced" Coordinates

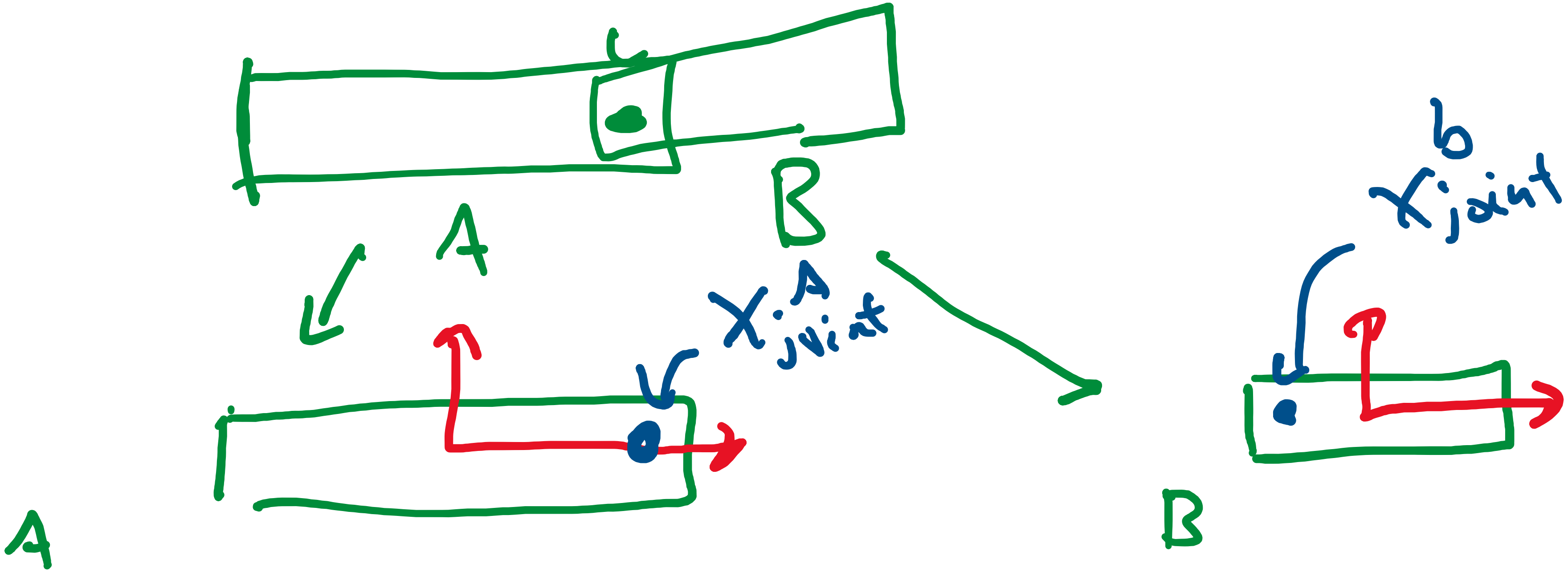
Rigid body has DoF that allow joint motion

Maximal Coordinates



The Simplest Joint: Spherical Joint





$$x^A(\bar{x}^A_{joint}) = R^A \bar{x}^A_{joint} + p^A$$

$$x^B(\bar{x}^B_{joint}) = R^B \bar{x}^B_{joint} + p^B$$

equal

$$C = R^A \bar{x}^A_{joint} - R^B \bar{x}^B_{joint} + p^A - p^B = 0$$

$$c(q^{t+1}) = 0$$

$$c(q^t + \Delta t\, \dot{q}^{t+1}) = 0$$

$$c(q^t) + \Delta t \frac{\partial c}{\partial q} \dot{q}^{t+1} = 0$$

Everything was fine

Constrained Equations of Motion

Step 1: Compute $\dot{q}$

Step 1.5: enforce $\frac{\partial c}{\partial q}\hat{\dot{q}} = 0$

Step 2: Update q

simultaneously

$$c_{spherical} = R^A \bar{x}^A_{joint} - R^B \bar{x}^B_{joint} + p^A - p^B$$

$$\frac{\partial c}{\partial q}\dot{q} = \frac{\partial c}{\partial q}\frac{\partial q}{\partial t} = \frac{dc}{dt}$$

$$\frac{dc}{dt} = \underbrace{R^A [\bar{x}^A_{joint}]^T R^{A^T} \bar{\omega}_A}_{\text{vel. in body space}} - R^B [\bar{x}^B_{joint}]^T R^{B^T}_B \omega_B + \dot{p}^A - \dot{p}^B$$

$$3\ \boxed{\overset{12}{\frac{\partial c}{\partial q} = G}}$$

$$\begin{bmatrix} \omega^A \\ p^A \\ \omega^B \\ p^B \end{bmatrix} \to \text{world space}$$

$$= G\dot{q} = 0$$

Solve at the Velocity Level

Exponential Euler

Fix this

$$R \underline{I} R^T \omega = \tau$$

$$m \dot{p} = F$$

$$=$$

$$\underbrace{\begin{bmatrix} R I R^T & 0 \\ 0 & mI \end{bmatrix}}_{M_A} \underbrace{\begin{bmatrix} \omega \\ \dot{p} \end{bmatrix}}_{\dot{q}_A} = \underbrace{\begin{bmatrix} \tau \\ F \end{bmatrix}}_{F_A} \ldots$$

$$\underbrace{\begin{bmatrix} M_A & 0 \\ 0 & M_B \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \omega_A \\ \dot{p}_A \\ \omega_B \\ \dot{p}_B \end{bmatrix}}_{\dot{q}^{t+1}} = \bar{\Delta t} \underbrace{\begin{bmatrix} \tau_A \\ F_A \\ \tau_B \\ F_B \end{bmatrix}}_{F} + M \dot{q}^t$$

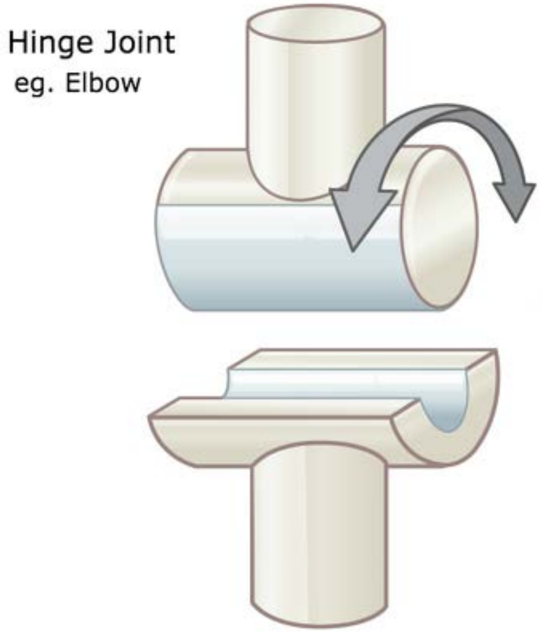
$$G \dot{q}^{t+1} = 0$$

$$M \dot{q}^{t+1} = \Delta t F + \underbrace{\Delta t G^{T} \lambda}_{\text{constraint force}} + M \dot{q}^{t} \qquad \lambda \in \mathbb{R}^{3}$$

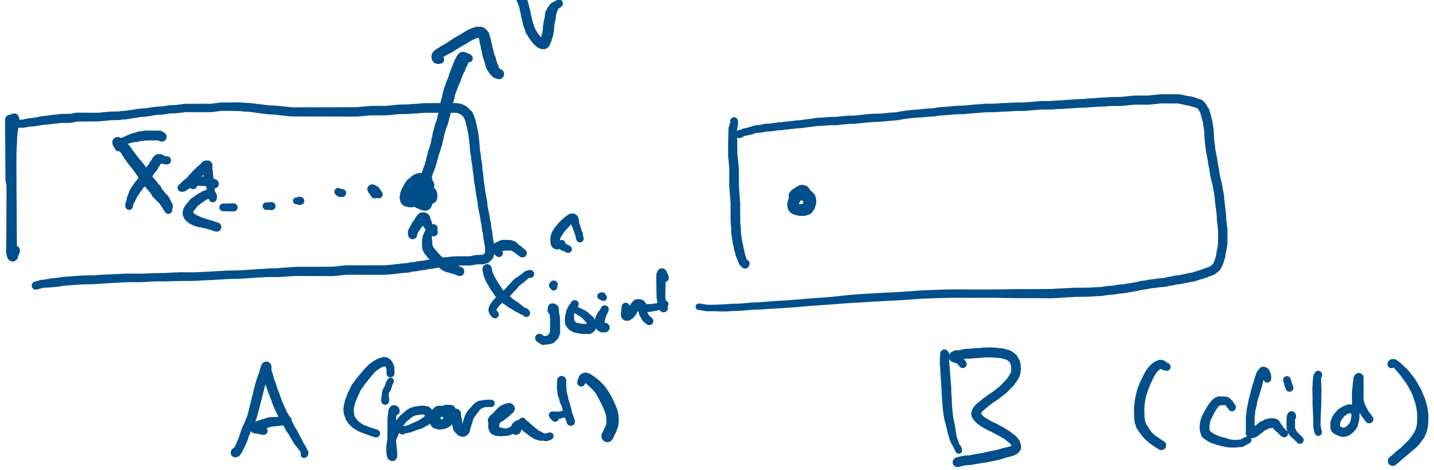
$$G \dot{q}^{t+1} = 0$$

$$\begin{bmatrix} M & G^{T} \\ G & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ -\lambda \Delta t \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \qquad \text{(KKT)}$$

Other Joint Types: Hinge Joint



$G_{spherical} \; \dot{q} = 0$

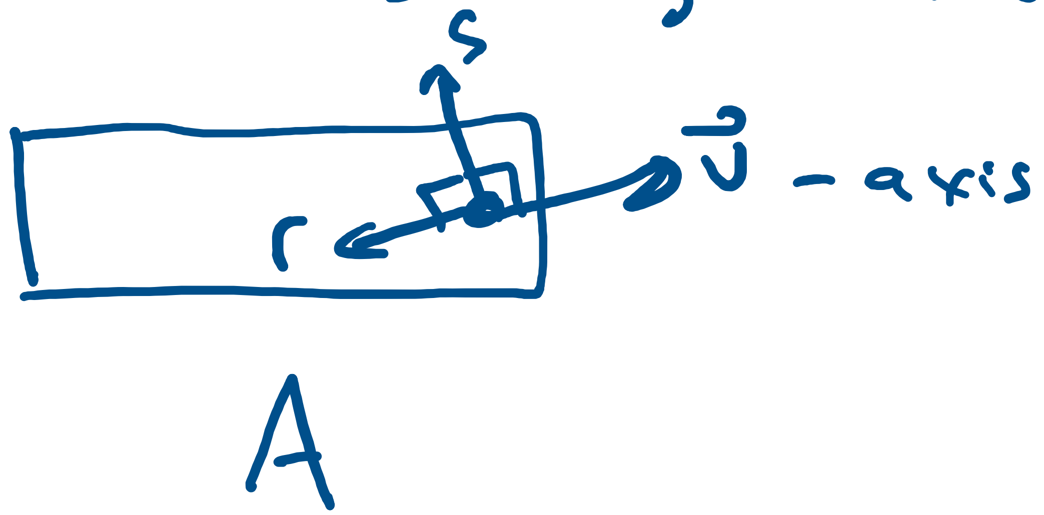


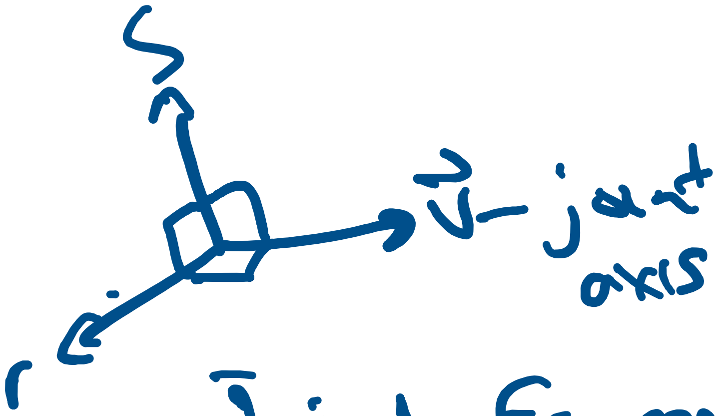
$$0 = R^A(v)\left(\bar{x}^A - \bar{x}^A_{joint}\right) + p^A - R^B(v)\left(\bar{x}^B - \bar{x}^B_{joint}\right) + p^B$$

any scalar

$$\frac{dL}{dt} = R^A [\alpha \hat{v}] \; \ldots \ldots \quad \ldots$$

Ω^A is aligned r/w $\vec{v}$





$$\Omega^{A^T} r = 0$$

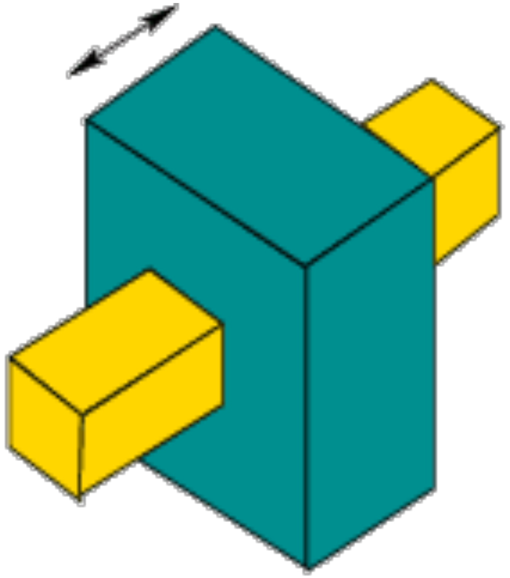
$$\Omega^{A^T} s = 0$$

$$\omega^{A^T} R^A r = 0$$

$$\omega^{A^T} R^A s = 0$$

Add rotations to convert to world spa.

Other Joint Types: Prismatic Joint



Prismatic
1 DOF

Maximal Method Pros and Cons

Pros:

1. Relatively easy to implement and solve (just use standard linear solvers more or less).

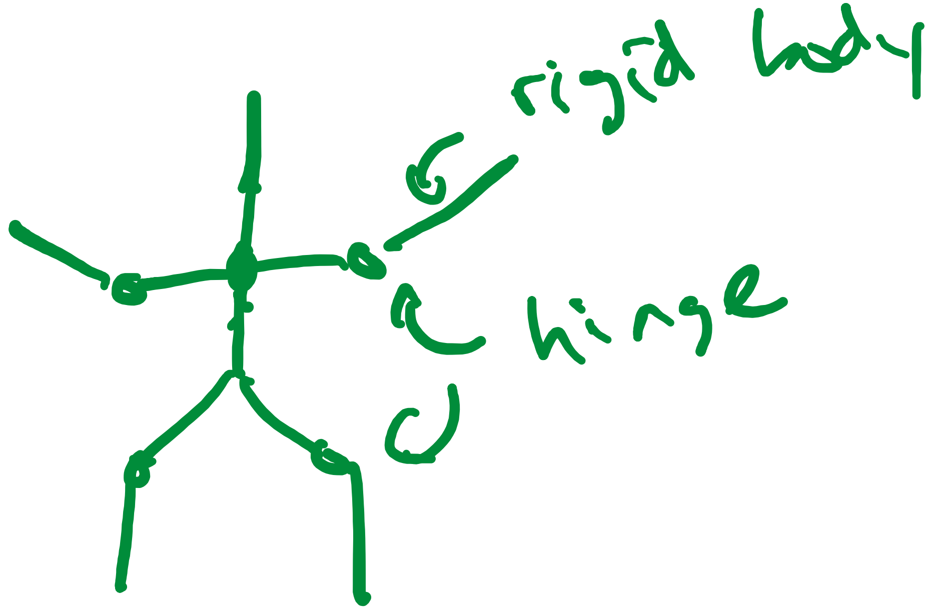
Cons:

1. Constraint Drift -- can be fixed with extra work like Baumgarte Stabilization or Post-Stabilization

2. The systems you solve get large. Every rigid body adds a rotation and translation to the proceedings, and every joint adds at least one constraint.

Joints using Reduced Coordinates

Remove constrained degrees of freedom from dynamics equations



Reduced Coordinates

Maximal Rotations map to the world

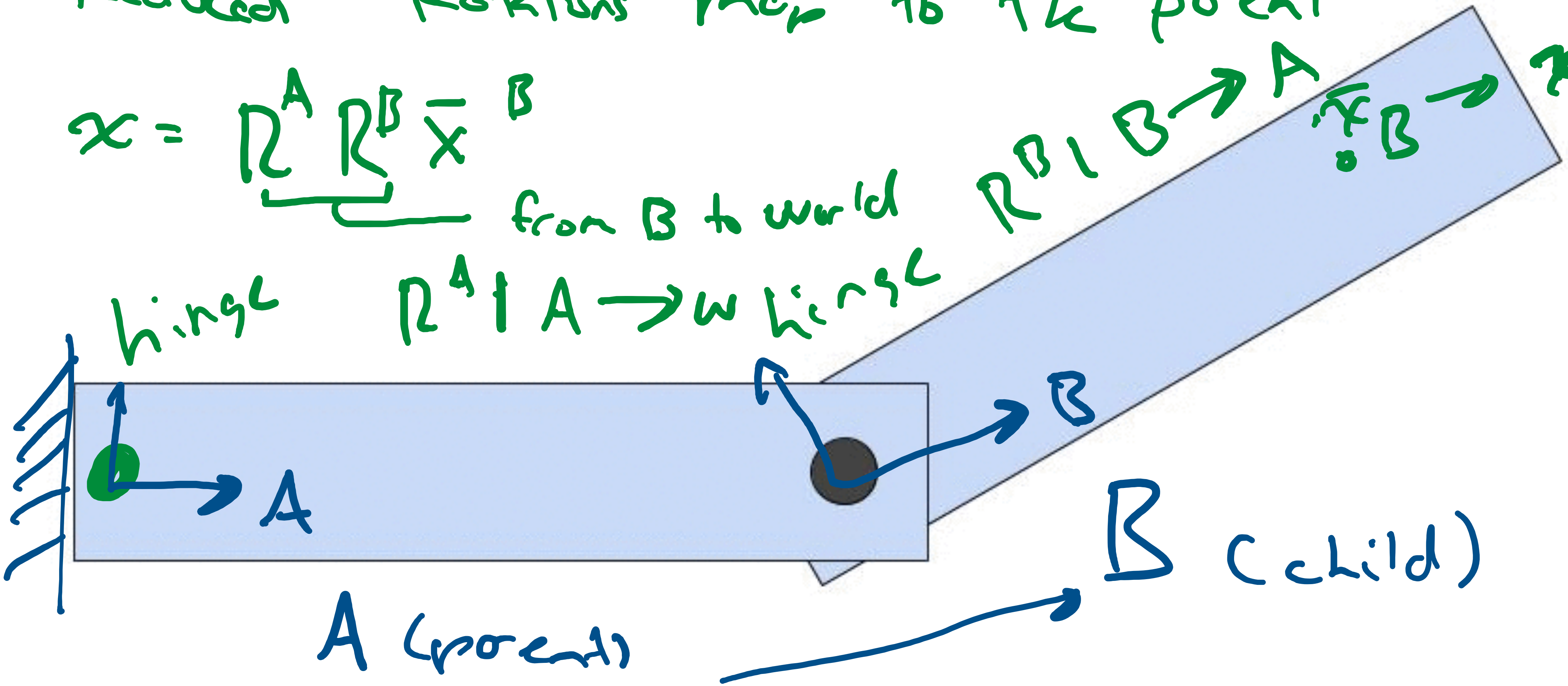
Reduced Rotations map to the parent

$$x = \underbrace{\underbrace{R^A R^B}\bar{x}^B}_{\text{from B to world}}$$

R^B | $B \to A$

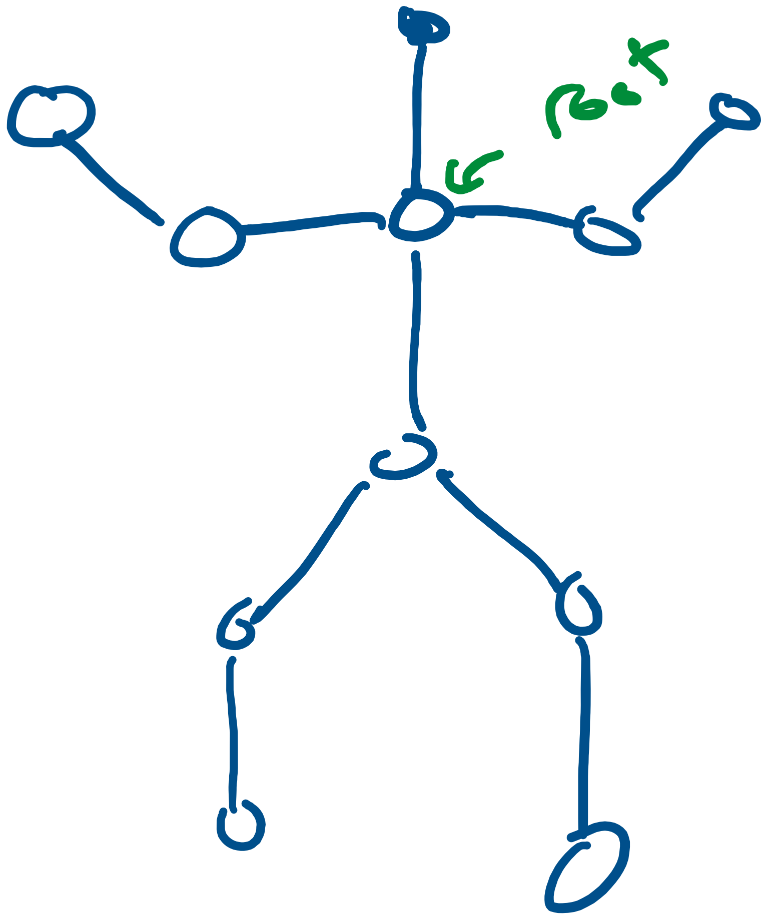
$\bar{x}B \to x$?

hinge R^A | $A \to w$ hinge



Simple Reduced Coordinate Example

The Joint Graph



o - joint

— body

$$x = R^A R^B \bar{x}^B$$

$$\frac{dx}{dt} = R^A [\Omega^A] R^B \bar{x}^B + R^A R^B [\Omega^B] \bar{x}^B$$

$$= R^A [R^B \bar{x}^B]^T R^{AT} \omega^A + R^A R^B [\bar{x}^B]^T R_B^T R_A^T \omega_B$$

$$= R^A R^B [\bar{x}^B]^T R_B^T R_A^T \omega^A + R^A R^B [\bar{x}^B]^T R_B^T R_A^T \omega_B$$

$$= R^A R^B [\bar{x}^B]^T R_B^T R_A^T (\omega^A + \omega^B)$$

$$\dot{q}_B = \underbrace{\begin{bmatrix} I & I & 0 & 0 \end{bmatrix}}_{L_B} \begin{bmatrix} \omega^A \\ \omega^B \\ \omega^C \\ \vdots \end{bmatrix}$$

$$T_B = \frac{1}{2} \int_{\Omega} \rho\, v^T v \, dV$$

${}^{w}_{B}R \rightarrow$ rotation from b $\rightarrow$ world

$$= \frac{1}{2} \dot{q}^T L_B^T \, {}^{w}_{B}R^T \underbrace{\int_{\Omega} \rho \, [\bar{X}_B] \, \underbrace{\cancel{{}^{w}_{B}R^T \, {}^{w}_{B}R}}_{I} \, [\bar{X}_B]^T dV}_{M_B} L_B \, \dot{q}$$

$$T = \sum_{i = \text{rigid bodies}} T_i = \frac{1}{2} \dot{q}^T \underbrace{L^T}_{\text{BLK Diag}} R^T \underbrace{\begin{bmatrix} M_A & & \\ & M_B & \\ & & M_c \end{bmatrix}}_{\text{BLK Diag}} \overbrace{R}^{\text{BLKDIAG}}{}^T L \, \dot{q}$$

$$L_B = [\, I \quad I \quad 0 \quad 0 \quad \dots \,]$$

$$L_A = [\, I \quad 0 \quad 0 \quad 0 \quad \dots \,]$$

$$L_C = [\, I \quad I \quad I \quad 0 \quad 0 \dots \,]$$

L is lower triangular

Solving Equations of Motion on the Joint Graph

$$L^T R^T M R L \ddot{q}^{t+1} = F^t$$

Featherstones

(1) $L^T y = F^t$

$$\begin{bmatrix} I & I & I \\ 0 & I & I \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$z = y_3 + y_2$ $\quad y_1 = F_1 - z$

$y_3 = F_3$, $\quad y_2 = F_2 - y_3$, $\quad y_1 = F_1 - y_2 - y_3$

Reduced Method Pros and Cons

Pros:

1. Constraints are always satisfied

Cons:

1. The linear algebra and the implementations get tricky

2. Basic methods are limited to structures with tree like topology

Next Week

Collision Resolution and Final Assignment