Motion Planning for Mobile Robots - HW4

peng00bo00

July 17, 2020

1. 代码文件可参见./src/grid_path_searcher/src/demo_node.cpp 及./src/grid_path_searcher/src/hw_tool.cpp，运行截图如 Fig.1所示

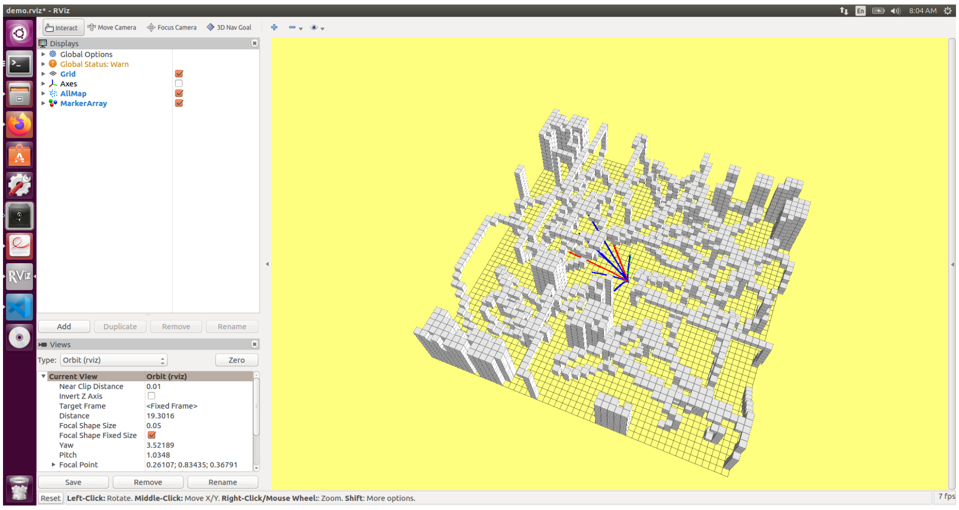

Figure 1: OBVP

2. OBVP 推导

以 x 轴为例，记 $\Delta x = p_{xf} - p_{x0}$，则系数可以表示为

$$
\begin{aligned}
\alpha &= -\frac{12}{T^3}(p_{xf} - v_{x0}T - p_{x0}) + \frac{6}{T^2}(v_{xf} - v_{x0}) \\
&= -\frac{12}{T^3}(\Delta x - v_{x0}T) - \frac{6}{T^2}v_{x0} \\
&= -\frac{12}{T^3}\Delta x + \frac{6}{T^2}v_{x0}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\beta &= \frac{6}{T^2}(p_{xf} - v_{x0}T - p_{x0}) - \frac{2}{T}(v_{xf} - v_{x0}) \\
&= \frac{6}{T^2}(\Delta x - v_{x0}T) + \frac{2}{T}v_{x0} \\
&= \frac{6}{T^2}\Delta x - \frac{4}{T}v_{x0}
\end{aligned}
\tag{2}
$$

将系数带入损失函数，得到

$$
\begin{aligned}
J &= T + \frac{1}{3}\alpha^2T^3 + \alpha\beta T^2 + \beta^2 T \\
&= T + \frac{T^3}{3}(\frac{144}{T^6}\Delta x^2 - \frac{144}{T^5}\Delta x v_{x0} + \frac{36}{T^4}v_{x0}^2) + T^2(-\frac{72}{T^5}\Delta x^2 + \frac{84}{T^4}\Delta x v_{x0} - \frac{24}{T^3}v_{x0}^2) \\
&+ T(\frac{36}{T^4}\Delta x^2 - \frac{48}{T^3}\Delta x v_{x0} + \frac{16}{T^2}v_{x0}^2) \\
&= T + \frac{12}{T^3}\Delta x^2 - \frac{12}{T^2}\Delta x v_{x0} + \frac{4}{T}v_{x0}^2
\end{aligned}
\tag{3}
$$

令 J 的导数为 0，有

$$
J' = 1 - \frac{36}{T^4}\Delta x^2 + \frac{24}{T^3}\Delta x v_{x0} - \frac{4}{T^2}v_{x0}^2 = 0 \tag{4}
$$

等价于求解关于 T 的多项式的根

$$
T^4 - 4v_{x0}^2T^2 + 24\Delta x v_{x0}T - 36\Delta x^2 = 0 \tag{5}
$$

该方程可通过计算对应的伴随矩阵特征值来求解。

将上述步骤推广到 3 个坐标轴得到整体损失函数为

$$
J = T + \frac{12}{T^3}(\Delta x^2 + \Delta y^2 + \Delta z^2) - \frac{12}{T^2}(\Delta x v_{x0} + \Delta y v_{y0} + \Delta z v_{z0}) + \frac{4}{T}(v_{x0}^2 + v_{z0}^2 + v_{z0}^2) \tag{6}
$$

对应的最优时间 T 为如下多项式的根

$$
T^4 - 4(v_{x0}^2 + v_{z0}^2 + v_{z0}^2)T^2 + 24(\Delta x v_{x0} + \Delta y v_{y0} + \Delta z v_{z0})T - 36(\Delta x^2 + \Delta y^2 + \Delta z^2) = 0 \tag{7}
$$