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Indirect inference in structural econometric models

Tong Li

Department of Economics, Vanderbilt University, United States

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ABSTRACT

This paper considers parametric inference in a wide range of structural econometric models. It illustrates how the indirect inference principle can be used in the inference of these models. Specifically, we show that an ordinary least squares (OLS) estimation can be used as an auxiliary model, which leads to a method that is similar in spirit to a two-stage least squares (2SLS) estimator. Monte Carlo studies and an empirical analysis of timber sale auctions held in Oregon illustrate the usefulness and feasibility of our approach.

1. Introduction

This paper considers parametric inference in a wide range of structural econometric models. These models include the job search models studied in Flinn and Heckman (1982), Wolpin (1987) among many others and empirical game-theoretic models such as structural auction models pioneered by Paarsch (1992) and subsequently further developed by Donald and Paarsch (1993, 1996), Laffont et al. (1995), Guerre et al. (2000), Li et al. (2000, 2002), and Hong and Shum (2003), to name a few. A common feature of these models is that while one's primary interest is the parameters that characterize the distribution of some latent variables such as the wage distribution in a job search model or the distribution of bidders' private values in an auction model, one only observes some variables that are related to but not identical to these latent variables. For example, in the job search case, the observed wages are those above the (unobserved) reservation wage, and in the case of auctions, we usually observe bids, which are generally different from private values. Thus, a researcher needs to rely on a structural model that defines a map between these latent and observed variables. This paper is motivated by two major complications arising from the structural inference. First, a consequence of the structural approach is that the support of the distribution of these observed dependent variables usually depends on the parameters of interest, and hence violates the regularity conditions for the consistency and asymptotic normality of the maximum likelihood (ML) estimator.[1] Second, since the relationship between the observed (equilibrium) outcomes and the latent variables defined by a structural model is complicated, it is usually difficult to derive explicit moment conditions and hence to implement Generalized Method of Moments (GMM) or Empirical Likelihood (EL) estimation.

To address the issues arising from the nonregular ML problem, Flinn and Heckman (1982) were the first to suggest employing the ML method subject to the restrictions imposed from economic theory including the condition for the support of the observed wage distribution by using the observed minimum wage as a superconsistent estimate of the (unobserved) reservation wage. Since then, progress has been made in the last two decades. For example, in considering a prototypical job search model with homogeneous samples, Christensen and Kiefer (1991) study the exact likelihood estimation. In empirical auction models, Donald and Paarsch (1993, 1996) first initiate the method of maximum likelihood estimation by considering the estimation as a constrained optimization problem with some constraints becoming binding as the sample size becomes large. Hong (1998) studies asymptotic properties of the nonregular maximum likelihood estimator. In particular, the estimator usually has a convergence rate at n, faster than the usual $\sqrt{n}$ rate, but with a nonstandard asymptotic distribution complicated and dependent

E-mail address: tong.li@vanderbilt.edu.

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[1] Such a problem in implementing the ML method in structural econometric models is first recognized in Flinn and Heckman (1982), while there is a longer history in the statistical literature studying a somewhat related but different problem that is the estimation of the lower (or upper) boundary parameter of a distribution (see, e.g., Woodroofe, 1972, 1974).

on nuisance parameters. More recently, Donald and Paarsch (2002) propose an estimation method using the extreme order statistics, a method that is simpler to implement in practice. They show that, with discrete covariates, their estimator is also superconsistent with convergence rate n.

Although the aforementioned estimators have in general a convergence rate n, they are difficult to calculate and have limiting distributions specific to whether the covariates are discrete or continuous, and also related to the exponential distribution. This makes the inference more involved. It is thus not surprising that the focus from the recent literature has been mainly on proposing easy-to-implement estimation procedures that are consistent but may not be efficient, because at issue is how to make the estimation of structural econometric models tractable.[2] See, e.g., Laffont et al. (1995), Bajari (1998), Hong and Shum (2003), among others. This paper follows this line of literature in aiming to provide a computationally convenient procedure for estimating the structural models discussed above.[3] Specifically, we show that the indirect inference principle recently developed by Smith (1993), Gourieroux et al. (1993), and Gallant and Tauchen (1996) can be used in proposing a computationally convenient estimator that has the standard root-n asymptotic normality. In particular, the estimation involves two steps. In a first step, an auxiliary model that is easy to estimate is estimated using the real data to get estimates for those auxiliary parameters, which are functions of the structural parameters of interest. In the second step, the structural model is simulated and the simulated data are used to estimate the auxiliary parameters. The estimates for the structural parameters are obtained when some criterion that measures the difference between these estimates for the auxiliary parameters obtained from the actual data at the first step and from the simulated data at the second step is minimized. While the principle of indirect inference is proposed by Smith (1993), Gourieroux et al. (1993), and Gallant and Tauchen (1996) as a general estimation principle in estimating otherwise intractable econometric models, at issue is how to choose an appropriate auxiliary model in a particular problem. Although in principle a pseudo maximum likelihood estimation (PMLE) can be used for the first-stage auxiliary model, where a possibly misspecified likelihood function can be maximized, we offer a particular choice for the auxiliary models for its intuitive appeal and attractive computational convenience. Specifically, we demonstrate that an ordinary least squares (OLS) estimation can be used as an auxiliary model. This is an interesting approach for several reasons. First, using OLS as an auxiliary model significantly enhances the computational efficiency. Second, nonlinear functions of the covariates that the structural model conditions on can be used as additional instruments in the OLS estimation; the resulting indirect inference estimator can be more efficient because the use of the additional instruments makes the structural model of interest over-identified. Third, the auxiliary parameters have the interpretation of reduced-form parameters, and hence our method makes the connection between the modern structural econometric models and the conventional Cowles Commission structural simultaneous equations models.[4] Lastly, noting that many empirical studies have used the reduced-form approach to conduct analysis given the complications of a structural analysis, our approach offers an effective way to bridge the gap between the reduced-form and structural approaches through the indirect inference principle.

Besides the advantage of computational convenience, our approach makes inference in structural models feasible. For example, model specification and hypothesis testing can be made readily within the framework of the indirect inference. In addition, since the encompassing method for selecting between two non-nested models have been extended to the case where an indirect inference estimator is used for either/both alternative models (see, e.g., Dhaene et al., 1998), our approach can be adapted to distinguishing between non-nested structural models such as two different equilibria, or two competing primitive models. Another advantage of our approach is that it can be less sensitive to the outliers than the extreme order statistics-based estimator and the constrained MLE, which both heavily rely on the extreme values of the observations.

We conduct Monte Carlo studies to demonstrate the usefulness and feasibility of our methods. Moreover, we apply our method to analyze timber sale auctions that were held in Oregon and organized by the Oregon Department of Forestry. We use the method proposed in the paper to provide a structural analysis to this new data set by estimating the distribution of private values accounting for the heterogeneity of the auctioned timber lots.

This paper is organized as follows. Section 2 is devoted to proposing a unified approach for using the indirect inference principle in structural econometric models. Section 3 presents Monte Carlo results that demonstrate the usefulness and feasibility of our approach. Section 4 is devoted to the empirical application. Section 5 concludes.

2. Indirect inference in structural econometric models

2.1. The canonical model

We present in this subsection the basic or canonical model that will be studied. In structural modeling, we are interested in estimating the distribution of a latent variable, say, v, with a known support on $[a, b]$. Suppose that the distribution of v conditioning on some covariates is parameterized by a vector of K parameters $\theta \in \Theta$, where Θ is a compact set in $\mathbb{R}^K$. Denote this distribution by $F(\cdot|\mathbf{x}, \theta)$, where $\mathbf{x} \in \Xi \subset \mathbb{R}^q$ is a vector of q variables that are used to control for the observed heterogeneity. The corresponding density is denoted by $f(\cdot|\mathbf{x}, \theta)$. The true value of the structural parameters is denoted by θ^0. While v is partially or fully unobserved, we observe a (dependent) variable y, which, as a result of the structural modeling, has a density also dependent on θ with a support on $[h_1(\mathbf{x}, \theta), h_2(\mathbf{x}, \theta)]$. We denote the distribution of y as $G(\cdot|\mathbf{x}, \theta)$ and the corresponding density as $g(\cdot|\mathbf{x}, \theta)$. Also, assume that n independent observations y_i and $\mathbf{x}_i$, $i = 1, \ldots, n$ are available for both y and $\mathbf{x}$. Note that, if v were observed, then estimation of θ could be done as a straightforward application of the ML method. However, we do not observe v. Therefore a main objective of the structural approach is to estimate the structural parameters θ^0 using the observations on both y and $\mathbf{x}$. We have the following assumption on the latent density $f(\cdot|\mathbf{x}, \theta)$.

A1: For each $\theta \in \Theta$, $f(\cdot|\cdot, \theta)$ is a Borel measurable function on $[a, b] \times \Xi$; and for each $(v, \mathbf{x}) \in [a, b] \times \Xi$, $f(v|\mathbf{x}, \cdot)$ is a continuous function on Θ.

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[2] In fact, Chernozhukov and Hong (2004) and Hirano and Porter (2003) show that for structural models with parameter-dependent support, the (constrained) ML estimator is generally inefficient, but the Bayes estimator is efficient according to the local asymptotic minmax criterion for standard loss functions such as the squared error loss.

[3] Some structural models such as the equilibrium search models (Eckstein and Wolpin, 1990; Hong and Shum, 2001) do not have the parameter-dependent support problem for the MLE. But the implementation of the MLE in these models can also be computationally demanding. The method proposed in this paper can be applied to these models as well.

[4] See Heckman (2001) for an insightful overview on the development of structural econometrics.

Before proceeding, we need to clarify several issues surrounding the canonical model. First, it is possible that some additional parameters such as bidders' risk aversion parameters that are not associated with the latent density can also affect the dependent variable y and hence enter both $g(\cdot)$ and the bounds of its support. If this is the case, estimating this additional set of parameters becomes part of the task as well. For ease of exposition and discussion, however, the canonical model does not include this case, and we will defer the discussion of this case to Section 2.2 when we discuss extensions of our approach. Second, in the canonical model, both bounds of the support of the density $g(\cdot|\mathbf{x}, \theta)$ for y can depend on θ. Thus this model is general enough to include those with only one bound depending on θ. Third, both $h_1(\mathbf{x}, \theta)$ and $h_2(\mathbf{x}, \theta)$ are required to be explicit functions of $\mathbf{x}$ and θ, or if not, they can be determined through some numerical procedure. Moreover, y is usually a function of v and $f(\cdot|\mathbf{x}, \theta)$, for example, in the job search case, $y = v$ for y above some range, and in the auction case, y is a strictly increasing function of v, which is given as a solution to some differential equation. Hence it is also required that such a function is either explicitly expressed or implicitly given and can be numerically determined for any value of θ, as is the case for the piecewise pseudo maximum likelihood estimation or constrained maximum likelihood estimation (Donald and Paarsch, 1993, 1996) and the estimation using the extreme order statistics (Donald and Paarsch, 2002).[5]

2.2. The indirect inference approach

We will focus in this subsection on how the estimation of the canonical model can be made tractable using the indirect inference principle. The method of indirect inference was initially used in Smith (1993) in estimating nonlinear time series models. It was later formulated as a general methodology in Gourieroux et al. (1993) using parameter calibration. Also, Gallant and Tauchen (1996) propose a similar method from an alternative point of view, that is to match scores of a quasi ML procedure by a GMM approach with simulation techniques if these scores are difficult to calculate. Thus the method of Gallant and Tauchen has been called the efficient method of moments (EMM).[6] Since then, the method of indirect inference has been applied mainly in financial time series econometrics; see, e.g., Broze et al. (1995), Pastorello et al. (2000) among others. This method, however, has been rarely applied in microeconometrics with only a few exceptions that use it to tackle the problems caused by the initial conditions problem (An and Liu, 2000) and the censoring problem of individual employment histories (Magnac et al., 1995) in labor market transitions.[7]

5 For numerical procedures that can be used to calculate the Nash–Bayesian equilibrium strategies in auctions when they have no explicit solutions, see Marshall et al. (1994).

6 While these two methods are asymptotically equivalent as shown by Gourieroux et al. (1993), which one to use is a practical question depending on the auxiliary model used in the procedure. The parameter calibration is preferred if the auxiliary model can be estimated routinely, while one can use the EMM when the auxiliary model has a closed-form score function. Moreover, an advantage of the method by Gourieroux et al. (1993) is that it does not require simulating the exogenous variables.

7 In discussing the potential applications of the indirect inference, Gourieroux et al. (1993) suggest using it in discrete-choice models with a large number of alternatives as a leading example in microeconometric applications. Gallant and Tauchen (1996) suggest using their EMM to estimate empirical auction models. They mention that a good approximation to the conditional distribution of bids given the covariates as a score generator or an auxiliary model can be used. However, a good approximation to the bids distribution can be difficult to find in practice given the complicated feature of the equilibrium strategy, leading often to an intractable bids distribution, not to mention the complication due to the problem of the parameter-dependent support of the bids distribution.

Since our approach makes use of auxiliary models that are easy to estimate in practice, we will follow the indirect inference based on parameter calibration. This method originates from the following idea. First, an auxiliary and possibly misspecified model is estimated using the observations leading to estimates for the auxiliary parameters whose dimension is at least that of the structural parameters of interest. Since in general the relationship between the auxiliary model and the structural model is complicated and hence does not have an explicit expression, the structural model is simulated and the simulated data are used to estimate the auxiliary model. The final estimates for the structural parameters are obtained when the estimates for the auxiliary parameters using the real data and the simulated data are closest according to some measure. It is clear that the auxiliary model plays a pivotal role in this method, as a good choice of auxiliary model is crucial for achieving the goal of the indirect inference, that is to make the estimation of some complicated models tractable. In particular, for estimating latent structural models such as option pricing models and auction models, Renault (1997) has suggested using PMLE as an auxiliary model by first maximizing the latent log-likelihood where the latent variable is replaced by the observed dependent variable.[8] In this paper we propose using an OLS as an auxiliary model.

To fix the idea, we first present the new procedure using the indirect inference principle, and then establish the asymptotic results. Let $\mathbf{z}$ denote a q-dimensional vector of variables, consisting of $\mathbf{x}$ and powers of some components of $\mathbf{x}$ such that $q \geq K - 1$. Let $\tilde{\mathbf{z}}$ denote $(1, \mathbf{z}')'$. Note that $\tilde{\mathbf{z}}$, whose dimension can be greater than that of the structural parameters of interest because of the inclusion of the nonlinear functions of the covariates $\mathbf{x}$, serves as "instruments" which make the structural model over-identified when $q > K - 1$, and exactly identified when $q = K - 1$. Now, in the first step, regress the n observations y_n^1 of the dependent variables on $\tilde{\mathbf{z}}_n^1$, and get an OLS estimate $\hat{\beta}_n$. In the second step, for any given value $\theta \in \Theta$, we consider S simulated paths: $[v_i^s(\mathbf{x}_i, \theta),\ i = 1, \dots, n]$, $s = 1, \dots, S$, where v_i^s are independent draws from $f(\cdot|\mathbf{x}_i, \theta)$, and then calculate from the structural model the corresponding values for the observed dependent variable $[\tilde{y}_i^s(\mathbf{x}_i, \theta),\ i = 1, \dots, n]$, $s = 1, \dots, S$. Now, for each $s = 1, \dots, S$, we regress $(\tilde{y}^s)_n^1$ on $\tilde{\mathbf{z}}_n^1$ and denote the resulting OLS estimate by $\hat{\beta}_n^s$. Note that $\hat{\beta}_n^s$ is a function of the trial value θ: $\hat{\beta}_n^s = \hat{\beta}_n^s(\theta)$. The indirect inference estimator $\widehat{\theta}_I^A$ for the structural parameters θ^0 is now defined as the solution to the following minimum distance problem:

$$\operatorname*{Min}_{\theta \in \Theta} \left[\hat{\beta}_n - \frac{1}{S} \sum_{s=1}^{S} \hat{\beta}_n^s(\theta) \right]' A \left[\hat{\beta}_n - \frac{1}{S} \sum_{s=1}^{S} \hat{\beta}_n^s(\theta) \right], \tag{1}$$

where A is a $(q + 1) \times (q + 1)$ positive definite matrix.

Note that first the indirect inference estimator defined above depends in general on the choice of the positive definite matrix A, which serves as a weighting matrix here. The optimal choice of A that minimizes the asymptotic covariance matrix of the indirect inference estimator will be given later. Second, the indirect inference estimator here is defined as a solution to the minimum distance problem as given in (1). This is because we have $q \geq K - 1$, and hence the number of the auxiliary parameters, which is $q + 1$, is at least the same as the number of the structural parameters. When $q = K - 1$, or the number of the auxiliary parameters is equal to the

8 See also applications of this method in Pastorello et al. (2000). Moreover, following the research agenda in Renault (1997), a more efficient procedure has been proposed by Pastorello et al. (2003). I am grateful to a referee for bringing Renault (1997) and Pastorello et al. (2003) to my attention.

number of the structural parameters, the problem of (1) is reduced to finding the indirect inference estimator $\widehat{\theta}_I$ as the solution to the following equation:

$$\hat{\beta}_n = \frac{1}{S}\sum_{s=1}^{S}\hat{\beta}_n^s(\widehat{\theta}_I).$$

In this case, the choice of A is irrelevant.

To give a theoretical justification of the approach suggested above, we need to investigate the asymptotic properties of the indirect inference estimator $\widehat{\theta}_I^A$. We will proceed in several steps. First, we note that the OLS estimator $\hat{\beta}_n$ can be viewed as the solution to maximizing the objective function

$$S_n(y_n^1, \mathbf{z}_n^1, \beta^a) = -\frac{1}{n}\sum_{i=1}^{n}(y_i - \tilde{\mathbf{z}}_i'\beta^a)^2, \tag{2}$$

for $\beta^a \in \mathbb{R}^{q+1}$. Here the β^a are referred to as the auxiliary parameters. It is clear that as the sample size increases, $\hat{\beta}_n$ does not converge to the true value of the structural parameters but to something else. In fact, it can be readily verified that $\hat{\beta}_n$ converges in probability to $\beta^* = (\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}'])^{-1}\mathrm{E}_{\mathbf{x},y;\theta^0}[\tilde{\mathbf{z}}'y]$ under the following assumption.

A2: $\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}']$ is non-singular.

It is now necessary to define a binding function to link the auxiliary parameters and the structural parameters. To this end, define $\tilde{b}(F, G, \theta) = (\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}'])^{-1}\int_{\Xi}\int_{h_1(\mathbf{x},\theta)}^{h_2(\mathbf{x},\theta)}\tilde{\mathbf{z}}'yg(y|\mathbf{x},\theta)f_{\mathbf{x}}(\mathbf{x})\mathrm{d}y\mathrm{d}\mathbf{x}$. Then $\beta^* = \tilde{b}(F_0, G_0, \theta^0)$. We make the following assumption.

A3: $\tilde{b}(F_0, G_0, \theta) \neq \tilde{b}(F_0, G_0, \theta^0)$ for $\theta \neq \theta^0$, and $\frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta)$ is of full column rank K for any $\theta \in \Theta$.

Note that A3 is made to ensure the identification of the underlying structural parameters. The validity of A3, however, may depend on the family of parametric distributions that is specified, as is the case for many nonlinear parametric models. We can then establish the consistency and asymptotic normality of $\widehat{\theta}_I^A$, following Gourieroux et al. (1993).

Consistency: *Assume* A1, A2 and A3. *For any fixed S, as the sample size* $n \to \infty$, $\widehat{\theta}_I^A$ *converges to* θ^0 *in probability*.

Asymptotic normality: *Assume* A1, A2, A3 *and the usual regularity conditions. Then* $\widehat{\theta}_I^A$ *is asymptotically normally distributed. Specifically, for fixed S and when n goes to infinity*,

$$\sqrt{n}(\widehat{\theta}_I^A - \theta^0) \to^d N[0, W_S],$$

where

$$\begin{aligned} W_S \;=\; & \left(1+\frac{1}{S}\right)\left(\frac{\partial\tilde{b}'}{\partial\theta}(F_0, G_0, \theta^0)A\frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta^0)\right)^{-1} \\ & \times\frac{\partial\tilde{b}'}{\partial\theta}(F_0, G_0, \theta^0)A(\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}'])^{-1}\tilde{H}(\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}'])^{-1} \\ & \times A\frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta^0)\left(\frac{\partial\tilde{b}'}{\partial\theta}(F_0, G_0, \theta^0)A\frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta^0)\right)^{-1}, \end{aligned}$$

$$\tilde{H} = \mathrm{E}[(y - \mathrm{E}_{\mathbf{x}}[y|\mathbf{x}])^2\tilde{\mathbf{z}}\tilde{\mathbf{z}}'].$$

Moreover, the optimal choice for A is $A_{opt} = \mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}']\tilde{H}^{-1}\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}']$, *and the corresponding asymptotic variance–covariance matrix for* $\widehat{\theta}_I^A$ *is*

$$\begin{aligned} W_{S,opt} = & \left(1+\frac{1}{S}\right) \\ & \times\left(\frac{\partial\tilde{b}'}{\partial\theta}(F_0, G_0, \theta^0)\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}']\tilde{H}^{-1}\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}']\frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta^0)\right)^{-1}. \end{aligned}$$

It is helpful to note that $\frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta^0)$ can be expressed as follows:

$$\begin{aligned} \frac{\partial\tilde{b}}{\partial\theta'}(F_0, G_0, \theta^0) = \; & (\mathrm{E}_{\mathbf{x}}[\tilde{\mathbf{z}}\tilde{\mathbf{z}}'])^{-1} \\ & \times\Bigg[\int_{\Xi}\int_{h_1(\mathbf{x},\theta^0)}^{h_2(\mathbf{x},\theta^0)}\tilde{\mathbf{z}}'y\frac{\partial g(y|\mathbf{x},\theta^0)}{\partial\theta'}f_{\mathbf{x}}(\mathbf{x})\mathrm{d}y\mathrm{d}\mathbf{x} \\ & + \int_{\Xi}\tilde{\mathbf{z}}'yg(h_2(\mathbf{x},\theta^0)|\mathbf{x},\theta^0)\frac{\partial h_2(\mathbf{x},\theta^0)}{\partial\theta'}f_{\mathbf{x}}(\mathbf{x})\mathrm{d}x \\ & - \int_{\Xi}\tilde{\mathbf{z}}'yg(h_1(\mathbf{x},\theta^0)|\mathbf{x},\theta^0)\frac{\partial h_1(\mathbf{x},\theta^0)}{\partial\theta'}f_{\mathbf{x}}(\mathbf{x})\mathrm{d}x\Bigg]. \end{aligned}$$

It is also worth noting that, in applications, the calculation of the asymptotic variance–covariance matrix for the indirect inference estimator as given above can be cumbersome as the approximation of derivatives of functions and integrations are needed. If this is the case, then one can use the bootstrap method to get an estimate for the asymptotic variance–covariance matrix.[9]

The case studied in this subsection and the estimation procedure proposed are of particular interest as they provide an estimation procedure that is based on the OLS and hence very easy to implement, and at the same time offer a link between the reduced-form approach and the structural approach. The reduced-form approach has been used in empirical analysis for a long time in economics.[10] Most of the work in using the structural approach also uses the reduced-form approach to conduct some preliminary studies and to gain some insight before conducting the structural analysis. The reduced-form and structural approaches, however, are not as integrated as they are in the conventional Cowles Commission approach in estimating simultaneous equations models. Our approach bridges such a gap and shows that the reduced-form approach can be not only used as a tool for a preliminary analysis, but also as an intermediate in the structural approach leading to a computationally convenient procedure thanks to the indirect inference principle. In fact, the estimation procedure proposed here resembles the conventional Cowles Commission structural econometrics spirit and can be viewed as an analog to the two-stage least squares (2SLS) estimator with the first stage estimating the reduced-form parameters and the second stage recovering the structural parameters based on simulation and parameter calibration. [11]

Another issue of interest, as mentioned earlier, is that, in some applications, it is possible that additional parameters such as bidders' risk aversion parameters that are not related to the latent density can affect the observed dependent variable y as well. While this case is not included in the canonical model described in Section 2.1 which has been the focus of the paper, it can also be resolved using the approach proposed in this subsection by using OLS as an auxiliary model. This is because with the OLS

9 As noted in Horowitz (2001) and Donald and Paarsch (2002), the validity of bootstrap is questionable for the MLE-based estimators in estimating structural econometric models studied in this paper because of the parameter-dependent support problem. On the other hand, the bootstrap is valid in our case as our estimators can be viewed as GMM estimators. See Horowitz (2001) for the bootstrap method in standard estimation methods including GMM. In addition, bootstrapping is not a demanding task in our case given that our estimators are easy to implement.

10 See, e.g., the pioneering work of Hendricks and Porter as surveyed in Porter (1995) in using the reduced-form analysis on the Outer Continental Shelf (OCS) auction data to test auction theory.

11 As pointed out by a referee, Broze et al. (1998) mention the analogy between the indirect inference method and the 2SLS method, and Dridi (1999) discusses the issue of indirect inference with over-identification of the auxiliary model. I am grateful to the referee for bringing these references to my attention.

as an auxiliary model, we can have the number of the auxiliary parameters at least the same as the number of the structural parameters including those associated with the underlying latent variable density and those additional parameters such as the bidders' risk aversion parameter. This can be achieved by using not only **x**, but also nonlinear functions of **x** as the independent variables in the first stage OLS regression.

2.3. Inference in structural econometric models

A considerable advantage of our indirect inference estimators as proposed in the last two subsections is that they make the inference in complicated structural econometric models feasible. Though the structural econometric models considered in this paper can be estimated by likelihood-based methods in principle, the inference based on these methods is complicated and challenging. The major problem here arises from the complication of the asymptotic distribution of the resulting estimators, not to mention the computational demand of obtaining the estimators alone. Thus, even getting standard errors and testing parameter significance could become a formidable task with using these estimators. Furthermore, it is often the case in the structural analysis that a researcher faces competing models to select, such as choosing between different parametric specifications of the model, or game theory leads to multiple equilibria. Therefore, one needs some testing procedures for non-nested models to decide which model to choose.[12]

Since our indirect inference estimator has a standard root-n asymptotic normality, the standard procedures for testing significance of or restrictions on parameters remain valid. In fact, Gourieroux et al. (1993) show that despite the use of the simulations for parameter calibration, the asymptotic equivalence among the Wald test, the score test, and the test based on the comparison of the constrained and unconstrained values of the objective function used in the second step is still valid.

For testing non-nested hypotheses with indirect inference estimators, Dhaene et al. (1998) establish that the usual asymptotic results for direct inference with non-nested hypotheses such as encompassing test carry over to indirect inference.[13] The test statistics as discussed in Dhaene et al. (1998) are computationally convenient with the use of the indirect inference estimator. Such a feature adds to the advantage in the inference of structural models, which often calls for testing non-nested hypotheses.

3. Monte Carlo results

This section focuses on Monte Carlo studies that investigate the small sample performance of our proposed estimators. As an illustration, we consider using our proposed estimators in the estimation of structural auction models.

We consider auctions within an independent private paradigm. To be more specific, we consider the auctions that are held as Dutch auctions in which only winning bids that are the highest bids are observed to the econometrician and hence have more limited data than first-price sealed-bid auctions with all bids observed. In our design, the number of bidders is fixed across all auctions and is chosen as $N = 6$. The sample size is $n = 100$, meaning that we consider 100 auctions, which is about the usual sample size for auctions. At the ℓ-th auction, $\ell = 1, \ldots, 100$, the bidders draw their private values from an exponential distribution with the density

$$f(v|x_\ell) = \frac{1}{\exp(\theta_0 + \theta_1 x_\ell)} \exp\left[-\frac{1}{\exp(\theta_0 + \theta_1 x_\ell)} v\right],$$

where x_ℓ, $\ell = 1, \ldots, 100$ are generated from the square of a Uniform variable on (0, 2) and are used to control for some observed heterogeneity of the auctioned objects, and $\theta' = (\theta_0, \theta_1) = (1, 0.5)$. It is worth noting that following Riley and Samuelson (1981) among others, at the ℓ-th auction, the Nash–Bayesian equilibrium strategy for the winning bidder can be written as

$$b_{w,\ell}(v_{w,\ell}) = v_{w,\ell} - \frac{1}{F^{N-1}(v_{w,\ell}|x_\ell)} \int_0^{v_{w,\ell}} F^{N-1}(u|x_\ell)\mathrm{d}u,$$

where $v_{w,\ell}$ is the private value of the winning bidder at the ℓ-th auction, $b_{w,\ell}(v_{w,\ell})$ is the corresponding equilibrium bid, and $F(\cdot|x_\ell)$ is the cumulative distribution associated with the density $f(\cdot|x_\ell)$.

In this Monte Carlo study, we evaluate the performance of both estimators using the PML as the auxiliary model and the OLS as the auxiliary model with 400 replications. Table 1 reports the Monte Carlo results on the estimates of the structural parameters using the indirect inference estimator with the PML as the auxiliary model by maximizing in the first stage $\sum_{\ell=1}^{n} \log f(b_{w,\ell}|x_\ell)/n$, which is suggested by Renault (1997). Table 2 reports the results from the indirect inference estimation using the OLS as the auxiliary model proposed in Section 2.2, where the first-stage OLS is conducted by regressing $b_{w,\ell}$ on a constant and x_ℓ. In both cases, the number of simulations used in the second step of the estimation is chosen to be 1, meaning that we choose the minimum number for the simulation. As it turns out, even with such a minimum number of simulation in the second step, the estimation in both cases performs remarkably well, as evidenced by both the small sizes of biases and small magnitudes of standard deviations in Tables 1 and 2.[14] Also, while the asymptotic variances for the indirect inference estimators using the PMLE and OLS as auxiliary models respectively are difficult to compare analytically, as indicated from Tables 1 and 2, the standard deviations from the two methods are quite close, with the one using the PMLE as the auxiliary model being a bit more efficient than the one using the OLS as the auxiliary model. Of course, this could be because of the specification of the structural model in our design. Moreover, all the programs are written in Gauss and run in a Pentium IV 1.7 GHz computer. In both cases, it takes less than one second to get the results for one replication, indicating that the estimators are computationally efficient. This constitutes a considerable advantage for our estimators.

Table 1
Estimates for structural parameters using PML as the auxiliary model.

Estimates	Mean	Median	Minimum	Maximum	Std. Dev.
$\hat{\theta}_0$	0.9692	0.9699	0.8470	1.0748	0.0436
$\hat{\theta}_1$	0.5039	0.5032	0.4355	0.5594	0.0227

Table 2
Estimates for structural parameters using OLS as the auxiliary model.

Estimates	Mean	Median	Minimum	Maximum	Std. Dev.
$\hat{\theta}_0$	0.9518	0.9473	0.7932	1.0756	0.0461
$\hat{\theta}_1$	0.5117	0.5113	0.4217	0.5862	0.0261

12 For testing non-nested hypotheses with estimators that are asymptotically normally distributed with rate $\sqrt{n}$, see the survey by Gourieroux and Monfort (1994).

13 For using the encompassing principle in testing non-nested models, see, e.g., Mizon and Richard (1986), and Wooldridge (1990), among many others.

14 We have also tried different numbers of simulations in the second step such as $S = 50$ and $S = 100$; the results are very similar to those reported in Tables 1 and 2, with only minor reductions in standard deviations of the estimates. These results are available from the author upon request.

Table 3
Summary for bootstrapped standard errors of estimates of structural parameters using PML as the auxiliary model.

	Mean	Median	Min.	Max.	Std. Dev.
Std. error of $\hat{\theta}_0$	0.0461	0.0462	0.0430	0.0507	0.0012
Std. error of $\hat{\theta}_1$	0.0275	0.0275	0.0257	0.0294	0.0007

Table 4
Summary for bootstrapped standard errors of estimates of structural parameters using OLS as the auxiliary model.

	Mean	Median	Min.	Max.	Std. Dev.
Std. error of $\hat{\theta}_0$	0.0531	0.0536	0.0397	0.0703	0.0047
Std. error of $\hat{\theta}_1$	0.0275	0.0278	0.0200	0.0468	0.0038

As mentioned previously, the bootstrap can be used as an alternative in estimating the asymptotic variance–covariance matrix of our estimators given that such a matrix is computationally complicated. In view of this, it would be useful to study the performance of the bootstrap when it is used for inference purposes. Since in this paper we consider parametric structural econometric models, in which the distribution of the latent variables is parametrically specified up to some finite-dimensional unknown parameters, parametric bootstrap is preferred to nonparametric bootstrap.[15] The results on the bootstrapped standard errors on the structural estimates with the PML as the auxiliary model and the OLS as the auxiliary model are reported in Tables 3 and 4, respectively. These results are obtained with 800 replications. As can be seen from Tables 3 and 4, in both cases, the mean and median of the bootstrapped standard errors are quite close to the standard deviations of the structural estimates reported in Tables 1 and 2. This offers a strong evidence in supporting the use of the bootstrap as a reliable alternative in inference.

4. An empirical application

This section is devoted to using our method to conduct a structural analysis of timber sale auctions organized by the Oregon Department of Forestry (ODF). Our data set contains the timber sale auctions that were held from May 2000 through October 2002. Note that while timber sale auctions have been analyzed in the empirical auction literature, the studies of the auctions that were held in the US have used the data from auctions that were organized by the US Forest Service.[16] Therefore, our data set is new and our empirical analysis contributes to the growing literature on studying timber auctions in addition to its merit as an illustration of our estimation method.

All auctions organized by the ODF are held as first-price sealed-bid auctions. Well before each auction, the ODF makes an announcement called a "notice of timber sale" with the information on the auctioned tract available to the public. The information includes the appraised volumes and quality of bid species, as well as the required minimum bids on these bid species. In effect, these minimum bids serve as the reserve prices because the bids submitted have to be above the minimum bids. The information also includes the region where the tract is located, as well as the time when the submitted bids will be opened. Once the sealed bids are opened, the tract is awarded to the highest bid.

4.1. Data

The data analyzed in this paper consist of the auctions in a period of more than two years between May 2000 and October 2002. It is worth noting that some tracts contain different bid species which can lead to "skewed bidding" as studied in Athey and Levin (2001). To avoid the complication arising from "skewed bidding", we only focus on those auctions with Douglas fir as the single species. This leads to 108 sales in our data set. For these 108 sales, we observe the following information: the appraised volumes (measured in thousand board feet or MBF) of timber at different log grades, the reserve prices, the regions where the tracts are located, the bids per MBF and the identities of the bidders who submitted the bids. Note that the log grades used by the ODF are ranks in letters according to whether the species are peeler or sawmill. To make these log grades operational in our analysis, we construct a variable for grades denoted "grade" by assigning numbers to each of the log grades from 1 to 7, with 1 indicating the lowest grade and 7 the highest grade, and then calculating a weighted grade for each tract using the information on the appraised volumes of the timber at each log grade. Also, the ODF divides the forest into four areas: region 1 includes districts of Astoria, Forest Grove, Clarkmas–Marion, Western Lane, and West Oregon; regions 2 and 3 are now combined and considered a single region including Coos Bay; region 4 includes Southwest and Grants Pass Unit; and region 5 includes Klamath Falls. Since no tract in our data set is located in region 5, we create two region dummy variables: "region1" = 1 if the tract is in region 1 and 0 otherwise; "regions2&3" = 1 if the tract is in regions 2 and 3 and 0 otherwise, thus leaving region 4 as the baseline region. Table 5 gives the summary statistics on these variables. As can be seen from Table 5, our data set has 108 tracts with 451 bids in total.

4.2. Structural analysis of the timber sales

To analyze our timber auction data, we will adopt the independent private value (IPV) paradigm, which has been used in most of the previous empirical work analyzing timber auctions. See, e.g., Paarsch (1997), Elyakime et al. (1994, 1997), Baldwin et al. (1997), and Li and Perrigne (2003).[17] In a symmetric IPV paradigm for first-price sealed-bid auctions with risk-neutral bidders, all potential bidders draw their private values independently from a common distribution, say $F(\cdot)$, knowing the number of potential bidders, say N. Relevant to our empirical analysis, those potential bidders whose private values are above the reservation prices (say p) submit their bids and the bidder with the highest bid wins the auction. As shown by Riley and Samuelson (1981) among others, the Nash–Bayesian equilibrium bid b_m for the m-th bidder with a private value v_m above p is

$$b_m = v_m - \frac{1}{(F(v_m))^{N-1}} \int_p^{v_m} F^{N-1}(x)\mathrm{d}x.$$

As a result, the bidding strategy depends on the number of potential bidders, which is assumed to be known to all potential bidders. However, when the reservation price is binding, those potential bidders whose private values are below the reservation price do not bid, resulting in an econometric problem to the econometrician as the number of potential bidders is unobserved to him/her. This is indeed the case for our data, as the number of submitted bids for each auction cannot be treated as the number of

[15] See, e.g., a detailed discussion in Horowitz (2001) on the superiority of the parametric bootstrap over the nonparametric bootstrap when the underlying distribution belongs to a known parametric family. We have also tried the nonparametric bootstrap, the results of which are far less accurate than the results of the parametric bootstrap, and hence are not reported here.

[16] See, e.g., Hansen (1986), Baldwin et al. (1997), Athey and Levin (2001), and Haile (2001) among others. On the other hand, Paarsch (1997) study British Columbian timber sales, and Li and Perrigne (2003) study timber sale auctions that were held in France.

[17] While it is possible that there are some common value components among bidders' private values and hence it could be more desirable to employ a more general structural model for timber auctions, for the purpose of illustrating our methods, we use the IPV model as a first approximation.

Table 5
Summary statistics of the timber sale data.

Variable	Number of observations	Mean	Std. Dev.	Min.	Max.
Bid	451	331.62	136.70	119.67	2578.3
Winning bid	108	382.5	231.13	157.86	2578.3
Reserve price	108	273.19	77.32	118.32	463.96
Volume	108	3165.22	2894.31	256.74	20211
Grade	108	2.1653	0.3837	1.2727	3.0199
Region1	108	0.8448	0.3625	0	1
Regions2&3	108	0.1397	0.3471	0	1
Number of submitted bids	108	4.1759	2.0178	1	10

Table 6
Estimates for reduced form parameters in timber sale auctions.

Parameter	γ_0^a	γ_1^a	γ_2^a	γ_3^a
Estimate	26.3596	143.0753	−1.2082	−25.2196

Table 7
Estimates for structural parameters using OLS as the auxiliary model in timber sale auctions.

Parameter	γ_0	γ_1	γ_2	γ_3
Estimate	4.9042	0.2642	0.2070	−0.0865
Standard error	0.3017	0.0962	0.1927	0.2203

Table 8
Summary statistics from simulations of auctions with optimal reserve prices with 800 replications.

Variable	Mean	Std. Dev.	Min.	Max.
Number of unsold tracts	24.67	3.73	15	37
Revenue	$177,799,550	$13271,883	$131,798,100	$210,114,310
Profit	$97438,437	$7607,193	$72414,316	$117,295,750

the potential bidders because of the existence of reservation prices. To resolve this issue, we assume that the number of potential bidders is $N = 10$, which is the maximum number of actual bidders in our data.[18]

In our econometric implementation, we consider 108 lots with different species grades and in different regions. To control for these observed heterogeneities, we assume that the private value density at the ℓ-th lot can be specified as

$$f_\ell(v_{m\ell}|z_\ell) = \frac{1}{\exp(\gamma_\ell)} \exp\left[-\frac{1}{\exp(\gamma_\ell)} v_{m\ell}\right],$$

where $v_{m\ell}$ is the private value for the m-th bidder at the ℓ-th auction, $\gamma_\ell \equiv \gamma_0 + \gamma_1 \text{grade}_\ell + \gamma_2 \text{region1}_\ell + \gamma_3 \text{regions2\&3}_\ell$, and z_ℓ denotes the heterogeneity vector consisting of variables such as "grade", "region1", and "regions2&3". Therefore, the objective of the structural approach reduces to estimating the parameters γ_0, γ_1, γ_2, and γ_3. Note that, because of our specification for the private value distribution, $\gamma_k = (1/E_\ell[v])\partial E_\ell[v]/\partial z_{k\ell}$ for $k = 1, 2, 3$. Thus these coefficients have the interpretation of being the percentage change of the expected value of the private value corresponding to a one unit change of $z_{k\ell}$.

We first regress the bid on a constant, grade, region1 and regions2&3. Table 6 reports the results from the reduced form OLS estimation. In view of our previous discussions, these OLS estimates can be treated as estimates for some auxiliary but not true structural parameters. We then use the indirect inference-based estimation procedure proposed in Section 2.2 to obtain the estimates for the structural parameters. Table 7 reports these estimates obtained with the number of simulations equal to 100. Note that the reported standard errors for these structural parameters' estimates are obtained using the parametric bootstrapping method with 800 replications. As Table 7 indicates, notably, the "grade" variable is positively significant, while both regional dummies are insignificant. Moreover, one unit increase in "grade" could increase the expected value of the private value by about 26%.

As is well known, the structural approach is useful in analyzing auction data as it can recover the distribution of private values and hence make it possible to address some interesting economic problems. For example, in an IPV paradigm with risk-neutral bidders, it has been shown by Laffont and Maskin (1980) and Riley and Samuelson (1981) among others that an optimal auction mechanism is reduced to finding the optimal reserve price which is determined as follows:

$$p_0 = v_0 + \frac{1 - F(p_0)}{f(p_0)}, \tag{3}$$

where p_0 is the optimal reserve price and v_0 is the seller's private value for the auctioned object. Since our structural analysis yields estimates for the structural parameters in $f(\cdot)$, it is possible to assess the optimality of the reserve prices used in timber sales. As we do not observe the seller's private values, to estimate the optimal reserve price that should be used, we assume that the reserve prices that were actually used in these auctions represent the seller's private values. Then the structural estimates from Table 7 enable us to use (3) to estimate the optimal reserve price for each auction. After getting these estimated optimal reserve prices, using the estimated structural parameters, we simulate the bidding behavior of bidders at each of the 108 auctions when facing the optimal reserve prices. Table 8 gives the summary statistics from our simulations with 800 replications using the estimates reported in Table 7. Table 8 presents some interesting findings. First, increasing reserve prices could lead to some tracts being unsold while increasing the magnitudes of the bids. This is confirmed from the results on the number of unsold tracts in Table 8, which indicates that if the optimal reserve prices were used, on average the number of unsold tracts would have been 25, about 23% of the total sales in our data set. Also, despite some tracts being unsold, the total revenue for the seller from implementing the optimal reserve prices would be expected to be $177,799,550, about 41 million dollars more than the actual revenue, which is $136,664,757. Furthermore, assuming that the seller's private value is the same as the announced reserve price, we find that the total actual profit for the seller resulting from 108 sales is $30,357,537. In contrast, as illustrated from Table 8, the expected total profit for the seller could be $97,438,437 if the optimal reserve prices were used. Such an analysis clearly demonstrates that it is likely that the reserve prices used in the timber auctions analyzed

[18] Note that the number of potential bidders can be considered as a structural parameter that is not identified from the model. See, e.g. Laffont et al. (1995). Here we follow Guerre et al. (2000) and use the maximum number of actual bidders as an estimate for the number of potential bidders, as Guerre et al. (2000) show that, assuming that the number of actual bidders is a constant across auctions, the maximum number of actual bidders is a superconsistent estimator of the number of potential bidders.

in this paper are not optimal, and the seller can increase his/her revenue/profit by implementing the optimal reserve prices.[19]

5. Conclusion

This paper provides a new approach to inference in structural econometric models. Building on the indirect inference principle, we show that the nonregularity associated with the structural models which breaks down the standard asymptotic theory and complicates the inference of these models can be overcome by our method. In particular, our method yields estimators that are asymptotically normally distributed, an attractive feature that makes the inference based on our method feasible. Computationally, our method is straightforward to implement because it relies on the estimation of OLS in the first stage, and the simulation and parameter calibration in the second stage. The applicability of the proposed method can also be extended to dynamic discrete-choice models and dynamic oligopoly models, in addition to the auction and job search models mentioned previously. We also demonstrate the good finite sample performance through a Monte Carlo study and the usefulness of our approach using an empirical example. Besides its usefulness as an illustration of the feasibility of our indirect inference method, the structural analysis of the timber sale auctions held in Oregon also contributes to the empirical auction literature in that it identifies the structural elements that determine the underlying private value distribution, and addresses the issues of the optimality of the auction mechanism. Moreover, our method offers an approach that integrates the reduced-form analysis and the structural model estimation in general structural econometric models, and hence unifies the inference of these models in the spirit of conventional Cowles Commission structural econometrics.

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[19] Note this conclusion is based on the assumption that the seller sets the reserve price at his/her private value. We use this assumption to conduct the counterfactual analysis because we do not know the seller's value. Thus the conclusion is not robust. Nevertheless, it demonstrates the advantage of the structural approach in drawing policy-related conclusions.

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