

1. This exercise shows the finite sample problem of weak instruments in the two-stage least squares. Please hand in five kernel density plots together with your code.  
Let the structural equation be

$$y_i = \beta_1 + \beta_2 x_i + \epsilon_i$$

while the reduced-form equation be

$$x_i = \gamma_1 + \gamma_2 z_i + e_i.$$

Let the sample be i.i.d., with  $z_i \sim N(0, 1)$ , and  $\begin{pmatrix} \epsilon_i \\ e_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}\right)$ .  
Set the true parameter values as  $\beta_1 = 1$ ,  $\beta_2 = 2$ ,  $\gamma_1 = 0$ .

For each  $n$  and  $\gamma_2$  combination below, please use the standard two-stage least square estimator to estimate  $\beta_2$  (Write down the estimator by yourself. Do not invoke any built-in function.) Repeat this experiment for 1000 times so that we have 1000 estimated  $\hat{\beta}_2$ 's. Plot the kernel density of these  $\hat{\beta}_2$ 's so that we can see the sampling distribution.

- (a)  $n = 100$  and  $\gamma_2 = 1$ .
- (b)  $n = 100$  and  $\gamma_2 = 0.1$ .
- (c)  $n = 100$  and  $\gamma_2 = 0.01$ .
- (d)  $n = 1000$  and  $\gamma_2 = 0.01$ .
- (e)  $n = 10000$  and  $\gamma_2 = 0.01$ .