Lecture 1: Basic R

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Basic R

Introduction

One cannot acquire a new programming language without investing numerous hours. R-Introduction is an official manual maintained by the R core team. It was the first document that I perused painstakingly when I began to learn R in 2005. After so many years, this is still the best starting point for you to have a taste.

This lecture quickly sketches some key points of the manual, while you should carefully go over R-Introduction after today's lecture.

Help System

The help system is the first thing we must learn for a new language. In R, if we know the exact name of a function and want to check its usage, we can either call `help(function_name)` or a single question mark `?function_name`. If we do not know the exact function name, we can instead use the double question mark `??key_words`. It will provide a list of related function names from a fuzzy search.

Example: `?seq`, `??sequence`

Vector

A *vector* is a collection of elements of the same type, say, integer, logical value, real number, complex number, characters or factor. Unlike C, R does not require explicit type declaration.

`<-` assigns the value on its right-hand side to a self-defined variable name on its left-hand side. `=` is an alternative for assignment, which I prefer.

`c()` combines two or more vectors into a long vector.

Binary arithmetic operations `+`, `-`, `*` and `\` are performed element by element. So are the binary logical operations `&` `|` `!=`.

Factor is a categorical number. *Character* is text.

Missing values in R is represented as `NA` (Not Available). When some operations are not allowed, say, `log(-1)`, R returns `NaN` (Not a Number).

Vector selection is specified in square bracket `a[ ]` by either positive integer or logical vector.

Example

Logical vector operation.

```
# logical vectors
logi_1 = c(T,T,F)
logi_2 = c(F,T,T)

logi_12 = logi_1 & logi_2
print(logi_12)
```

```
## [1] FALSE  TRUE FALSE
```

Array and Matrix

An array is a table of numbers.
A matrix is a 2-dimensional array.

- array arithmetic: element-by-element. Caution must be exercised in binary operations involving two objects of different length. This is error-prone.
- `%*%`, `solve`, `eigen`

Example

OLS estimation with one x regressor and a constant. Graduate textbook expresses the OLS in matrix form

$$\hat{\beta} = (X'X)^{-1}X'y.$$

To conduct OLS estimation in R, we literally translate the mathematical expression into code.

Step 1: We need data Y and X to run OLS. We simulate an artificial dataset.

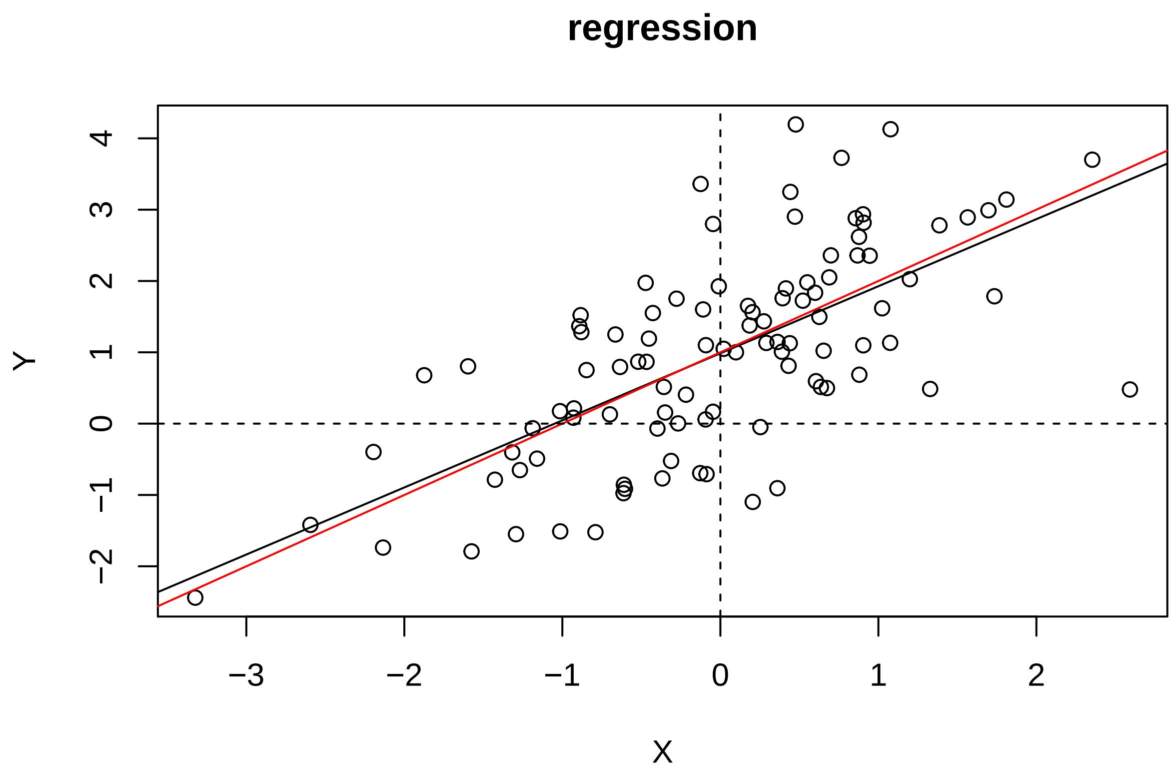
```
# simulate data
rm(list = ls( ) )
set.seed(111) # can be removed to allow the result to change

# set the parameters
n = 100
b0 = matrix(1, nrow = 2 )

# generate the data
e = rnorm(n)
X = cbind( 1, rnorm(n) )
Y = X %*% b0 + e
```

Step 2: translate the formula to code

```
# OLS estimation
bhat = solve( t(X) %*% X, t(X)%*% Y )
```

Step 3 (additional): plot the regression graph with the scatter points and the regression line. Further compare the regression line (black) with the true coefficient line (red).

```
# plot
plot( y = Y, x = X[,2], xlab = "X", ylab = "Y", main = "regression")
abline( a= bhat[1], b = bhat[2])
abline( a = b0[1], b = b0[2], col = "red")
abline( h = 0, lty = 2)
abline( v = 0, lty = 2)
```



Step 4: In econometrics we are often interested in hypothesis testing. The t-statistic is widely used. To test the null $H_0 : \beta_2 = 1$, we compute the associated t-statistic. Again, this is a translation.

$$t = \frac{\hat{\beta}_2 - \beta_{02}}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{\hat{\beta}_2 - \beta_{02}}{\sqrt{[(X'X)^{-1}\hat{\sigma}^2]_{22}}}.$$

where $[\cdot]_{22}$ is the (2,2)-element of a matrix.

```
# calculate the t-value
bhat2 = bhat[2] # the parameter we want to test
e_hat = Y - X %*% bhat
sigma_hat_square = sum(e_hat^2)/ (n-2)
Sigma_B = solve( t(X) %*% X  ) * sigma_hat_square
t_value_2 = ( bhat2 - b0[2]) / sqrt( Sigma_B[2,2] )
print(t_value_2)
```

```
## [1] -0.5615293
```

Package

A pure clean installation of R is small, but R has an extensive ecosystem of add-on packages. This is the unique treasure for R users, and other languages like Python or MATLAB are not even close. Most packages are hosted on CRAN. A common practice today is that statisticians upload a package to CRAN after they write or publish a paper with a new statistical method. They promote their work via CRAN, and users have easy access to the state-of-the-art methods.

A package can be installed by `install.packages("package_name")` and invoked by `library(package_name)`.

Applied Econometrics with R by Christian Kleiber and Achim Zeileis is a useful book. It also has a companion package `AER` that contains popular econometric methods such as instrumental variable regression and robust variance.

Before we can "knit" in R-studio the Rmd file to produce the pdf document you are reading at this moment, we have to install several packages such as knitr and those it depends on.

Mixed Data Types

A vector only contains one type of elements. *list* is a basket for objects of various types. It can serve as a container when a procedure returns more than one useful object. For example, when we invoke `eigen`, we are interested in both eigenvalues and eigenvectors, which are stored into `$value` and `$vector`, respectively.

data.frame is a two-dimensional table that stores the data, similar to a spreadsheet in Excel. A matrix is also a two-dimensional table, but it only accommodates one type of elements. Real world data can be a collection of integers, real numbers, characters, categorical numbers and so on. Data frame is the best way to organize data of mixed type in R.

Example

This is a data set in a graduate-level econometrics textbook. We load the data into memory and display the first 6 records.

```
library(AER)
```

```
## Warning: package 'AER' was built under R version 3.4.3

## Warning: package 'car' was built under R version 3.4.3

## Warning: package 'lmtest' was built under R version 3.4.2
```

```
data("CreditCard")
head(CreditCard)
```

```
##   card reports      age income       share expenditure owner selfemp
## 1  yes       0 37.66667 4.5200 0.033269910  124.983300   yes      no
## 2  yes       0 33.25000 2.4200 0.005216942    9.854167    no      no
## 3  yes       0 33.66667 4.5000 0.004155556   15.000000   yes      no
## 4  yes       0 30.50000 2.5400 0.065213780  137.869200    no      no
## 5  yes       0 32.16667 9.7867 0.067050590  546.503300   yes      no
## 6  yes       0 23.25000 2.5000 0.044438400   91.996670    no      no
##   dependents months majorcards active
## 1          3     54          1     12
## 2          3     34          1     13
## 3          4     58          1      5
## 4          0     25          1      7
## 5          2     64          1      5
## 6          0     54          1      1
```

Input and Output

Raw data is often saved in ASCII file or Excel. I discourage the use of Excel spreadsheet in data analysis, because the underlying structure of an Excel file is too complicated for statistical software to read. I recommend the use of `csv` format, a plain ASCII file format.

`read.table()` or `read.csv()` imports data from an ASCII file into an R session. `write.table()` or `write.csv()` exports the data in an R session to an ASCII file.

Example

Besides loading a data file on the local hard disk, We can directly download data from internet. Here we show how to retrieve the daily stock price of *Hong Kong Exchange Limited* from *Yahoo Finance*, and save the dataset locally.

```
# Update in Jan 2018: the following link is no longer valid.
# Yahoo changes the format of the inquiry to forbid massive data downloading
# We need to figure out a new way
HEX = read.csv("http://ichart.finance.yahoo.com/table.csv?s=0388.HK")
print(head(HEX))
write.csv(HEX, file = "HEX.csv")
```

Statistics

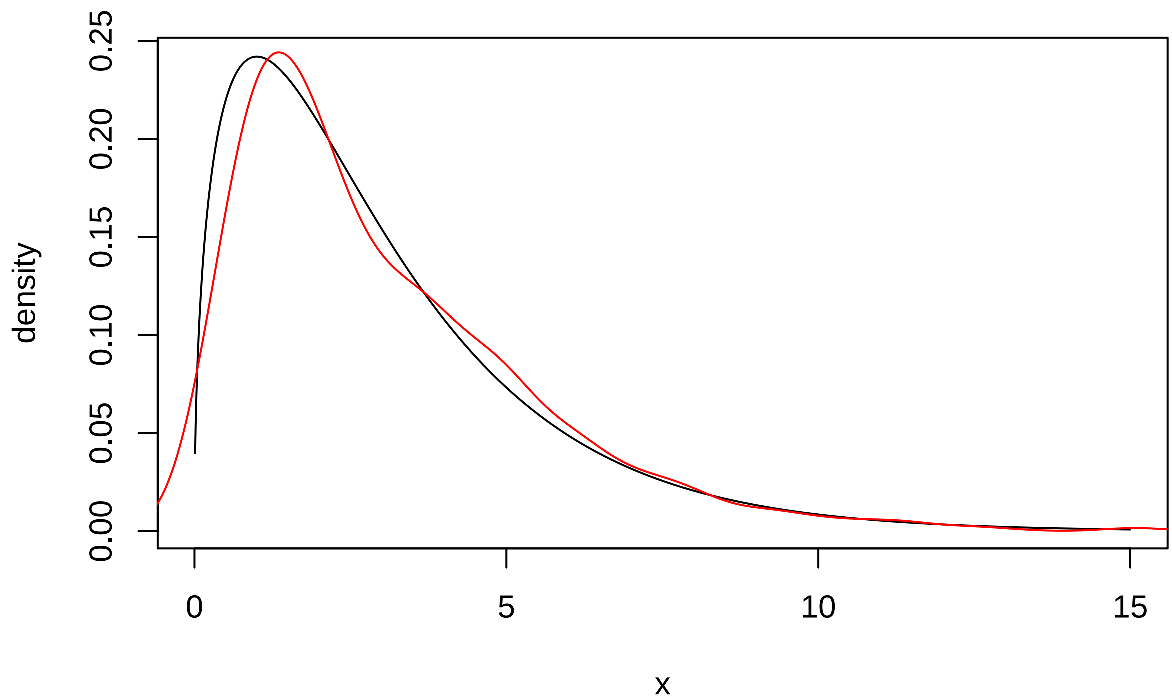
R is a language created by statisticians. It has elegant built-in statistical functions. `p` (probability), `d` (density for a continuous random variable, or mass for a discrete random variable), `q` (quantile), `r` (random variable generator) are used ahead of the name of a probability distribution, such as `norm` (normal), `chisq` (χ^2), `t` (t), `weibull` (Weibull), `cauchy` (Cauchy), `binomial` (binomial), `pois` (Poisson), to name a few.

Example

This example illustrates the sampling error.

1. Plot the density of $\chi^2(3)$ over an equally spaced grid system `x_axis = seq(0.01, 15, by = 0.01)` (black line).
2. Generate 1000 observations from $\chi^2(3)$ distribution. Plot the kernel density, a nonparametric estimation of the density (red line).
3. Calculate the 95-th quantile and the empirical probability of observing a value greater than the 95-th quantile. In population, this value should be 5%. What is the number in this experiment?

```
set.seed(888)
x_axis = seq(0.01, 15, by = 0.01)

y = dchisq(x_axis, df = 3)
plot(y = y, x=x_axis, type = "l", xlab = "x", ylab = "density")
z = rchisq(1000, df = 3)
lines( density(z), col = "red")
```




```
crit = qchisq(.95, df = 3)

mean( z > crit )
```

```
## [1] 0.047
```

User-defined Function

R has numerous built-in functions. However, in practice we will almost always have some DIY functionality to be used repeatedly. It is highly recommended to encapsulate it into a user-defined function. There are important advantages:

1. In the developing stage, it allows us to focus on a small chunk of code. It cuts an overwhelmingly big project into manageable pieces.
2. A long script can have hundred or thousand of variables. Variables defined inside a function are local. They will not be mixed up with those outside of a function. Only the input and the output of a function have interaction with the outside world.
3. If a revision is necessary, We just need to change one place. We don't have to repeat the work in every place where it is invoked.

The format of a user-defined function in R is

```
function_name = function(input) {
  expressions
  return(output)
}
```

Example

If a central limit theorem is applicable, then we can calculate the 95% two-sided asymptotic confidence interval as

$$\left(\hat{\mu} - \frac{1.96}{\sqrt{n}}\hat{\sigma}, \hat{\mu} + \frac{1.96}{\sqrt{n}}\hat{\sigma}\right)$$

from a given sample. It is an easy job, but I am not aware there is a built-in function in R to do this.

```
# construct confidence interval

CI = function(x){
  # x is a vector of random variables

  n = length(x)
  mu = mean(x)
  sig = sd(x)
  upper = mu + 1.96/sqrt(n) * sig
  lower = mu - 1.96/sqrt(n) * sig
  return( list( lower = lower, upper = upper) )
}
```

Flow Control

Flow control is common in all programming languages. `if` is used for choice, and `for` or `while` is used for loops.

Example

Calculate the empirical coverage probability of a Poisson distribution of degrees of freedom 2. We conduct this experiment for 1000 times.

```
Rep = 1000
sample_size = 100
capture = rep(0, Rep)


pts0 = Sys.time() # check time
for (i in 1:Rep){
  mu = 2
  x = rpois(sample_size, mu)
  bounds = CI(x)
  capture[i] = ( ( bounds$lower <= mu  ) & (mu <= bounds$upper) )
}
mean(capture) # empirical size
```

```
## [1] 0.938
```

```
pts1 = Sys.time() - pts0 # check time elapse
print(pts1)
```

```
## Time difference of 0.1686199 secs
```

Statistical Model

Statistical models are formulated as `y~x`, where `y` on the left-hand side is the dependent variable, and `x` on the right-hand side is the explanatory variable. The built-in OLS function is `lm`. It is called by `lm(y~x, data = data_frame)`.

All built-in regression functions in R share the same structure. Once one type of regression is understood, it is easy to extend to other regressions.

A Linear Regression Example

Add a toy example by Zhan Gao for ECON3121D tutorial.

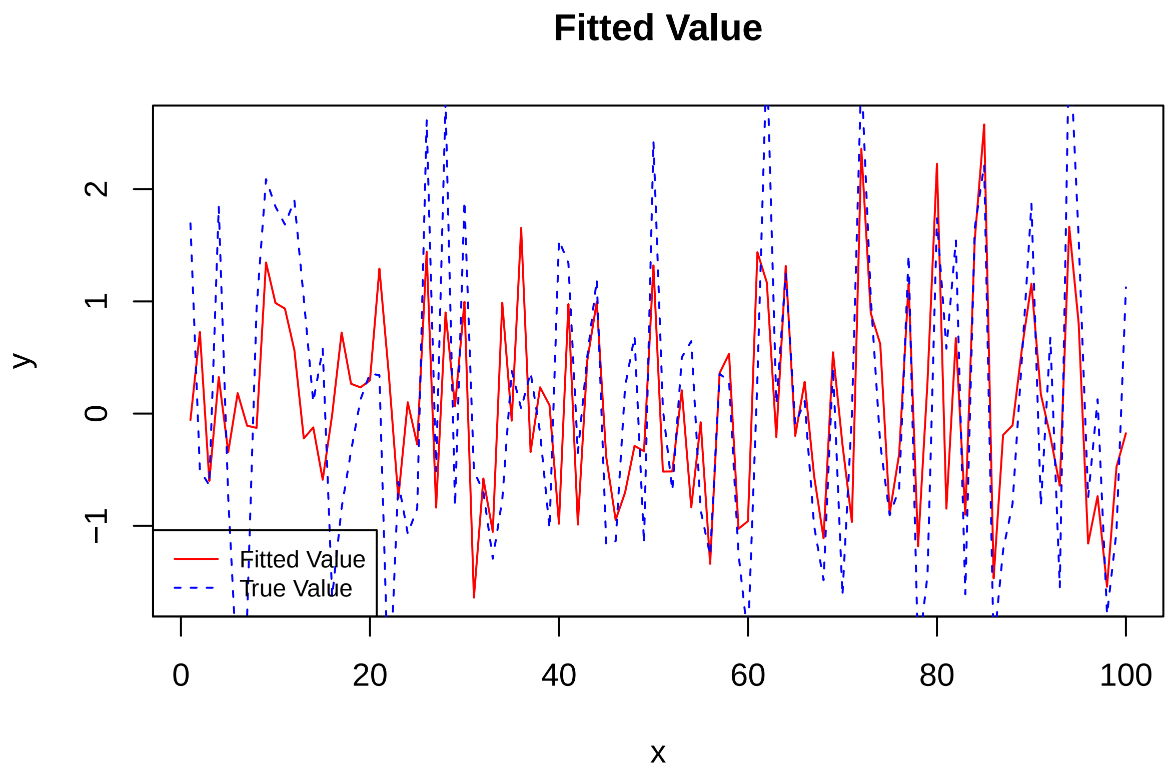
```
T = 100
p = 1

b0 = 1
# Generate data
x = matrix( rnorm(T*p), T, 1)
y = x %*% b0 + rnorm(T)

# Linear Model
result = lm(y ~ x)
summary(result)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.96282 -0.70247 -0.02817  0.75194  2.52548
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.04445    0.10221   0.435    0.665
## x            0.84427    0.09604   8.791 5.06e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.021 on 98 degrees of freedom
## Multiple R-squared:  0.4409, Adjusted R-squared:  0.4352
## F-statistic: 77.29 on 1 and 98 DF,  p-value: 5.06e-14
```

The `result` object is a list containing the regression results. As shown in the results, we can easily read the estimated coefficients, t-test results, F-test results, and the R-sqaure.

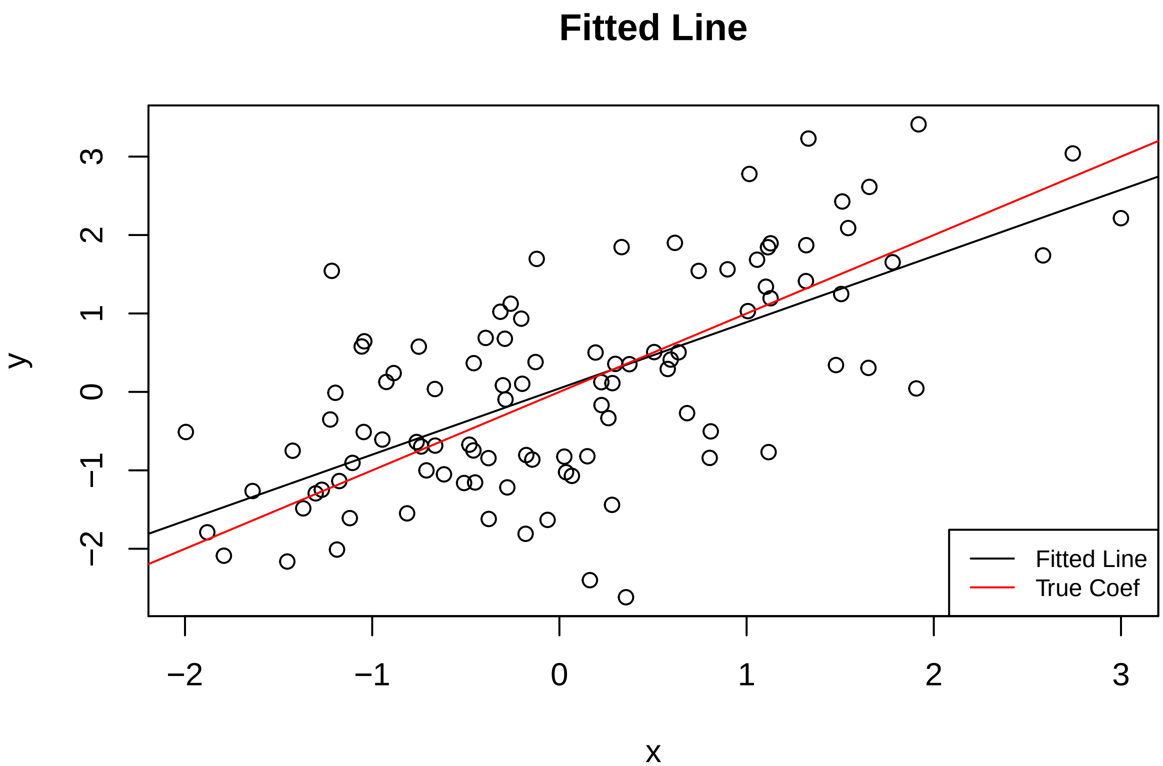
We can plot the true value of y and fitted value to examine whether the regression model fit the data well.

```
plot(result$fitted.values, col = 'red', type = 'l', xlab = 'x', ylab = 'y',
     main = 'Fitted Value')
lines(y, col = 'blue', type = 'l', lty = 2)
legend('bottomleft', legend=c("Fitted Value", "True Value"),
       col=c("red", "blue"), lty=1:2, cex = 0.75)
```



Then we plot the best fitted line.

```
plot( y = y, x = x, xlab = "x", ylab = "y", main = "Fitted Line")
abline( a= result$coefficients[1], b = result$coefficients[2])
abline( a = 0, b = b0, col = "red")

legend('bottomright', legend=c("Fitted Line", "True Coef"),
       col=c("black", "red"), lty=c(1,1), cex = 0.75)
```



Fitted Line
y
x
−2
−1
0
1
2
3
Fitted Line
True Coef