Moment-Based Automatic Modulation Classification: FSKs and Pre-Matched-Filter QAMs

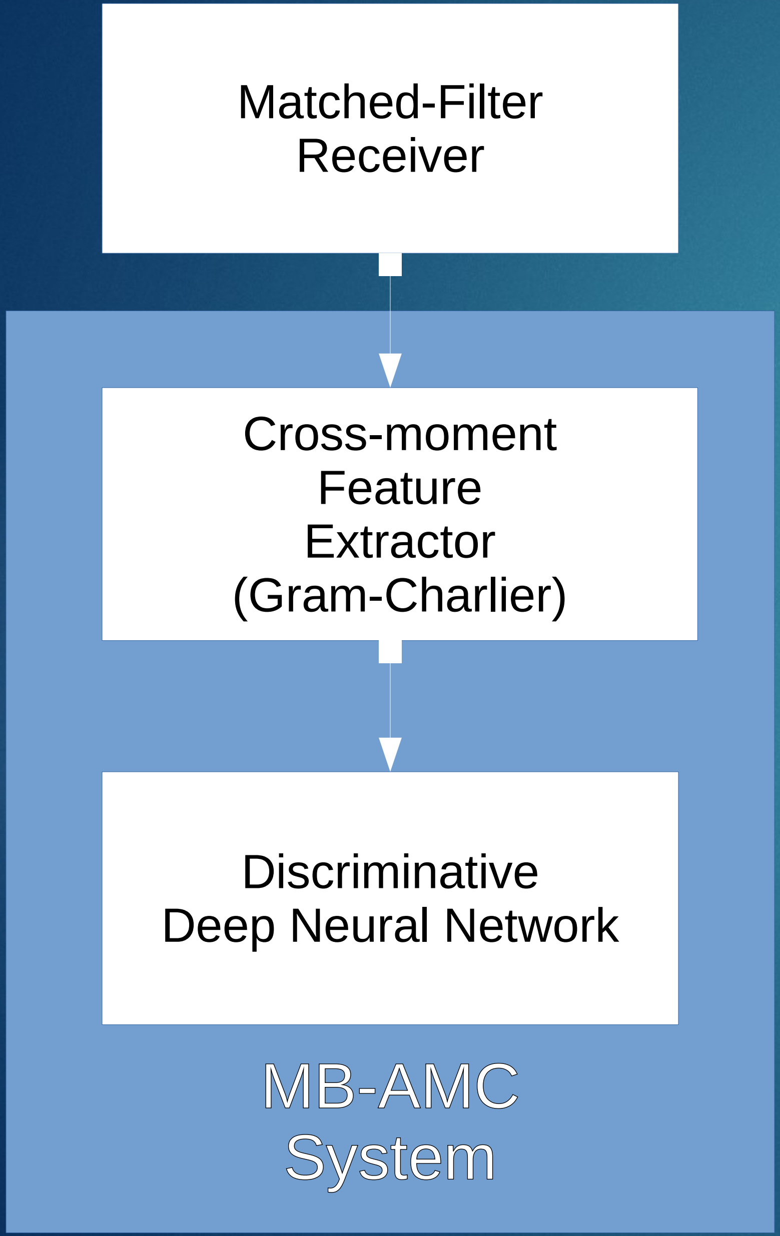
Darek Kawamoto, Bob McGwier
VT Hume Center
HawkEye 360

MB-AMC – GRCon 2016 Paper

Kawamoto, McGwier (2017) – *Rigorous Moment-Based Automatic Modulation Classification*

- Showed comparable performance between Moment-Based Automatic Modulation Classification (**MB-AMC**) and a likelihood-based approach.
- Linked moments of input symbols to a Hilbert Space using complex-domain Gram-Charlier series ("Fourier analysis" expansion of probability density functions by Hermite polynomials).
- "*Finally, these authors fully expect that these techniques can be applied, with slight modification and an appropriate decrease in performance, directly to pre-receiver symbols.*"

MB-AMC – Overview

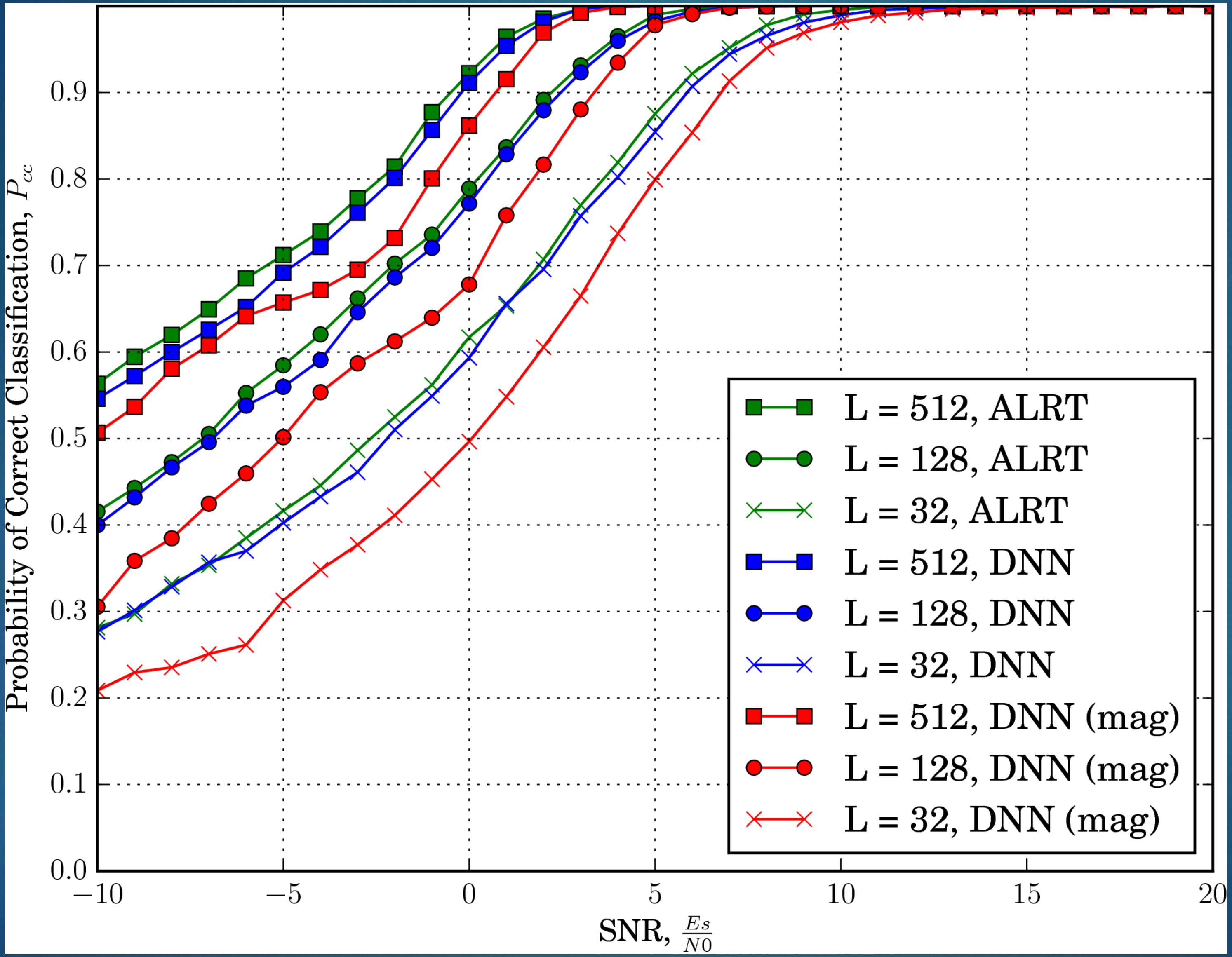


- I/Q samples in
- I/Q symbols out

- Calculates cross-moments of input symbols
- Related to Gram-Charlier series expansion
- Output features belong to a Euclidean space

- Implements a non-linear decision region slicer
- During training, automatically identifies modulation class clusters
- During execution, outputs soft-decision classifications

MB-AMC – GRCon 2016 Paper



MB-AMC – Shortcomings and FSKs

- Utility is somewhat limited, in the sense that inputs were post-receiver output symbols. Required prior time, frequency, and phase synchronization.
- Because of the 1 sample per symbol constraint, MB-AMC is limited to classifying linear modulations due to its inability to examine the pulse shapes of the various modulations.
- In particular, MB-AMC typically suffers against FSKs, whose signals may be non-linear transformations of pulses.
- Chicken and egg problem: MB-AMC operates on post-receiver symbols, but the optimal receiver depends on the modulation.

Improved MB-AMC – Approach

- We'll extend the MB-AMC by performing classification in the pre-receiver domain (assuming prior knowledge of the baud-rate and SNR, but <u>not</u> frequency offset!)
- In order to mitigate the carrier frequency offset (CFO), we introduce a Delay-Conjugate-Multiply (DCM) operation in order to turn frequency offsets into phase offsets in the transformed output I/ Q constellation.
- We'll call this **DCM-MB-AMC**
- This work extends MB-AMC in the direction of cyclostationary analysis (see, for example, *The Cumulant Theory of Cyclostationary Time-Series Parts I and II*, by Spooner and Gardner, 1994).

Quick Math Review of MB-AMC

- The MB-AMC formulation presented last year treated input symbols as independent random variables.
- The cross-moments of these input symbols are used to approximate the probability density function the symbols came from. This is the Gram-Charlier series expansion.
- The series expansion coefficients are based on expected values of complex-valued polynomials H(z) which are computed using the cross-moments of the input symbols.

$$f_Z(z) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \frac{E[\overline{H_{p,q}(z)}]}{\sqrt{p!q!}} \frac{H_{p,q}(z)}{\sqrt{p!q!}} \frac{1}{\pi} e^{-z\bar{z}}$$

Quick Math Review of MB-AMC

- Some complex Hermite polynomials (*Orthogonal Polynomials of Several Variables,* Dunkl & Xu, 2014):

$$H_{0,0}(z) = 1,$$

$$H_{1,0}(z) = z,$$

$$H_{1,1}(z) = |z|^2 - 1,$$

$$H_{2,1}(z) = z^2 z^* - 2z,$$

$$H_{2,2}(z) = |z|^4 - 4|z|^2 + 2$$

Quick Math Review of MB-AMC

- Letting $h_{p,q} = \frac{E[\overline{H_{p,q}(z)}]}{\sqrt{p!q!}}$, we can completely describe these density functions by the infinite sequence of these coefficients.
- The way the math works out, the distance between two density functions (or coefficient sequences) can be computed easily,

$$d(f_{Z_1}, f_{Z_2}) = \sqrt{\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} |h_{p,q}(f_{Z_1}) - h_{p,q}(f_{Z_2})|^2}.$$

- This is Euclidean distance, and is where the “rigor” of “rigorous MB-AMC” comes from.

DCM-MB-AMC

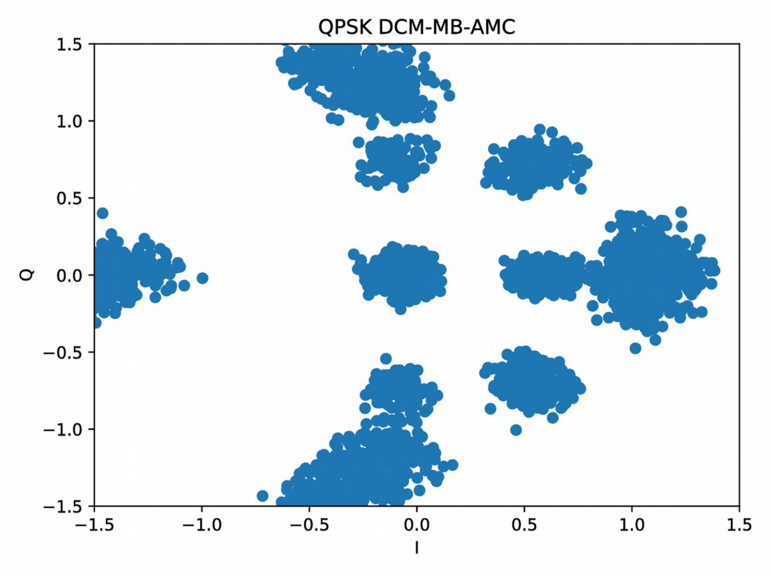
- A significant trade-off of the MB-AMC method is that there is no time-dependence captured in the formulation.
- In order to classify FSKs, we'd like to capture the non-zero-crossing nature in our features, as well as introduce time-dependencies to capture the various phase-increments associated with each frequency.
- In order to achieve this, we take Z to be pre-receiver samples, and compute a transformed version,

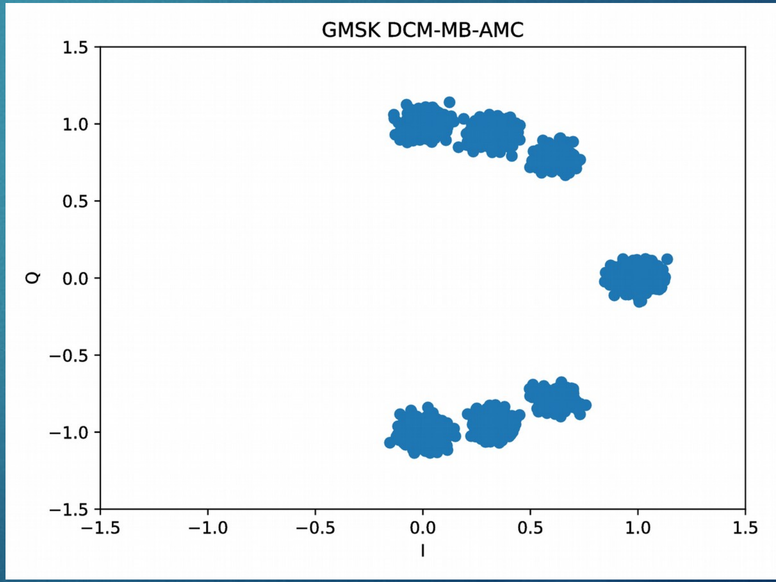
$$\tilde{Z} := Z(t)Z^*(t-\tau)$$

where τ is some delay parameter (typically one symbol period).

DCM-MB-AMC

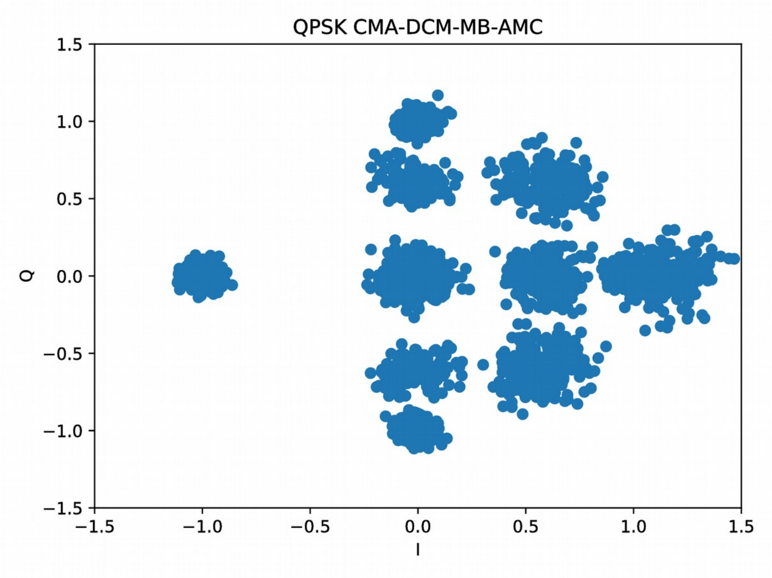
- These transformed samples are fed into the typical MB-AMC algorithm and a new DNN is trained on these features.
- The idea here is that the delay captures time dependencies necessary to properly discriminate between FSK and QAM, while the conjugation mitigates CFO.

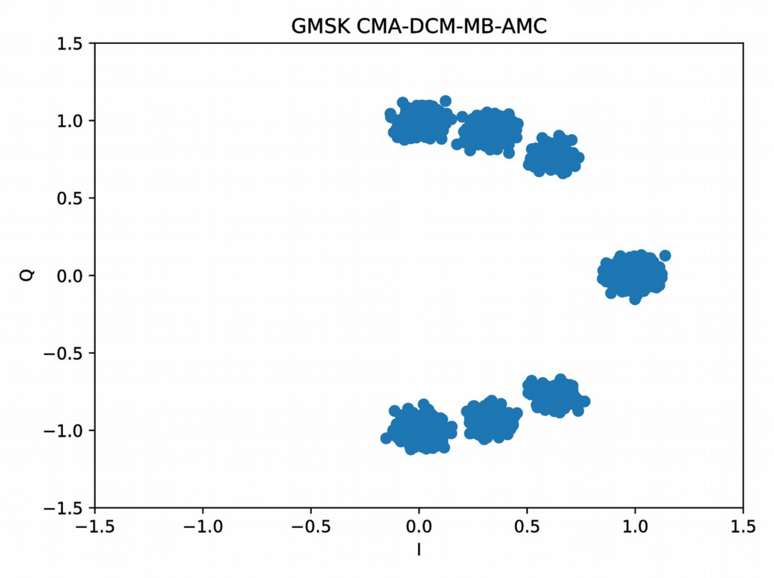




CMA-DCM-MB-AMC

- One objection to operating in the pre-receiver domain is the SNR loss associated with operating in a higher sample rate domain (and without the matched filter recovery). We can apply a blind equalizer (such as the Constant Modulus Algorithm) to partially-mitigate this SNR loss in QAMs while leaving the FSKs untouched.





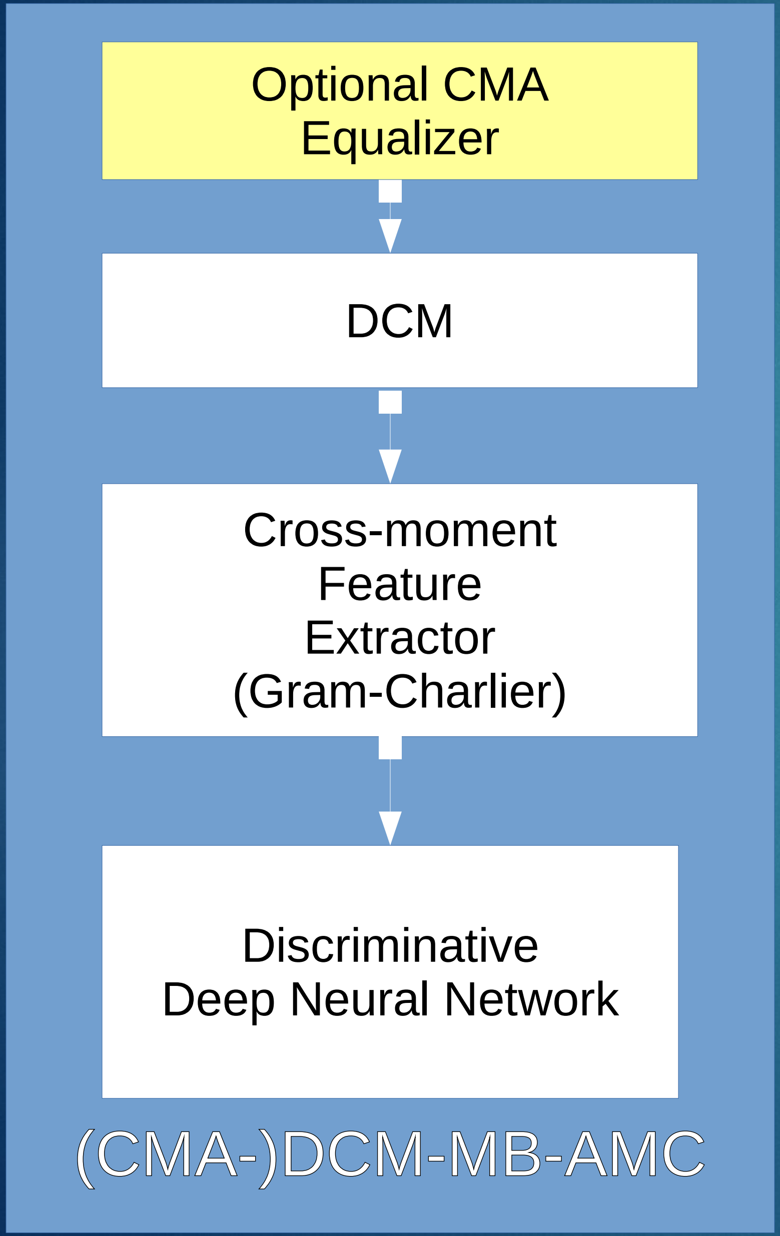
Connection: Cyclostationary Analysis

- In cyclostationary analysis, the delay product

$$L_x(t, \tau_n)_n = \prod_{j=1}^{n} Z^{(*)_j}(t - \tau_j)$$

  plays a huge role. The DCM transformation is a particular delay product, and it seems that the DCM-MB-AMC uses a specific subset of features from the cyclostationary arsensal.
- Further work will include a more thorough exploration of this connection.

Experiments

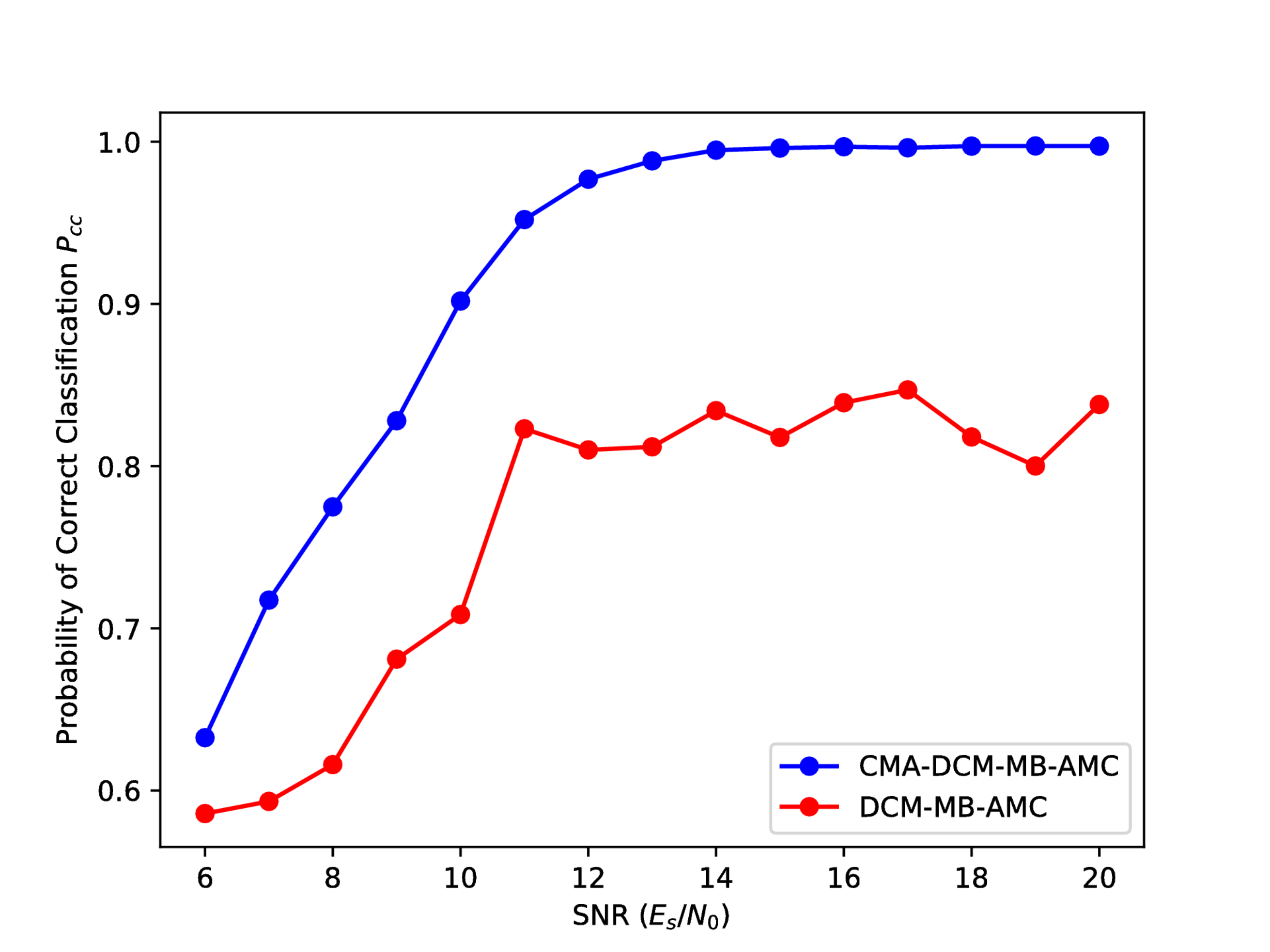


- Extended the MB-AMC system by simply adding the DCM operation (and optional CMA).
- Input is raw I/Q, 2 samples per symbol
- 10 modulations: 2ASK, 4ASK, BPSK, QPSK, 8PSK, 16QAM, 2FSK (rect), 4FSK (rect), GFSK (BT=0.5, h=0.7), and GMSK (BT=0.5).
- 4 layer DNN, widths 400, 400, 400, 100. Re-trained the DNN using simulation data.

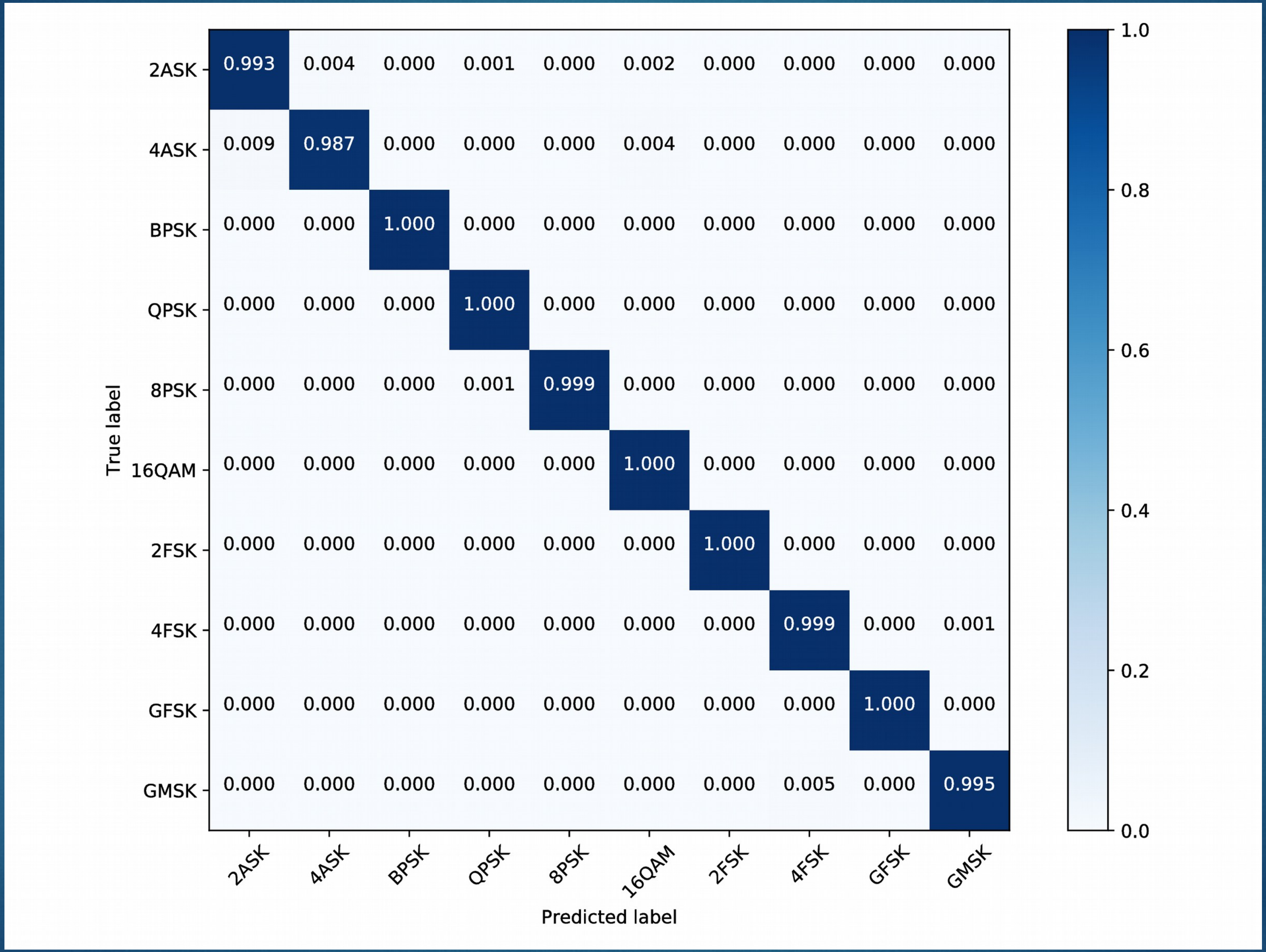
Experiments

- The simulation modulated random data with random time offsets (any fractional symbol offset) and frequency offsets (within a quarter baud-rate).
- Signals were modulated at 2 samples per symbol; 500 samples (250 symbols) were forwarded on to the AMC system.
- After training, 1000 of each modulation were run through the system to test performance.
- Probability of Correct Classification (Pcc) and confusion matrices shown next...

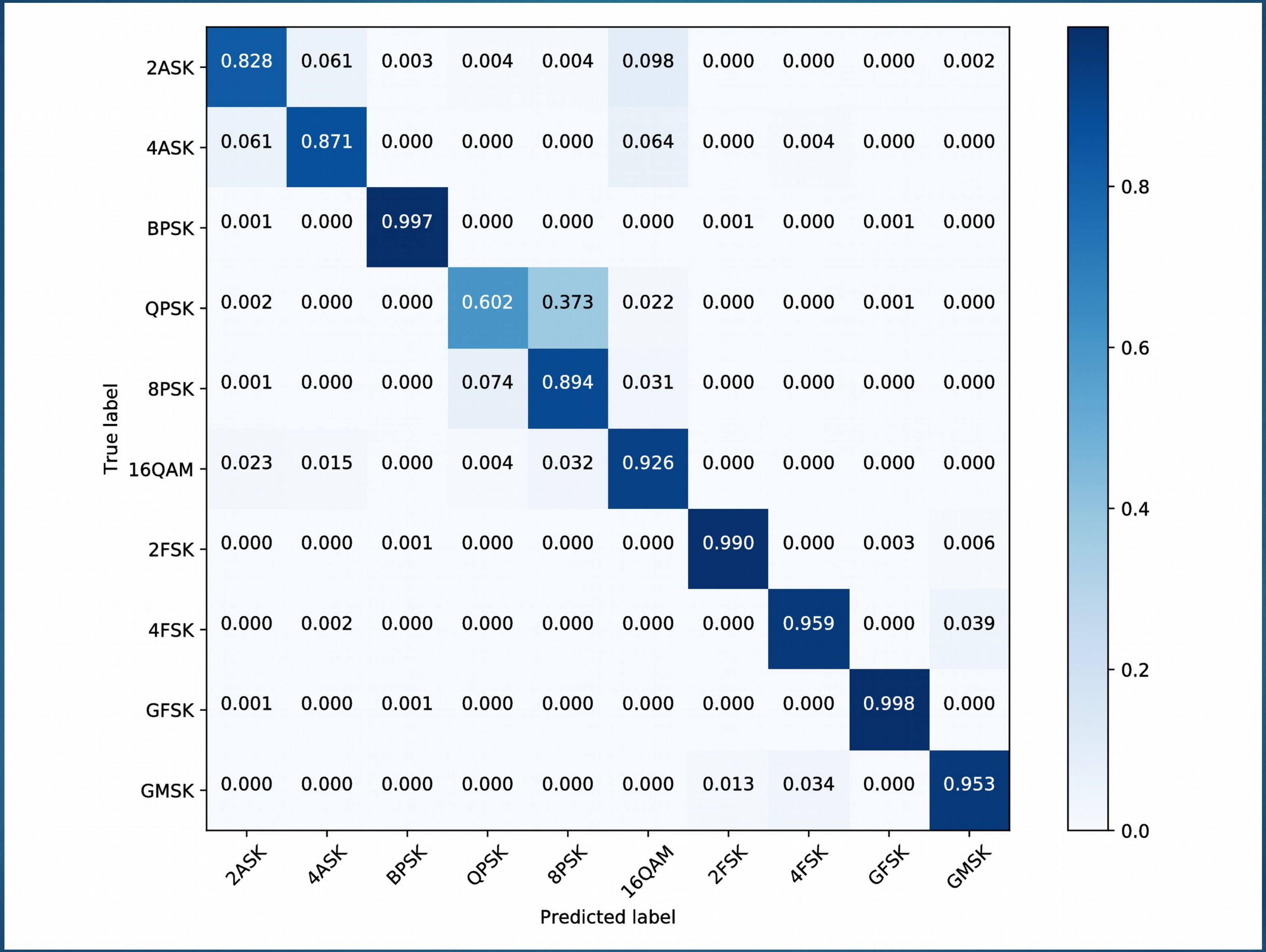
Results – Pcc vs SNR



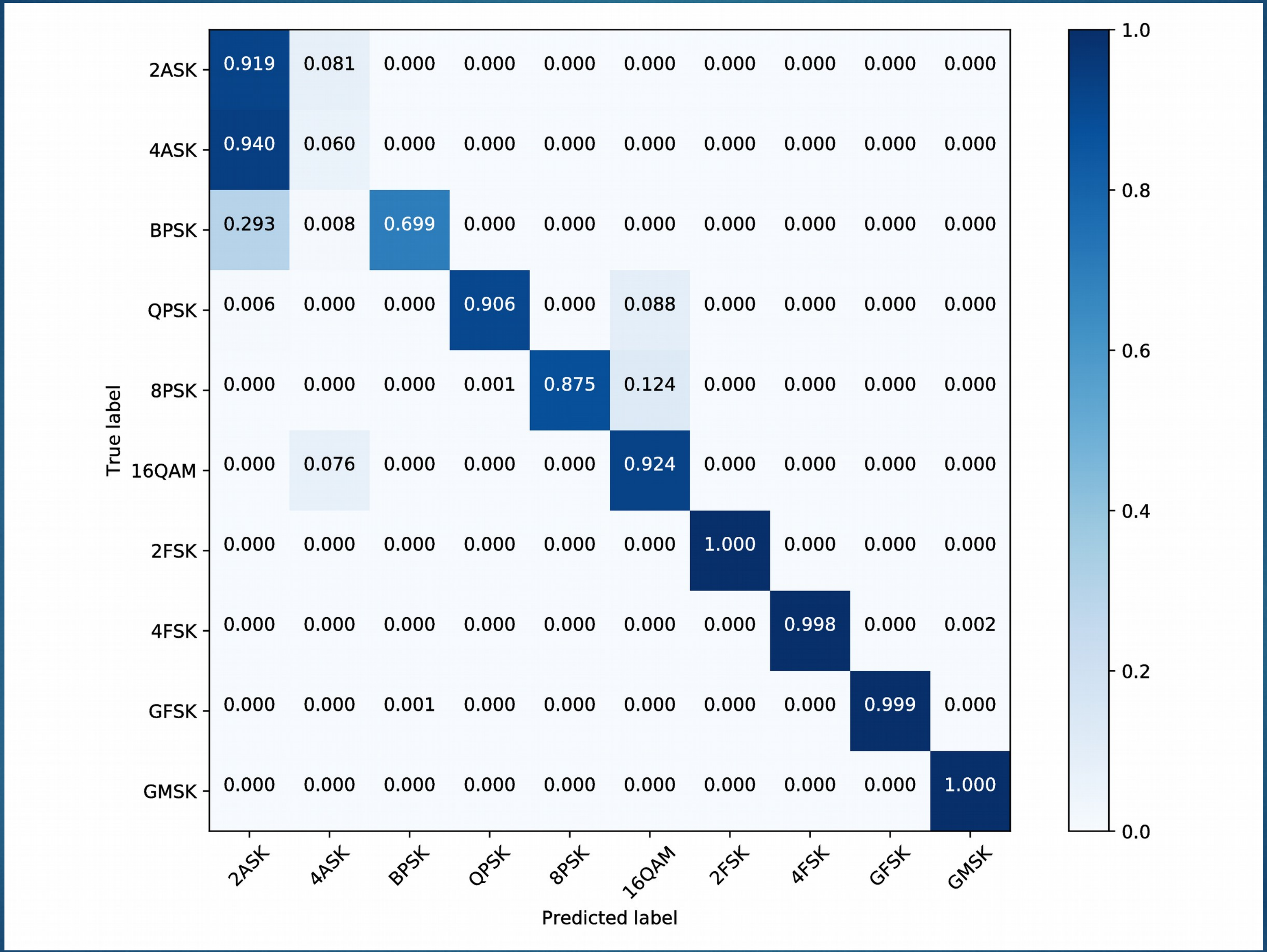
Results – CMA-DCM-MB-AMC 20 dB



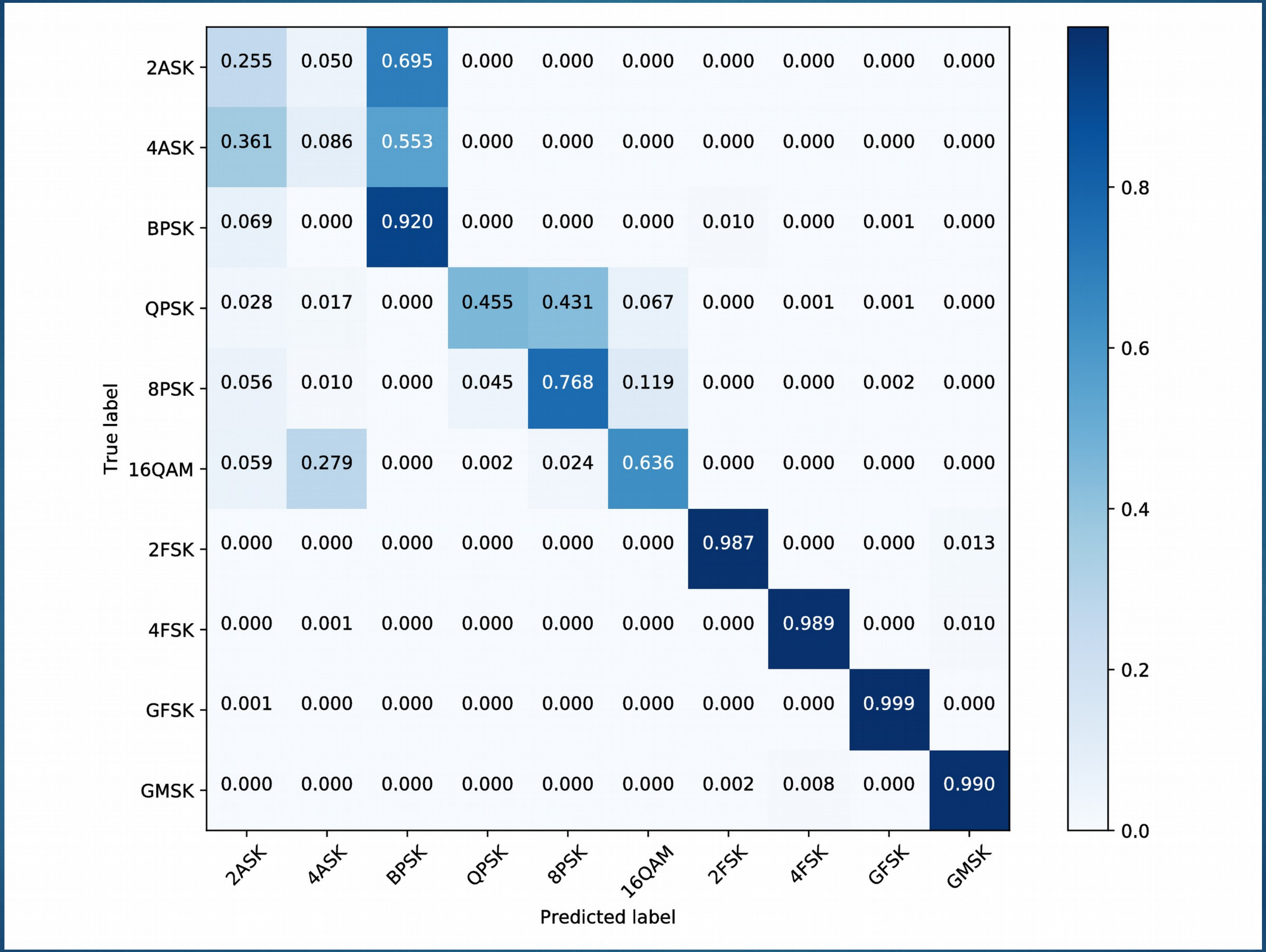
Results – CMA-DCM-MB-AMC 10 dB



Results – DCM-MB-AMC 20 dB



Results – DCM-MB-AMC 10 dB



Summary / Conclusion

- We’ve extended the 1 sample per symbol MB-AMC to operate in the pre-receiver domain in a computationally efficient manner. The main cost of this extension has been the corresponding increase in the input signal’s required SNR to maintain similar performance to post-receiver MB-AMC.
- The DCM was introduced to mitigate CFO and to incorporate short-term time-dependencies into the classifier.
- The CMA was introduced in order to improve SNR and sharpen up the I/Q constellation.
- Further work will explore the connection with cyclostationary analysis!