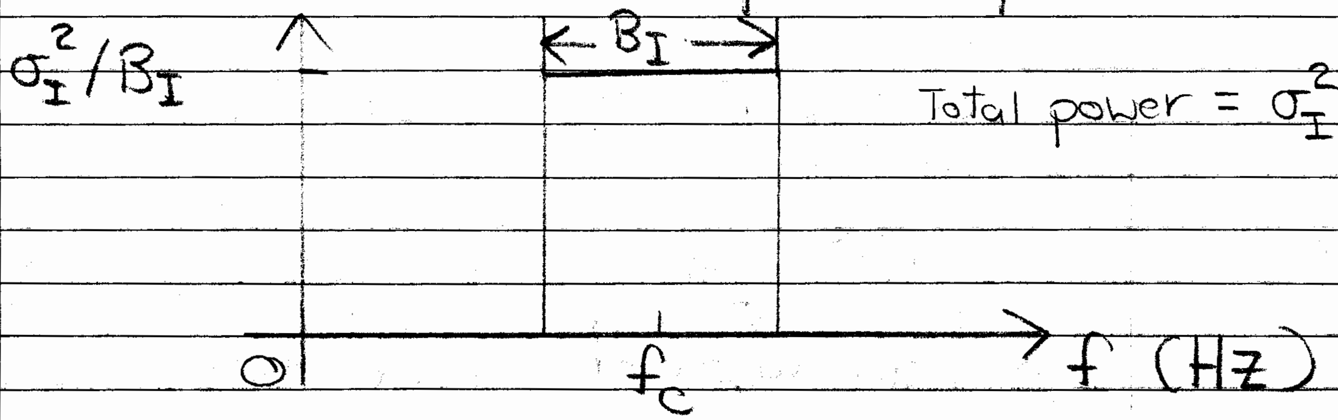
6.6.13 Broadband Interference Sources

Consider a 10-element linear array designed for frequency f_c

A narrowband plane wave signal arrives from $u = u_s$.

There are two plane wave interferers from $u_1 = 0.3$ and $u_2 = 0.5$. They have broadband temporal spectra:



Let $B_{fI} = B_I / f_c$ (normalized)

(a) Design a MVDR beamformer using the assumption that the interferers are narrowband and evaluate array gains for several values of B_{fI}.

Solution:

I need to determine the actual $\underline{S}_n(\omega)$ in order to solve the problem.

For a single broadband source, it can be shown that the (n, m)th element of the array covariance matrix is:

$[\underline{S}_x(\omega)]_{n,m} =$

$$\sigma_I^2 \exp(j\pi(n-m)u_I)\,\mathrm{sinc}\left(\frac{\pi(n-m)B_{fI}u_I}{2}\right)$$

If $B_{fI} = 0$, $\mathrm{sinc}(0) = 1$ and $\underline{S}_x(\omega)$ collapses to that of a rank one narrowband interference source

To solve part (a), I did the following:

(1) Generated an assumed $\underline{S}_n(\omega)$, making the assumption, that the interferers are narrowband, rank-one sources

(2) Built $\underline{w} = \underline{S}_n^{-1}\underline{v}_s / (\underline{v}_s^H \underline{S}_n^{-1} \underline{v}_s)$

assuming $u_s = 0$ and using the narrowband $\underline{S}_n(\omega)$

(3) Evaluated array gain with the expression

$$AG = |\underline{w}^H \underline{v}_s|^2 / (\underline{w}^H \rho_n \underline{w})$$

where ρ_n is derived from the actual $\underline{S}_n(\omega)$

(a) The beampattern is the same for each value of B_{fI} because the narrowband assumption was made in each case. It is attached.

$B_{fI} = B_I / f_c$	Array Gain (dB)
0	32.74 (upper bound)
0.1	31.27
0.2	28.70
0.4	24.76

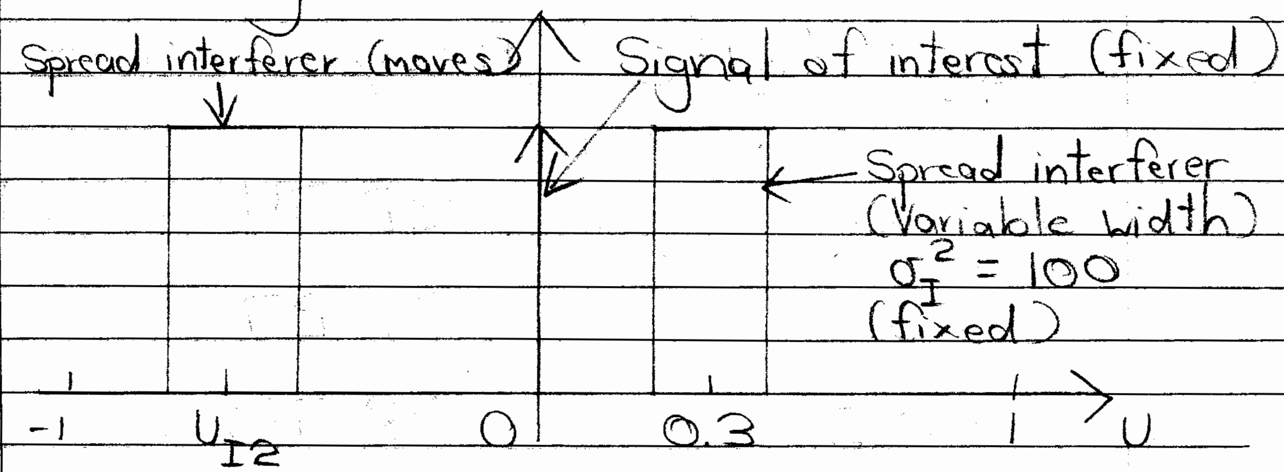
The trend is expected. As the interferer bandwidth increases, the mismatch between actual and assumed $\underline{S}_n(\omega)$ increases and the processor is mismatched to the environment. That results in performance degradation.

(b) Let the DOA and INR of the second interferer be variables.

Plot array gain vs. u_{I2} for several values of INR.

Solution:

I wrote a script to implement the following scenario:



(a) The second spread interferer has variable width, but same width as the first one.

(b) $-1 \leq u_{I2} \leq 1$

(c) Signal of interest from $u_s = 0$

(d) The INR of the second interferer varies

(e) The white noise floor is set to one.

The beamformer is designed assuming <u>narrowband</u> interferers :)

INR of interferer #1 = 20 dB

INR of second interferer is variable

The DOA of the second interferer varies
$(-1 \le u_{I2} \le 1)$

I have plotted array gain vs u_{I2} for INR's = -10, 0, 10 and 20 db and $B_{fI} = 0$ and 0.1.

Plots are attached.

(c) Design a MVDR beamformer using the actual $S_n(\omega)$. Plot beam patterns and compute array gains.

Solution:

I included this in the code I used to solve part (a). Beampatterns are attached and array gains tabulated below.

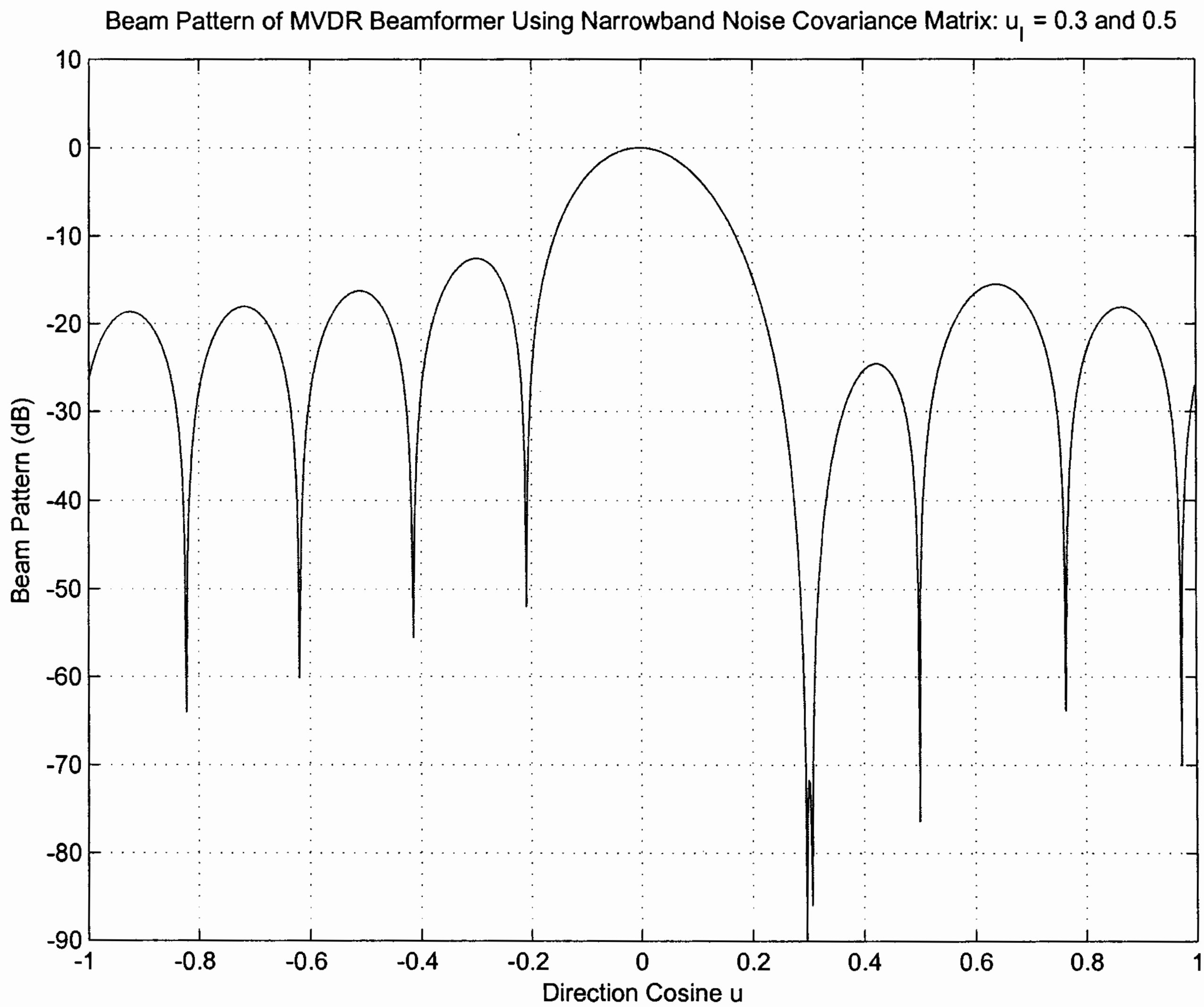
$B_{fI} = B_I/f_c$	Gain w/ Correct $S_n(\omega)$ (dB)
0	32.74 (32.74)
0.1	32.37 (31.27)
0.2	32.12 (28.7)
0.4	31.61 (24.7)

The numbers in parantheses are the gains provided by the MVDR processor that assumes $B_{fI} = 0$ for both interferers.

As the bandwidth increases, the performance degradation becomes more apparent.

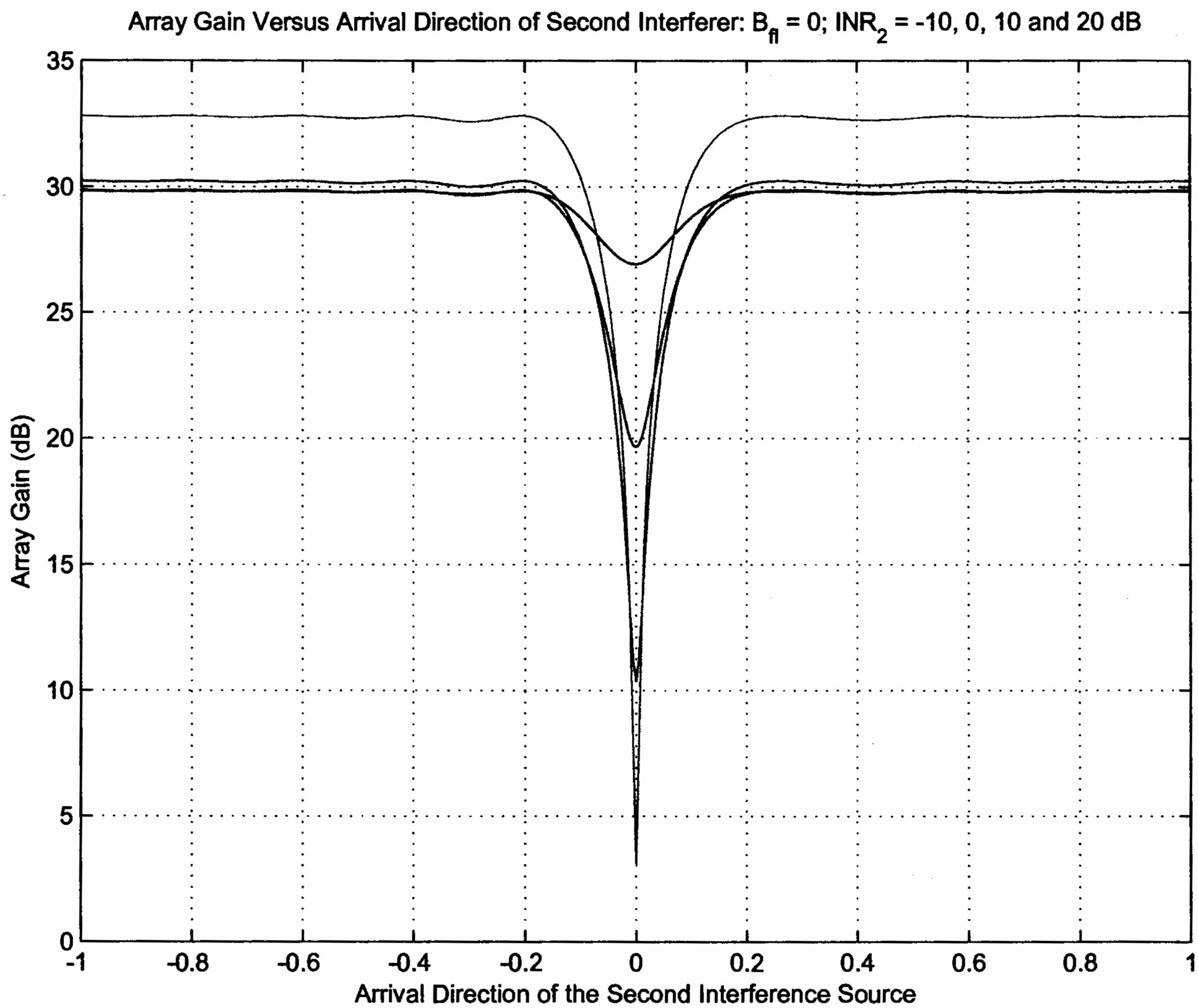
Beam pattern plots follow.

Part (a)

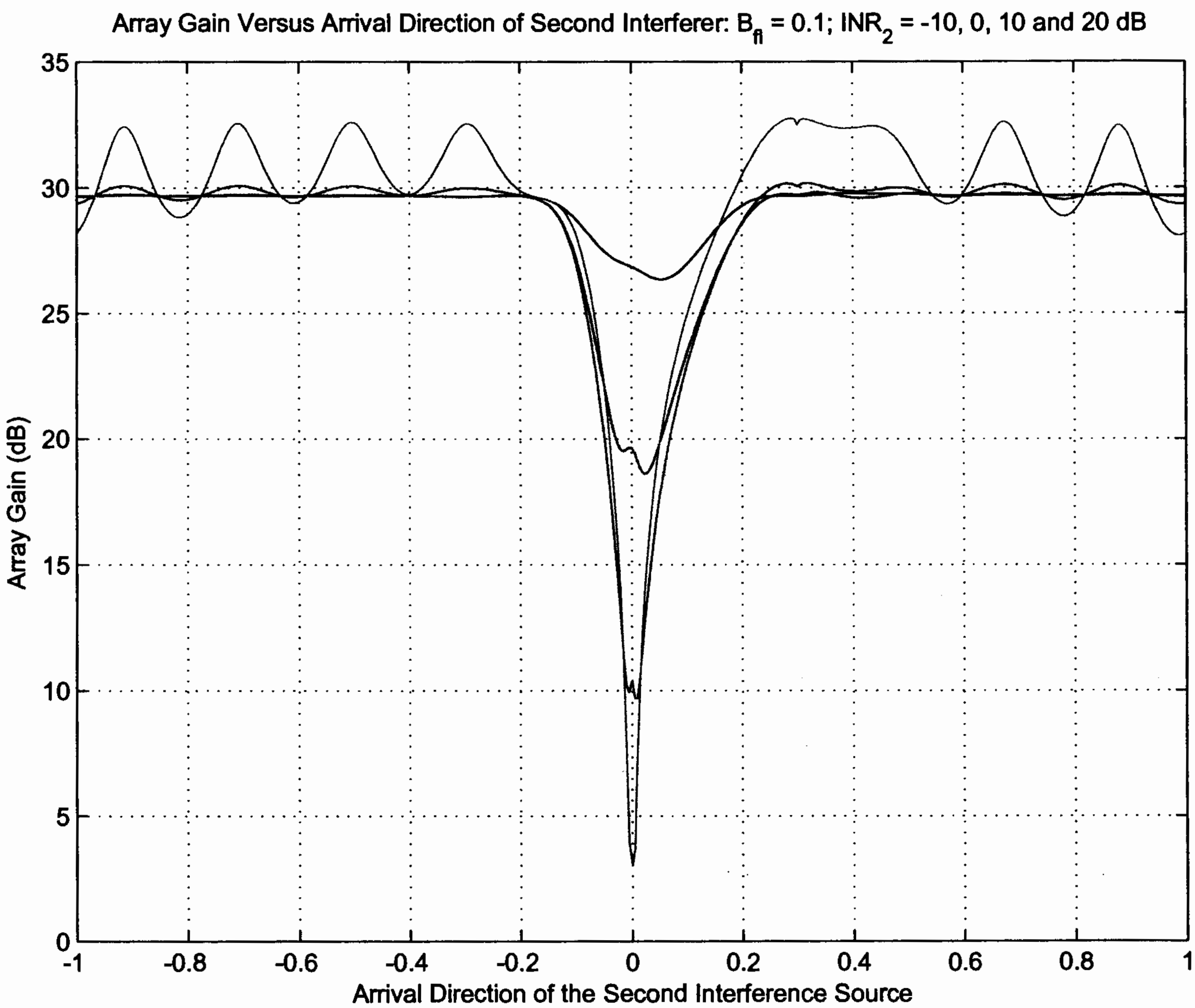


Beam Pattern of MVDR Beamformer Using Narrowband Noise Covariance Matrix: u_I = 0.3 and 0.5
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Beam Pattern (dB)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Direction Cosine u

Part (b)

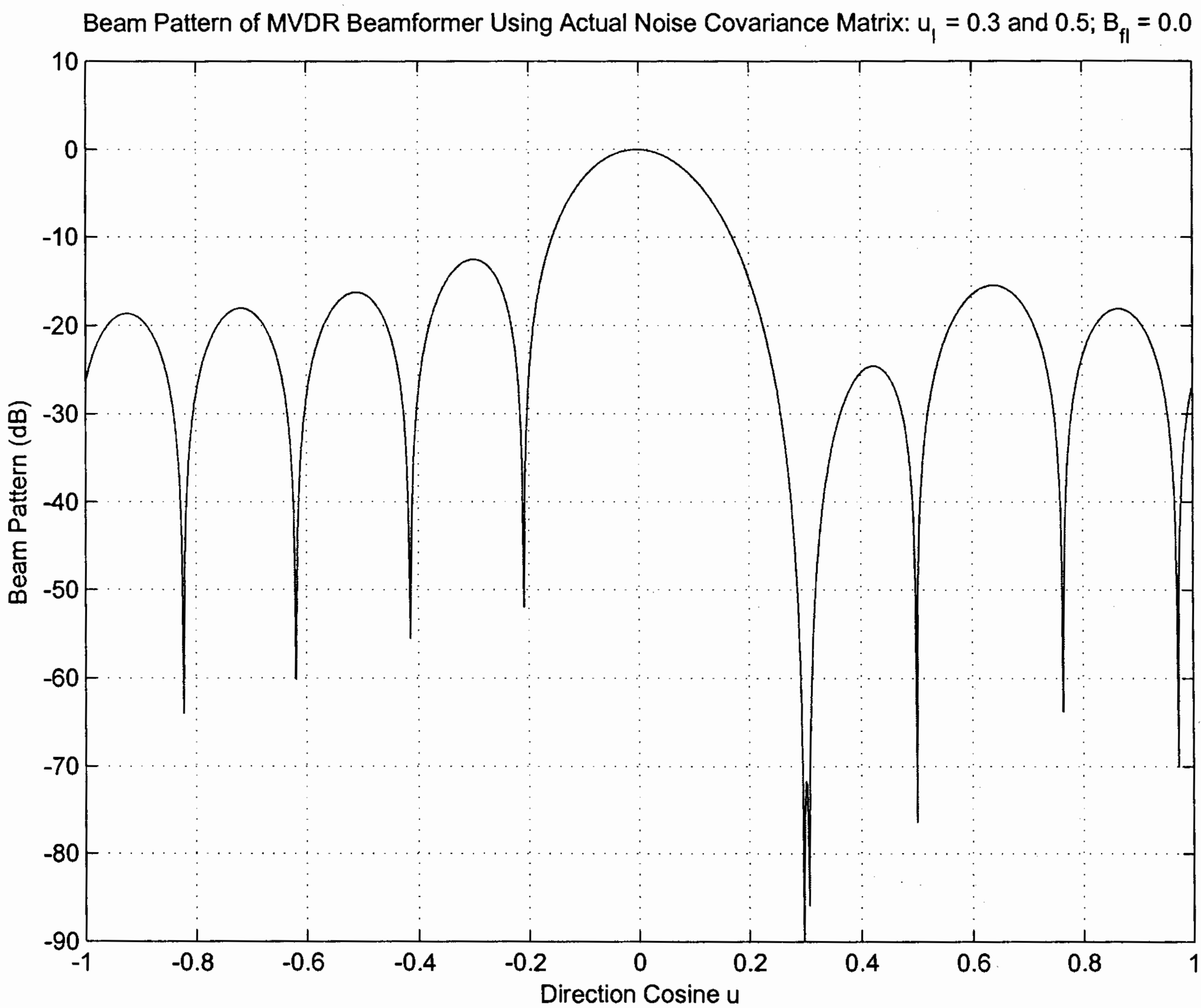

Array Gain Versus Arrival Direction of Second Interferer: B_{fi} = 0; INR_2 = -10, 0, 10 and 20 dB
35
30
25
20
15
10
5
0
Array Gain (dB)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Arrival Direction of the Second Interference Source

Part (b)



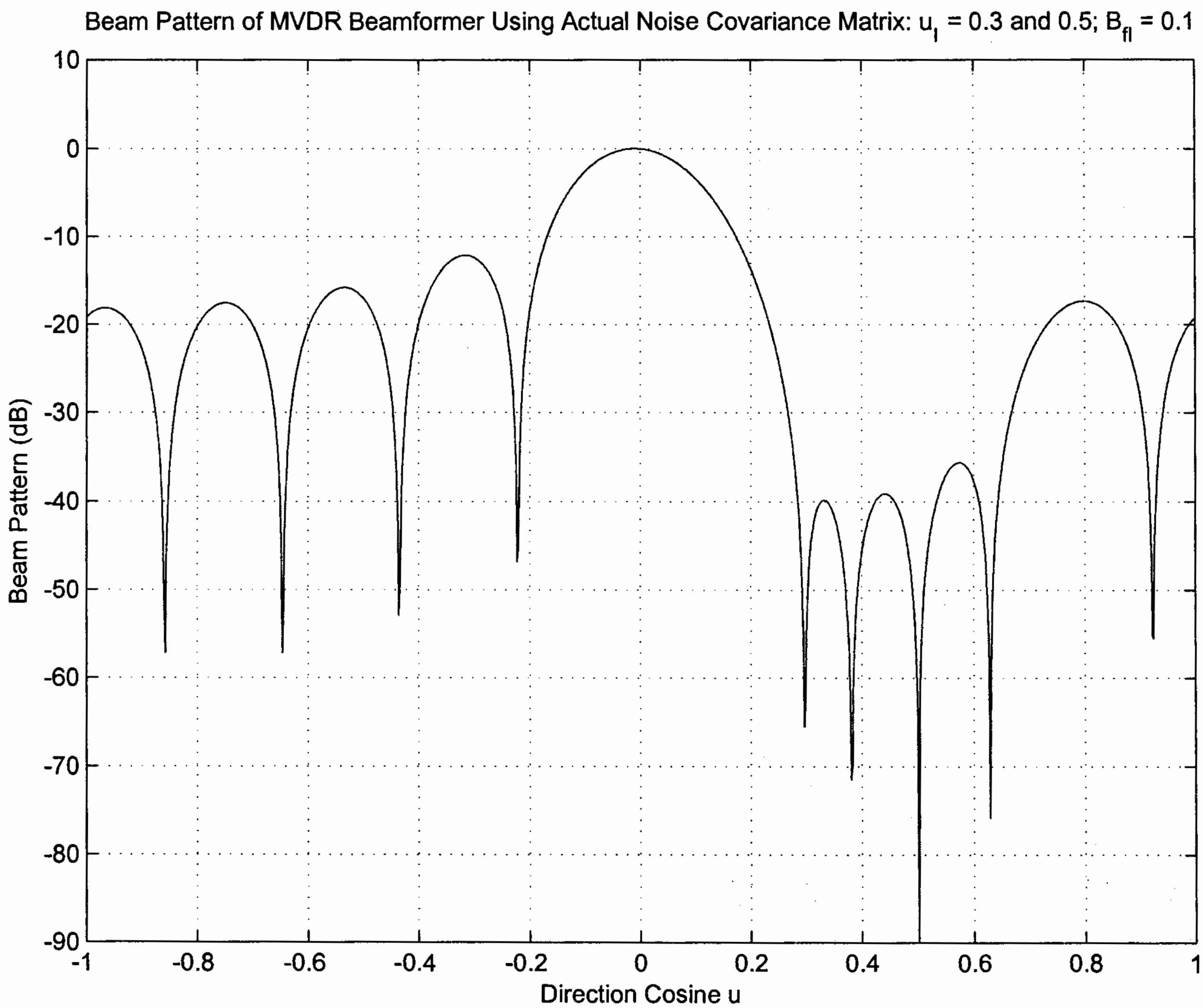
Array Gain Versus Arrival Direction of Second Interferer: B_{fl} = 0.1; INR_2 = -10, 0, 10 and 20 dB
Array Gain (dB)
Arrival Direction of the Second Interference Source
35
30
25
20
15
10
5
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Part (c)



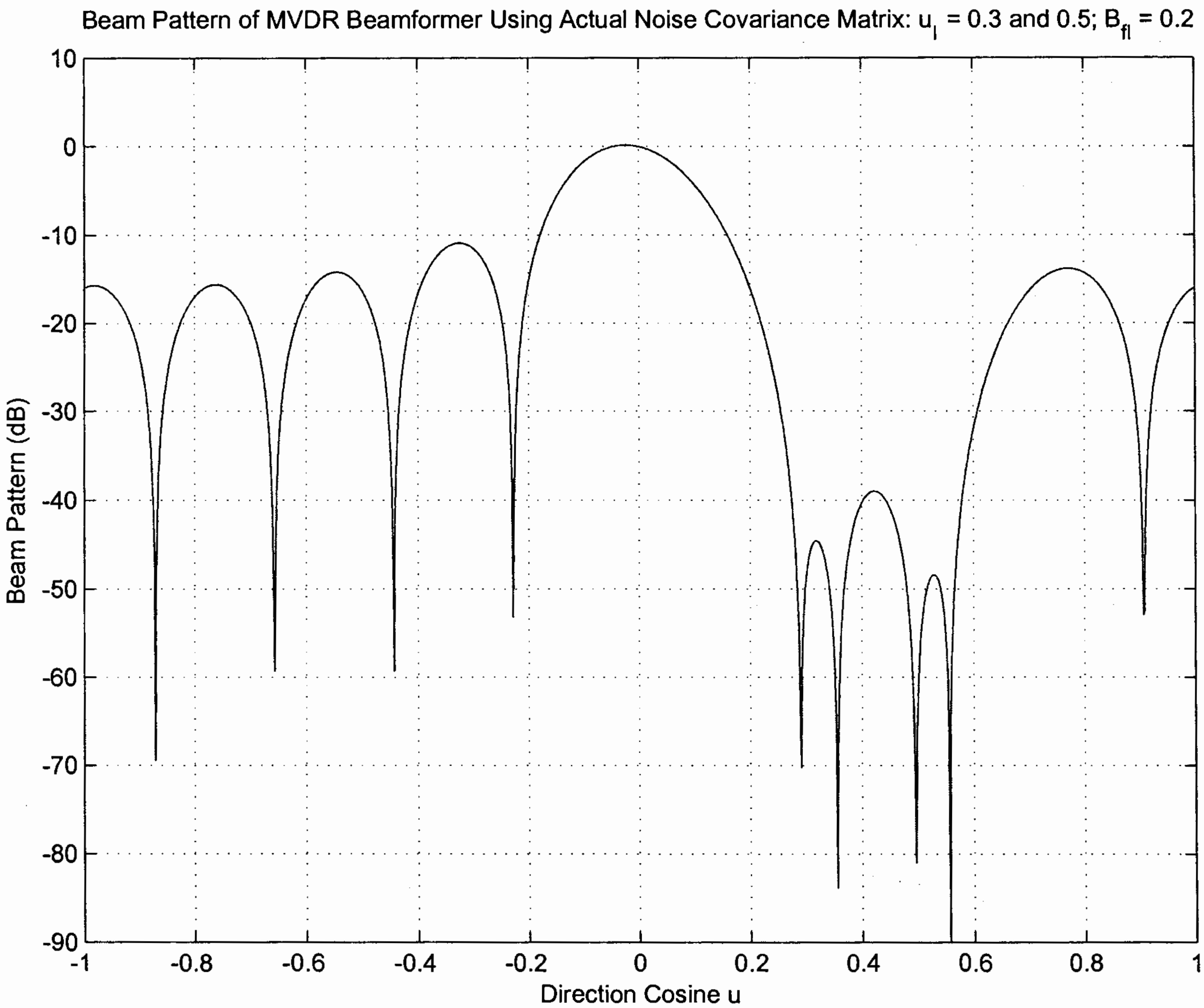
Beam Pattern of MVDR Beamformer Using Actual Noise Covariance Matrix: u_I = 0.3 and 0.5; B_{fI} = 0.0
Beam Pattern (dB)
Direction Cosine u
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Part (c)



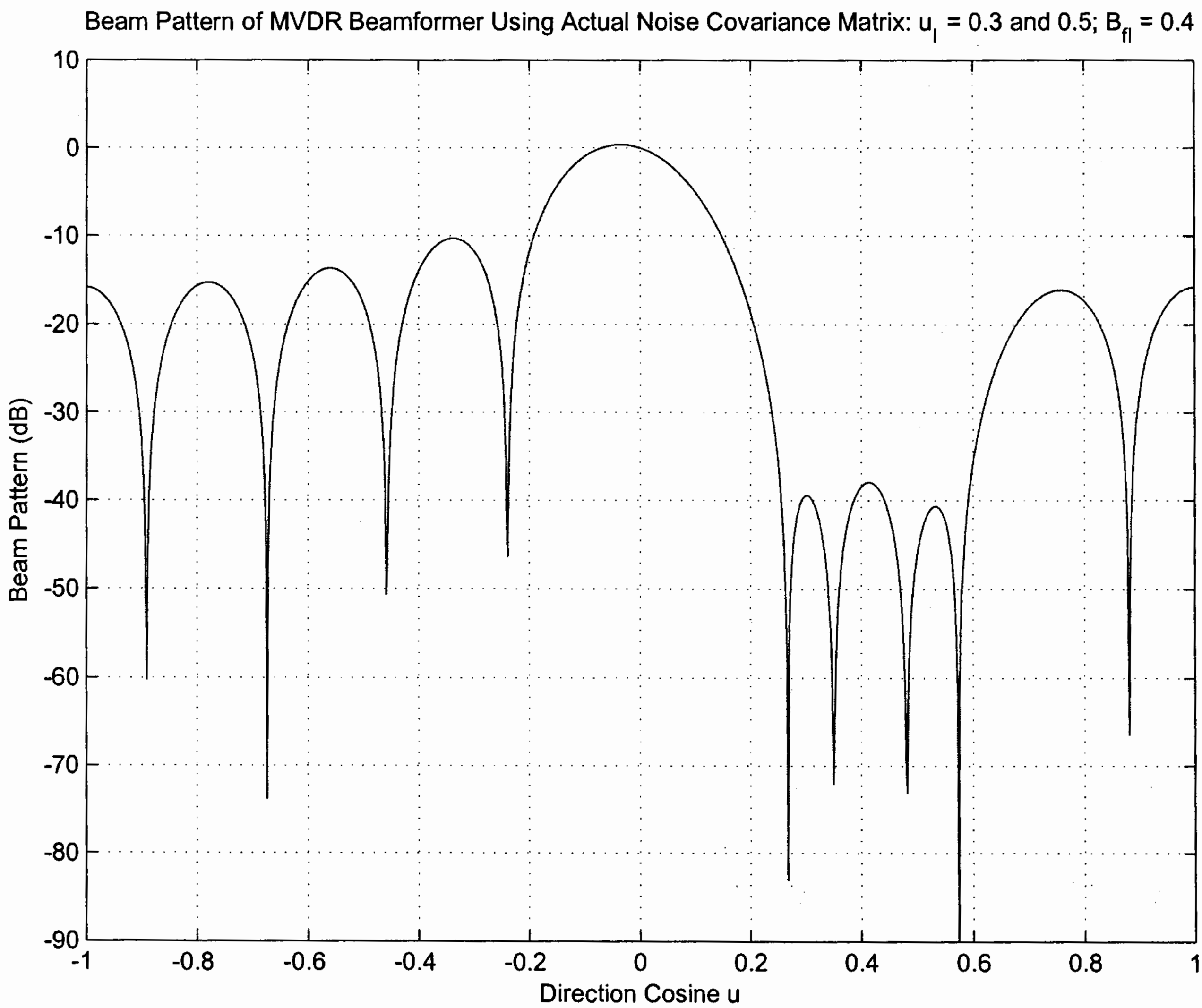
Beam Pattern of MVDR Beamformer Using Actual Noise Covariance Matrix: u_I = 0.3 and 0.5; B_{fI} = 0.1
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Beam Pattern (dB)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Direction Cosine u

Part (c)



Beam Pattern of MVDR Beamformer Using Actual Noise Covariance Matrix: u_I = 0.3 and 0.5; B_{fI} = 0.2
Beam Pattern (dB)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Direction Cosine u

Part (c)



Beam Pattern of MVDR Beamformer Using Actual Noise Covariance Matrix: u_I = 0.3 and 0.5; B_{fI} = 0.4
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Beam Pattern (dB)
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Direction Cosine u