6.4.3 assume $\omega = \omega_0$ corresponds to $\lambda_0 = 2d$, $N=10$

$$\underline{S}_{n\,k,l} = S_x(\omega_0 : p_l - p_k) + \sigma_n^2 \delta_{k,l}$$

$$S_x(\omega_0; \Delta p) = S_0(\omega_0)\left\{\mathrm{sinc}(k_0|\Delta p|) + \alpha\left[\left(\frac{3}{(k_0|\Delta p|)^2} - 1\right)\mathrm{sinc}(k_0|\Delta p|) - \frac{3\cos(k_0|\Delta p|)}{(k_0|\Delta p|)^2}\right]\right\}$$

for Δp along z-axis $[\cos(2\theta_z) = 1]$

- $S_0(\omega_0) = INR \cdot \sigma_n^2$
- $k_0 = 2\pi/\lambda_0$
- $|\Delta p| = |l-k|\,d = |l-k|\,\lambda_0/2$ for standard ULA
- $k_0|\Delta p| = \pi|l-k|$

$$\underline{S}_{n\,k,l} = \sigma_n^2\left[INR\left\{\mathrm{sinc}(\pi|l-k|) + \alpha\left[\left(\frac{3}{\pi^2(l-k)^2} - 1\right)\mathrm{sinc}(\pi|l-k|) - \frac{3\cos(\pi|l-k|)}{\pi^2(l-k)^2}\right]\right\} + \right.$$

$$S_n(\omega_0) = S_{n,k=l} = \sigma_n^2(INR+1)$$

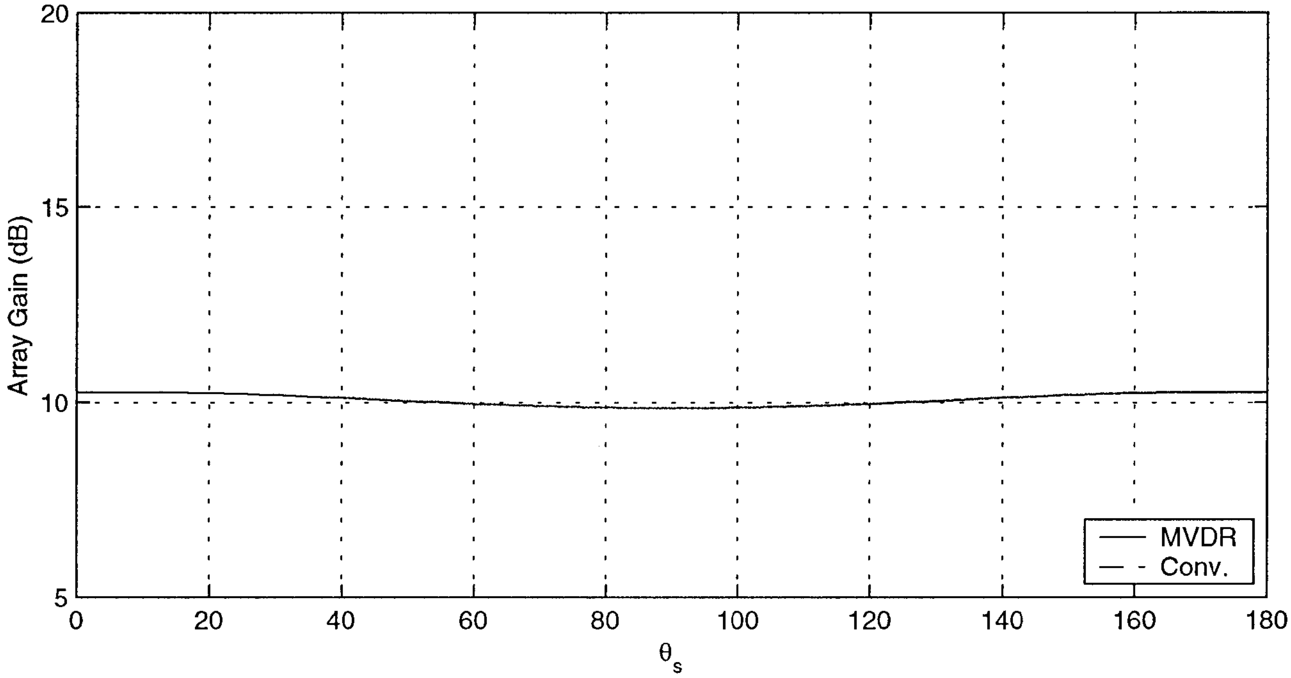
$$\rho_{n\,k,l} = \underline{S}_{n,k,l} / \sigma_n^2(INR+1)$$

$$A_{MVDR} = v(\theta_s)^H \rho_n^{-1} v(\theta_s)$$

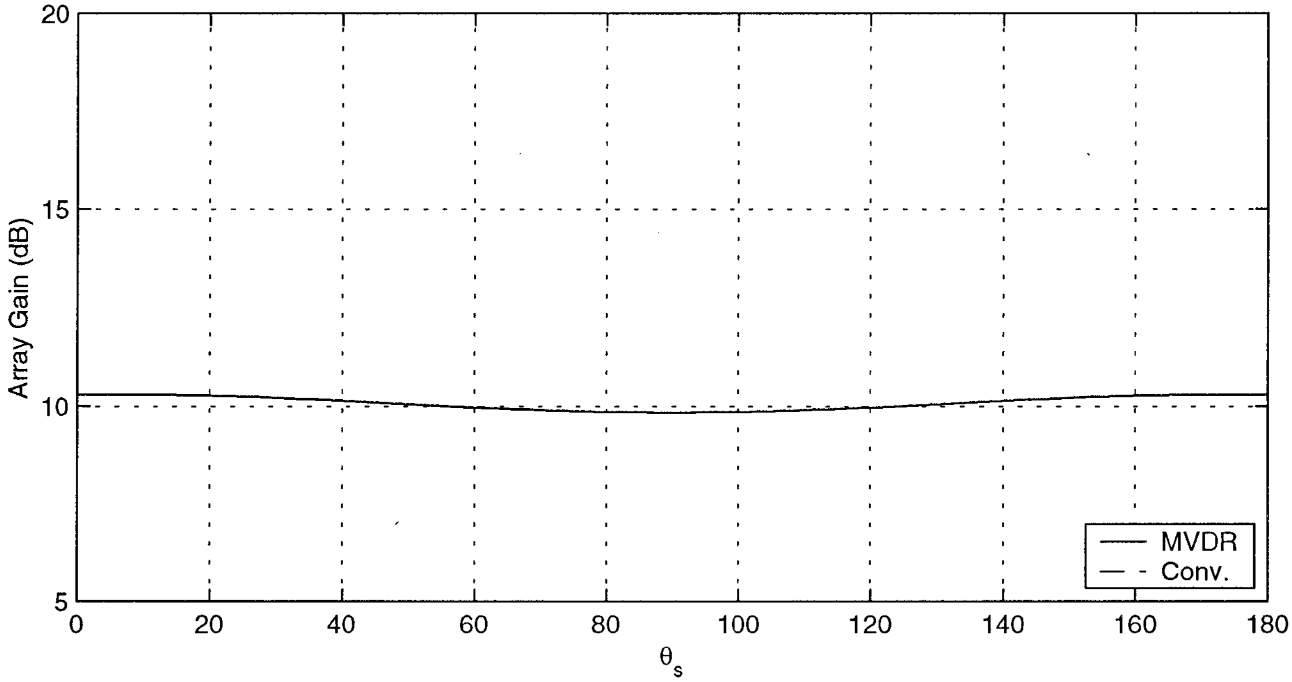
$$A_{conv} = N^2 / v(\theta_s)^H \rho_n v(\theta_s)$$

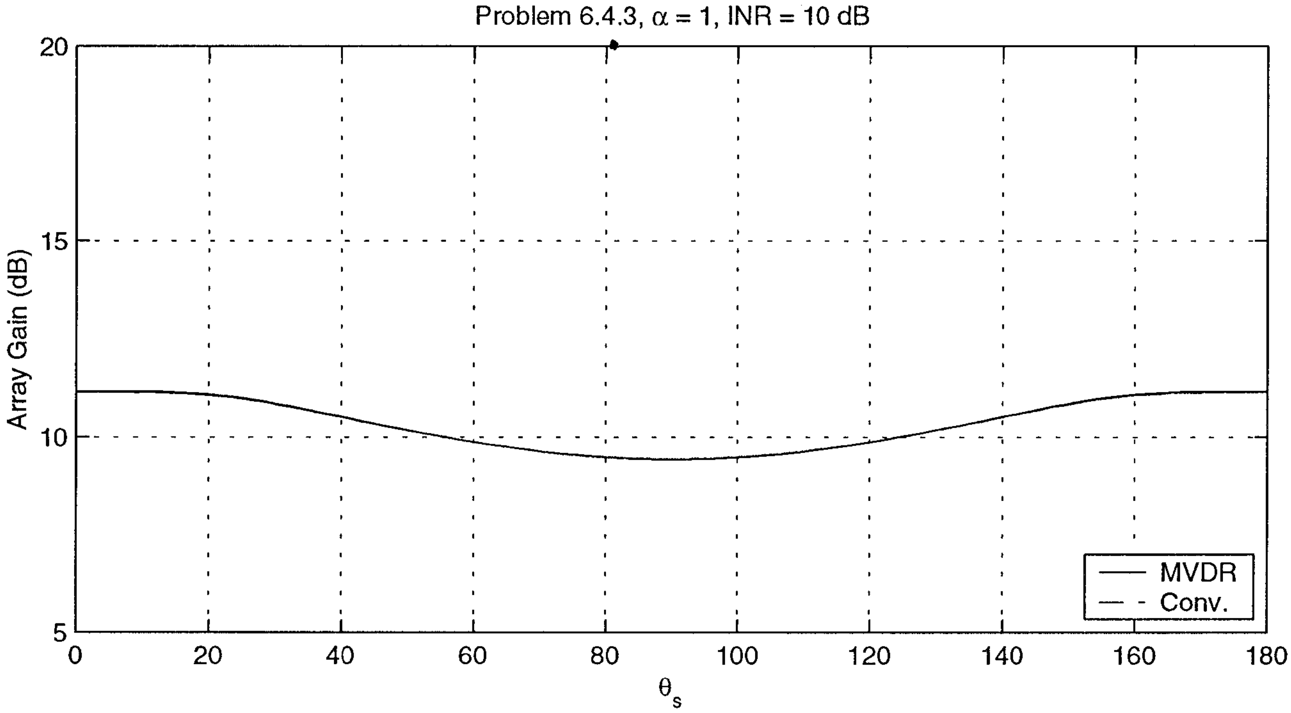
for $\alpha = 1$, get slightly reduced array gain at broadside than endfire. MVDR and conventional beamformers have practically the same performance.

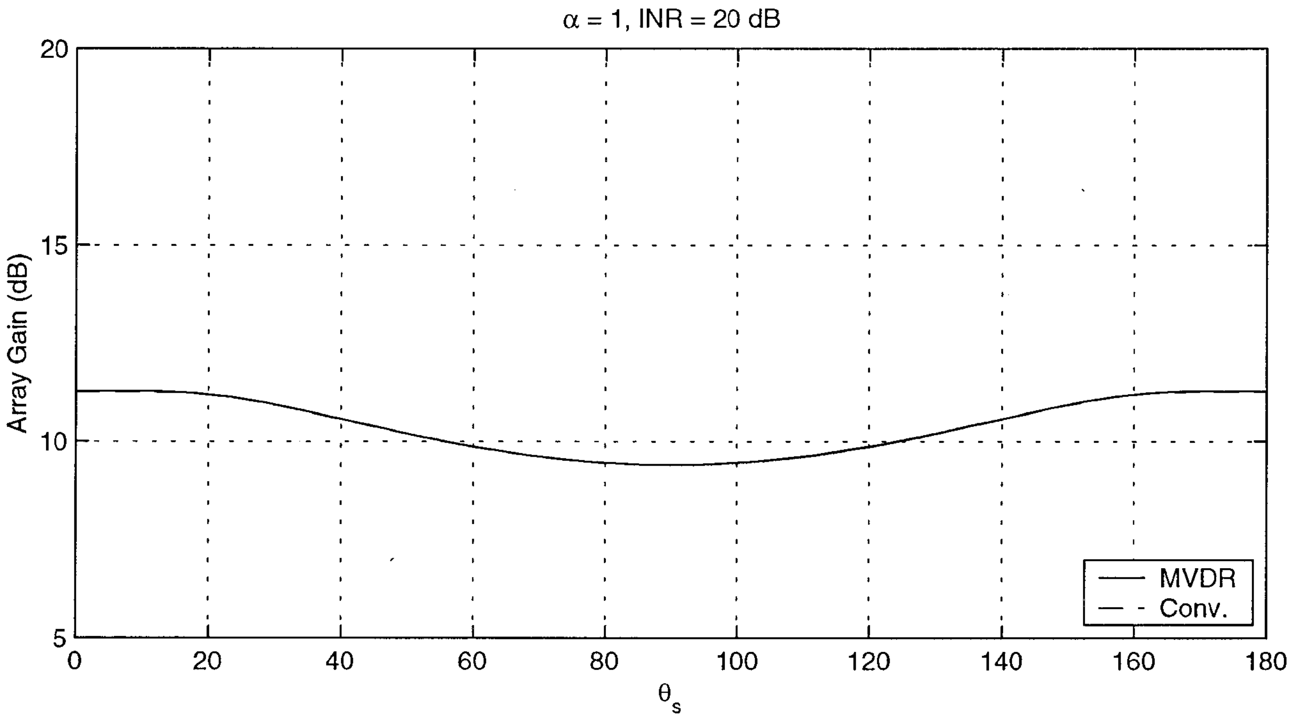
Problem 6.4.3, $\alpha = 0.25$, INR = 10 dB

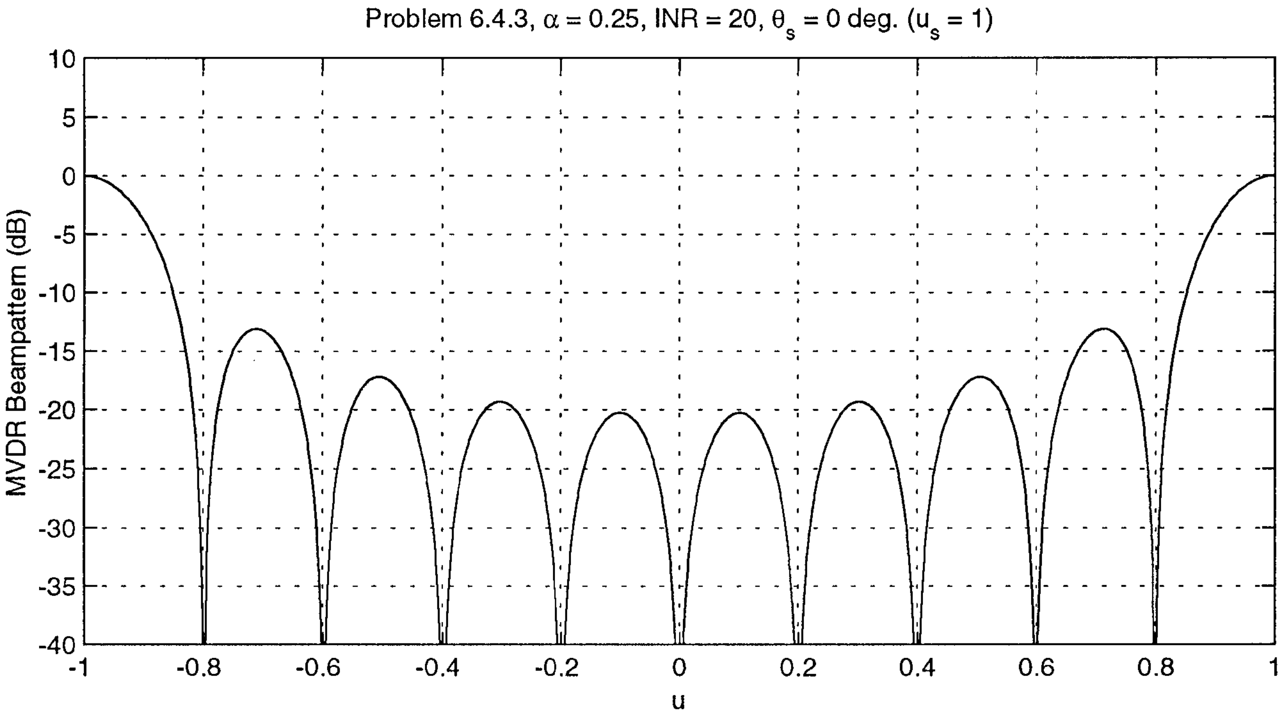


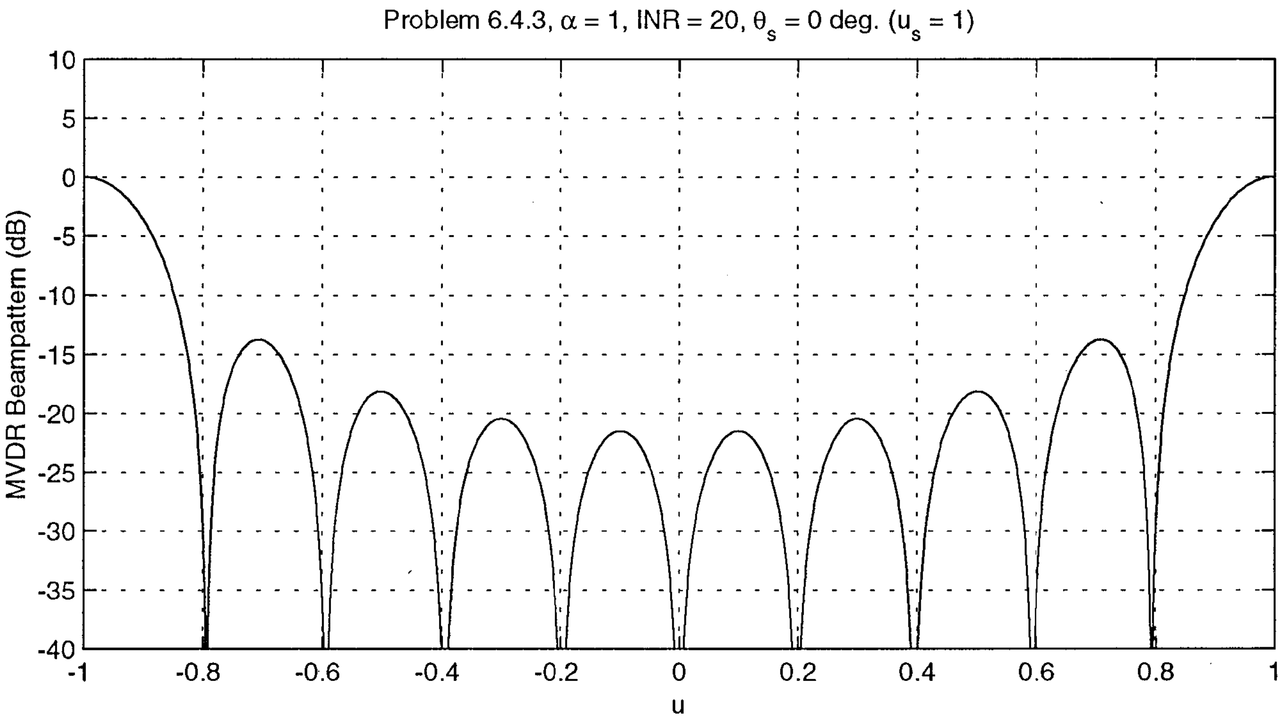
$\alpha = 0.25$, INR = 20 dB



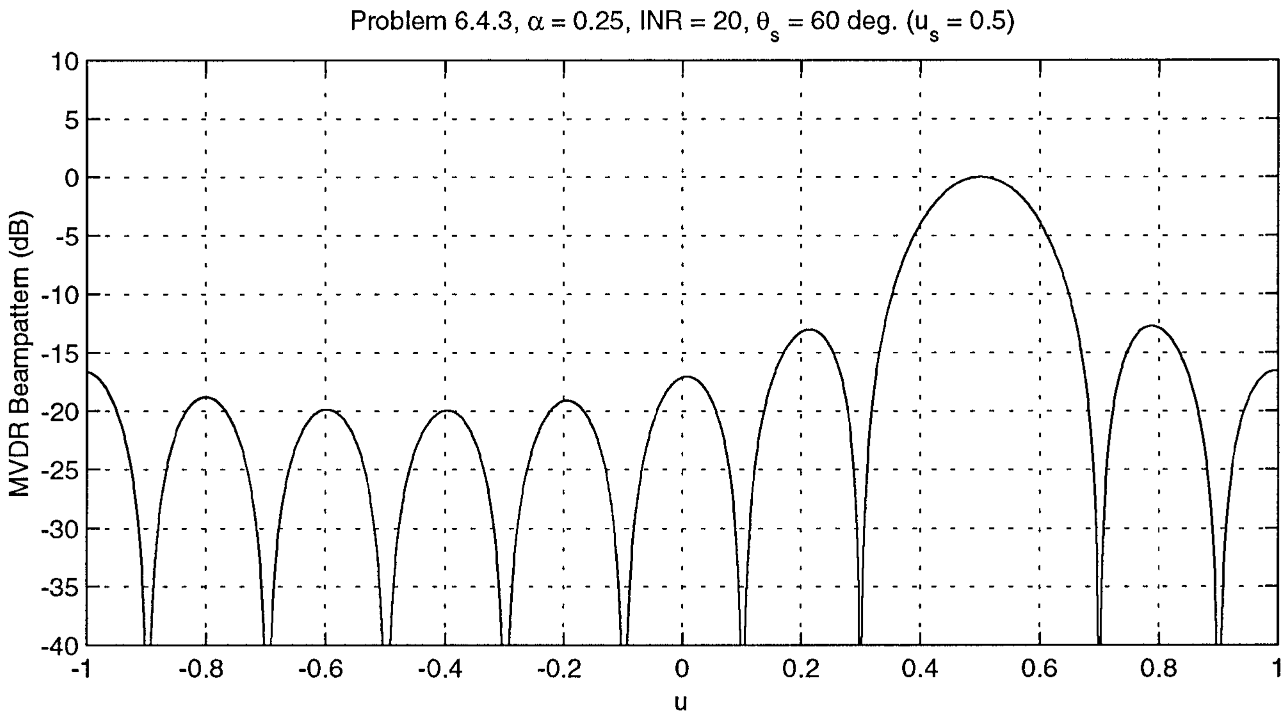

Problem 6.4.3, α = 1, INR = 10 dB
Array Gain (dB)
20
15
10
5
0
20
40
60
80
100
120
140
160
180
θs
MVDR
Conv.


α = 1, INR = 20 dB
Array Gain (dB)
20
15
10
5
0
20
40
60
80
100
120
140
160
180
θs
MVDR
Conv.

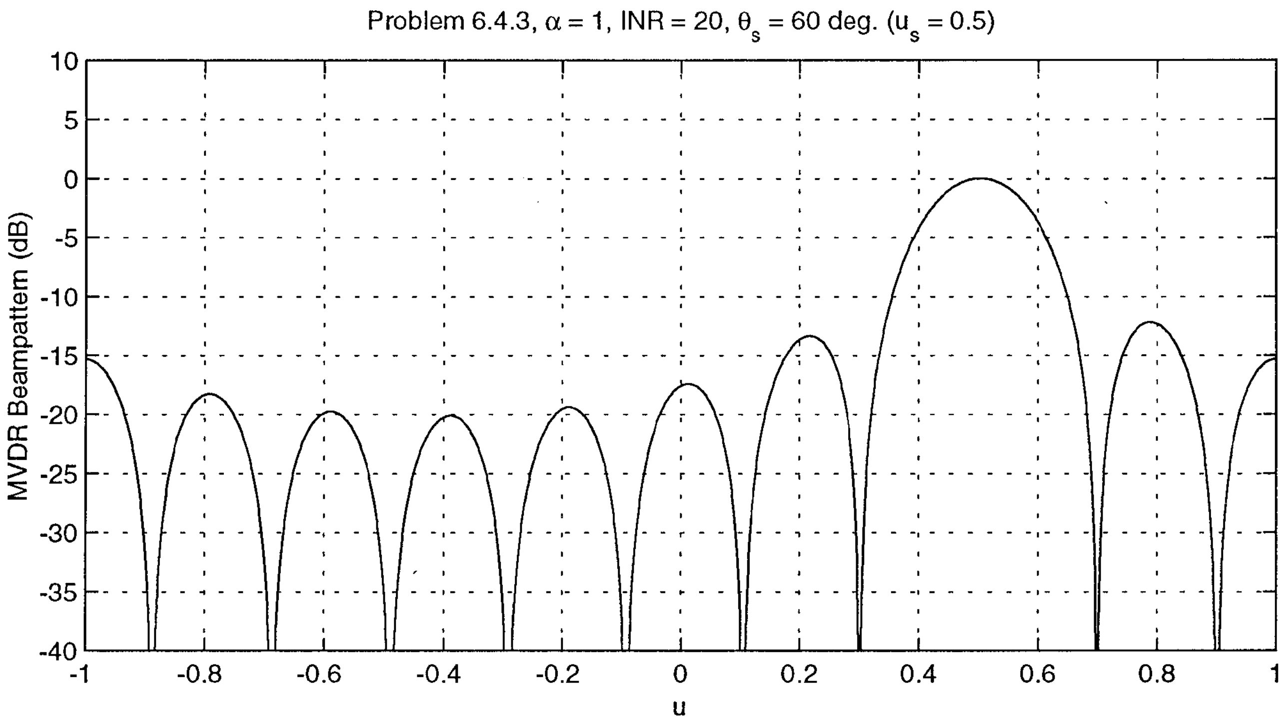

Problem 6.4.3, α = 0.25, INR = 20, θ_s = 0 deg. (u_s = 1)
MVDR Beampattern (dB)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
u


Problem 6.4.3, α = 1, INR = 20, θ_s = 0 deg. (u_s = 1)
MVDR Beampattern (dB)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
u

6.4.3 5/8

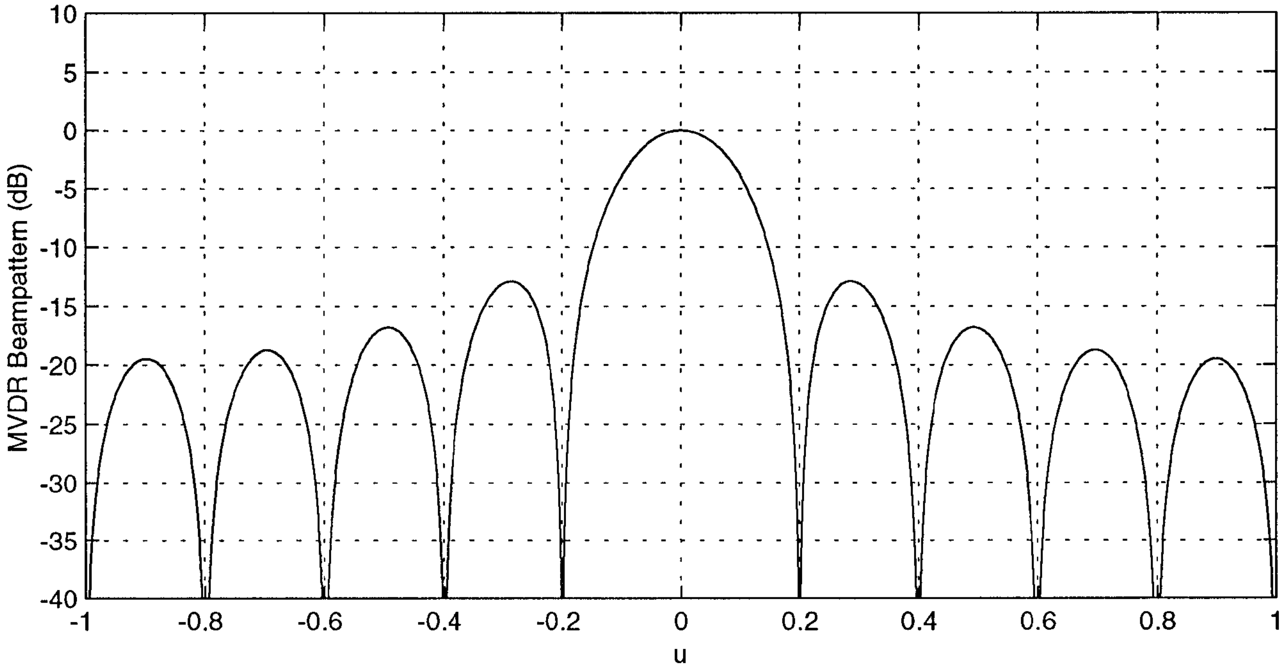


Problem 6.4.3, α = 0.25, INR = 20, θ_s = 60 deg. (u_s = 0.5)
MVDR Beampattern (dB)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
u

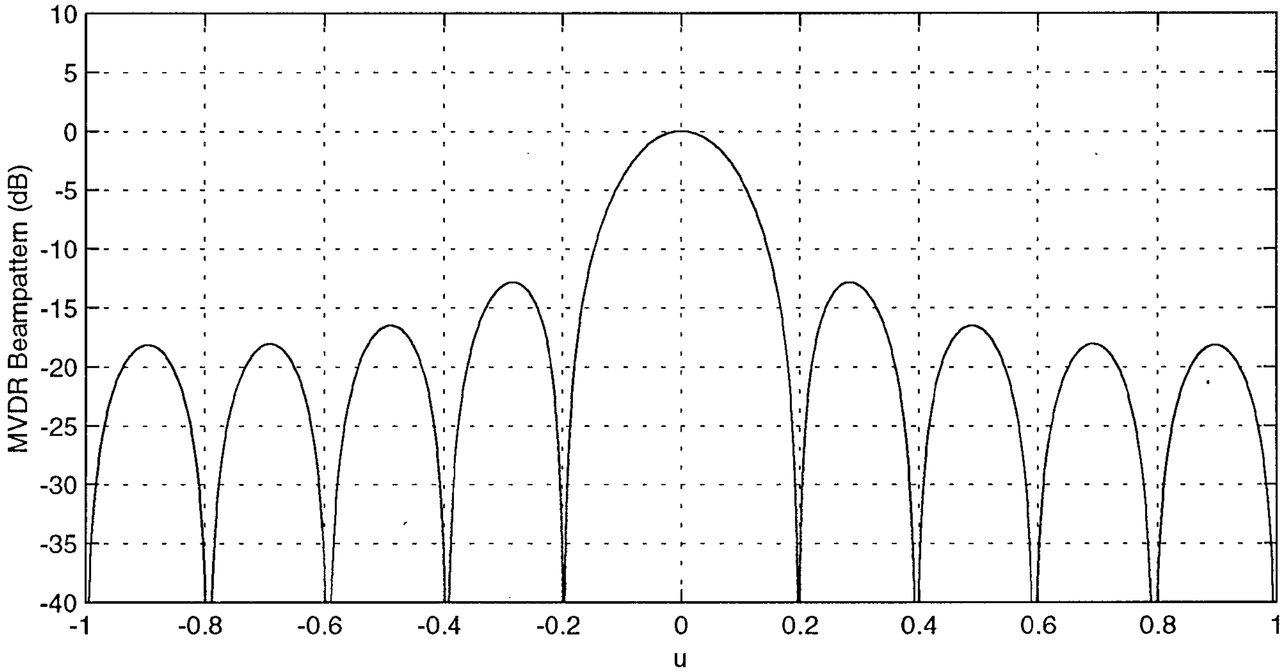


Problem 6.4.3, α = 1, INR = 20, θ_s = 60 deg. (u_s = 0.5)
MVDR Beampattern (dB)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
u

Problem 6.4.3, $\alpha = 0.25$, INR = 20, $\theta_s = 90$ deg. ($u_s = \cancel{6.1232e-017}$ 0)



Problem 6.4.3, $\alpha = 1$, INR = 20, $\theta_s = 90$ deg. ($u_s = \cancel{6.1232e-017}$ 0)



6.4.3 7/8

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% problem 6.4.3
% K. Bell 11/27/00
% Function called: sinc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all
close all

%************************
% Array
%************************
N = 10;                                   % Elements in array
d = 0.5;                                  % sensor spacing half wavelength wrt wc
D = [-(N-1)/2:1:(N-1)/2].';
BWNN = 2/(N*d);
u=[-1:0.001:1];
nu=length(u);
vv = exp(j*2*pi*d*D*u);

%************************
% Source
%************************
theta_s = [0 1/3 0.5]*pi;
us = cos(theta_s);
AS = exp(j*2*pi*d*D*us);
ns = length(us);

INR = 10.^([10 20]/10);
nI = length(INR);

alpha = [0.25 1];
for n=1:ns
   figure
   for a = 1:2
      p = [0:1:N-1];
      pI = [1:1:N-1];
      r = sinc(p*2*d)+[0 alpha(a)*((3./(pi*pI*2*d).^2)-1).*(sinc(pI*2*d))-
alpha(a)*cos(pi*2*d*pI)./(pi*pI*2*d).^2];
      Sn = INR(2)*toeplitz(r,conj(r))+eye(N);
      Sninv = inv(Sn);
      w = Sninv*AS(:,n)/real(AS(:,n)'*Sninv*AS(:,n));
      set(gcf,'Paperposition',[0.25 1 8 9])
      subplot(2,1,a)
      B = w'*vv;
      plot(u,10*log10(abs(B).^2));
      hold on
      xlabel('u')
      ylabel('MVDR Beampattern (dB)')
      title(['Problem 6.4.3, \alpha = ' num2str(alpha(a)) ', INR = '
num2str(10*log10(INR(2))) ', \theta_s = ' num2str(theta_s(n)*180/pi) ' deg. (u_s =
'num2str(us(n)) ')'])
      grid on
      axis([-1 1 -40 10])
      hold off
   end
end

theta_s = [0:0.01:1]*pi;
us = cos(theta_s);
AS = exp(j*2*pi*d*D*us);
ns = length(us);
for a = 1:2
   A = zeros(nI,ns);
   Ac = zeros(nI,ns);
   for n=1:ns
```

```
      for q=1:nI
         p = [0:1:N-1];
         pI = [1:1:N-1];
         r = sinc(p*2*d)+[0 alpha(a)*((3./(pi*pI*2*d).^2)-1).*(sinc(pI*2*d))-
alpha(a)*cos(pi*2*d*pI)./(pi*pI*2*d).^2];
         Sn = INR(q)*toeplitz(r,conj(r))+eye(N);
         Sninv = inv(Sn);
         Ac(q,n) =N*N*Sn(1,1)/ real(AS(:,n)'*Sn*AS(:,n));
         A(q,n) = real(AS(:,n)'*Sninv*AS(:,n))*Sn(1,1);
      end
   end
   figure
   subplot(2,1,1)
   h1=plot(theta_s*180/pi,10*log10(A(1,:)),'-');
   hold on
   h2=plot(theta_s*180/pi,10*log10(Ac(1,:)),'--');
   legend('MVDR', 'Conv.',4)
   xlabel('\theta_s')
   ylabel('Array Gain (dB)')
   title(['Problem 6.4.3, \alpha = ' num2str(alpha(a)) ', INR = '
num2str(10*log10(INR(1))) ' dB'])
   grid on
   hold off
   axis([0 180 5 20])
   subplot(2,1,2)
   h1=plot(theta_s*180/pi,10*log10(A(2,:)),'-');
   hold on
   h2=plot(theta_s*180/pi,10*log10(Ac(2,:)),'--');
   legend('MVDR', 'Conv.',4)
   xlabel('\theta_s')
   ylabel('Array Gain (dB)')
   title(['\alpha = ' num2str(alpha(a)) ', INR = ' num2str(10*log10(INR(2))) ' dB'])
   grid on
   hold off
   axis([0 180 5 20])
   set(gcf,'Paperposition',[0.25 1 8 9])
end
```