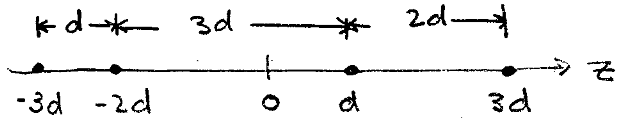
Problem 2.4.6



$d = \lambda/2$

$$B(\varphi) = \frac{1}{4}\left\{ e^{-j3\varphi} + e^{-j2\varphi} + e^{j\varphi} + e^{j3\varphi} \right\}$$

This can be written as

(i) $$B(\varphi) = \frac{1}{4}\left\{ e^{-j3\varphi} + e^{j3\varphi} + e^{-j2\varphi} + \frac{1}{2}e^{j2\varphi} - \frac{1}{2}e^{j2\varphi} + e^{j\varphi} + \frac{1}{2}e^{-j\varphi} - \frac{1}{2}e^{-j\varphi} \right\}$$

$$\boxed{B(\varphi) = \frac{1}{4}\left\{ 2\cos(3\varphi) + \cos(2\varphi) + \cos(\varphi) - j\sin(2\varphi) + j\sin(\varphi) \right\}}$$

or (ii) $$B(\varphi) = \frac{1}{4}\left\{ e^{-j3\varphi} + e^{j3\varphi} \right\} + \frac{e^{-j\varphi/2}}{4}\left\{ e^{-j3\varphi/2} + e^{j3\varphi/2} \right\}$$

$$\boxed{B(\varphi) = \frac{1}{2}\cos(3\varphi) + \frac{1}{2}e^{-j\varphi/2}\cos\left(\frac{3}{2}\varphi\right)}$$

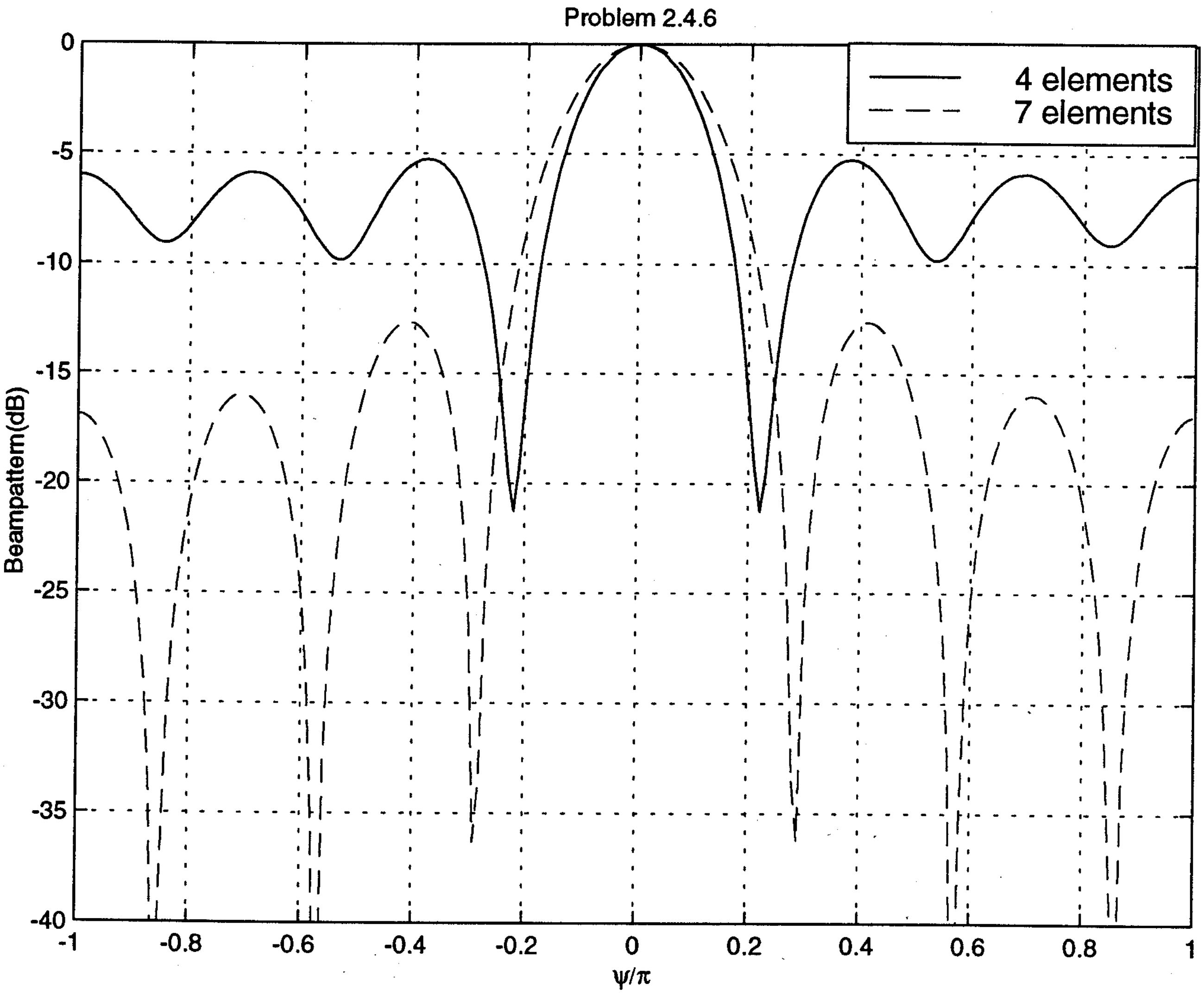
The beampattern does not have true nulls ($B(\varphi)=0$), therefore the "null-to-null" beamwidth is really the "minimum-to-minimum" beamwidth. The minima occur at $\varphi_m = \pm 0.2205\pi = \pm 0.6927$ with $B(\varphi_m) = 0.0862$. Thus $\boxed{BW_{mm} = 0.4410\,\pi = 1.3854}$

b) For 7-element uniform linear array, nulls occur at $\varphi_N = \pm \frac{2}{7}\pi = 0.2857\pi = 0.8976$, with

$BW_{NN} = \frac{4}{7}\pi = 0.5714\,\pi = 1.7952$.

(i) 4-element array has narrower main beam

(ii) 4-element array has much higher sidelobes and shallow "nulls".


Problem 2.4.6
Beampattern(dB)
ψ/π
4 elements
7 elements
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1