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Problem 6.7.9

Analyze the performance of a LCMP-GSC beamformer in which fixed nulls are placed in the direction of strong interferers whose directions have been previously determined.

Implement it as a generalized sidelobe canceller.

Solution:

(1) Set up the $\underline{C}$ and g matrices

(a) $\underline{C} = [\, \underline{v}(0) \quad \underline{v}(0.3) \quad \underline{v}(0.5) \quad \underline{v}(0.7) \quad \underline{v}(-0.5) \,]$

(b) $g = [\, 1 \quad 0 \quad 0 \quad 0 \quad 0 \,]^T$

(2) Determine the upper and lower branches of the GSC:

(a) Upper branch:

$\underline{w}_q = \underline{C}(\underline{C}^H \underline{C})^{-1} g \qquad (20 \times 1)$

$\underline{C} = 20 \times 5$ matrix

(b) Lower branch:

Calculate the SVD of $\underline{C}$:

$\underline{C} = \underline{U}\,\underline{\Sigma}\,\underline{V}^H$

$\Rightarrow$ $\underline{B} = [\underline{u}_6 \ \underline{u}_7 \ \ldots \ \underline{u}_{20}]$

(Columns extracted from $\underline{U}$)

Can show that $\underline{C}^H \underline{B} = \underline{O}$

$\Rightarrow$ $\underline{w}_a = (\underline{B}^H \underline{S}_x \underline{B})^{-1} \underline{B}^H \underline{S}_x^H \underline{w}_q$

(3) I also evaluated the performance of a MPDR beamformer:

$\underline{w} = \underline{S}_x^{-1} \underline{v}(0) / (\underline{v}(0)^H \underline{S}_x^{-1} \underline{v}(0))$

The results are plotted and are identical.

(4) The constrained beam patterns are essentially the same as the conventional beam former w/uniform shading. The 5 interferers all fall in the natural nulls of the conventional beamformer, so there is no need to move nulls

(b) Assume random errors in the estimated DOA's and assess performance loss as a function σ_e

<u>Solution</u>:

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I added Gaussian distributed random variables to each nominal DOA.

I estimated array gain of the LCMP-GSC and averaged over 1000 individual Monte Carlo trials.

I used 3 standard deviations:

$\sigma_e = 0.1, \; 0.2$ and 0.3

I plotted the results as a function of σ_e and included the result from part (a) (known DOA's) to benchmark the results.

I was "disappointed" because the loss was not as large as I expected.

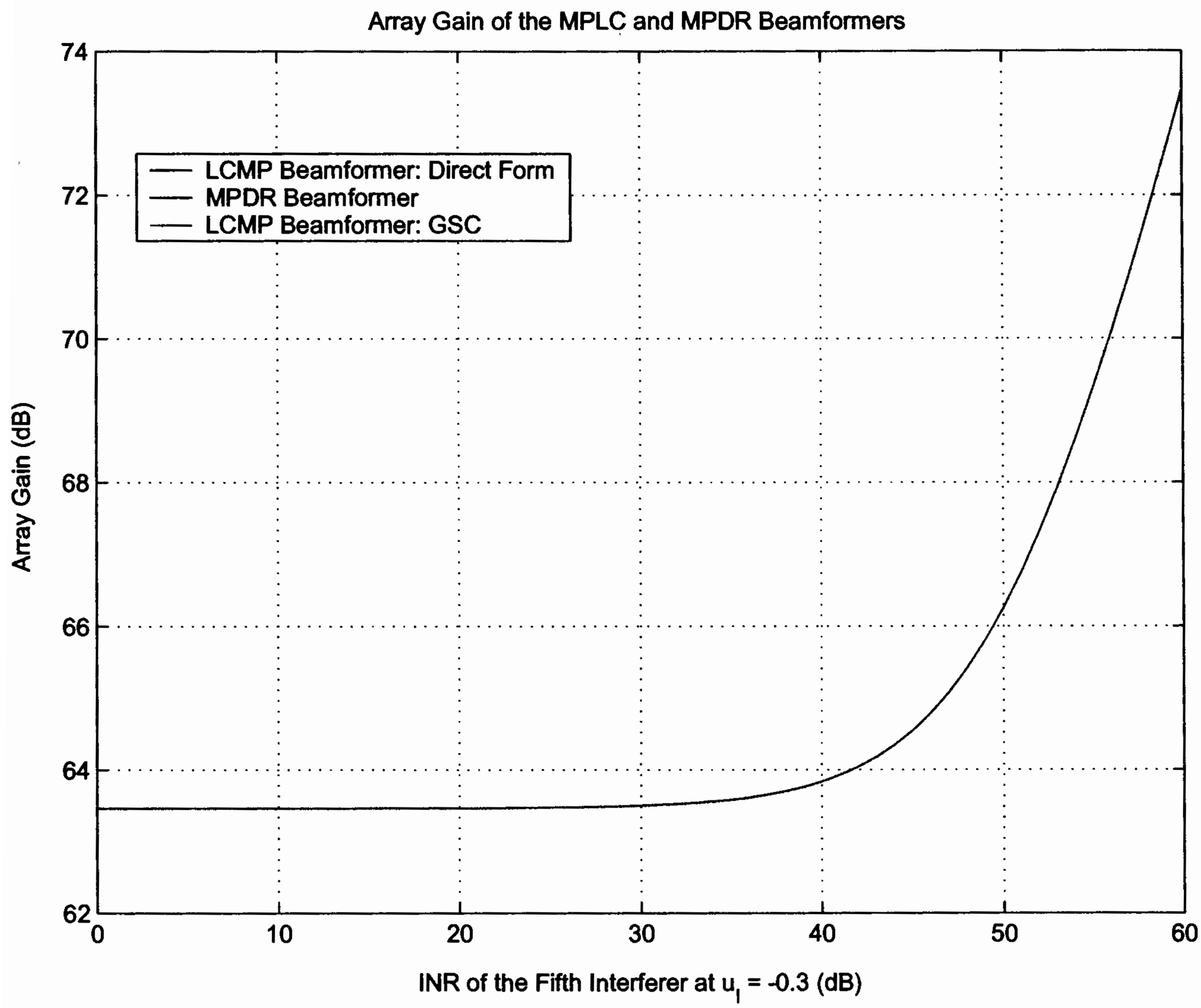
To check the results I did a couple of things:

(1) Repeated for N = 10. The performance loss was more pronounced. I think the reason is that the interferers do not fall into the natural nulls of the unconstrained conventional beam pattern.

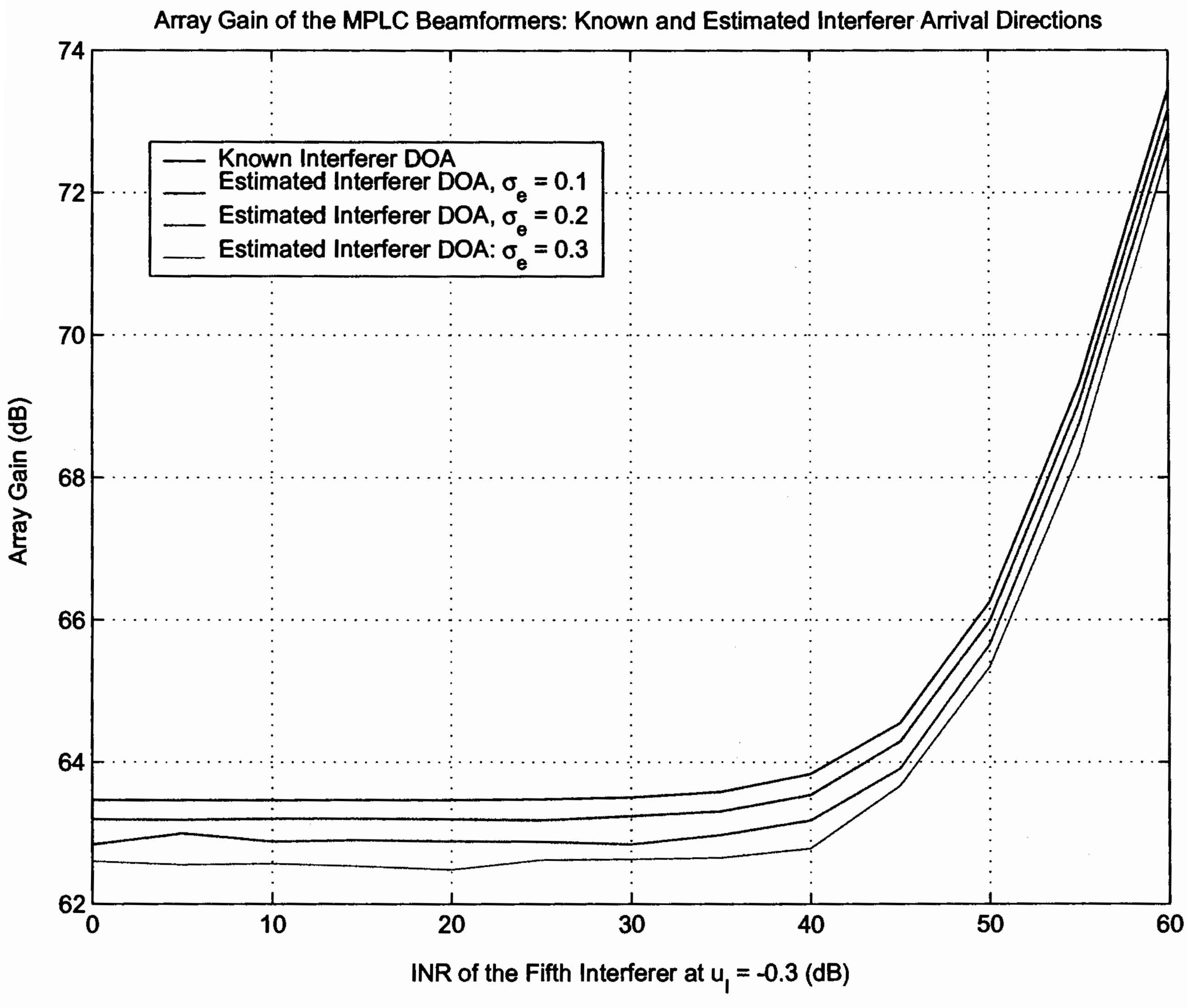
(2) Examined the beam patterns of the

LCMP-GSC including random errors. The beam patterns are significantly impacted; however, I found in most cases, there were nulls on or close to the actual interferer DOA's. I believe this is why performance loss was less than what I expected.

How to improve robustness: I would add a derivative constraint at the estimated DOA to broaden out the nulls. Of course, the cost is fewer adaptive degrees of freedom that the processor has available to null out unmodeled interferers.



Array Gain of the MPLC and MPDR Beamformers
LCMP Beamformer: Direct Form
MPDR Beamformer
LCMP Beamformer: GSC
74
72
70
68
66
64
62
0
10
20
30
40
50
60
Array Gain (dB)
INR of the Fifth Interferer at u_I = -0.3 (dB)



Array Gain of the MPLC Beamformers: Known and Estimated Interferer Arrival Directions
Known Interferer DOA
Estimated Interferer DOA, $\sigma_e = 0.1$
Estimated Interferer DOA, $\sigma_e = 0.2$
Estimated Interferer DOA: $\sigma_e = 0.3$
Array Gain (dB)
INR of the Fifth Interferer at $u_I = -0.3$ (dB)
74
72
70
68
66
64
62
0
10
20
30
40
50
60