Problem 2.8.4

$N_x = 10$, $N_z = 10$, $d_x = d_z = \lambda/2$, uniform weighting $w_n = \frac{1}{100}$

$$B(\underline{k}) = \sum_{n=0}^{N-1} w_n^* e^{-j \underline{k}^T \underline{p}_n} = \sum_{n=0}^{N-1} \frac{1}{100} e^{-j(k_x p_x + k_z p_z)} =$$

$$= \sum_{n=0}^{10} \sum_{m=0}^{10} \frac{1}{100} e^{-j k_x (n - \frac{N-1}{2}) d} \; e^{-j k_z (n - \frac{N-1}{2}) d}$$

$$= \left(\sum_{n=0}^{10} \frac{1}{10} e^{-j k_x (n - \frac{N-1}{2}) d} \right) \left(\sum_{m=0}^{10} \frac{1}{10} e^{-j k_z (n - \frac{N-1}{2}) d} \right)$$

$$\varphi_x = -k_x d = \frac{2\pi}{\lambda} \sin\theta \cos\phi \, d \quad ; \quad \varphi_z = -k_z d = \frac{2\pi}{\lambda} \cos\theta \, d$$

$$= \frac{\sin(5\varphi_x)}{10 \sin(\frac{1}{2}\varphi_x)} \cdot \frac{\sin(5\varphi_z)}{10 \sin(\varphi_z)}$$

$$B(\theta, \phi) = \frac{\sin(5\pi \sin\theta\cos\phi)}{10 \sin(\pi/2 \sin\theta\cos\phi)} \cdot \frac{\sin(5\pi \cos\theta)}{10 \sin(\pi/2 \cos\theta)}$$

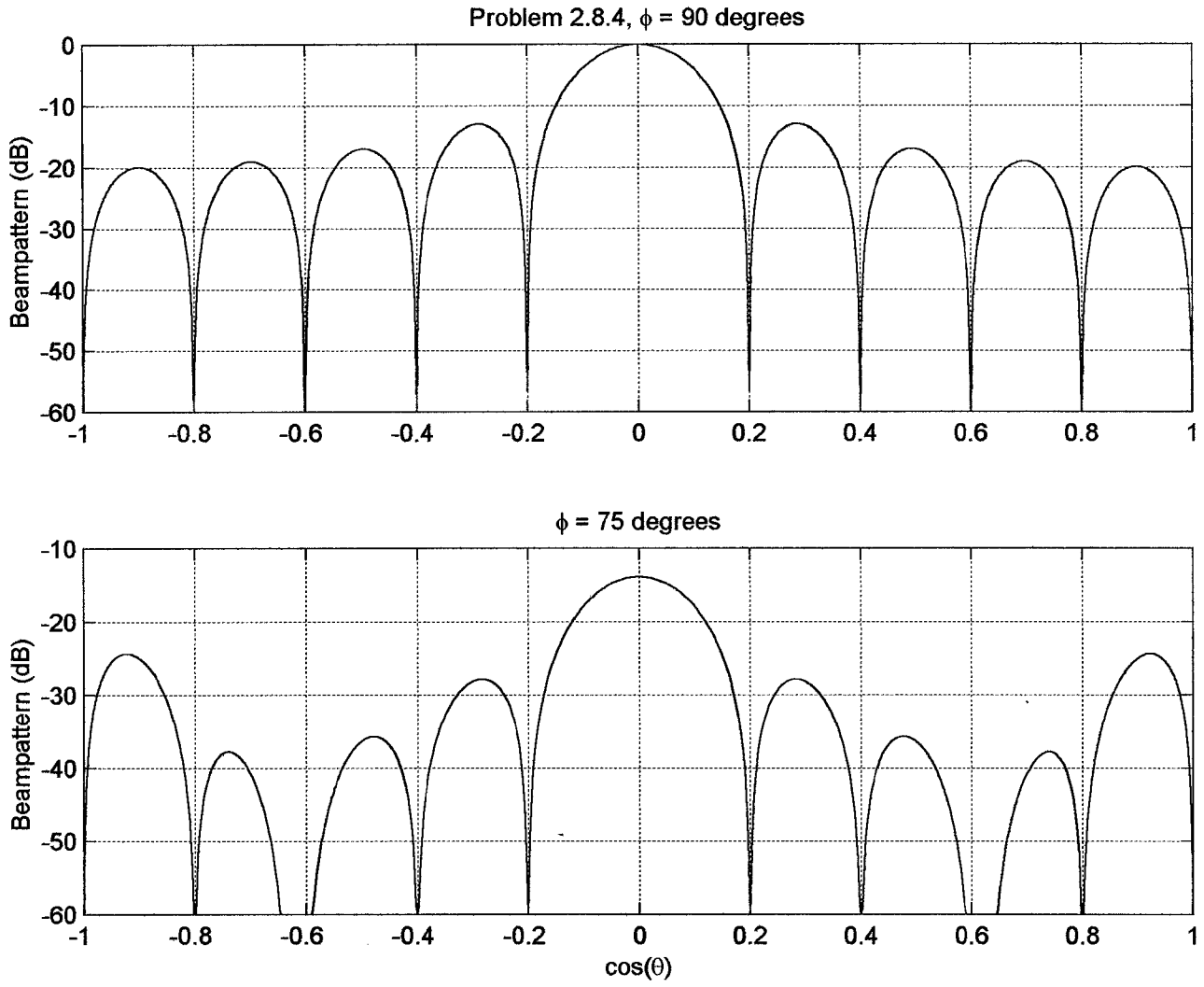
see plots

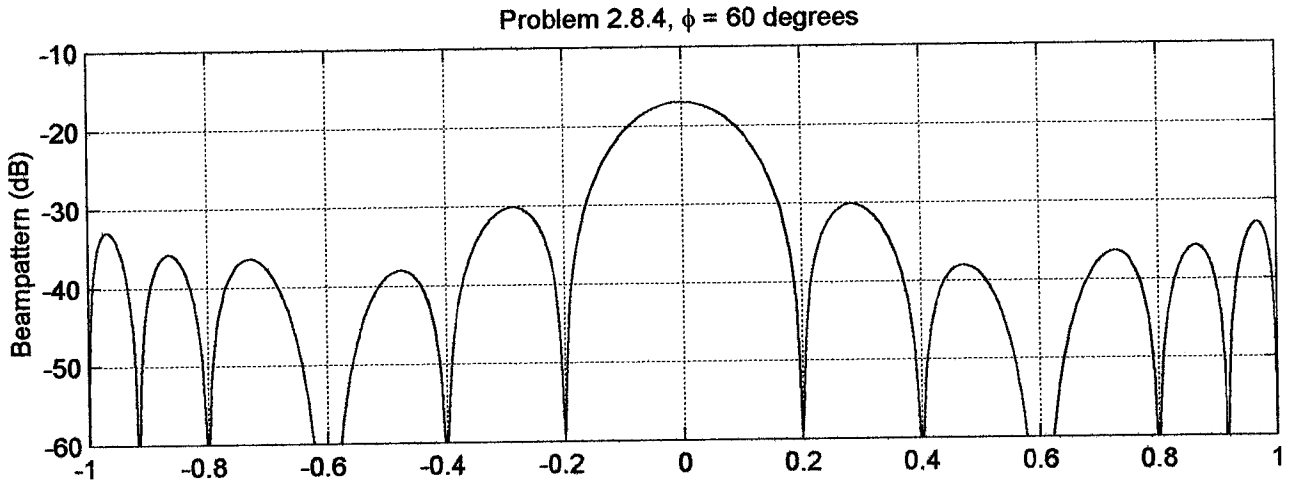
in terms of $u_x = \varphi_x/\pi$, $u_z = \varphi_z/\pi$ when $d = \lambda/2$

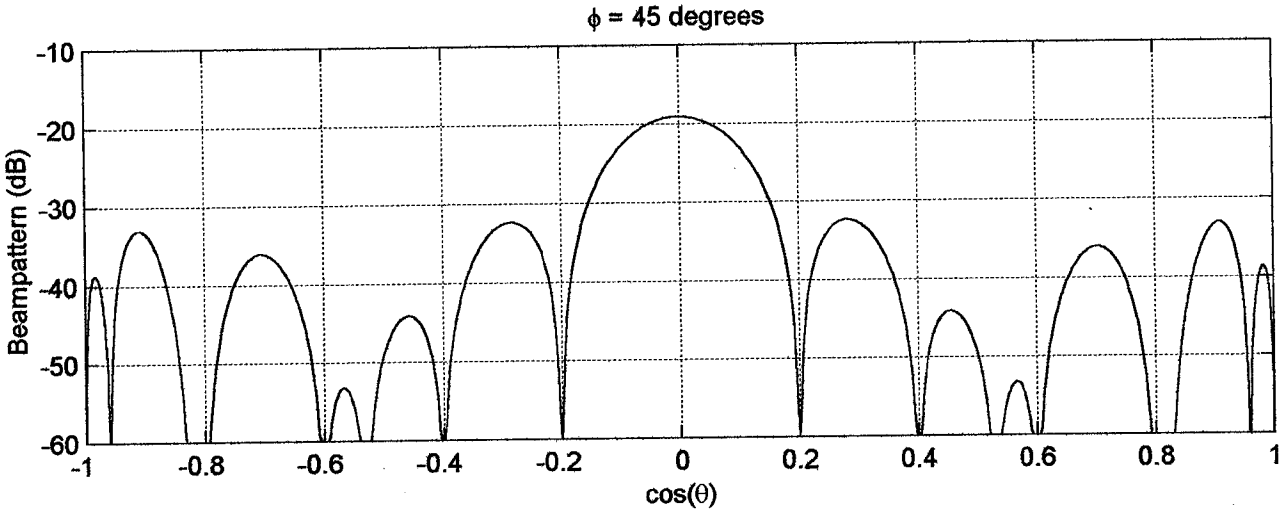
$$B(u_x, u_z) = \frac{\sin(5u_x)}{10 \sin(u_x/2)} \cdot \frac{\sin(5u_z)}{10 \sin(u_z/2)}$$

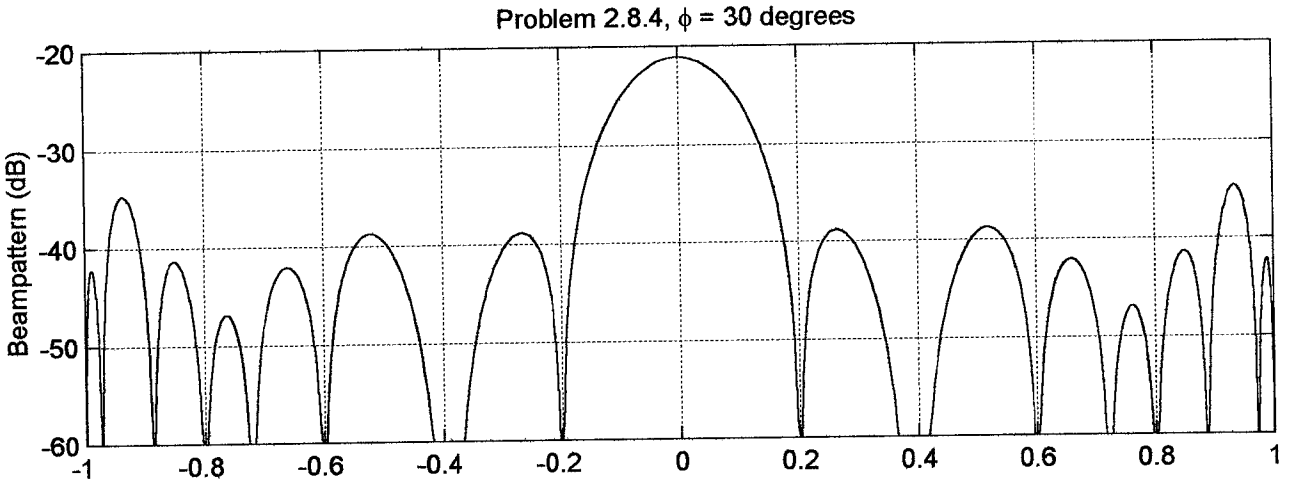
see Figures 4.10-4.14

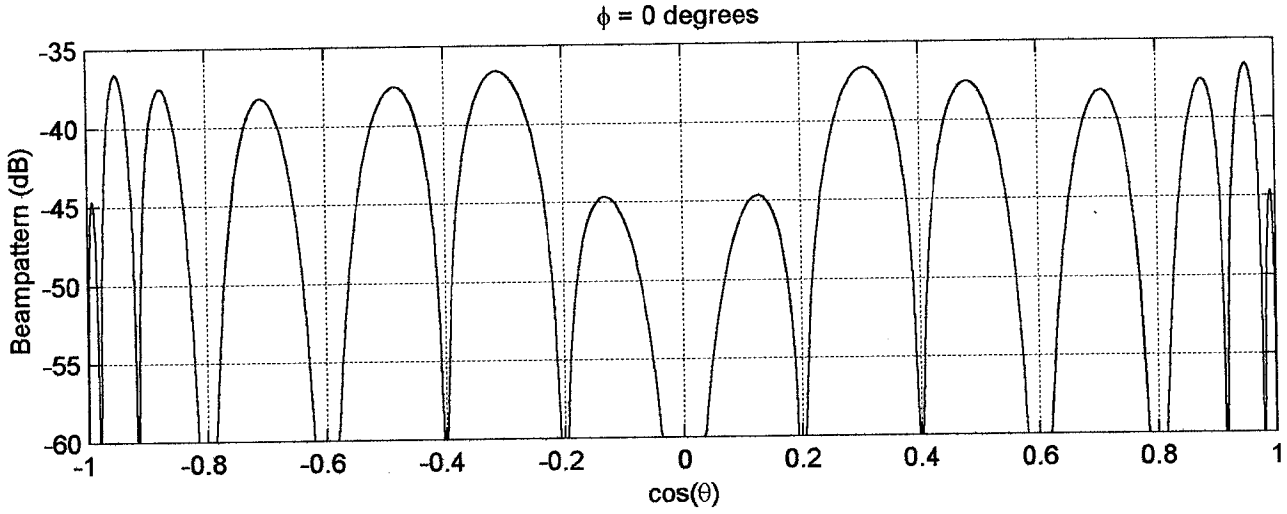
2.8.4 ②


Problem 2.8.4, ϕ = 90 degrees
Beampattern (dB)
0
-10
-20
-30
-40
-50
-60
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
ϕ = 75 degrees
Beampattern (dB)
-10
-20
-30
-40
-50
-60
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
cos(θ)

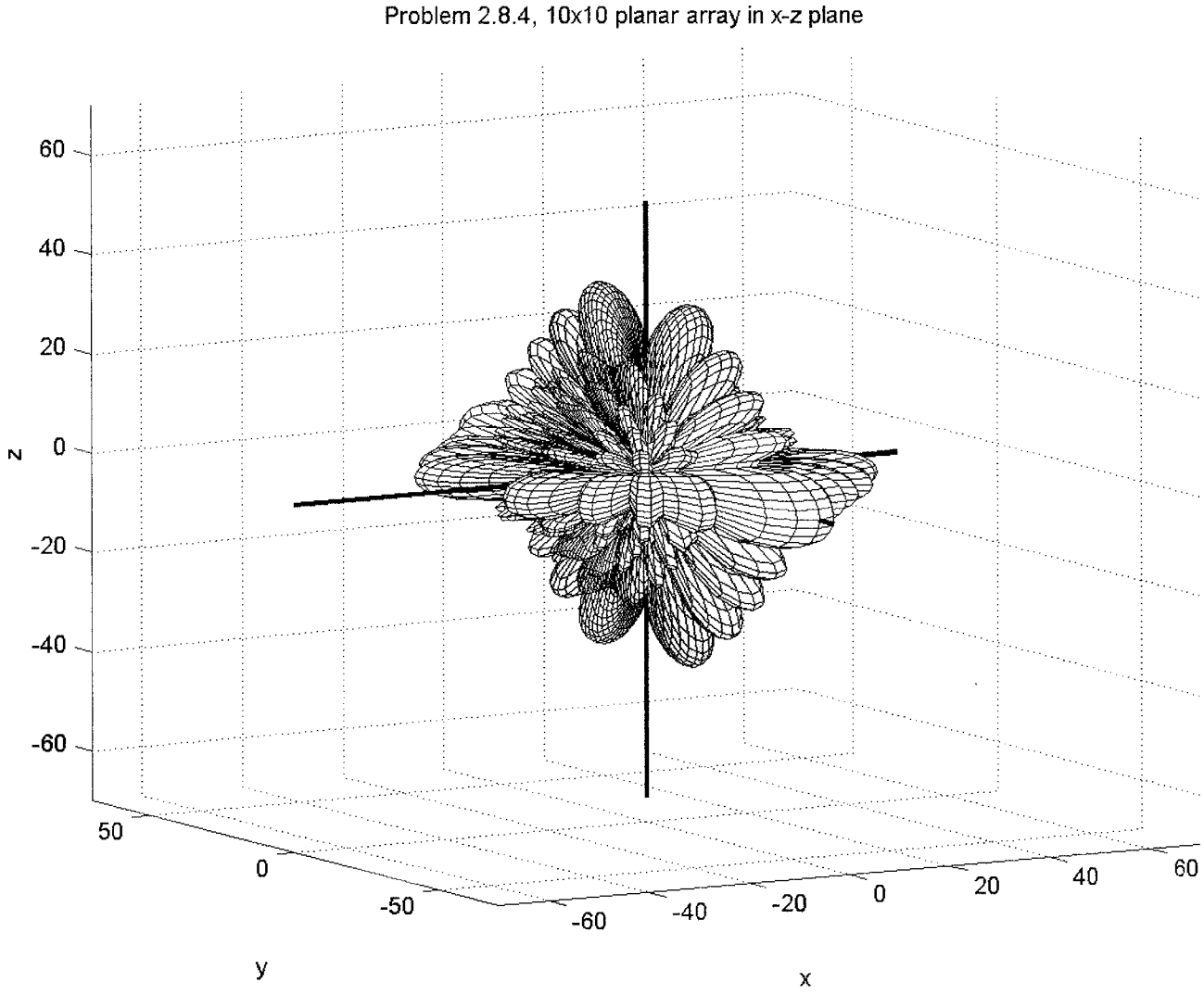

Problem 2.8.4, ϕ = 60 degrees
Beampattern (dB)
-10
-20
-30
-40
-50
-60
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


ϕ = 45 degrees
Beampattern (dB)
-10
-20
-30
-40
-50
-60
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
cos(θ)


Problem 2.8.4, ϕ = 30 degrees
Beampattern (dB)
-20
-30
-40
-50
-60
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


ϕ = 0 degrees
Beampattern (dB)
-35
-40
-45
-50
-55
-60
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
cos(θ)

2.8.4 (5)


Problem 2.8.4, 10x10 planar array in x-z plane
60
40
20
0
-20
-40
-60
z
50
0
-50
y
-60
-40
-20
0
20
40
60
x