3.10.2 $B_{bs}^{\#} = H_w B_{no}^{\#}$

a) We need to show that if $H_w = \left[B_{no}^{\#} B_{no}\right]^{-1/2}$, then B_{bs} satisfies the orthonormality condition

$$B_{bs}^{\#} = \left[B_{no}^{\#} B_{no}\right]^{-1/2} B_{no}^{\#}$$

$$B_{bs}^{\#} B_{bs} = \left[B_{no}^{\#} B_{no}\right]^{-1/2} B_{no}^{\#} B_{no} \left[B_{no}^{\#} B_{no}\right]^{-1/2}$$

$$= \left[B_{no}^{\#} B_{no}\right]^{-1/2} (B_{no}^{\#} B_{no})^{1/2} (B_{no}^{\#} B_{no})^{1/2} \left[B_{no}^{\#} B_{no}\right]^{-1/2}$$

$$= I \checkmark$$

H_w is a square root matrix of the positive definite Hermitian matrix $B_{no}^{\#} B_{no}$. It is not unique. One way to construct H_w is by decomposing

$$B_{no}^{\#} B_{no} = \sum_{i=1}^{N_{bs}} \lambda_i u_i u_i^{\#}$$

where λ_i and u_i $i=1 \dots N_{bs}$ are the eigenvalues and eigenvectors of $B_{no}^{\#} B_{no}$. Then

$$H_w = \sum_{i=1}^{N_{bs}} \frac{1}{\sqrt{\lambda_i}} u_i u_i^{\#}$$

- In matlab, you can also use the sqrtm function. It appears to give the same result.

b) in matlab

c) The price for orthogonalization seems to be increasing the first sidelobe back to about -13 dB !