Problem 2.4.4

- If the n^{th} sensor fails $\Rightarrow$ $w_n = 0$, $P_n(F) = \frac{1}{10}(1-\alpha)$
- At most one sensor fails at a time, therefore

$P(\text{more than 1 failure}) = 0$

$P(\text{no failure}) = 1 - \frac{N}{10}(1-\alpha)$

- $$B(\psi)\Big|_{n \text{ fails}} = \frac{1}{N}\sum_{m=0}^{N-1} e^{j\left(m-\frac{N-1}{2}\right)\psi} - \frac{1}{N} e^{j\left(n-\frac{N-1}{2}\right)\psi}$$

$$= \frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin(\psi/2)} - \frac{1}{N} e^{j\left(n-\frac{N-1}{2}\right)\psi}$$

- $$E\{B(\psi)\} = P(\text{no } F)\, B(\psi)\Big|_{\text{no } F} + \sum_{n=0}^{N-1} P_n(F)\, B(\psi)\Big|_{n \text{ fails}}$$

$$= \left(1-\frac{N}{10}(1-\alpha)\right)\frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin(\psi/2)} + \sum_{n=0}^{N-1}\frac{1}{10}(1-\alpha)\left\{\frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin(\psi/2)} - \frac{1}{N}e^{j\left(n-\frac{N-1}{2}\right)\psi}\right\}$$

$$= \frac{\sin\left(\frac{N}{2}\psi\right)}{N\sin(\psi/2)} - \frac{1}{10}(1-\alpha)\frac{1}{N}\sum_{n=0}^{N-1} e^{j\left(n-\frac{N-1}{2}\right)\psi}$$

$$= \left\{1-\frac{1}{10}(1-\alpha)\right\}\frac{\sin\left(N/2\,\psi\right)}{N\sin(\psi/2)}$$

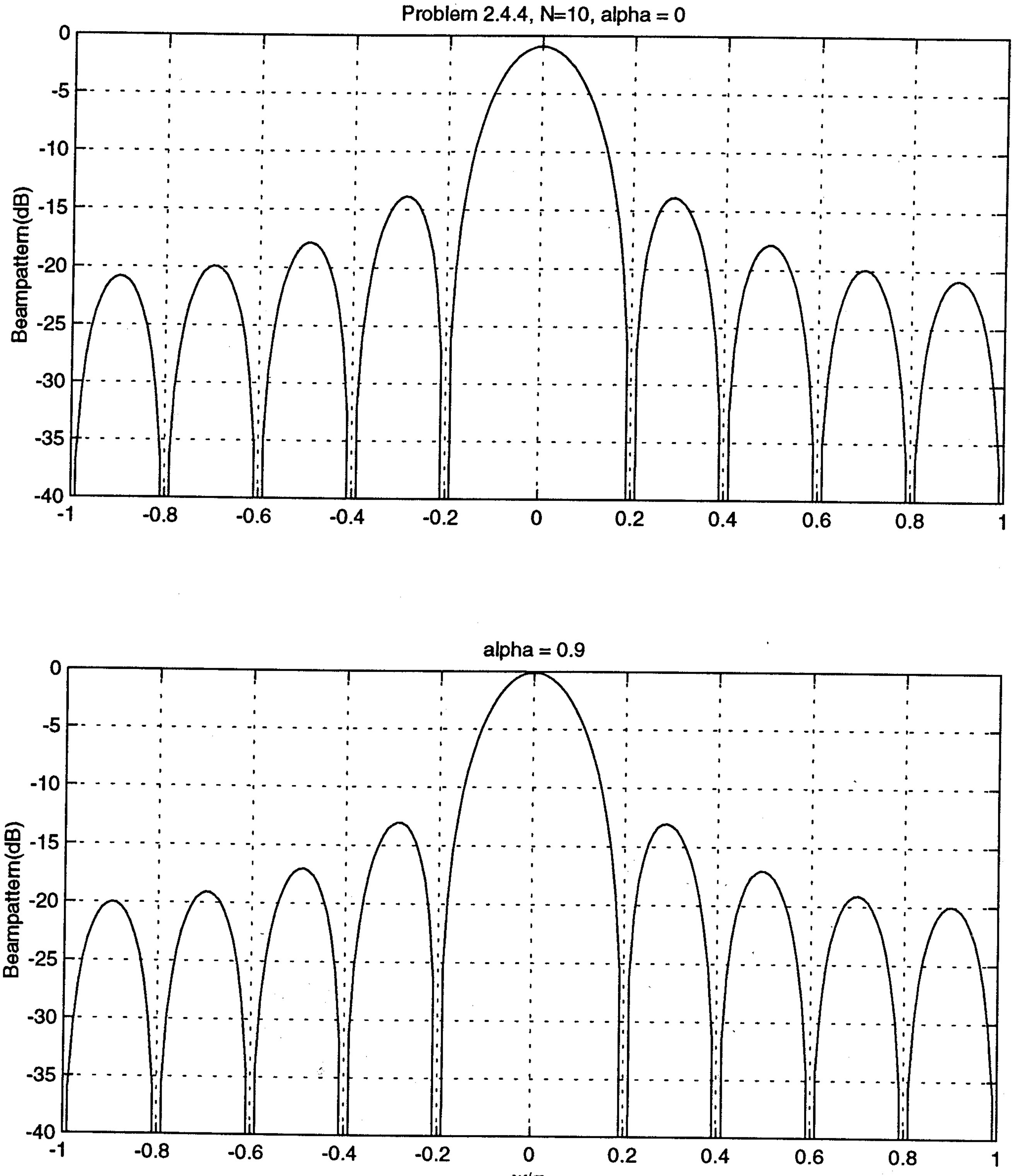
$$\boxed{E\{B(\psi)\} = (0.9 + 0.1\alpha)\frac{\sin(N/2\,\psi)}{N\sin(\psi/2)} \qquad 0 \le \alpha \le 1}$$

scaled version of conventional beampattern

b) $N=10$, $\alpha=0$ $\quad E\{B(\psi)\} = 0.09\,\dfrac{\sin(5\psi)}{\sin(\psi/2)}$

$N=10$, $\alpha=0.9$ $\quad E\{B(\psi)\} = 0.099\,\dfrac{\sin(5\psi)}{\sin(\psi/2)}$

②


Problem 2.4.4, N=10, alpha = 0
Beampattern(dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
alpha = 0.9
Beampattern(dB)
ψ/π