6.6.10 Consider a standard 10-element linear array with $d = \lambda_c/2$

A MPDR beamformer is steered to $\theta_m = 30°$.

(a) Assume $\theta_a = 30°$ but its frequency is $f_c + f_a$. Plot array gain vs. f_a for several SNR's.

Solution:

The nth element of the array manifold vector is

$$[\underline{v}(\theta)]_n = \exp\left\{ j\left(n - \frac{N-1}{2}\right) \frac{2\pi}{\lambda} d \cos\theta \right\}$$

Now $2\pi/\lambda = 2\pi f/c$ and

$$2\pi/\lambda_c = 2\pi f_c/c$$

Next, express f_a in normalized units (normalized by f_c):

$$f = f_c + f_a = f_c(1 + f_a/f_c)$$

$$= f_c(1+\Delta)$$

Then $\dfrac{2\pi}{\lambda} d = \dfrac{2\pi f_c}{c}(1+\Delta)\dfrac{\lambda_c}{2}$

$$= \frac{2\pi(1+\Delta)}{\lambda_c}\frac{\lambda_c}{2}$$

$$= \pi(1+\Delta)$$

$$\boxed{[\underline{v}(\theta)]_n = \exp\left\{j\left(n - \frac{N-1}{2}\right)\pi(1+\Delta)\cos\theta\right\}}$$

I wrote a script to plot the array gain of a MPDR beamformer as a function of $\Delta = f_a / f_c$ for several SNR's. The plot is attached.

(b) Plot array gain vs SNR for $f_a = 0.2 f_c$ and $0.4 f_c$.

Solution:

$f_a = 0.2 f_c \Rightarrow \Delta = 0.2$

$f_a = 0.4 f_c \Rightarrow \Delta = 0.4$

I wrote a script to plot array gain for both values of Δ as a function of SNR.

The array gain degradation, as a function of SNR, doesn't change very much between $\Delta = 0.2$ and $\Delta = 0.4$, but the trend is what I expect. I played with various values of Δ, large and small, and the trends matched my expectations.

(c) Assume $f_a \sim U(-f_1, f_1)$ for

$f_1 = 0.04 f_c$

$f_1 = 0.1 f_c$

$f_1 = 0.2 f_c$

Plot $E\{AG\}$ as a function of SNR for each case.

Solution:

I used the formula

$$AG = \frac{N|\rho|^2}{1+(2M+M^2)(1-|\rho|^2)}$$

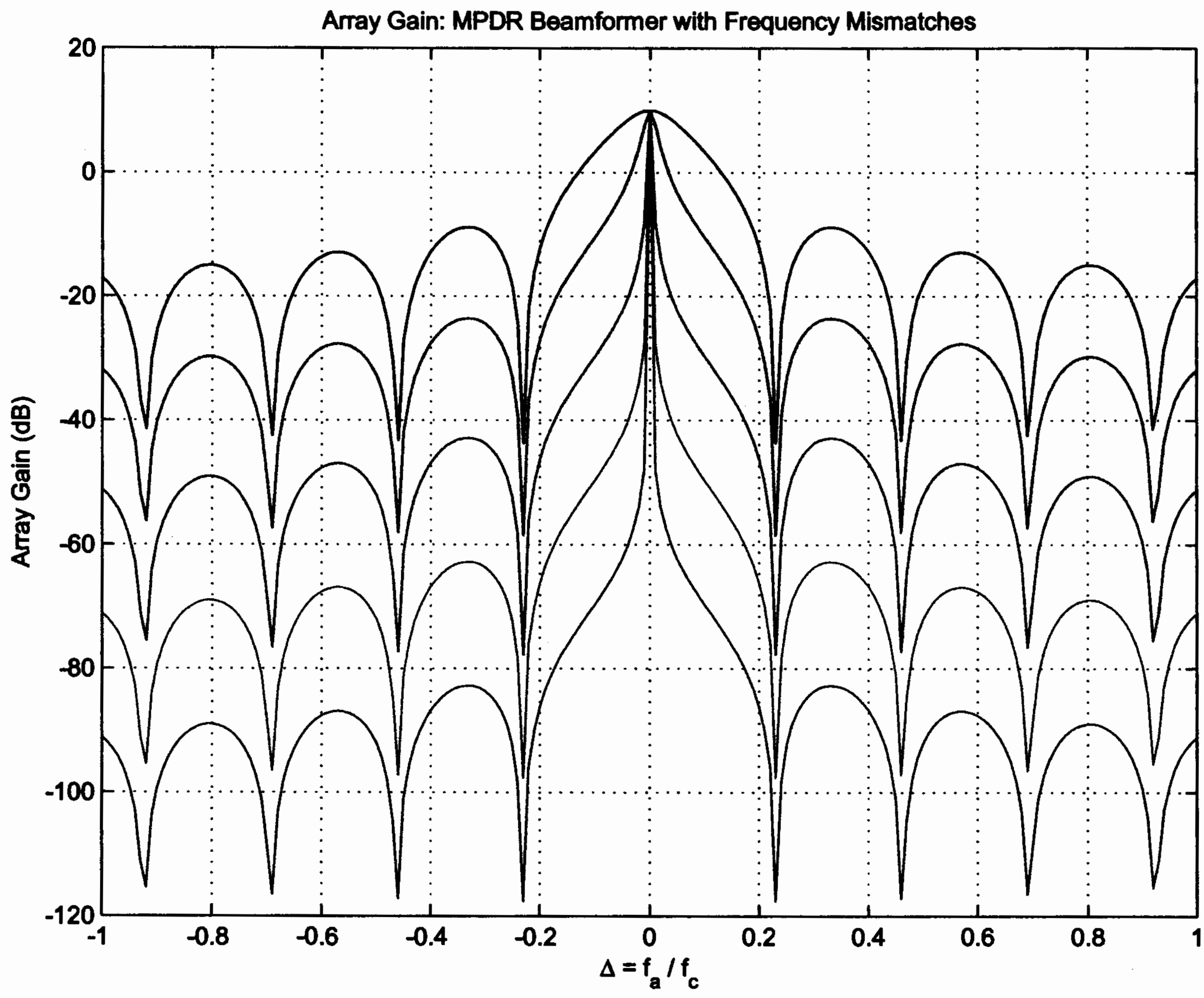
to estimate $E\{AG\}$ w/Monte Carlo trials.

Now $\rho = \underline{v}_a^H \underline{v}_m / N$

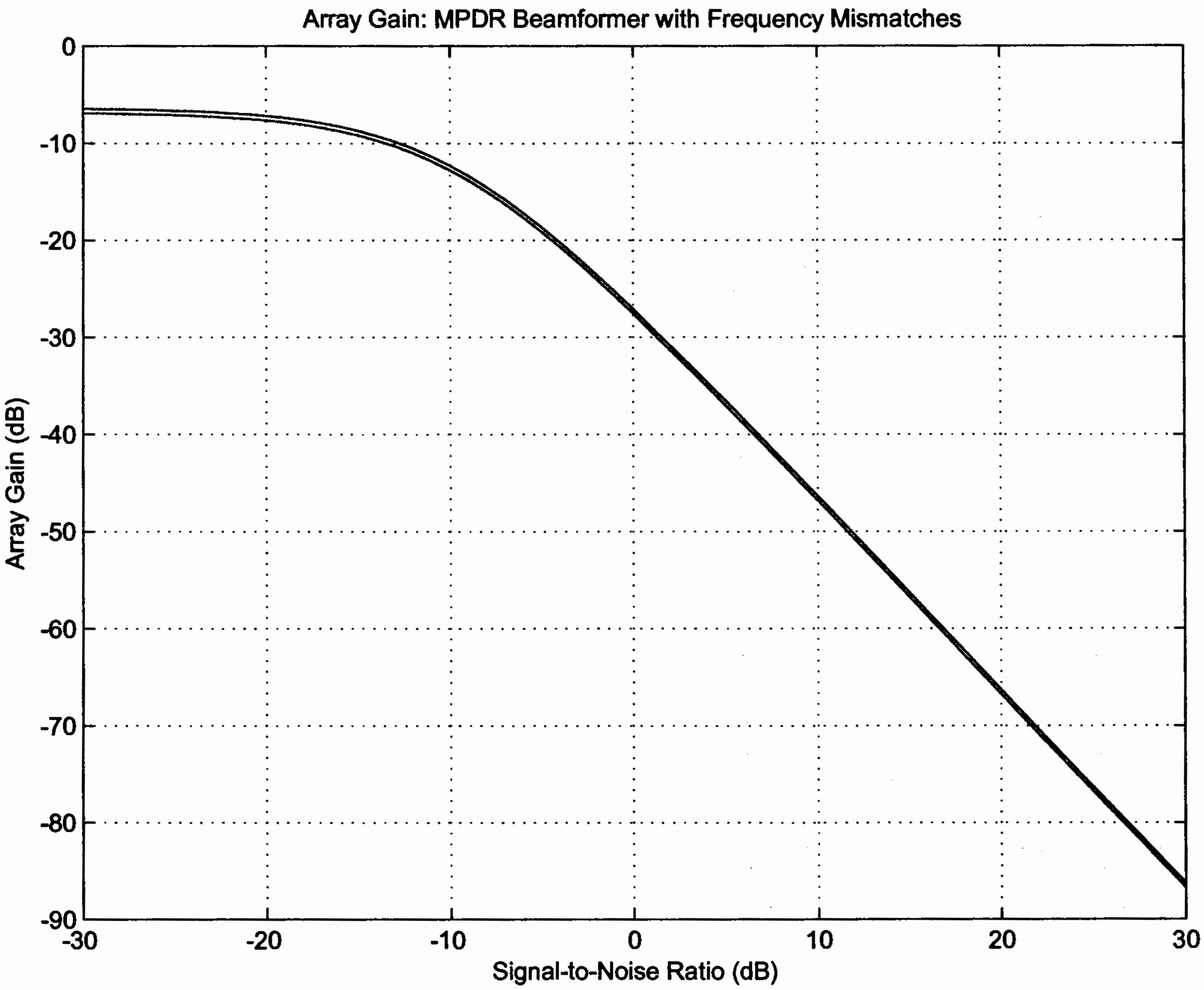
and the nth element of $\underline{v}_a$ is:

$$[\underline{v}_a(\theta)]_n = \exp\left\{ j\left(n - \frac{N-1}{2}\right)\pi(1+\Delta)\cos\theta \right\}$$

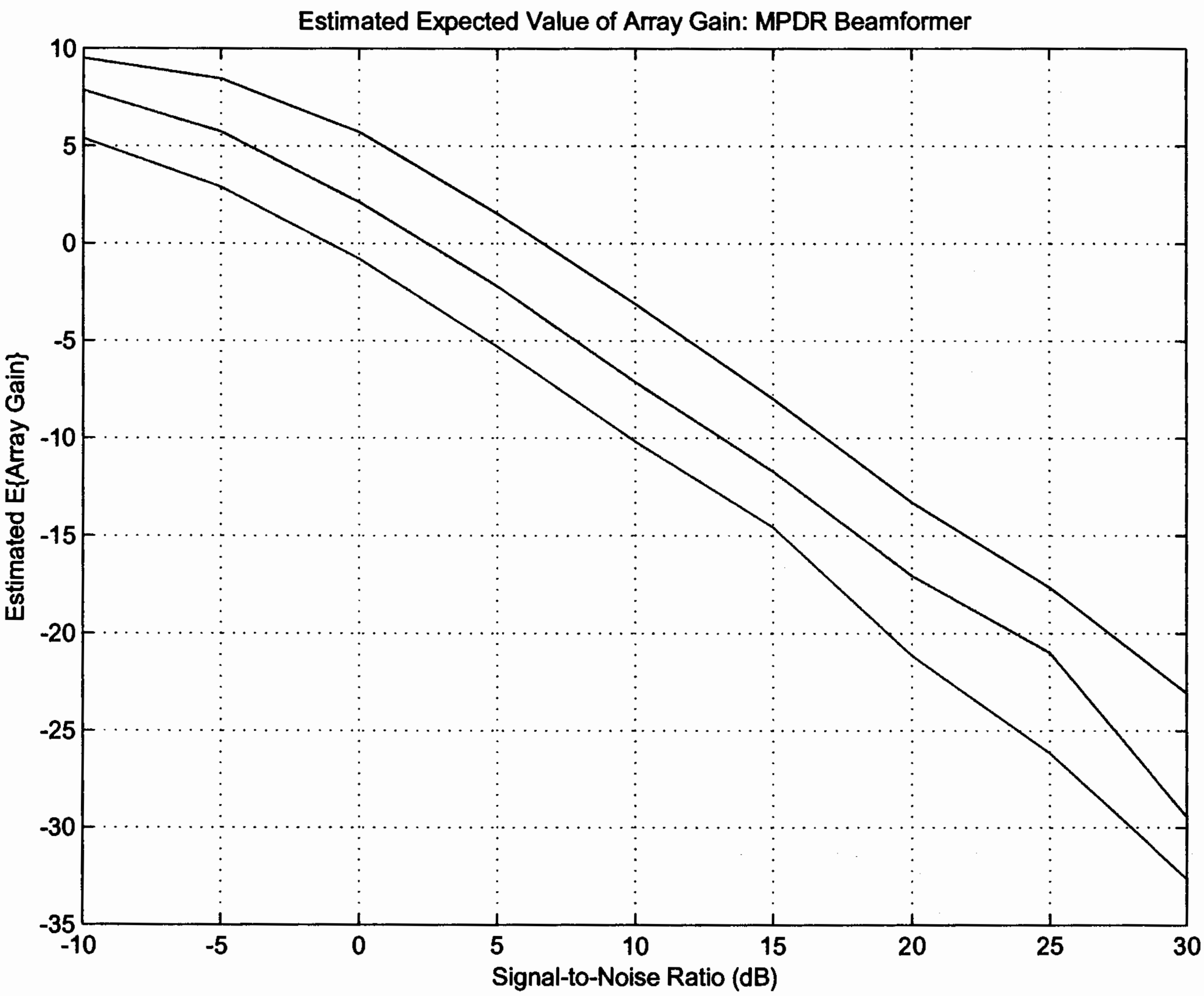
To solve the problem I generated uniformly distributed random variables for Δ, calculated the resulting array gain, and averaged over 20,000 iterations at each SNR and each Δ. A plot is attached.



Array Gain: MPDR Beamformer with Frequency Mismatches
Array Gain (dB)
20
0
-20
-40
-60
-80
-100
-120
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
$\Delta = f_a / f_c$



Array Gain: MPDR Beamformer with Frequency Mismatches
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Array Gain (dB)
-30
-20
-10
0
10
20
30
Signal-to-Noise Ratio (dB)



Estimated Expected Value of Array Gain: MPDR Beamformer
Estimated E{Array Gain}
Signal-to-Noise Ratio (dB)
10
5
0
-5
-10
-15
-20
-25
-30
-35
-10
-5
0
5
10
15
20
25
30