6.4.1 assume $\omega = \omega_0$ corresponds to $\lambda_0 = 2d$, $N = 10$

$$\underline{S}_{n\,k,l} = S_x(\omega_0 : p_l - p_k) + \sigma_n^2 \delta_{kl}$$

* There is apparently a mistake in the derivation of the result in Problem 5.3.1. $S_x(\omega_0, \Delta p)$ should be as given in Example 6.4.1

$$S_x(\omega_0; \Delta p) = S_0(\omega_0)\left\{ \text{sinc}(k_0|\Delta p|) + \frac{j\alpha\cos\theta_p}{k_0|\Delta p|}\left[\text{sinc}(k_0|\Delta p|) - \cos(k_0|\Delta p|)\right]\right\}$$

- $S_0(\omega_0) = INR \cdot \sigma_n^2$
- $k_0 = 2\pi/\lambda_0$
- $|\Delta p| = |l-k|\, d = |l-k|\,\lambda_0/2$ for standard ULA
- $k_0|\Delta p| = \pi\,|l-k|$
- $\cos\theta_p = \cos 0 = 1$ if $\Delta p > 0$ $(l > k)$

  $\cos\theta_p = \cos\pi = -1$ if $\Delta p < 0$ $(l < k)$

  $\Rightarrow \dfrac{\cos\theta_p}{k_0|\Delta p|} = \dfrac{1}{k_0 \Delta p} = \dfrac{1}{\pi(l-k)}$

$$\underline{S}_{n\,k,l} = \sigma_n^2\left[INR\left\{\text{sinc}(\pi|l-k|) + \frac{j\alpha}{\pi(l-k)}\left[\text{sinc}(\pi|l-k|) - \cos(\pi|l-k|)\right]\right\} + \delta_{kl}\right]$$

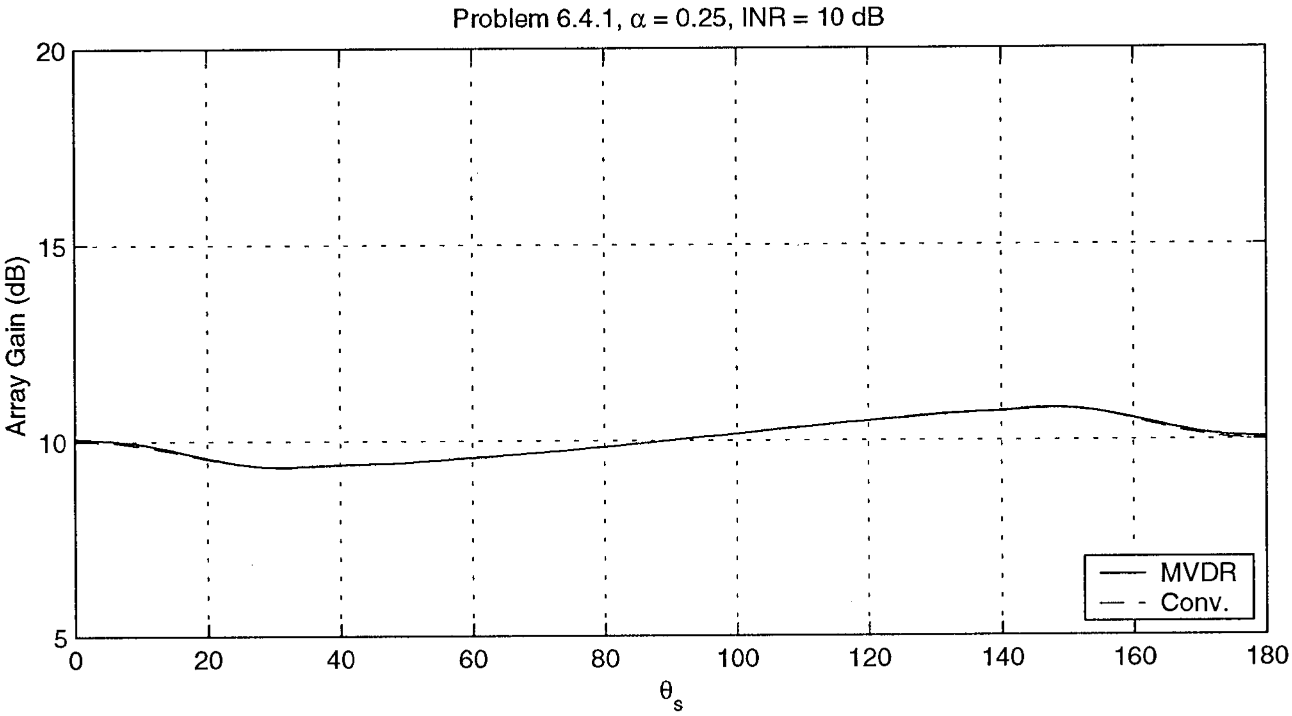
$$S_n(\omega_0) = S_{n,k=l} = \sigma_n^2(INR+1)$$

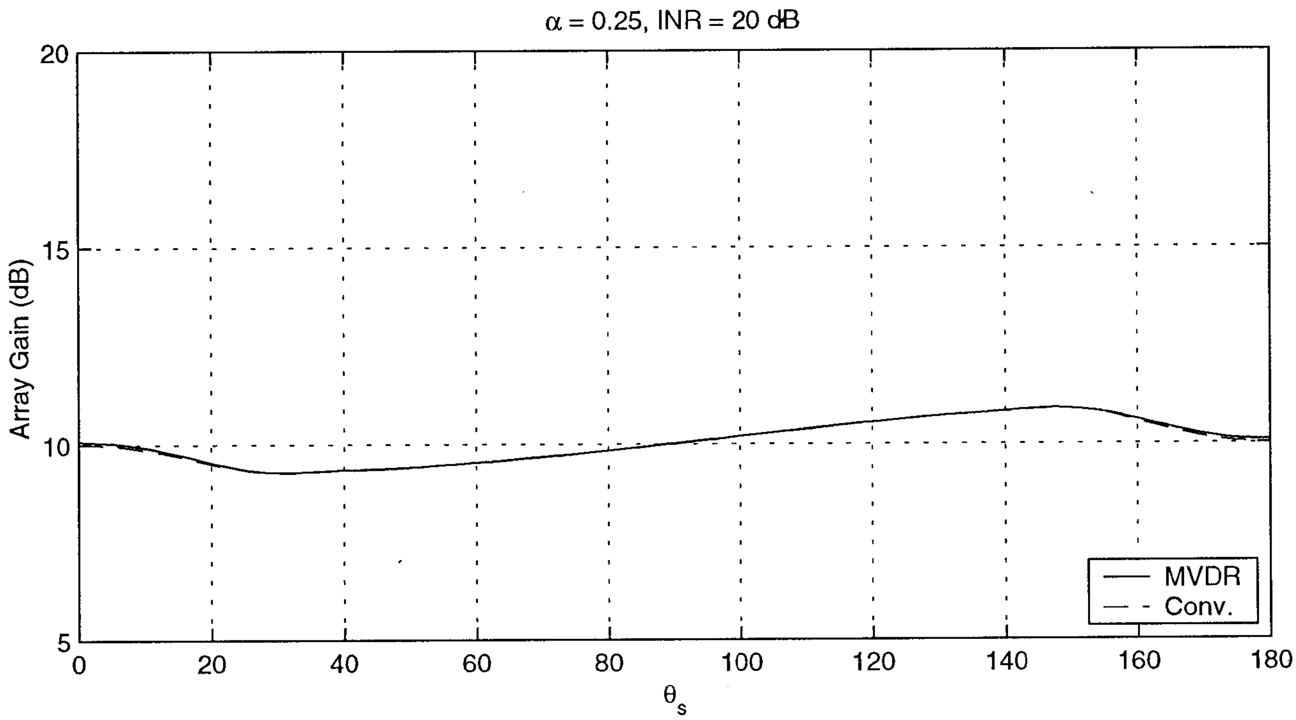
$$\rho_{n\,k,l} = \underline{S}_{n,k,l} / \sigma_n^2(INR+1)$$

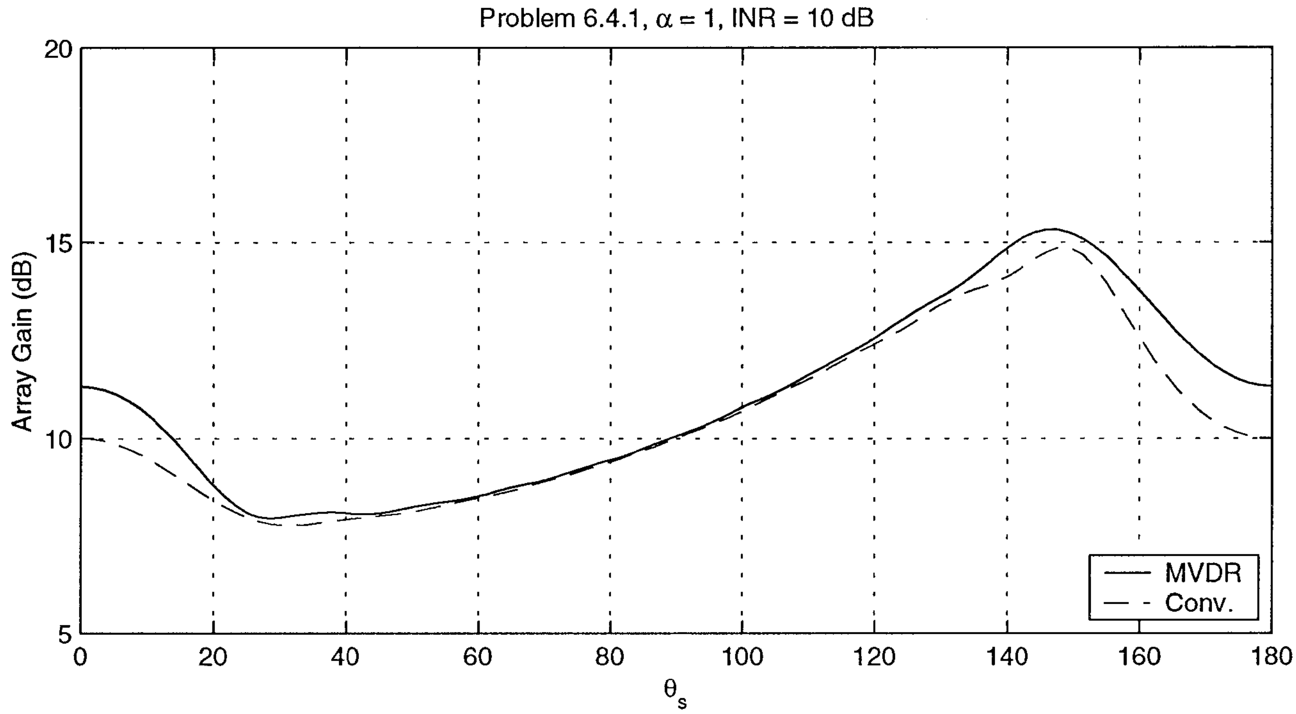
$$A_{MVDR} = v(\theta_s)^{\#}\rho_n^{-1}v(\theta_s)$$

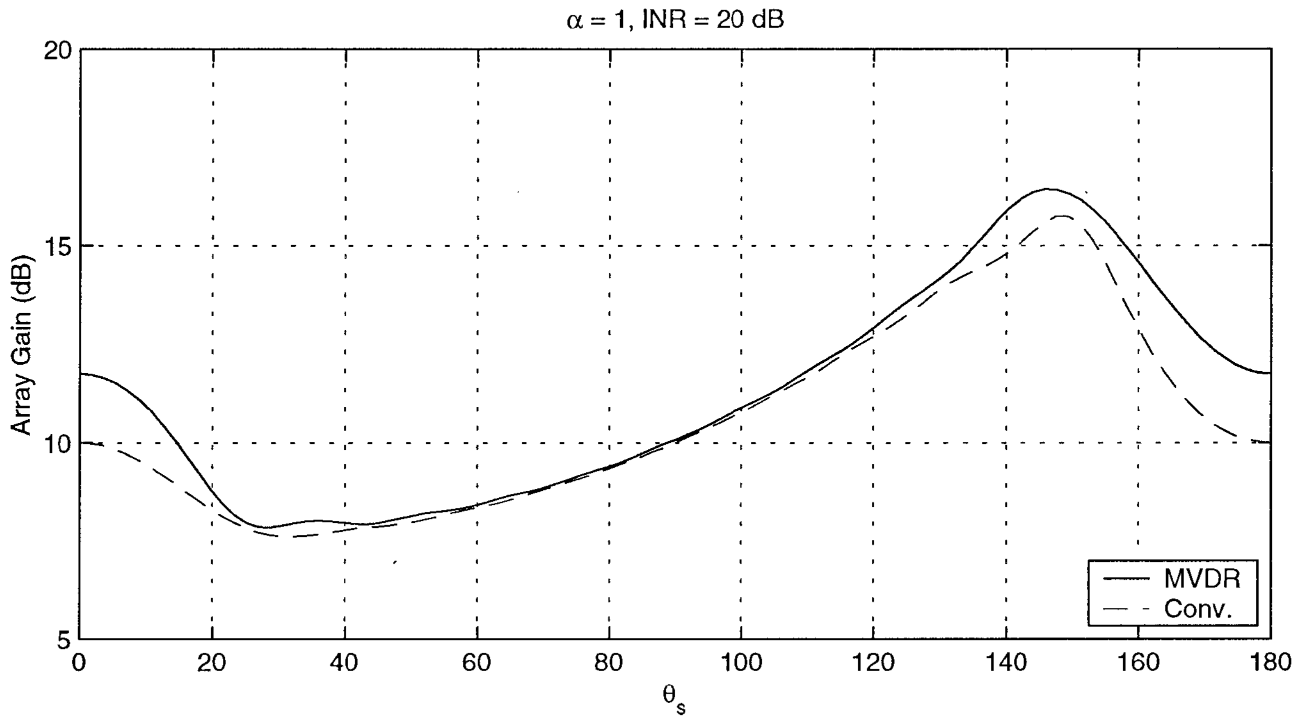
$$A_{CONV} = N^2 / v(\theta_s)^{\#}\rho_n\, v(\theta_s)$$

- for $\alpha = 1$ (surface noise), get reduced array gain for θ_s near 0 (surface) and improved array

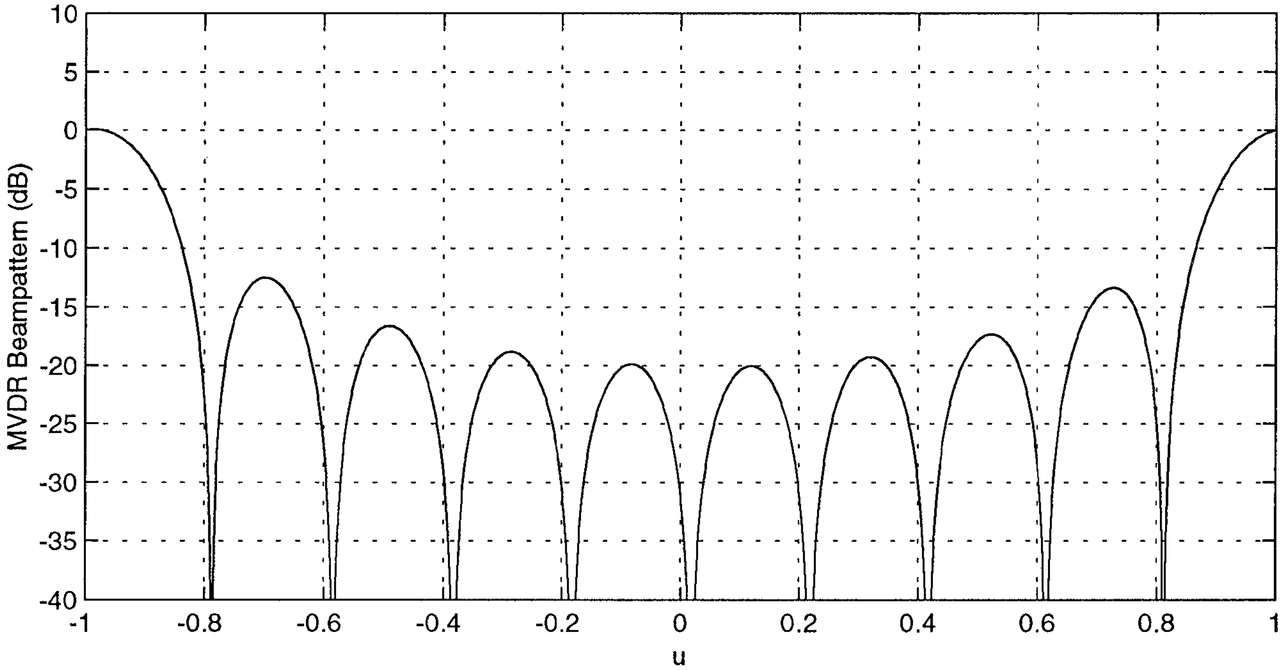

Problem 6.4.1, α = 0.25, INR = 10 dB
Array Gain (dB)
20
15
10
5
0
20
40
60
80
100
120
140
160
180
θs
MVDR
Conv.


α = 0.25, INR = 20 dB
Array Gain (dB)
20
15
10
5
0
20
40
60
80
100
120
140
160
180
θs
MVDR
Conv.

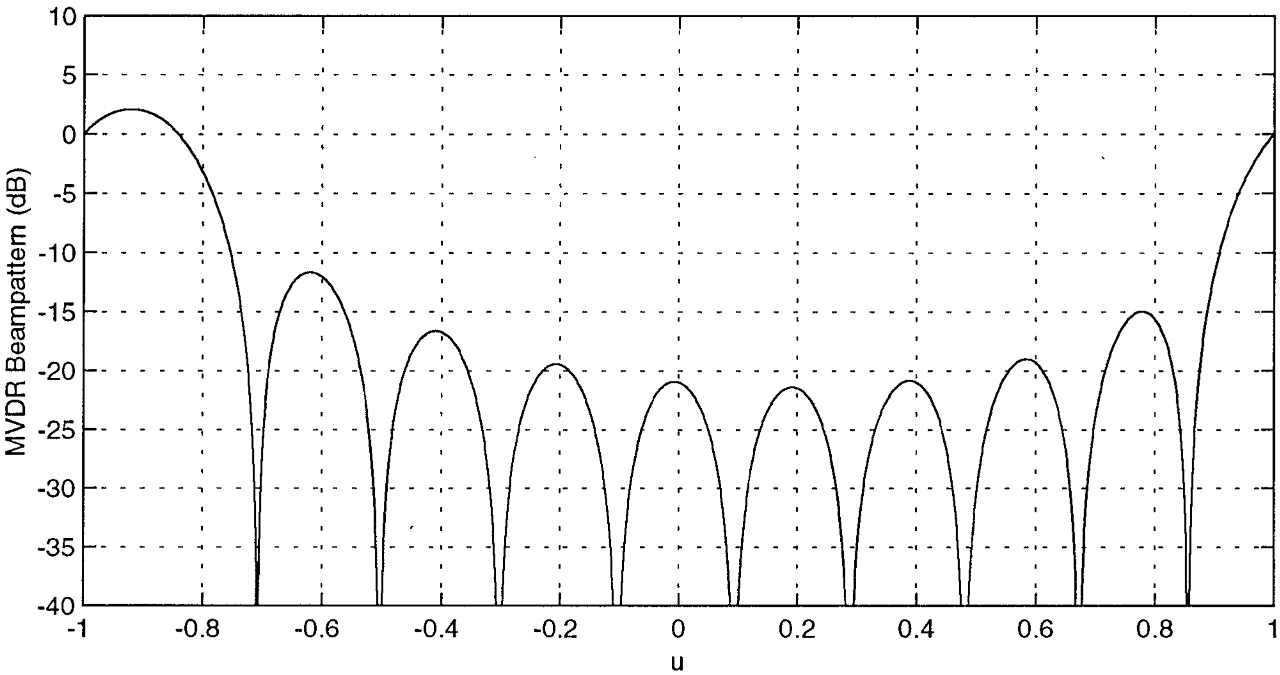

Problem 6.4.1, α = 1, INR = 10 dB
Array Gain (dB)
20
15
10
5
0
20
40
60
80
100
120
140
160
180
θs
MVDR
Conv.


α = 1, INR = 20 dB
Array Gain (dB)
20
15
10
5
0
20
40
60
80
100
120
140
160
180
θs
MVDR
Conv.

Problem 6.4.1, $\alpha = 0.25$, INR = 20, $\theta_s = 0$ deg. ($u_s = 1$)

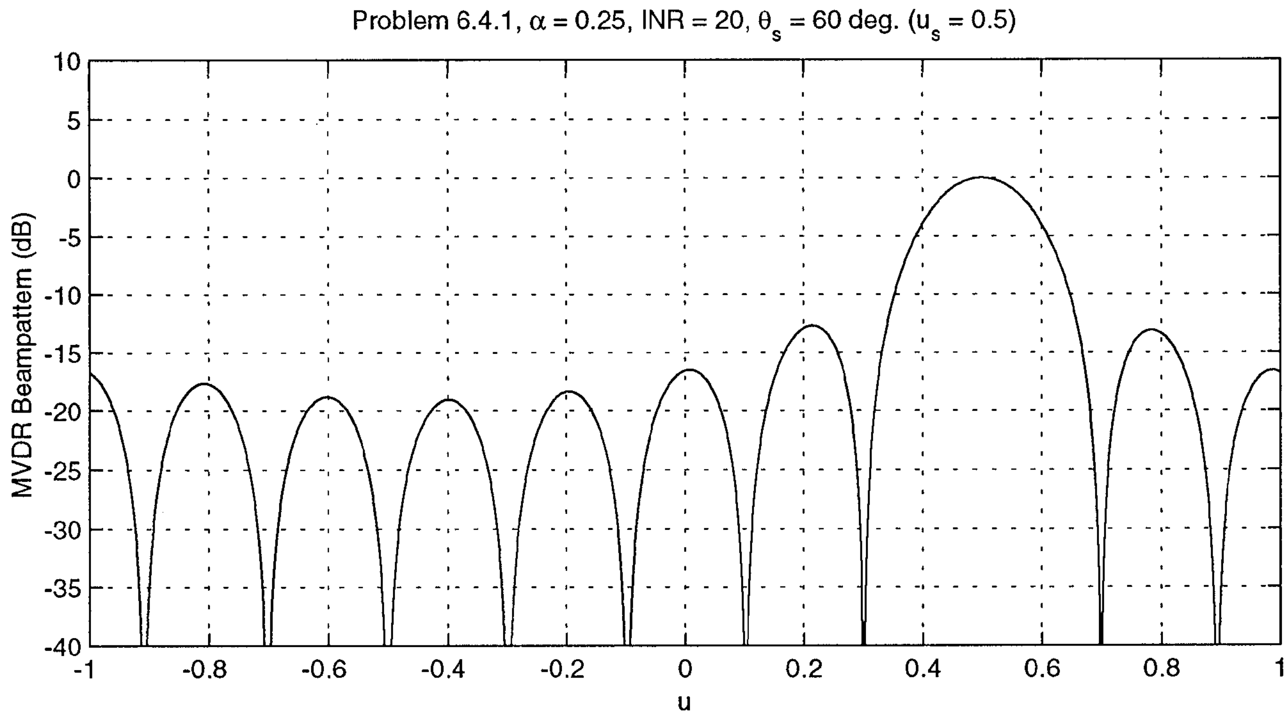


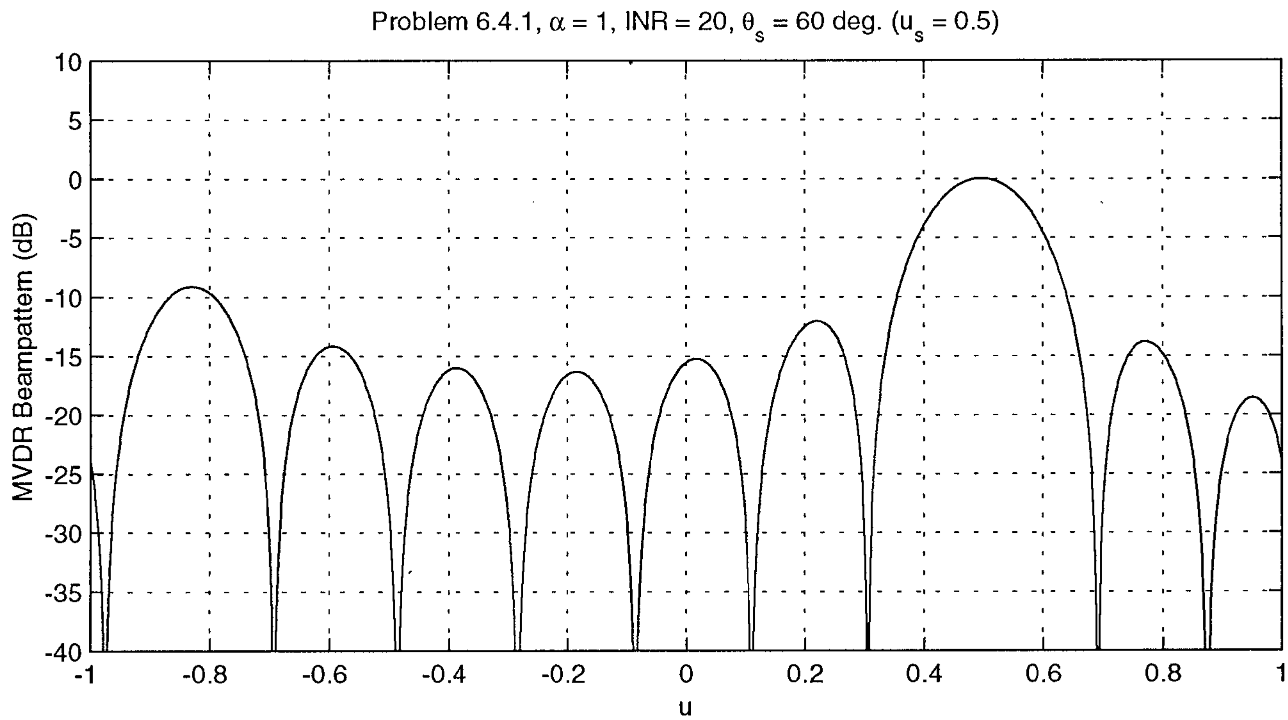
Problem 6.4.1, $\alpha = 1$, INR = 20, $\theta_s = 0$ deg. ($u_s = 1$)



for $\alpha = 1$, we see narrower mainlobe in direction of interference, larger backlobe

6.4.1 5/10



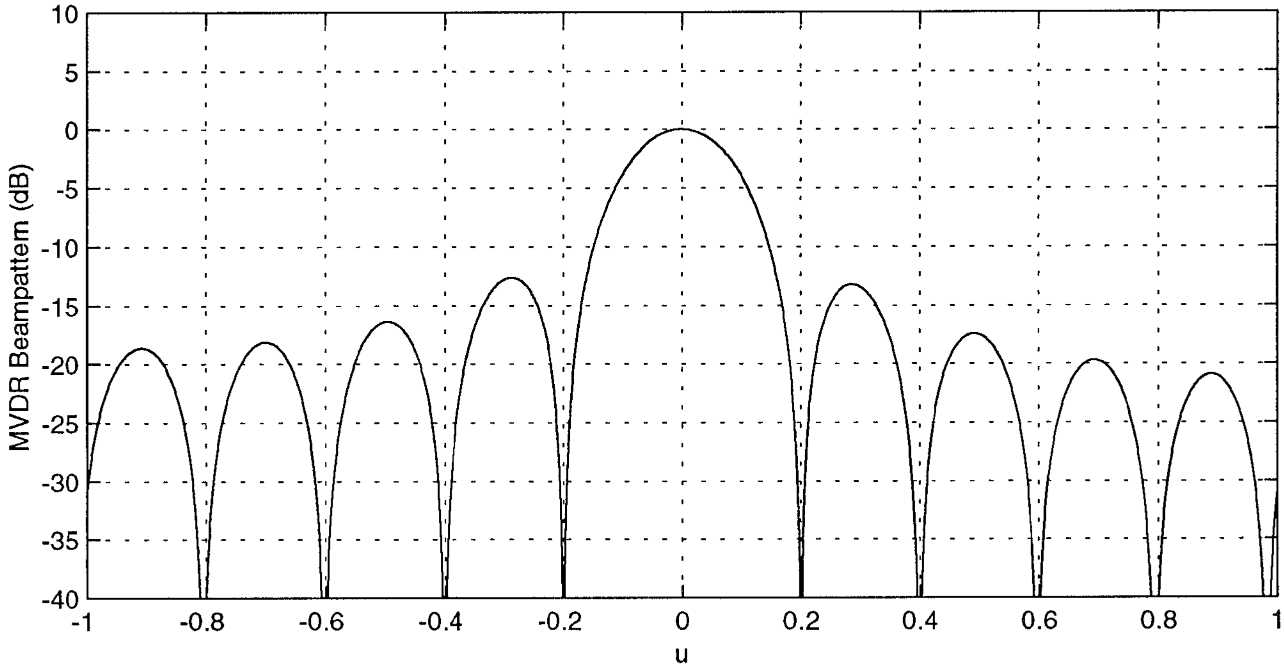


for $\alpha = 1$, we see smaller sidelobes in direction of interference.

6.4.1 6/10

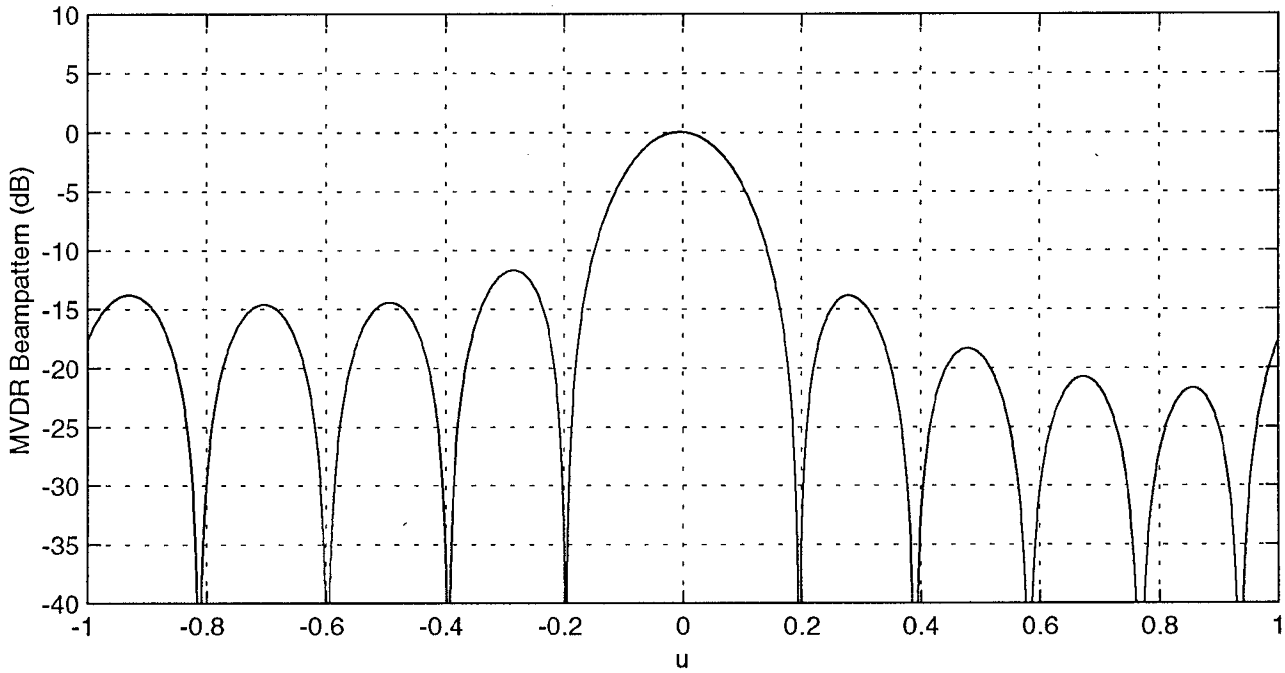
0

Problem 6.4.1, $\alpha = 0.25$, INR = 20, $\theta_s = 90$ deg. ($u_s =$ ~~6.1232e-017)~~

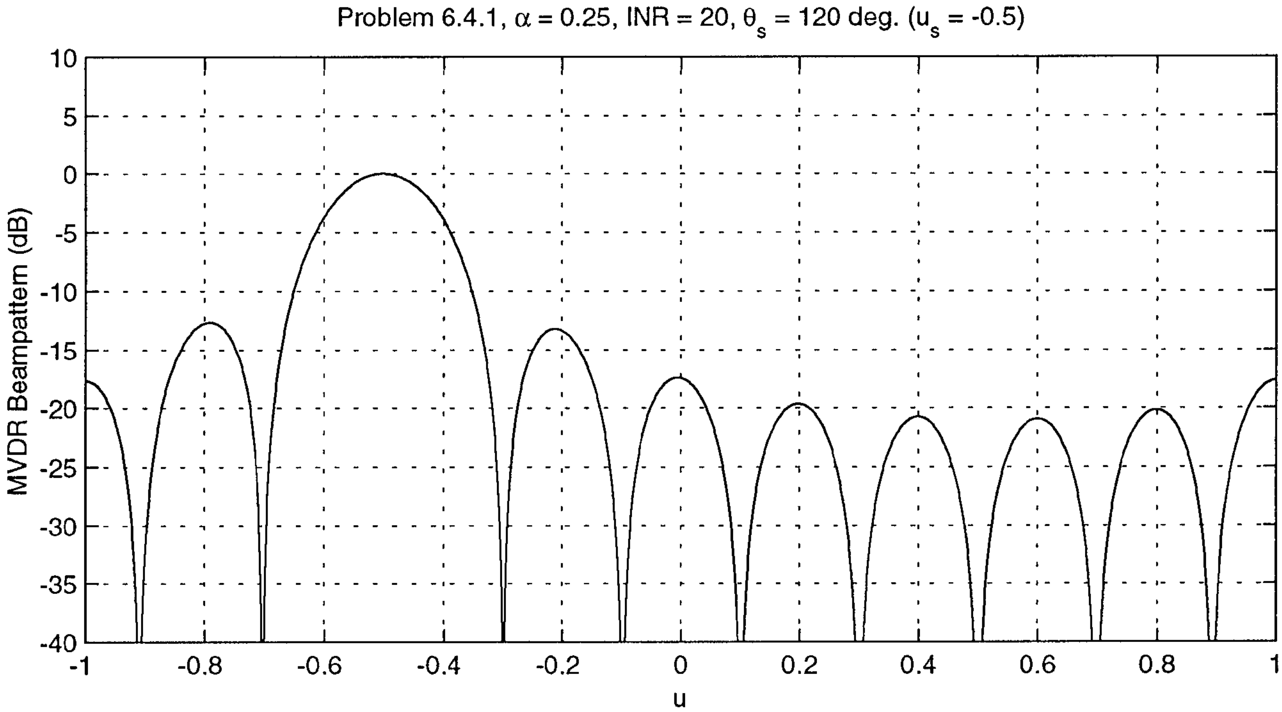


$u_s = 0$

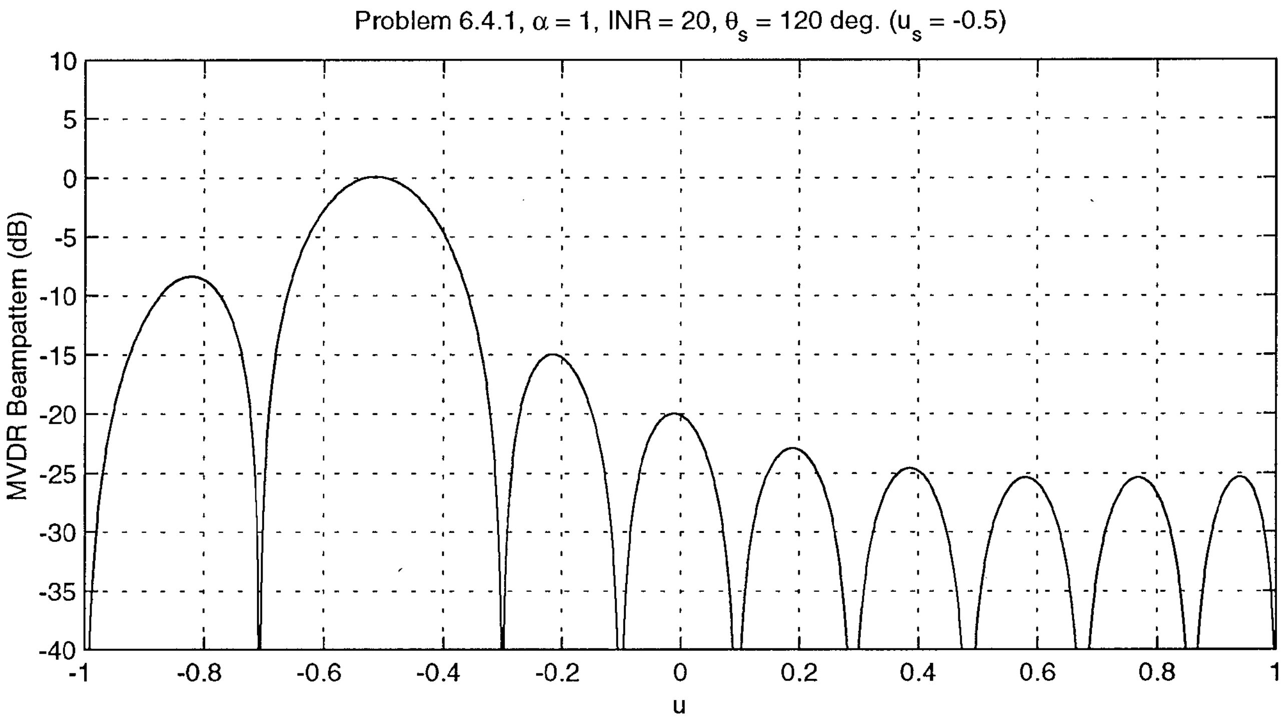
Problem 6.4.1, $\alpha = 1$, INR = 20, $\theta_s = 90$ deg. ~~(u_s = 6.1232e-017)~~



6.4.1 7/10



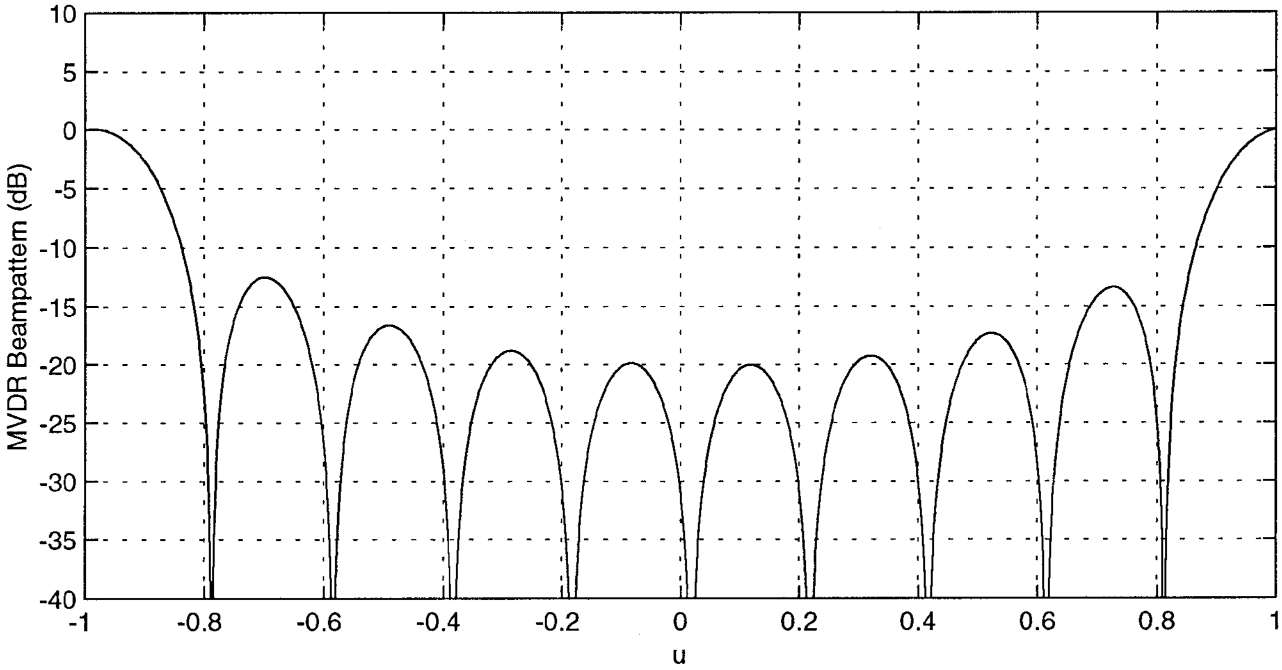
Problem 6.4.1, α = 0.25, INR = 20, θ_s = 120 deg. (u_s = -0.5)
MVDR Beampattern (dB)
u
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1



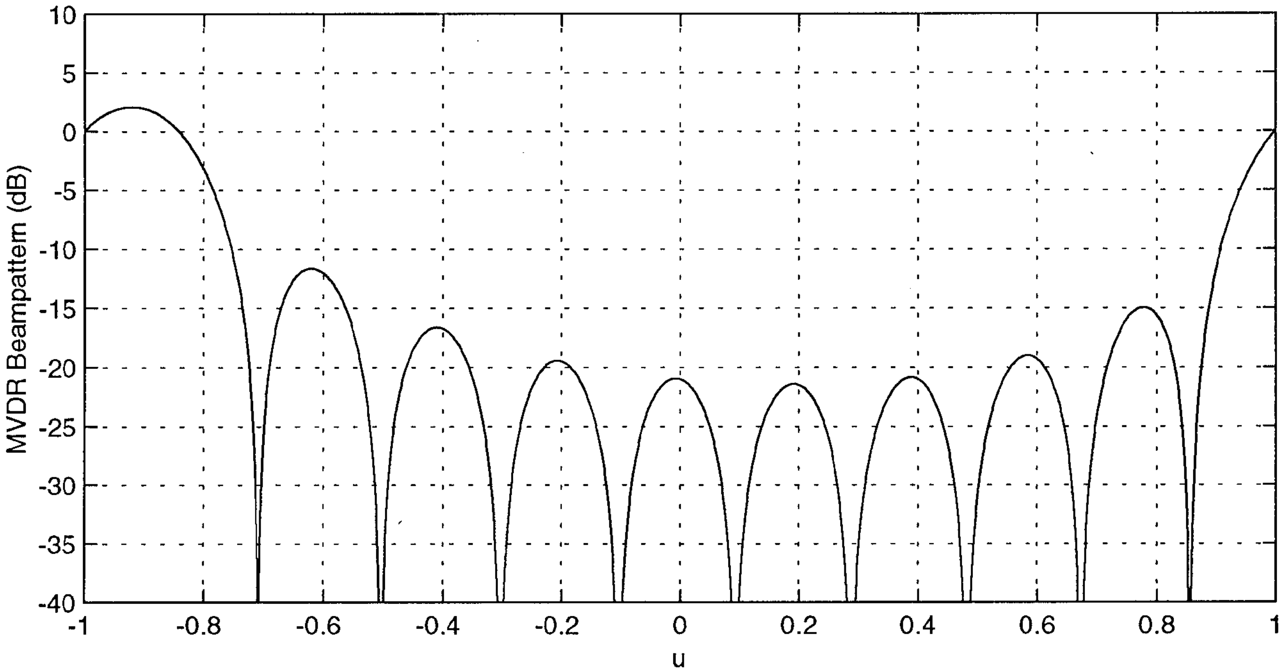
Problem 6.4.1, α = 1, INR = 20, θ_s = 120 deg. (u_s = -0.5)
MVDR Beampattern (dB)
u
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

$\boxed{6.4.1 \quad 8/10}$

Problem 6.4.1, $\alpha = 0.25$, INR = 20, θ_s = 180 deg. (u_s = -1)



Problem 6.4.1, $\alpha = 1$, INR = 20, θ_s = 180 deg. (u_s = -1)



6.4.1 9|10

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% problem 6.4.1
% K. Bell 11/27/00
% Function called: sinc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all
close all

%************************
% Array
%************************
N = 10;                                  % Elements in array
d = 0.5;                                 % sensor spacing half wavelength wrt wc
D = [-(N-1)/2:1:(N-1)/2].';
BWNN = 2/(N*d);
u=[-1:0.001:1];
nu=length(u);
vv = exp(j*2*pi*d*D*u);

%************************
% Source
%************************
theta_s = [0 1/3 0.5 2/3 1]*pi;
us = cos(theta_s);
AS = exp(j*2*pi*d*D*us);
ns = length(us);

INR = 10.^([10 20]/10);
nI = length(INR);

alpha = [0.25 1];
for n=1:ns
   figure
   for a = 1:2
      p = [0:1:N-1];
      pI = [1:1:N-1];
      r = sinc(p*2*d)+[0 ((j*alpha(a))./(pi*pI*2*d)).*(sinc(pI*2*d)-cos(pi*2*d*pI))];
      Sn = INR(2)*toeplitz(r,conj(r))+eye(N);
      Sninv = inv(Sn);
      w = Sninv*AS(:,n)/real(AS(:,n)'*Sninv*AS(:,n));
      set(gcf,'Paperposition',[0.25 1 8 9])
      subplot(2,1,a)
      B = w'*vv;
      plot(u,10*log10(abs(B).^2));
      hold on
      xlabel('u')
      ylabel('MVDR Beampattern (dB)')
      title(['Problem 6.4.1, \alpha = ' num2str(alpha(a)) ', INR = '
num2str(10*log10(INR(2))) ', \theta_s = ' num2str(theta_s(n)*180/pi) ' deg. (u_s =
'num2str(us(n)) ')'])
      grid on
      axis([-1 1 -40 10])
      hold off
   end
end

theta_s = [0:0.01:1]*pi;
us = cos(theta_s);
AS = exp(j*2*pi*d*D*us);
ns = length(us);
for a = 1:2
   A = zeros(nI,ns);
   Ac = zeros(nI,ns);
   for n=1:ns
      for q=1:nI
```

```
        p = [0:1:N-1];
        pI = [1:1:N-1];
        r = sinc(p*2*d)+[0 ((j*alpha(a))./(pi*pI*2*d)).*(sinc(pI*2*d)-cos(pi*2*d*pI))];
        Sn = INR(q)*toeplitz(r,conj(r))+eye(N);
        Sninv = inv(Sn);
        Ac(q,n) =N*N*Sn(1,1)/ real(AS(:,n)'*Sn*AS(:,n));
        A(q,n) = real(AS(:,n)'*Sninv*AS(:,n))*Sn(1,1);
      end
    end
    figure
    subplot(2,1,1)
    h1=plot(theta_s*180/pi,10*log10(A(1,:)),'-');
    hold on
    h2=plot(theta_s*180/pi,10*log10(Ac(1,:)),'--');
    legend('MVDR', 'Conv.',4)
    xlabel('\theta_s')
    ylabel('Array Gain (dB)')
    title(['Problem 6.4.1, \alpha = ' num2str(alpha(a)) ', INR = '
num2str(10*log10(INR(1))) ' dB'])
    grid on
    hold off
    axis([0 180 5 20])
    subplot(2,1,2)
    h1=plot(theta_s*180/pi,10*log10(A(2,:)),'-');
    hold on
    h2=plot(theta_s*180/pi,10*log10(Ac(2,:)),'--');
    legend('MVDR', 'Conv.',4)
    xlabel('\theta_s')
    ylabel('Array Gain (dB)')
    title(['\alpha = ' num2str(alpha(a)) ', INR = ' num2str(10*log10(INR(2))) ' dB'])
    grid on
    hold off
    axis([0 180 5 20])
    set(gcf,'Paperposition',[0.25 1 8 9])
end
```