5.4.1 We have the following 3-D Fourier Transform relationships

- $$S_x(\omega:\Delta p) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_x(\omega:\underline{k})\, e^{-j\underline{k}^T \Delta \underline{p}}\, \frac{d\underline{k}}{(2\pi)^3} \qquad (5.95)$$
- $$P_x(\omega:\underline{k}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_x(\omega:\Delta \underline{p})\, e^{j\underline{k}^T \Delta \underline{p}}\, d\Delta \underline{p} \qquad (5.94)$$
- $$R_W(\omega:\Delta \underline{p}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |B(\omega:\underline{k})|^2\, e^{j\underline{k}^T \Delta \underline{p}}\, \frac{d\underline{k}}{(2\pi)^3} \qquad (5.184)$$

Then

$$S_y(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |B(\omega:\underline{k})|^2\, P_x(\omega:\underline{k})\, \frac{d\underline{k}}{(2\pi)^3} \qquad (5.182)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |B(\omega:\underline{k})|^2 \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_x(\omega:\Delta \underline{p})\, e^{j\underline{k}^T \Delta \underline{p}}\, d\Delta \underline{p} \right\} \frac{d\underline{k}}{(2\pi)^3}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_x(\omega:\Delta \underline{p}) \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |B(\omega:\underline{k})|^2 e^{j\underline{k}^T \Delta \underline{p}}\, \frac{d\underline{k}}{(2\pi)^3} \right\} d\Delta p$$

$$\boxed{S_y(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_x(\omega:\Delta \underline{p})\, R_W(\omega:\Delta \underline{p})\, d\Delta \underline{p}}$$