

2.3.2

 N is odd.

$$\underline{V}_p(\varphi) = \begin{bmatrix} V_{p,1}(\varphi) \\ 1 \\ \underline{J} V_{p,1}^*(\varphi) \end{bmatrix}$$

$$\underline{W}^H = \frac{1}{N} [1 \dots 1] \quad 1 \times N$$

$$\underline{W}^H = [\underline{W}_1^H \quad \frac{1}{N} \quad \underline{W}_1^T \underline{J}]$$

$$B_p(\varphi) = \underline{W}^H \underline{V}_p(\varphi) = [\underline{W}_1^H \quad \frac{1}{N} \quad \underline{W}_1^T \underline{J}] \begin{bmatrix} V_{p,1}(\varphi) \\ 1 \\ \underline{J} V_{p,1}^*(\varphi) \end{bmatrix}$$

$$= \underline{W}_1^H V_{p,1}(\varphi) + \frac{1}{N} + \underline{W}_1^T V_{p,1}^*(\varphi)$$

$$= 2 \operatorname{Re} \{ \underline{W}_1^H V_{p,1}(\varphi) \} + \frac{1}{N}$$

$$= \frac{2}{N} \operatorname{Re} \left\{ [1 \dots 1] \begin{bmatrix} e^{-j \frac{N-1}{2} \varphi} \\ \vdots \\ e^{-j \varphi} \end{bmatrix} \right\} + \frac{1}{N}$$

$$= \frac{2}{N} \operatorname{Re} \left\{ e^{-j \varphi} \sum_{m=0}^{N-1} e^{-j m \varphi} \right\} + \frac{1}{N}$$

$$= \frac{2}{N} \operatorname{Re} \left\{ e^{-j \varphi} \frac{1 - e^{-j \frac{N-1}{2} \varphi}}{1 - e^{-j \varphi}} \right\} + \frac{1}{N}$$

$$= \frac{2}{N} \operatorname{Re} \left\{ e^{-j \frac{N+1}{4} \varphi} \frac{\sin(\frac{N-1}{4} \varphi)}{\sin \frac{\varphi}{2}} \right\} + \frac{1}{N}$$

$$= \frac{2}{N} \frac{\cos(\frac{N+1}{4} \varphi) \sin(\frac{N-1}{4} \varphi)}{\sin \frac{\varphi}{2}} + \frac{1}{N}$$

$$= \frac{1}{N} \frac{2 \cos(\frac{N+1}{4} \varphi) \sin(\frac{N-1}{4} \varphi) + \sin \frac{\varphi}{2}}{\sin \frac{\varphi}{2}}$$

$$= \frac{1}{N} \frac{\sin(\frac{N}{2} \varphi) - \sin \frac{\varphi}{2} + \sin \frac{\varphi}{2}}{\sin \frac{\varphi}{2}}$$

$$= \frac{1}{N} \frac{\sin(\frac{N}{2} \varphi)}{\sin \frac{\varphi}{2}}$$

$$-\frac{2\pi d}{\lambda} \leq \varphi \leq \frac{2\pi d}{\lambda}$$