Problem 2.6.12

Perturbations only in sensor positions in y-direction

$$p_i = \begin{bmatrix} 0 \\ 0 \\ (i-\frac{N-1}{2})d \end{bmatrix} \qquad \Delta p_i = \begin{bmatrix} 0 \\ \Delta p_i \\ 0 \end{bmatrix}$$

$$B(\underline{k}) = \sum_{i=0}^{N-1} w_i^* e^{-jk^T(p_i^n - \Delta p_i)}$$

$$E\{|B(\underline{k})|^2\} = E\left\{\sum_{i=0}^{N-1}\sum_{l=0}^{N-1} w_i^* w_l \, e^{-jk^T(p_i^n + \Delta p_i - p_l^n - \Delta p_l)}\right\}$$

$$= \sum_{i=0}^{N-1}\sum_{l=0}^{N-1} w_i^* w_l \, e^{-jk^T(p_i^n - p_l^n)} E\left\{e^{-jk^T(\Delta p_i - \Delta p_l)}\right\}$$

let $\beta_{il}(k) = E\left\{e^{-jk^T(\Delta p_i - \Delta p_l)}\right\} = E\left\{e^{-jk_y(\Delta p_i - \Delta p_l)}\right\}$

$$= \begin{cases} 1 & i = l \\ e^{-\sigma_p^2 k_y^2} & i \neq l \end{cases}$$

$$\sigma_p^2 k_y^2 = \sigma_p^2\left(\frac{2\pi}{\lambda}\right)^2(\sin\theta \sin\phi)^2 = \underbrace{\left(\frac{2\pi}{\lambda}\sigma_p\right)^2}_{\sigma_\lambda^2} \sin^2\theta \sin^2\phi$$

$$= \sigma_\lambda^2 \sin^2\theta \sin^2\phi$$

$$E\{|B(\underline{k})|^2\} = \sum_{\substack{i=0 \\ i\neq l}}^{N-1}\sum_{l=0}^{N-1} w_i^* w_l \, e^{-jk^T(p_i^n - p_l^n)} e^{-\sigma_\lambda^2 \sin^2\theta \sin^2\phi}$$

$$+ \sum_{i=0}^{N-1}|w_i|^2 + \sum_{i=0}^{N-1}|w_i|^2 e^{-\sigma_\lambda^2\sin^2\theta\sin^2\phi} - \sum_{i=0}^{N-1}|w_i|^2 e^{-\sigma_\lambda^2\sin^2\theta\sin^2\phi}$$

$$= |B^{(n)}(k)|^2 e^{-\sigma_\lambda^2\sin^2\theta\sin^2\phi} + \sum_{i=0}^{N-1}|w_i|^2\left(1 - e^{-\sigma_\lambda^2\sin^2\theta\sin^2\phi}\right)$$

$$\boxed{E\{|B(\underline{k})|^2\} = \left|\sum_{n=0}^{N-1} w_i^* e^{j(n-\frac{N-1}{2})\frac{2\pi d}{\lambda}\cos\theta}\right|^2 e^{-\sigma_\lambda^2\sin^2\theta\sin^2\phi} + \sum_{i=0}^{N-1}|w_i|^2\left(1-e^{-\sigma_\lambda^2\sin^2\theta\sin^2\phi}\right)}$$

For 10 element, uniform weighting, $d = \lambda/2$

$$E\{|B(\underline{k})|^2\} = \frac{\sin^2(10\pi\cos\theta)}{100\sin^2(\pi\cos\theta)} e^{-\sigma_\lambda^2 \sin^2\theta \sin^2\phi} + \frac{1}{10}\left(1 - e^{-\sigma_\lambda^2 \sin^2\theta \sin^2\phi}\right)$$

<u>Degradation depends on both θ and ϕ</u>

(i) if $\phi = 0, \pi$ (Signal in x-z plane) or $\theta = 0, \pi$ (Endfire) $\Rightarrow$ no degradation

(ii) if $\phi = \pi/2, 3\pi/2$ (signal in y-z plane) $\Rightarrow$ most degradation

(iii) degradation increases for signals near broadside ($\theta = \pi/2$)

Plots shown for $\phi = \pi/2$ vs. $u = \cos\theta$ $\Rightarrow \sin^2\theta = 1 - u^2$

$$\boxed{E\{|B(u)|^2\} = \frac{\sin^2(10\pi u)}{100\sin^2(\pi u)} e^{-\sigma_\lambda^2(1-u^2)} + \frac{1}{10}\left(1 - e^{-\sigma_\lambda^2(1-u^2)}\right)}$$

for $\sigma_\lambda^2 = (2\pi(0.01))^2,\ (2\pi(0.05))^2,\ (2\pi(0.1))^2,\ (2\pi(0.5))^2$

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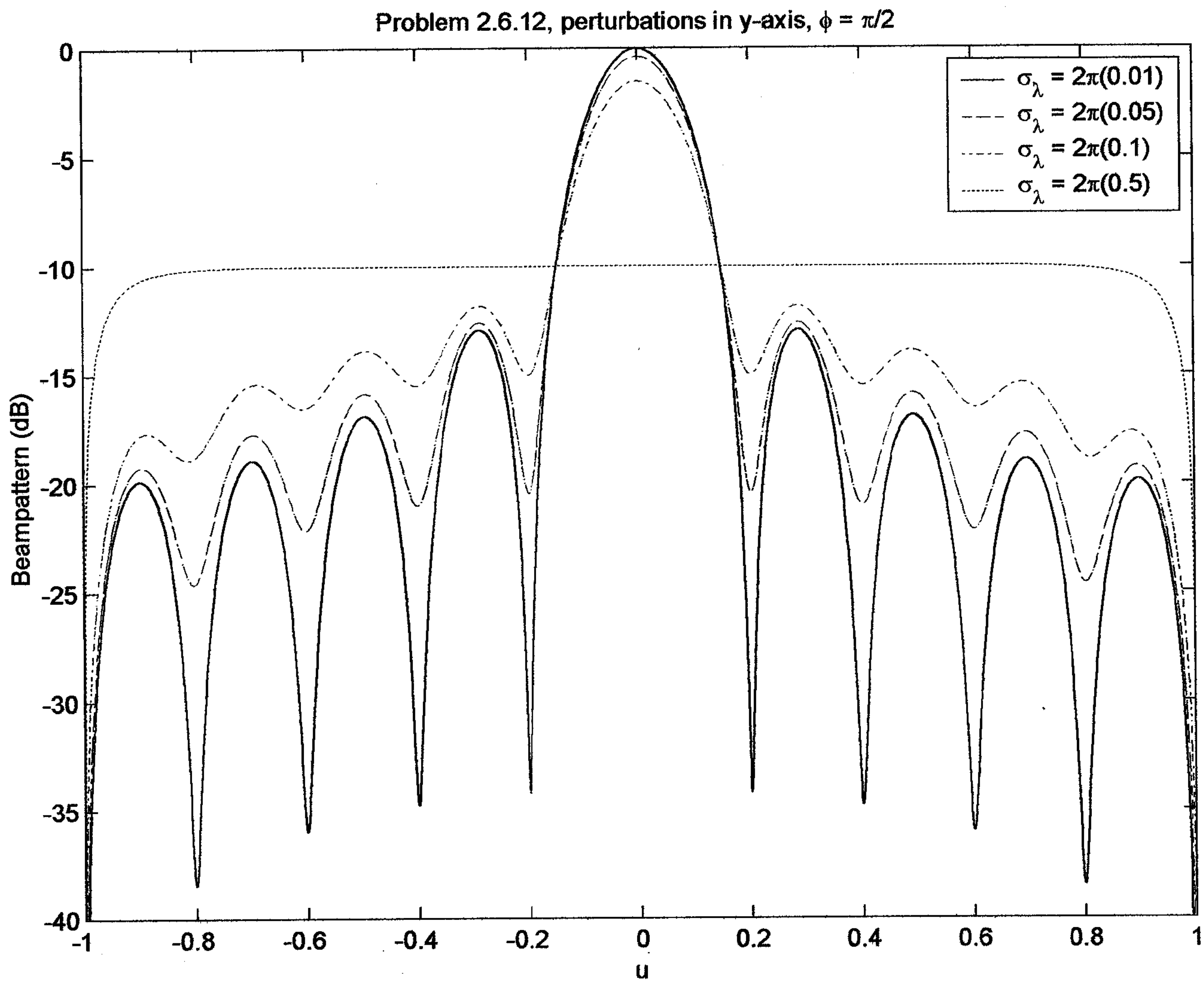
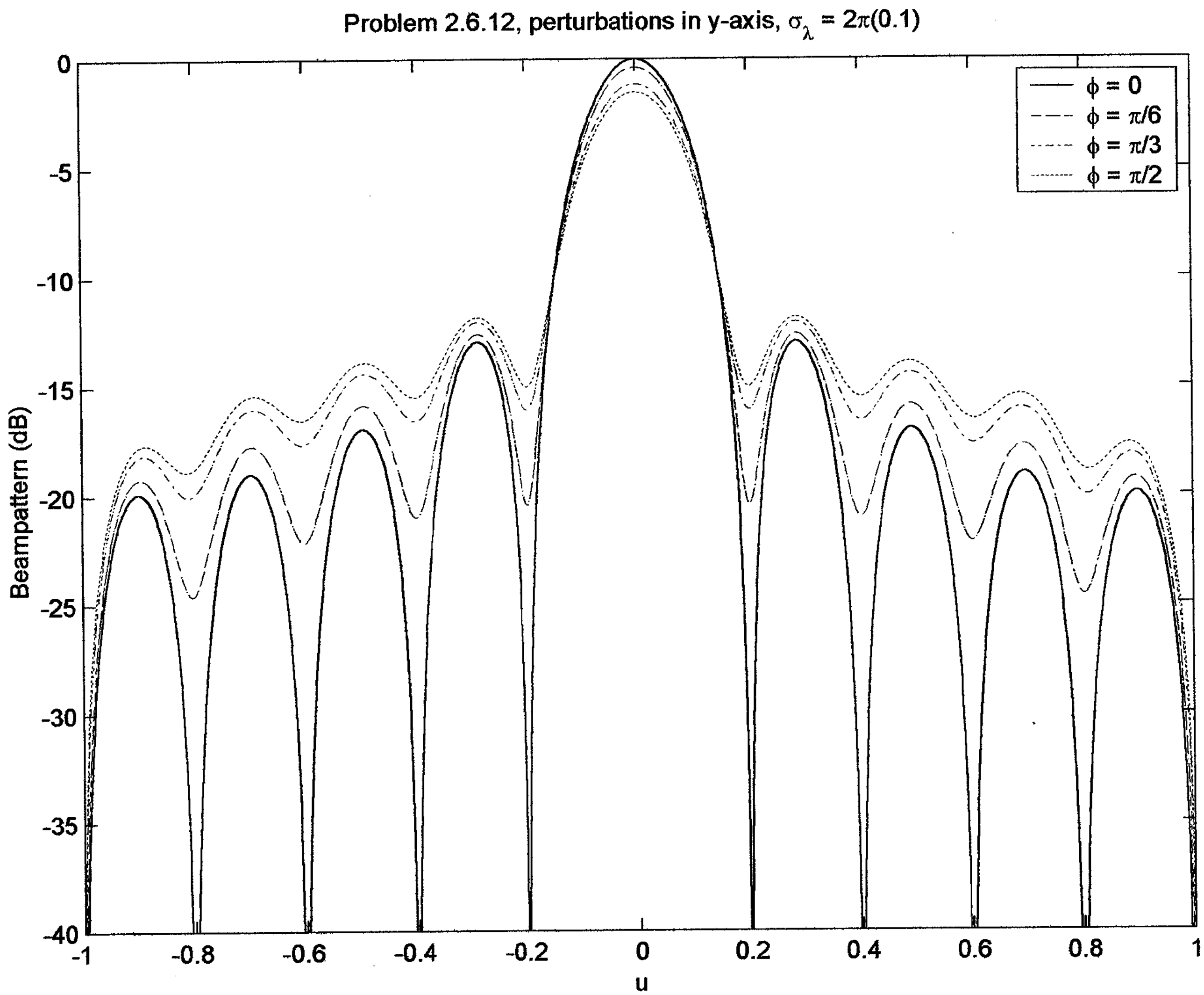


Problem 2.6.12, perturbations in y-axis, ϕ = π/2
Beampattern (dB)
u
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
σλ = 2π(0.01)
σλ = 2π(0.05)
σλ = 2π(0.1)
σλ = 2π(0.5)

2.6.12 (4)



Problem 2.6.12, perturbations in y-axis, $\sigma_\lambda = 2\pi(0.1)$
Beampattern (dB)
u
0
-5
-10
-15
-20
-25
-30
-35
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
$\phi = 0$
$\phi = \pi/6$
$\phi = \pi/3$
$\phi = \pi/2$