Math 55 - Section
Homework 10
Thursday, December 1, 2016

1, 2, 3, 4, 5

1 Show that in a non-empty connected graph, a maximal acyclic subgraph is a spanning tree.

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2 Show that if a connected graph with $n \geq 1$ vertices contains exactly $n-1$ edges, then it is acyclic (and thus a tree).

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3

(a) Show that if $0 < \lambda < 1$, then $(\lambda a + (1-\lambda)b)^2 \leq \lambda a^2 + (1-\lambda)b^2$. (That is, the square of an average is at most the average of the squares.)

(b) Show that

$$\left(\frac{1}{n}\sum_{1\leq k\leq n} a_k\right)^2 \leq \frac{1}{n}\sum_{1\leq k\leq n} a_k^2.$$

(Hint: Use induction and Part (a).)

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4

(a) Show that if $G = (V, E)$ is a graph with n vertices containing no K_3, and $\{v, w\} \in E$, then $\deg(v) + \deg(w) \leq n$.

(b) Show that in any graph

$$\sum_{\{v,w\} \in E} \deg(v) + \deg(w) = \sum_{v \in V} \deg(v)^2.$$

(c) Conclude that a graph with n vertices containing no K_3 can have at most $n^2/4$ edges.

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5 Show that if n in even, then there is a graph with n vertices and $n^2/4$ edges that contains no K_3.

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