Math 55 - Section —
Homework 1
Thursday, September 8, 2016

1, 2, 3, 4, 5, 6, 7, 8, 9

1 Starting from Washington DC, how many ways can you visit 5 of the 50 state capitals and return to Washington DC? (The order that you visit the capitals matters.)

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2 Find the number of 20-digit integers in which no two consecutive digits are the same. (We do not allow such a number to begin with zero.)

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3 How many ways can you seat 12 people at two distinct tables, where table 1 has 5 people and table 2 has 7 people? How does the answer change if both tables are circular (where arrangement ABCDE is the same as BCDEA)? (Give a numerical answer to each problem.)

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4 Let B be a subset of A, where $|A| = n$, and $|B| = k$. What is the number of subsets of A whose intersection with B has exactly 1 element?

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5 How many different "words" can you get by arranging all the letters of the word MISSISSIPPI? (Give a numerical answer.)

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6 You want to send postcards to 12 friends. In the shop, there are only 3 kinds of postcards. In how many ways can you send the postcards if:

(a) there is a large number of each kind of postcard, and you want to send one card to each friend;

(b) there is a large number of each kind of postcard, and you are willing to send one or more postcards to each friend (but no one should get two identical postcards);

(c) the shop has only 4 of each kind of postcard, and you want to send one card to each friend?

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7 The number of *ternary* words of length n is 3^n since each digit in the word has 3 choices (from the set $\{0, 1, 2\}$). How many ternary words of length n exist where the digits 0, 1, and 2 each appear at least once? Also, give a numerical answer when $n = 4$.

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8 Provide a combinatorial proof that for $n \geq 2$ and $0 \leq k \leq n$.

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}.$$

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9 In the game of *Octopoker* (just invented) a hand consists of 8 cards, where order is not important. Among the $\binom{52}{8}$ hands, count how many that have (numerical answer not required, but appreciated by grader):

(a) at least one spade.

(b) 2 cards of each suit.

(c) every suit appears 1 or 2 or 3 times.

(d) exactly 3 different values. (Hint: break into 3 cases.)

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