Math 55 - Section
Homework 5
Thursday, October 13, 2016

1, 2, 3, 4, 5, 6, 7, 8

1 Prove the I.C. theorem from class: If $d|a$ and $d|b$ then $d|ax + by$ for all integers x and y.

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2 Prove that n can be expressed as an integer combination of a and b if and only if n is a multiple of (a, b). (Recall that an if and only if proof requires proving two directions.)

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3

a) Without using prime factorizations, prove that for integers a, b, m: $(a, m) = 1$ and $(b, m) = 1$ if and only if $(ab, m) = 1$.

b) Do the same problem with unique factorization.

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4 Use the Euclidean algorithm to find:

a) gcd(754, 221),

b) an integer combination of 754 and 221 yielding the gcd.

c) integers x and y satisfying $87x + 28y = 200$.

d) another solution to c) where the signs of x and y are reversed.

(Note: All of your answers must utilize Euclid's algorithm. No credit for brut forcing or lucky guessing.)

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5 Use Euclid's algorithm to find an integer x such that $97x$ is one more than a multiple of 1234. (This is called the *multiplicative inverse* of 97, mod 1234.)

(Hint: 97 and 1234 are relatively prime.)

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6 For Fibonacci numbers F_n (with $F_0 = 0$ and $F_1 = 1$),

a) Prove that $(F_{n+1}, F_n) = 1$ for $n \geq 0$.

b) Prove for all $n \geq 1, F_{m+n} = F_{m+1}F_n + F_mF_{n-1}$.

(Hint: You can either give a combinatorial argument (recall $F_k = f_{k-1}$) or by fixing m and inducting on n.)

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7 Prove that if $m|n$ then $F_m|F_n$. (Hint: Prove F_{km} is a multiple of F_m by inducting on k and using part b) of Problem 6).

(In fact, Problems 6a and 7 are special cases of a more amazing theorem which says that if $(m, n) = g$, then $(F_m, F_n) = F_g$. For the proof of this, take Math 175!)

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8 Prove the theorem "If $(b, c) = 1$, then $(ab, c) = (a, c)$" in two different ways:

a) by showing that any common divisor of one pair is a common divisor of the other, and

b) by showing that any integer combination of one pair is an integer combination of the other.

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