Math 55 - Section
Homework 6
Thursday, October 27, 2016

1, 2, 3, 4, 5, 6, 7

1 Use Euclid's algorithm to find an integer x such that $846x$ is one more than a multiple of 929, i.e., find the multiplicative inverse of 846, mod 929.

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2 Using prime factorizations, prove: For $a, b, c \geq 0$,

$$\gcd(a, b, c) \cdot \operatorname{lcm}(ab, bc, ca) = abc.$$

$\blacksquare$

3 Using prime factorizations, prove the following:

a) If $a|bc$, and $(a, b) = 1$, then $a|c$.

b) If $(a, b) = 1$ then $(a^2, b^2) = 1$.

■

4

a) Prove that if p is prime then for all $0 < k < p$, $\binom{p}{k}$ is a multiple of p. (Hint: Begin with the identity $k\binom{p}{k} = p\binom{p-1}{k-1}$.)

b) Using a) and the Binomial Theorem applied to $(1+1)^p$, prove that if p is prime then $p|(2^p - 2)$.

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5 Prove that if $a^2 \equiv_p b^2$, where p is prime, then $a \equiv_p b$ or $a \equiv_p -b$.

$\blacksquare$

6

a) Prove that x is a multiple of 2^n iff its last n digits are divisible by 2^n.

b) Prove that x is a multiple of 11 iff when we alternately subtract and add its digits, we end up with a multiple of 11.

c) Determine the integer $r \in \{0, 1, \ldots, 10\}$ satisfying $r \equiv_{11} 31415926535897$.

∎

7

a) Here is a novel divisibility test for 7s. Prove that $10a + b$ is divisible by 7 iff $a - 2b$ is divisible by 7. For example, to determine if 2358 is divisible by 7, we reduce this number to 235 - 16 = 219. This number is reduced to 21 - 18 = 3, which is not a multiple of 7, and therefore neither is 2358. (Hint: Show $10a + b \equiv_7 0$ iff $a - 2b \equiv_7 0$. Multiplicative inverses might come in handy too.)

b) Now create a similar divisibility test for 29, and prove your result.

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Extra Credit *Note:* This problem is optional. If you decide to do it, then return it directly to Professor Benjamin.

An alien appears before number theorists Samantha and Peter and says "I am thinking of 2 integers X and Y such that $3 \leq X \leq Y \leq 97$ and I'll tell Samantha the sum and I will tell Peter the product". Then the alien goes away. The following conversation transpires:

Samantha: You do not know what X and Y are.

Peter: That was true, but I know them now.

Samantha: And now I know the numbers as well.

Determine X and Y.

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