## 1, 2, 3, 4, 5

1

- (a) How many subgraphs are there of the complete graph  $K_n$ ? (Your answer may involve a summation.)
- (b) How many of these are induced subgraphs?

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2

- (a) How many automorphisms are there of  $K_n$ , where  $n \ge 0$ ?
- (b) How many automorphisms are there of the path  $P_n$ , where  $n \ge 1$ ?
- (c) How many automorphisms are there of the cycle  $C_n$ , where  $n \geq 3$ ? (Be careful! Small cases sometimes differ from the general formula.)

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- **3** The *d*-dimensional cube  $Q_d$  ( $d \ge 0$ ) has  $2^d$  vertices labelled by the  $2^d$  lists of 0s and 1s of length d, with edges joining each pair of vertices whose lists differ in exactly one position.
  - (a) How many edges are there in  $Q_d$ ?
  - (b) For  $0 \le c \le d$ , how many induced subgraphs of  $Q_d$  are isomorphic to  $Q_c$ ? (The answer to part (a) should emerge as a special case of part (b).)

4 How many automorphisms are there of  $Q_d$ ?

5 Write a formula for the number of automorphisms of  $K_a + K_b$ . (Be careful! There are two cases to consider.)