

2.3.{24, 33, 40, 42}, 2.4.{5, 17, 23, 29a}

2.3.24 Find the gradient $\nabla f(\mathbf{a})$, where f and \mathbf{a} are given in Exercises 18-25.

$$f(x, y, z) = \cos z \ln(x + y^2), \quad \mathbf{a} = (e, 0, \pi/4)$$

■

2.3.33 In Exercises 26-33, find the matrix $D\mathbf{f}(\mathbf{a})$ of partial derivatives, where \mathbf{f} and \mathbf{a} are as indicated.

$$\mathbf{f}(s, t) = (s^2, st, t^2), \quad \mathbf{a} = (1, 1)$$

■

2.3.40 Find equations for the planes tangent to $z = x^2 - 6x + y^3$ that are parallel to the plane $4x - 12y + z = 7$.

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2.3.42 Suppose that you have the following information concerning a differentiable function f :

$$f(2,3) = 12, \quad f(1.98,3) = 12.1, \quad f(2,3.01) = 12.2.$$

- (a) Give an approximate equation for the plane tangent to the graph of f at $(2,3,12)$.
- (b) Use the result of part (a) to estimate $f(1.98,2.98)$.

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2.4.5 Verify the product and quotient rules (Proposition 4.2) for the pairs of functions given in Exercises 5-8.

$$f(x, y) = x^2y + y^3, \quad g(x, y) = \frac{x}{y}$$

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2.4.17 For the functions given in Exercises 9-17 determine all second-order partial derivatives (including mixed partials).

$$f(x, y) = x^2 e^y + e^{2z}$$

■

2.4.23 Let $f(x, y) = ye^{3x}$. Give general formulas for $\partial^n f / \partial x^n$ and $\partial^n f / \partial y^n$, where $n \geq 2$.

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2.2.29a The three-dimensional heat equation is the partial differential equation

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \frac{\partial T}{\partial t}$$

where k is a positive constant. It models the temperature $T(x, y, z, t)$ at the point (x, y, z) and time t of a body in space.

- (a) We examine a simplified version of the heat equation. Consider a straight wire “coordinatized” by x . Then the temperature $T(x, t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}.$$

Show that the function $T(x, t) = e^{-kt} \cos x$ satisfies this equation. Note that if t is held constant at value t_0 , then $T(x, t_0)$ shows how the temperature varies along the wire at time t_0 . Graph the curves $z = T(x, t_0)$ for $t_0 = 0, 1, 10$, and use them to understand the graph of the surface $z = T(x, t)$ for $t \geq 0$. Explain what happens to the temperature of the wire after a long period of time.

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