Math 60
HW 12
Thursday, June 2, 2016

7.2.{6, 14, 22, 28}, 7.3.{4, 6}, T/F(pg.522){6, 10, 18}

Colley 7.2.6 Find $\iint_S (x^2 + y^2)\, dS$, where S is the lateral surface of the cylinder of radius a and height h whose axis is the z-axis.

■

Colley 7.2.14 In Exercises 10-18, let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 4$, and lateral surface given by the equation $x^2 + y^2 = 9$. Orient S with outward normals. Determine the indicated scalar and vector surface integrals.

$$\iint_S (x\,\mathbf{i} + y\,\mathbf{j}) \cdot d\mathbf{S}$$

■

Colley 7.2.22 In Exercises 19-22, find the flux of the given vector field $\mathbf{F}$ across the upper hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$. Orient the hemisphere with an upward-pointing normal.

$$\mathbf{F} = x^2\,\mathbf{i} + xy\,\mathbf{j} + xz\,\mathbf{k}$$

∎

Colley 7.2.28 The glass dome of a futuristic greenhouse is shaped like the surface $z = 8 - 2x^2 - 2y^2$. The greenhouse has a flat dirt floor at $z = 0$. Suppose that the temperature T, at points in and around the greenhouse, varies as

$$T(x, y, z) = x^2 + y^2 + 3(z - 2)^2.$$

Then the temperature gives rise to a **heat flux density field H** given by $\mathbf{H} = -k\nabla T$. (Here k is a positive constant that depends on the insulating properties of the particular medium.) Find the total heat flux outward across the dome and the surface of the ground if $k = 1$ on the glass and $k = 3$ on the ground.

■

Colley 7.3.4 In Exercises 1-4, verify Stokes's theorem for the given surface and vector field.

$$S \text{ is defined by } x^2 + y^2 + z^2 = 4,\ z \leq 0, \text{ oriented by downward normal;}$$
$$\mathbf{F} = (2y - z)\,\mathbf{i} + (x + y^2 - z)\,\mathbf{j} + (4y - 3x)\,\mathbf{k}$$

∎

Colley 7.3.6 In Exercises 6-9, verify Gauss's theorem for the given three-dimensional region D and vector field $\mathbf{F}$.

$$\begin{aligned} \mathbf{F} &= x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}, \\ D &= \left\{ (x, y, z) \mid 0 \leq z \leq 9 - x^2 - y^2 \right\} \end{aligned}$$

■

Colley TRUE/FALSE PAGE 522

6. If S is the unit sphere centered at the origin, then $\iint_S x^3 \, dS = 0$.

10. $\iint_S (-y\mathbf{i} + x\mathbf{j}) \cdot d\mathbf{S} = 0$, where S is the cylinder $x^2 + y^2 = 9$, $0 \leq z \leq 5$.

18. $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ has the same value for all piecewise smooth, oriented surfaces S that have the same boundary curve C. *Hint:* think about what the field is doing.

■