Math 60
HW 5
Monday, May 23, 2016

# 3.3.24, 3.4.{4, 7, 12, 13, 16, 23}

**Colley 3.3.24** Consider the vector field $\mathbf{F} = 2x\mathbf{i} + 2y\mathbf{j} - 3\mathbf{k}$.

(a) Show that $\mathbf{F}$ is a gradient field.

(b) Describe the equipotential surfaces of $\mathbf{F}$ in words and with sketches.

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**Colley 3.4.4** Calculate the divergence of the vector fields given in Exercises 1-6.

$$\mathbf{F} = z\cos(e^{y^2})\,\mathbf{i} + x\sqrt{z^2+1}\,\mathbf{j} + e^{2y}\sin 3x\,\mathbf{k}$$

■

**Colley 3.4.7** Find the curl of the vector fields given in Exercises 7-11.

$$\mathbf{F} = x^2\,\mathbf{i} - xe^y\,\mathbf{j} + 2xyz\,\mathbf{k}$$

$\blacksquare$

**Colley 3.4.12**

(a) Consider again the vector field in Exercise 8 and its curl. Sketch the vector field and use your picture to explain geometrically why the curl is as you calculated.

(b) Use geometry to determine $\nabla \times \mathbf{F}$, where $\mathbf{F} = \dfrac{(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})}{\sqrt{x^2 + y^2 + z^2}}$.

(c) For $\mathbf{F}$ as in part (b), verify your intuition by explicitly computing $\nabla \times \mathbf{F}$.

$\blacksquare$

**Colley 3.4.13** Can you tell in what portions of $\mathbb{R}^2$, the vector fields shown in Figures 3.43-3.46 have positive divergence? Negative divergence?

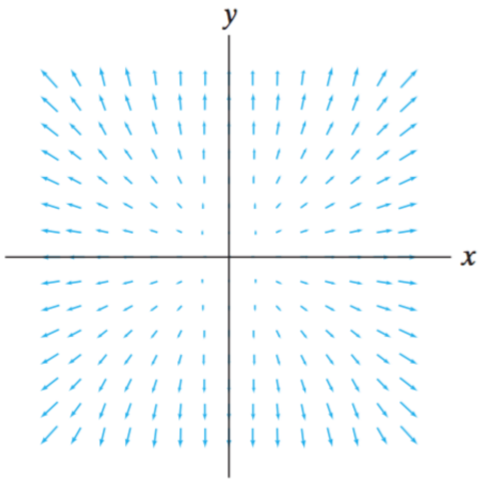


**Figure 3.43** Vector field for Exercise 13(a).

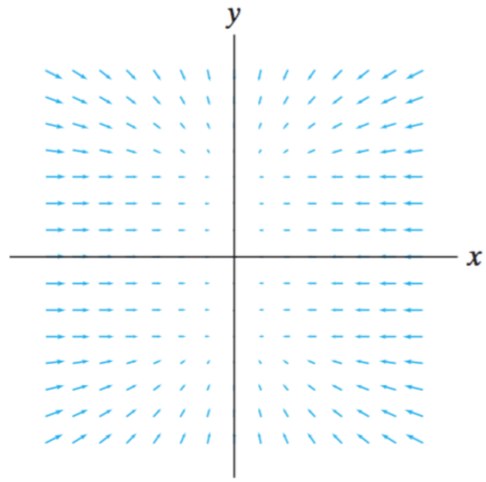


**Figure 3.44** Vector field for Exercise 13(b).

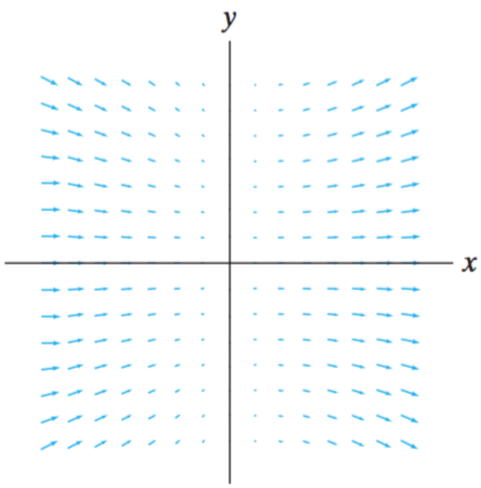


**Figure 3.45** Vector field for Exercise 13(c).

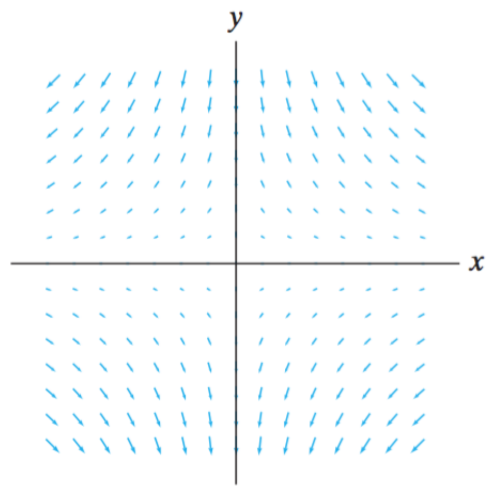


**Figure 3.46** Vector field for Exercise 13(d).

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**Colley 3.4.16 THEOREM 4.4:** Let $\mathbf{F} : X \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field of class $C^2$. Then $\text{div}\,(\text{curl}\,\mathbf{F}) = 0$. That is, curl $\mathbf{F}$ is an incompressible vector field.

Prove Theorem 4.4.

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**Colley 3.4.23** In Exercises 21-25, establish the given identities. (You may assume that any functions and vector fields are appropriately differentiable.)

$$\nabla \cdot (f\mathbf{F}) = f\nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla f$$

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