Math 60
HW 3
Thursday, May 19, 2016

2.5.{6, 14, 24, 36}, 2.6.{6, 12, 18}

**2.5.6** A rectangular stick of butter is placed in the microwave oven to melt. When the butter's length is 6 in and its square cross section measures 1.5 in on a side, its length is decreasing at a rate of 0.25 in/min and its cross-sectional edge is decreasing at a rate of 0.125 in/min. How fast is the butter melting (i.e., at what rate is the solid volume of butter turning to liquid) at that instant?

■

**2.5.14** Suppose that $z = f(x+y, x-y)$ has continuous partial derivatives with respect to $u = x+y$ and $v = x-y$. Show that

$$\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = \left(\frac{\partial z}{\partial u}\right)^2 - \left(\frac{\partial z}{\partial v}\right)^2.$$

■

**2.5.24** In Exercises 19-27, calculate $D(\mathbf{f} \circ \mathbf{g})$ in two ways: (a) by first evaluating $\mathbf{f} \circ \mathbf{g}$ and (b) by using the chain rule and the derivative matrices $D\mathbf{f}$ and $D\mathbf{g}$.

$\mathbf{f}(x, y, z) = (x^2y + y^2z, xyz, e^z)$,
$\mathbf{g}(t) = (t - 2, 3t + 7, t^3)$

∎

**2.5.36** Suppose that you are given an equation of the form

$$F(x,y,z) = 0,$$

for example, something like $x^3 z + y \cos z + (\sin y)/z = 0$. Then we may consider $z$ to be defined implicitly as a function $z(x,y)$.

(a) Use the chain rule to show that if $F$ and $z(x,y)$ are both assumed to be differentiable, then

$$\frac{\partial z}{\partial x} = -\frac{F_x(x,y,z)}{F_z(x,y,z)}, \quad \frac{\partial z}{\partial y} = -\frac{F_y(x,y,z)}{F_z(x,y,z)}.$$

(b) Use part (a) to find $\partial z/\partial x$ and $\partial z/\partial y$ where $z$ is given by the equation $xyz = 2$. Check your result by explicitly solving for $z$ and then calculating the partial derivatives.

■

**2.6.6** In Exercises 2-8, calculate the directional derivative of the given function $f$ at the point $\mathbf{a}$ in the direction parallel to the vector $\mathbf{u}$.

$f(x, y, z) = xyz, \mathbf{a} = (-1, 0, 2), \mathbf{u} = \dfrac{2\mathbf{k} - \mathbf{i}}{\sqrt{5}}$

■

**2.6.12** A ladybug (who is very sensitive to temperature) is crawling on graph paper. She is at the point $(3,7)$ and notices that if she moves in the $\mathbf{i}$-direction, the temperature *increases* at a rate of 3 deg/cm. If she moves in the $\mathbf{j}$-direction, she finds that her temperature *decreases* at a rate of 2 deg/cm. In what direction should the ladybug move if

(a) she wants to warm up most rapidly?

(b) she wants to cool off most rapidly?

(c) she desires her temperature *not* to change?

■

**2.6.18** In Exercises 16-19, find an equation for the tangent plane to the surface given by the equation at the indicated point $(x_0, y_0, z_0)$.

$2xz + yz - x^2y + 10 = 0, (x_0, y_0, z_0) = (1, -5, 5)$

■