Math 60
HW 11
Wednesday, June 1, 2016

6.2.{13, 15}, 6.3.{1, 3, 4, 25, 33}

Colley 6.2.13 Evaluate $\oint_C (x^4y^5 - 2y)\,dx + (3x + x^5y^4)\,dy$, where C is the oriented curve pictured in Figure 6.29.

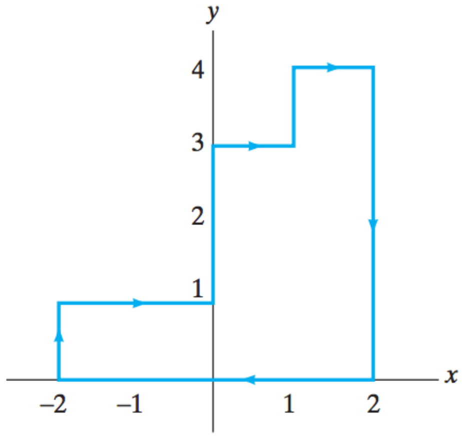

Figure 6.29 The oriented curve C of Exercise 13.

∎

Colley 6.2.15

(a) Sketch the curve given parametrically by $\mathbf{x}(t) = (1 - t^2, t^3 - t)$.

(b) Find the area inside the closed loop of the curve.

■

Colley 6.3.1 Consider the line integral $\int_C z^2\, dx + 2y\, dy + xz\, dz$.

(a) Evaluate this integral, where C is the line segment from $(0,0,0)$ to $(1,1,1)$.

(b) Evaluate this integral, where C is the path from $(0,0,0)$ to $(1,1,1)$ parametrized by $\mathbf{x}(t) = (t, t^2, t^3)$, $0 \le t \le 1$.

(c) Is the vector field $\mathbf{F} = z^2\,\mathbf{i} + 2y\,\mathbf{j} + xz\,\mathbf{k}$ conservative? Why or why not?

■

Colley 6.3.3 In Exercises 3-17, determine whether the given vector field **F** is conservative. If it is, find a scalar potential function for **F**.

$$\mathbf{F} = e^{x+y}\,\mathbf{i} + e^{xy}\,\mathbf{j}$$

■

Colley 6.3.4 In Exercises 3-17, determine whether the given vector field $\mathbf{F}$ is conservative. If it is, find a scalar potential function for $\mathbf{F}$.

$$\mathbf{F} = 2x \sin y \, \mathbf{i} + x^2 \cos y \, \mathbf{j}$$

■

Colley 6.3.25 Let $\mathbf{F} = x^2\,\mathbf{i} + \cos y \sin z\,\mathbf{j} + \sin y \cos z\,\mathbf{k}$.

(a) Show that $\mathbf{F}$ is conservative and find a scalar potential function f for $\mathbf{F}$.

(b) Evaluate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ along the path $\mathbf{x} : [0,1] \to \mathbb{R}^3,\ \mathbf{x}(t) = (t^2+1, e^t, e^{2t})$.

$\blacksquare$

Colley 6.3.33

(a) Determine where the vector field

$$\mathbf{F} = \frac{x + xy^2}{y^2}\,\mathbf{i} - \frac{x^2+1}{y^3}\,\mathbf{j}$$

is conservative.

(b) Determine a scalar potential for $\mathbf{F}$.

(c) Find the work done by $\mathbf{F}$ in moving a particle along the parabolic curve $y = 1 + x - x^2$ from $(0,1)$ to $(1,1)$.

■