Math 60
HW 2
Wednesday, May 18, 2016

2.3.{24, 33, 40, 42}, 2.4.{5, 17, 23, 29a}

2.3.24 Find the gradient $\nabla f(\mathbf{a})$, where f and $\mathbf{a}$ are given in Exercises 18-25.

$f(x,y,z) = \cos z \ln(x+y^2), \quad \mathbf{a} = (e, 0, \pi/4)$

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2.3.33 In Exercises 26-33, find the matrix D**f(a)** of partial derivatives, where **f** and **a** are as indicated.

$\mathbf{f}(s,t) = (s^2, st, t^2), \quad \mathbf{a} = (-1, 1)$

■

2.3.40 Find equations for the planes tangent to $z = x^2 - 6x + y^3$ that are parallel to the plane $4x - 12y + z = 7$.

■

2.3.42 Suppose that you have the following information concerning a differentiable function f:

$$f(2,3) = 12, \quad f(1.98,3) = 12.1, \quad f(2,3.01) = 12.2.$$

(a) Give an approximate equation for the plane tangent to the graph of f at (2,3,12).

(b) Use the result of part (a) to estimate $f(1.98, 2.98)$.

■

2.4.5 Verify the product and quotient rules (Proposition 4.2) for the pairs of functions given in Exercises 5-8.

$$f(x,y) = x^2y + y^3, \quad g(x,y) = \frac{x}{y}$$

■

2.4.17 For the functions given in Exercises 9-17 determine all second-order partial derivatives (including mixed partials).

$f(x, y, z) = x^2 e^y + e^{2z}$

■

$\boxed{\textbf{2.4.23}\ \text{ Let } f(x,y) = ye^{3x}\text{. Give general formulas for } \partial^n f/\partial x^n \text{ and } \partial^n f/\partial y^n\text{, where } n \geq 2.}$

■

2.2.29a The three-dimensional heat equation is the partial differential equation

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \frac{\partial T}{\partial t}$$

where k is a positive constant. It models the temperature $T(x, y, z, t)$ at the point (x, y, z) and time t of a body in space.

(a) We examine a simplified version of the heat equation. Consider a straight wire "coordinatized" by x. Then the temperature $T(x, t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}.$$

Show that the function $T(x, t) = e^{-kt} \cos x$ satisfies this equation. Note that if t is held constant at value t_0, then $T(x, t_0)$ shows how the temperature varies along the wire at time t_0. Graph the curves $z = T(x, t_0)$ for $t_0 = 0, 1, 10$, and use them to understand the graph of the surface $z = T(x, t)$ for $t \geq 0$. Explain what happens to the temperature of the wire after a long period of time.

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