Math 60
HW 5
Monday, May 23, 2016

3.3.24, 3.4.{4, 7, 12, 13, 16, 23}

Colley 3.3.24 Consider the vector field $\mathbf{F} = 2x\mathbf{i} + 2y\mathbf{j} - 3\mathbf{k}$.

(a) Show that $\mathbf{F}$ is a gradient field.

(b) Describe the equipotential surfaces of $\mathbf{F}$ in words and with sketches.

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Colley 3.4.4 Calculate the divergence of the vector fields given in Exercises 1-6.

$$\mathbf{F} = z\cos(e^{y^2})\,\mathbf{i} + x\sqrt{z^2+1}\,\mathbf{j} + e^{2y}\sin 3x\,\mathbf{k}$$

■

Colley 3.4.7 Find the curl of the vector fields given in Exercises 7-11.

$$\mathbf{F} = x^2\,\mathbf{i} - xe^y\,\mathbf{j} + 2xyz\,\mathbf{k}$$

$\blacksquare$

Colley 3.4.12

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{3.4.8}$$

(a) Consider again the vector field in Exercise 8 and its curl. Sketch the vector field and use your picture to explain geometrically why the curl is as you calculated.

(b) Use geometry to determine $\nabla \times \mathbf{F}$, where $\mathbf{F} = \dfrac{(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})}{\sqrt{x^2 + y^2 + z^2}}$.

(c) For $\mathbf{F}$ as in part (b), verify your intuition by explicitly computing $\nabla \times \mathbf{F}$.

$\blacksquare$

Colley 3.4.13 Can you tell in what portions of $\mathbb{R}^2$, the vector fields shown in Figures 3.43-3.46 have positive divergence? Negative divergence?

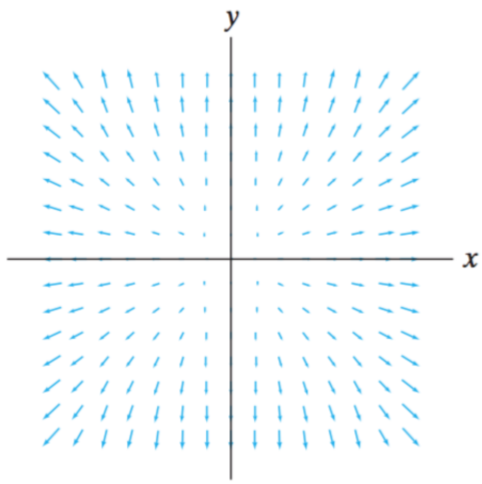

Figure 3.43 Vector field for Exercise 13(a).

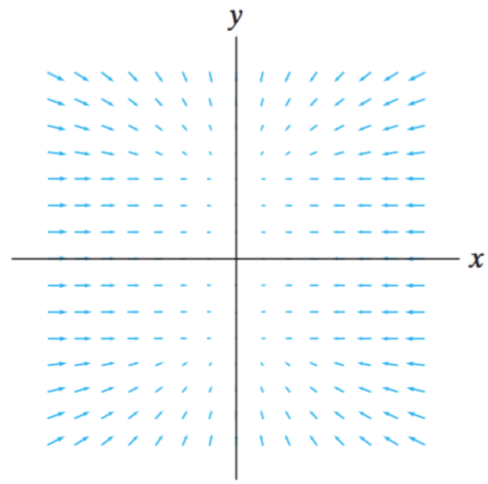

Figure 3.44 Vector field for Exercise 13(b).

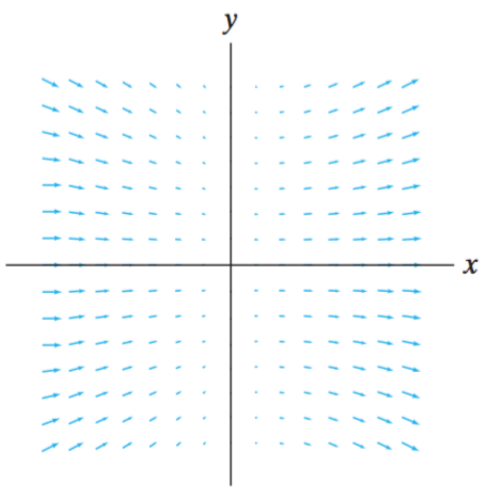

Figure 3.45 Vector field for Exercise 13(c).

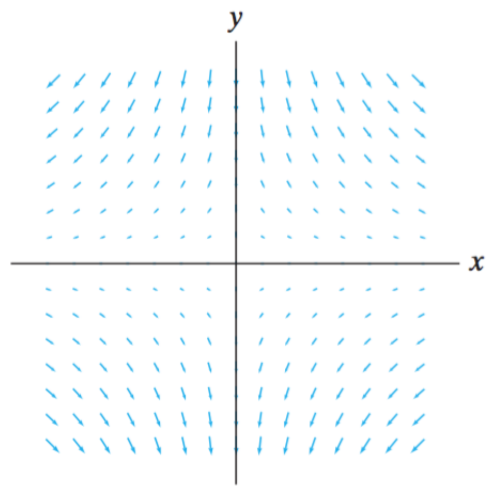

Figure 3.46 Vector field for Exercise 13(d).

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Colley 3.4.16 THEOREM 4.4: Let $\mathbf{F} : X \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field of class C^2. Then div (curl $\mathbf{F}$) $= 0$. That is, curl $\mathbf{F}$ is an incompressible vector field.

Prove Theorem 4.4.

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Colley 3.4.23 In Exercises 21-25, establish the given identities. (You may assume that any functions and vector fields are appropriately differentiable.)

$$\nabla \cdot (f\mathbf{F}) = f\nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla f$$

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