Math 65
HW 3
Thursday, May 19, 2016

6.4.{1, 2, 8, 9, 15, 27}, 6.5.{3, 7, 14, 22}

6.4.1 In Exercises 1-12, determine whether T is a linear transformation.

$T : M_{22} \to M_{22}$ defined by

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+b & 0 \\ 0 & c+d \end{bmatrix}$$

■

6.4.2 In Exercises 1-12, determine whether T is a linear transformation.

$T : M_{22} \to M_{22}$ defined by

$$T\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & w-z \\ x-y & 1 \end{bmatrix}$$

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6.4.8 In Exercises 1-12, determine whether T is a linear transformation.

$T : \mathscr{P}_2 \to \mathscr{P}_2$ defined by $T(a + bx + cx^2) = (a + 1) + (b + 1)x + (c + 1)x^2$

■

6.4.9 In Exercises 1-12, determine whether T is a linear transformation.

$T : \mathscr{P}_2 \to \mathscr{P}_2$ defined by $T(a + bx + cx^2) = a + b(x + 1) + b(x + 1)^2$

■

6.4.15 Let $T : \mathbb{R}^2 \to \mathscr{P}_2$ be a linear transformation for which

$$T\begin{bmatrix}1\\1\end{bmatrix} = 1 - 2x \quad \text{and} \quad T\begin{bmatrix}3\\-1\end{bmatrix} = x + 2x^2$$

Find $T\begin{bmatrix}-7\\9\end{bmatrix}$ and $T\begin{bmatrix}a\\b\end{bmatrix}$.

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6.4.27 Define linear transformations $S : \mathscr{P}_n \to \mathscr{P}_n$ and $T : \mathscr{P}_n \to \mathscr{P}_n$ by

$$S(p(x)) = p(x+1) \quad \text{and} \quad T(p(x)) = p'(x)$$

Find $(S \circ T)(p(x))$ and $(T \circ S)(p(x))$. [*Hint*: Remember the Chain Rule.]

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6.5.3 Let $T : \mathscr{P}_2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(a + bx + cx^2) = \begin{bmatrix} a - b \\ b + c \end{bmatrix}$$

(a) Which, if any, of the following polynomials are in $\ker(T)$?
(i) $1 + x$ **(ii)** $x - x^2$ **(iii)** $1 + x - x^2$

(b) Which, if any, of the following polynomials are in $\text{range}(T)$?

(i) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) Describe $\ker(T)$ and $\text{range}(T)$.

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6.5.7 In Exercises 5-8, find bases for the kernel and range of the linear transformations T in the indicated exercises. In each case, state the nullity and rank of T and verify the Rank Theorem.

Exercise 3

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6.5.14 In Exercises 9-14, find either the nullity or the rank of T and then use the Rank Theorem to find the other.

$T : M_{33} \to M_{33}$ defined by $T(A) = A - A^T$

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6.5.22 In Exercises 21-26, determine whether V and W are isomorphic. If they are, give an explicit isomorphism $T : V \to W$.

$V = S_3$ (symmetric 3×3 matrices), $W = U_3$ (upper-triangular 3×3 matrices)

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