Math 65
HW 5
Monday, May 23, 2016

6.1.47, 6.3.16, 6.4.{4, 21}, 6.5.12, 6.6.2, 4.4.22, EC: 6.2.33

Intructor's Note: I recommend that you also look at the Chapter Review on pages 527-528 of Poole and skim the problems to see if there are any concepts or problems that seem challenging to you. Try some of these problems for more practice.

Poole 6.1.47 Let V be a vector space with subspaces U and W. Give an example with $V = \mathbb{R}^2$ to show that $U \cup W$ need not be a subspace of V.

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Poole 6.3.16 Let $\mathcal{B}$ and $\mathcal{C}$ be bases for $\mathscr{P}_2$. If $\mathcal{B} = \{x, 1+x, 1-x+x^2\}$ and the change-of-basis matrix from $\mathcal{B}$ to $\mathcal{C}$ is

$$P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

find $\mathcal{C}$.

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Poole 6.4.4 In Exercises 1-12, determine whether T is a linear transformation.

$$T : M_{nn} \to M_{nn} \text{ defined by } T(A) = AB - BA, \text{ where } B \text{ is a fixed } n \times n \text{ matrix}$$

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Poole 6.4.21 Prove Theorem 6.14(b).

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Poole 6.5.12 In Exercises 9-14, find either the nullity or the rank of T and then use the Rank Theorem to find the other.

$$T : M_{22} \to M_{22} \text{ defined by } T(A) = AB - BA, \text{ where } B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

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Poole 6.6.2 In Exercises 1-12, find the matrix $[T]_{\mathcal{C}\leftarrow\mathcal{B}}$ of the linear transformation $T : V \to W$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$ of V and W, respectively. Verify Theorem 6.26 for the vector $\mathbf{v}$ by computing $T(\mathbf{v})$ directly and using the theorem.

$$\begin{aligned}
&T : \mathscr{P}_1 \to \mathscr{P}_1 \text{ defined by} \\
&T(a+bx) = b - ax, \\
&\mathcal{B} = \{1+x, 1-x\}, \\
&\mathcal{C} = \{1, x\}, \\
&\mathbf{v} = p(x) = 4 + 2x
\end{aligned}$$

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Poole 4.4.22 In Exercises 16-23, use the method of Example 4.29 to compute the indicated power of the matrix.

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}^k$$

(assume that k is a positive integer)

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Extra Credit: Poole 6.2.33 Let $\{\mathbf{u}_1, ..., \mathbf{u}_m\}$ be a set of vectors in an n-dimensional vector space V and let $\mathcal{B}$ be a basis for V. Let $S = \{[\mathbf{u}_1]_\mathcal{B}, ..., [\mathbf{u}_m]_\mathcal{B}\}$ be on the set of coordinate vectors of $\{\mathbf{u}_1, ..., \mathbf{u}_m\}$ with respect to $\mathcal{B}$. Prove that $\text{span}(\mathbf{u}_1, ..., \mathbf{u}_m) = V$ if and only if $\text{span}(S) = \mathbb{R}^n$.

(Remember that to prove an if-and-only-if theorem, you need to prove both directions.)

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