

6.1. {11, 13, 29, 34, 46, 60, 62}, Additional Problem #1

**6.1.11** In Exercises 1-11, determine whether the given set, together with the specified operations of addition and scalar multiplication, is a vector space. If it is not, list all of the axioms that fail to hold.

**DEF:** A square matrix is called *skew-symmetric* if  $A^T = -A$ .

The set of all skew-symmetric  $n \times n$  matrices, with the usual matrix addition and scalar multiplication.

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**6.1.13** Finish verifying that  $\mathcal{F}$  is a vector space (see Example 6.4).

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**6.1.29** In Exercises 24-45, use Theorem 6.2 to determine whether  $W$  is a subspace of  $V$ .

$$V = M_{22}, W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad \geq bc \right\}$$

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**6.1.34** In Exercises 24-45, use Theorem 6.2 to determine whether  $W$  is a subspace of  $V$ .

$$V = \mathcal{P}_2, W = \{bx + cx^2\}$$

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**6.1.46** Let  $V$  be a vector space with subspaces  $U$  and  $W$ . Prove multiplication that  $U \cap W$  is a subspace of  $V$ .

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**6.1.60** Is  $M_{22}$  spanned by  $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ ?

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**6.1.62** Is  $\mathcal{P}_2$  spanned by  $1 + x + 2x^2, 2 + x + 2x^2, -1 + x + 2x^2$ ?

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**Additional Problem #1** For each square matrix below, calculate its eigenvalues and eigenvectors. Then verify that  $PDP^{-1}$  is equal to the original matrix, where  $D$  is a diagonal matrix with your eigenvalues along its diagonal and  $P$  is a matrix with your eigenvectors as its columns.

(a)  $\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$

(b)  $\begin{bmatrix} 0 & -4 \\ 1 & 0 \end{bmatrix}$

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