Math 65
HW 2
Wednesday, May 18, 2016

6.2.{20, 25, 28, 38}, 6.3.{7, 18, 21}, Additional Problem #1

6.2.20 In Exercises 18-25, determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$V = M_{22}, \mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$

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6.2.25 In Exercises 18-25, determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$V = \mathscr{P}_2, \mathcal{B} = \{1, 2 - x, 3 - x^2, x + 2x^2\}$

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6.2.28 Find the coordinate vector of $p(x) = 1 + 2x + 3x^2$ with respect to the basis $\mathcal{B} = \{1 + x, 1 - x, x^2\}$ of $\mathscr{P}_2$.

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6.2.38 In Exercises 34-39, find the dimension of the vector space V and give a basis for V.

$V = \{A \text{ in } M_{22} : A \text{ is skew-symmetric}\}$

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6.3.7 In Exercises 5-8, follow the instructions for Exercises 1-4 using $p(x)$ instead of $\mathbf{x}$.

$p(x) = 1 + x^2, \mathcal{B} = \{1 + x + x^2, x + x^2, x^2\}, \mathcal{C} = \{1, x, x^2\}$ in $\mathscr{P}_2$

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6.3.18 Express $p(x) = 1 + 2x - 5x^2$ as a Taylor polynomial about $a = -2$.

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6.3.21 Let $\mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ be bases for a finite-dimensional vector space V. Prove that

$$P_{\mathcal{D} \leftarrow \mathcal{C}} P_{\mathcal{C} \leftarrow \mathcal{B}} = P_{\mathcal{D} \leftarrow \mathcal{B}}$$

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Additional Problem #1 For each square matrix below, calculate its eigenvalues and eigenvectors. Then verify that PDP^{-1} is equal to the original matrix, where D is a diagonal matrix with your eigenvalues along its diagonal and P is a matrix with your eigenvectors as its columns.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -13 & -4 \\ 0 & -3 & 0 \\ 1 & 13 & 0 \end{bmatrix}$

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