CS301 Computer Architecture

DR GAYATHRI ANANTHANARAYANAN

gayathri@iitdh.ac.in

Materials in these slides are borrowed from textbooks and existing Architecture courses

What does a Computer Understand ?

* Computers do not understand natural human languages, nor programming languages
* They only understand the language of bits

Bit	0 or 1
Byte	8 bits
Word	4 bytes
kiloByte	1024 bytes
megaByte	10^6 bytes

McGraw Hill Education

Representing Integers

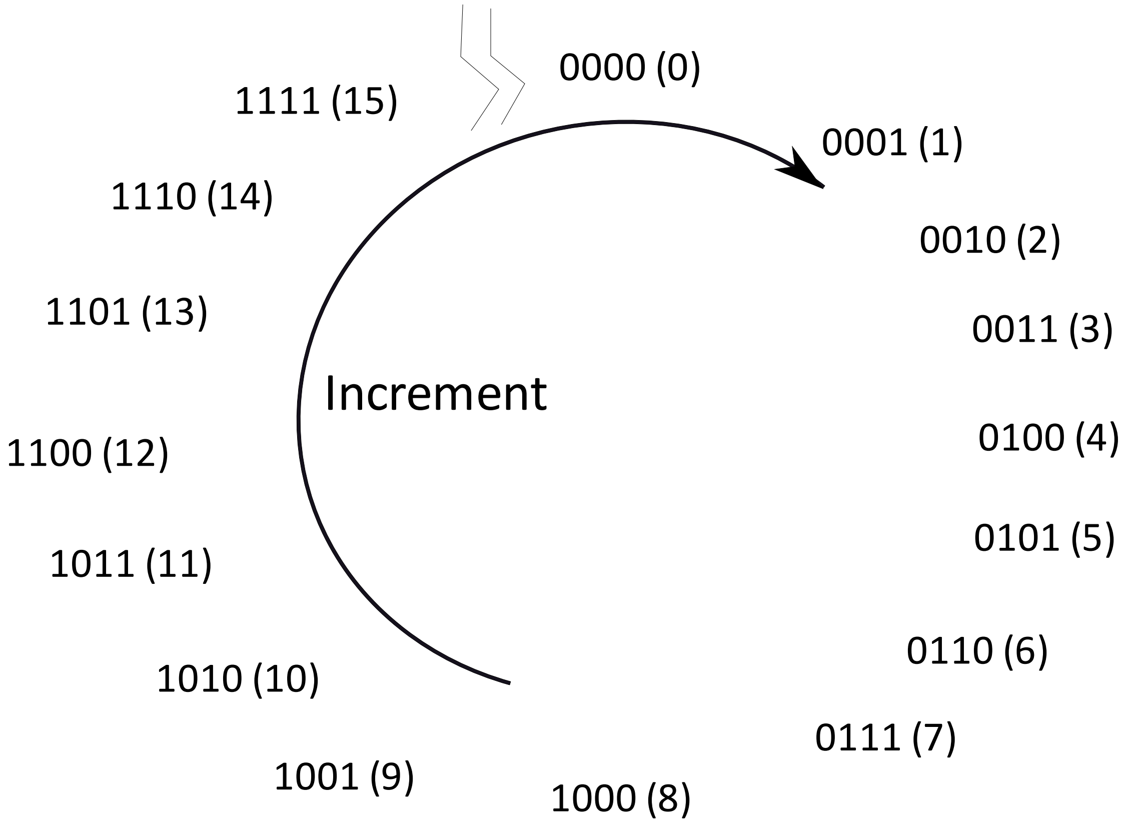
Decimal Value	Target Notation	Value
5	4-bit unsigned	0101
10	4-bit unsigned	1010
5	4-bit signed	0101
10	4-bit signed	can't be represented!
-5	4-bit signed	1011
-10	4-bit signed	can't be represented!

Given n bits,

- unsigned representation allows values from 0 to 2^(n)-1
- signed representation allows values from -2^(n-1) to 2^(n-1)-1

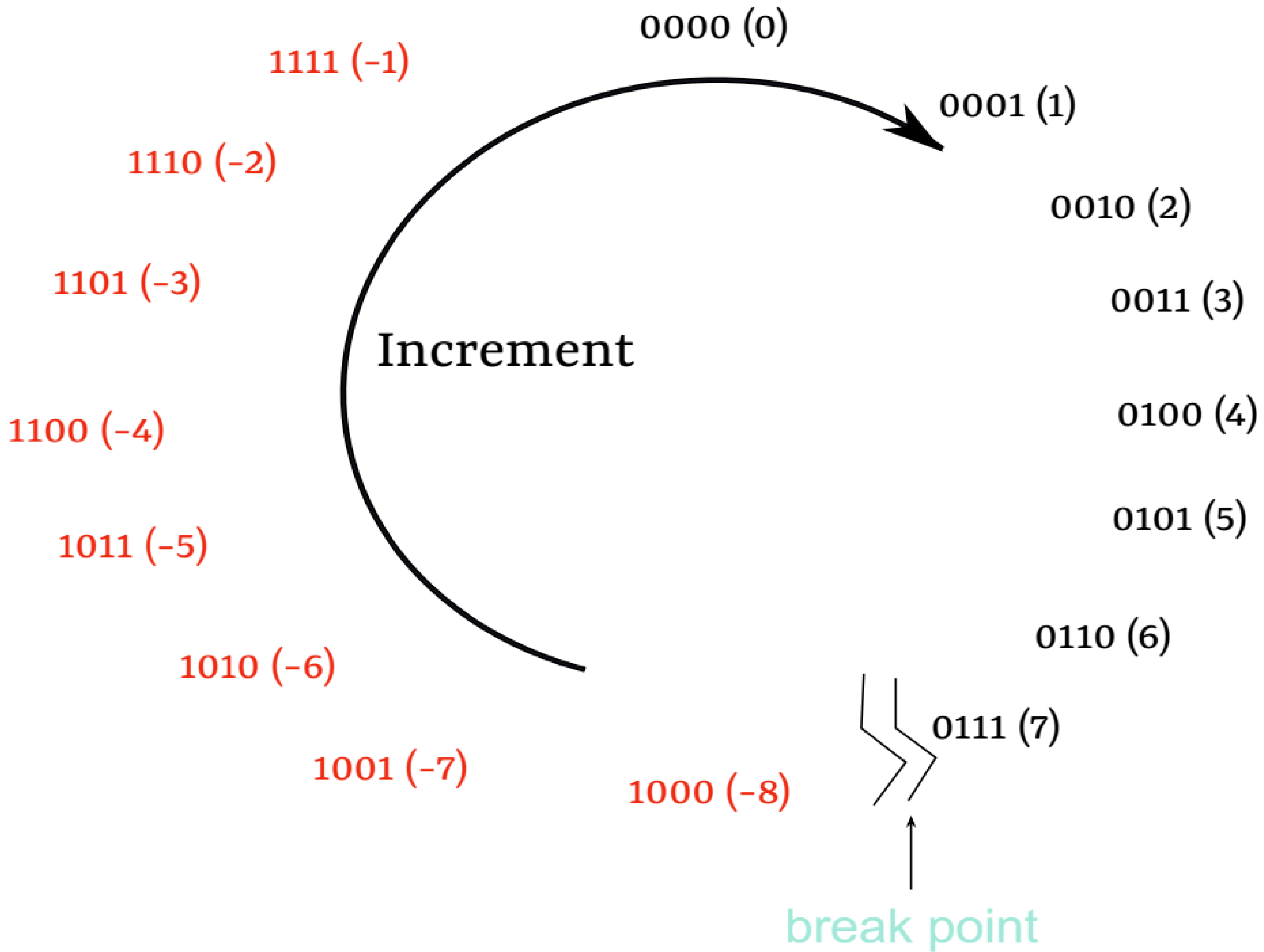
Mc Graw Hill Education

The Number Circle



Clockwise: increment
Anti-clockwise: decrement

Number Circle with Negative Numbers



Using the Number Circle

* To add M to a number, N
  * locate N on the number circle
  * If M is +ve
    * Move M steps clockwise
  * If M is -ve
    * Move M steps anti-clockwise, or 2^n – M steps clockwise
  * If we cross the break-point
    * We have an overflow
    * The number is too large/ too small to be represented

MSB and LSB

* MSB (Most Significant Bit) → The leftmost bit of a binary number. E.g., MSB of 1110 is 1
* LSB (Least Significant Bit) → The rightmost bit of a binary number. E.g., LSB of 1110 is 0

Floating-Point Numbers

* What is a floating-point number ?
  * 2.356
  * 1.3e-10
  * -2.3e+5
* What is a fixed-point number ?
  * Number of digits after the decimal point is fixed
  * 3.29, -1.83

Generic Form for Positive Numbers

- Generic form of a number in base 10

$$A = \sum_{i=-n}^{n} x_i 10^i$$

- Example :
  - $3.29 = 3 * 10^0 + 2*10^{-1} + 9*10^{-2}$

Generic Form in Base 2

* Generic form of a number in base 2

$$A = \sum_{i=-n}^{n} x_i 2^i$$

Number	Expansion
0.375	$2^{-2} + 2^{-3}$
1	2^{0}
1.5	$2^{0} + 2^{-1}$
2.75	$2^{1} + 2^{-1} + 2^{-2}$
17.625	$2^{4} + 2^{0} + 2^{-1} + 2^{-3}$

Binary Representation

* Take the base 2 representation of a floating-point (FP) number
* Each coefficient is a binary digit

Number	Expansion	Binary Representation
0.375	$2^{-2} + 2^{-3}$	0.011
1	2^0	1.0
1.5	$2^0 + 2^{-1}$	1.1
2.75	$2^1 + 2^{-1} + 2^{-2}$	10.11
17.625	$2^4 + 2^0 + 2^{-1} + 2^{-3}$	10001.101

Encoding Floats

L (16 bits)	R (16 bits)
L15 L14 L13 ….. L0	R15 R14 R13 …. R0

Required number, $A = L15 * 2^{(15)} + L14 * 2^{(14)} + \ldots + L0 * 2^{(0)} + R15 * 2^{(-1)} + R14 * 2^{(-2)} + \ldots + R0 * 2^{(-16)}$

- Range?
- Precision?

The smallest change that can be represented in floating point representation (the distance between successive numbers in the representation)

Encoding Floats

L (16 bits)	R (16 bits)
L15 L14 L13 ….. L0	R15 R14 R13 …. R0

Required number, A = L15 * 2^(15) + L14 * 2^(14) + ... + L0 * 2^(0) + R15 * 2^(-1) + R14 * 2^(-2) + ... + R0 * 2^(-16)

- Range? 0 to ~2^(16)
- Precision? 2^(-16)

Normalized Form

- Let us create a standard form of all floating point numbers

$$A = (-1)^S * P * 2^X, (P = 1 + M, 0 \le M < 1, X \in Z)$$

- S → sign bit, P → significand
- M → mantissa, X → exponent, **Z** → set of integers

McGraw Hill Education

IEEE 754 Format

De facto standard for representing floating point numbers in binary.

- General Principles
  - The significand is of the form : 1.xxxxx
  - No need to waste 1 bit representing (1.) in the significand
  - We can just save the mantissa bits
  - Need to also store the sign bit (S), exponent (X)

McGraw Hill Education

IEEE 754 Format - II

Sign(S)	Exponent(X)	Mantissa(M)
1	8	23

* sign bit – 0 (+ve), 1 (-ve)
* exponent, 8 bits
* mantissa, 23 bits

Representation of the Exponent

* Biased representation
  * bias = 127
  * E = X + bias
* Range of the exponent
  * 0 – 255 ⟷ -127 to +128
* Examples :
  * X = 0, E = 127
  * X = -23, E = 104
  * X = 30 , E = 157

Normal FP Numbers

- Have an exponent between -126 and +127
- Let us leave the exponents : -127, and +128 for special purposes.

$$A = (-1)^{S} * P * 2^{E-bias}$$

$$(P = 1 + M, 0 \le M < 1, X \in Z, 1 \le E \le 254)$$

S	E (8 bits)	M (23 bits)

M22 M21 M20 ....

Exponent = $E - 2^{(8-1)} = E - 127$

Mantissa = $M22 * 2^{(-1)} + M21 * 2^{(-2)} + \ldots + M0 * 2^{(-23)}$

Required number, A = $(-1)^S * (1 + \text{Mantissa}) * 2^{(\text{Exponent})}$

- Range? (approx) $-2^{(128)}$ to $+2^{(128)}$ (E=0 and E=255 are used for special purposes)
- Precision? $\sim 2^{(-126)}$

Known as the IEEE 754 Standard Format for Floating Point Numbers

IEEE 754 Format: Example

IEEE 754 format of 4.5 = ?

IEEE 754 Format: Example

IEEE 754 Format of 4.5 = 0x40900000

Can you try for

- -0.25
- 1.6

Decimal	IEEE 754 Format
4.5	0x4090000
-0.25	0xbe800000
1.6	0x3fcccccd

Floating Point Arithmetic is approximate!

How do we represent 0.0?

S = 0

M = 0

E = 1 (remember we agreed on not using E = 0)

(-1) ^ S * (1 + M) * 2^(E-127) = 2^(-126)

- Not perfectly zero!

The Number Line

Need for some special numbers to increase the range

Special Floating Point Numbers

E	M	Value
255	0	∞ if $S = 0$
255	0	$-\infty$ if $S = 1$
255	$\neq 0$	NAN (Not a number)
0	0	0
0	$\neq 0$	Denormal number

- NAN + x= NAN   1/0 = ∞
- 0/0 = NAN   -1/0 = -∞
- $\sin^{-1}(5)$ = NAN

Denormal Numbers

```
f = 2^(-126);
g = f/2;
if (g == 0)
  print ("error");
```

- Should this code print "error" ?
- How to stop this behaviour ?

McGraw Hill Education

Denormal Numbers - II

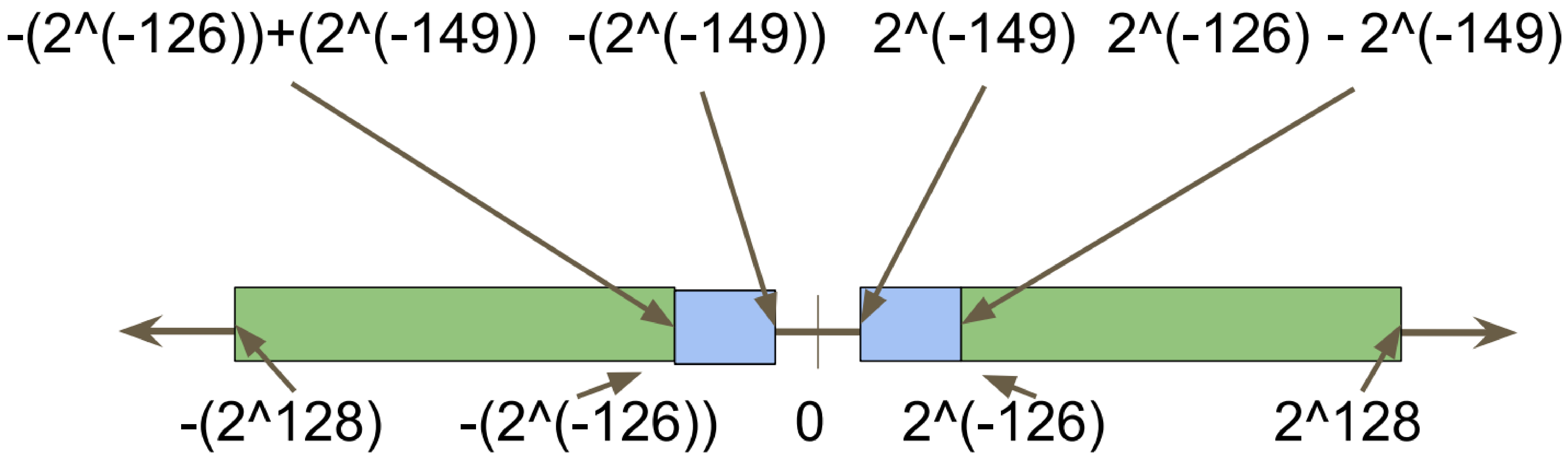
$$A = (-1)^{S} * P * 2^{-126}$$

$$(P = 0 + M, 0 \text{ £ } M < 1)$$

- Significand is of the form : 0.xxxx
- E = 0, X = -126 (why not -127?)
- Smallest +ve normal number : 2^{-126}
- Largest denormal number :
  - $0.11...11 * 2^{-126} = (1 - 2^{-23}) * 2^{-126}$
    - $= 2^{-126} - 2^{-149}$

Denormal Numbers

- $E = 0$, $M \neq 0$
- $A = (-1)^S * (0 + M) * 2^{(-126)}$
- Range?
  - From $2^{(-149)}$ to $2^{(-126)} - 2^{(-149)}$


-(2^(-126))+(2^(-149))
-(2^(-149))
2^(-149)
2^(-126) - 2^(-149)
-(2^128)
-(2^(-126))
0
2^(-126)
2^128

Double Precision Numbers

Field	Size(bits)
S	1
E	11
M	52

- Approximate range of doubles
  - $\pm\ 2^{1023} = \pm\ 10^{308}$
  - This is a lot !!!

ASCII Character Set

* ASCII – American Standard Code for Information Interchange
* It has 128 characters
* First 32 characters (control operations)
  * backspace (8)
  * line feed (10)
  * escape (27)
* Each character is encoded using 7 bits

ASCII Character Set

Character	Code	Character	Code	Character	Code
a	97	A	65	0	48
b	98	B	66	1	49
c	99	C	67	2	50
d	100	D	68	3	51
e	101	E	69	4	52
f	102	F	70	5	53
g	103	G	71	6	54
h	104	H	72	7	55
i	105	I	73	8	56
j	106	J	74	9	57
k	107	K	75	!	33
l	108	L	76	#	35
m	109	M	77	$	36
n	110	N	78	%	37
o	111	O	79	&	38
p	112	P	80	(	40
q	113	Q	81	)	41
r	114	R	82	*	42
s	115	S	83	+	43
t	116	T	84	,	44
u	117	U	85	.	46
v	118	V	86	;	59
w	119	W	87	=	61
x	120	X	88	?	63
y	121	Y	89	@	64
z	122	Z	90	^	94

Unicode Format

* UTF-8 (Universal character set Transformation Format)
  * UTF-8 encodes 1,112,064 characters defined in the Unicode character set. It uses 1-6 bytes for this purpose. E.g.अ आ क ख, ௹□ஜ ஸ
  * UTF-8 is compatible with ASCII. The first 128 characters in UTF-8 correspond to the ASCII characters. When using ASCII characters, UTF-8 requires just one byte. It has a leading 0.
  * Most of the languages that use variants of the Roman script such as French, German, and Spanish require 2 bytes in UTF-8. Greek, Russian (Cyrillic), Hebrew, and Arabic, also require 2 bytes.

Mc Graw Hill Education

UTF-16 and 32

- Unicode is a standard across all browsers and operating systems
- UTF-8 has been superseded by UTF-16, and UTF-32
- UTF-16 uses 2 byte or 4 byte encodings (Java and Windows)
- UTF-32 uses 4 bytes for every character (rarely used)