

Module 1 : Counting
CS 203: Discrete Structures
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INSTRUCTIONS: The following are the practice problems to improve your understanding of the concepts in basic counting. Try to solve all problems. You do not have to submit the solution.

1. There are two restaurants in a street - A and B. A offers 5 kinds of fries and 3 kinds of burgers. B offers 4 kinds of pastas and 4 kinds of shakes. How many ways you can eat or drink
 - (a) One item only.
 - (b) Two different items from shop A and two different items from shop B.
2. You have to pick one factor from 8100 and one factor from 1024. But they should be different. How many ways you can do that ?
3. In how many ways you can arrange the letters of the word EDUCATION such that A comes between D and O ?
4. In how many ways you can arrange the letters of the word EDUCATION such that vowels occur in the positions divisible by 3 ?
5. how many ways you can arrange the letters of the word EXECUTION such that vowels occur together but they occur between two consonants (Like - X(EEUIO)CTN) ?
6.
 - (a) What is the value of $\binom{99}{98} + \binom{99}{97}$?
 - (b) Simplify: $\binom{14}{6} + \binom{11}{6} + \binom{10}{7} + \binom{15}{6} + \binom{13}{6} + \binom{10}{6} + \binom{12}{6}$?
7. One student has 6 math books, another has 10 math books. All books are different. How many ways are there to exchange 3 books belonging to the first student with 3 books belonging to the second ?
8. How many ways are there to divide 10 boys into two basketball teams of 5 boys each ?
9. In how many ways can the letters of 'MILLIMICRON' be arranged?
10.
 - (a) Coefficient of x^6y^3 in $(x - y)^9$ is ?
 - (b) What is the middle term in the expansion of $(3x + 2y)^5$?
11. Prove the following identity :

$$\sum_{t=0}^n \binom{t}{k} = \binom{n+1}{k+1}.$$

12. How many sequences $x_k, k \geq 0$ non-negative integers satisfy $x_1 + x_2 + \dots + x_k = n$?
13. How many paths are totally present from top left to bottom right of a $m \times n$ matrix?

14. How many paths are there from $(0,0)$ to $(7,7)$ without crossing the diagonal?
15. Suppose you have n pairs of parentheses and you would like to form valid groupings of them, where “valid” means that each open parenthesis has a matching closed parenthesis. For example, “ $((()))$ ” is valid, but “ $()()()$ ” is not. How many groupings are there for each value of n ?
16. Prove that one can choose evenly many objects from a collection of n objects in 2^{n-1} ways.
17. Six boxes are numbered 1 through 6. How many ways are there to put 20 identical balls into these boxes so that
 - (a) none of them is empty ?
 - (b) some of them can be empty ?
18. A train with m passengers must make n stops. How many ways are there for passengers to get off the train at stops.
19. In how many ways can three people divide among themselves six identical apples, one banana, one orange and one mango ?
20. There are 12 books on a shelf. How many ways are there to choose five of them so that no two of the chosen books stand next to each other ?
21. What is the coefficient of x^2yzw^2 in $(x + y + z + w)^6$? What is the sum of all the coefficients of the expansion $(x + y + z + w)^6$?
22. What is the sum of the numbers in the 20^{th} row of Pascal’s triangle ? Prove that sum of 6^{th} and 7^{th} number in the 11^{th} row of Pascal’s triangle is equal to 7^{th} number in 12^{th} row of the triangle.