Inclusion-Exclusion, Recurrence Relation and Generating Functions
CS 203: Discrete Structures
Course Instructor : Prof. Prabuchandran K J
Teaching Assistant : Sagartanu Pal

INSTRUCTIONS: The following are the practice problems to improve your understanding of the concepts in basic counting. Try to solve all problems. You do not have to submit the solution.

1. Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time. What are the initial conditions? In how many ways can this person climb a flight of eight stairs?

2. Find a recurrence relation for the number of bit strings of length n that contain a pair of consecutive 0s. What are the initial conditions? How many bit strings of length seven contain two consecutive 0s?

3. Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive 0s. What are the initial conditions? How many ternary strings of length six do not contain two consecutive 0s?

4. Find a recurrence relation for the number of bit strings of length n that do not contain three consecutive 0s. What are the initial conditions? How many bit strings of length seven do not contain three consecutive 0s?

5. Solve the recurrence relation for $a_n = 5a_{n-1} - 6a_{n-2}$ where $n \geq 2, a_0 = 1, a_1 = 0$.

6. Using inclusion and exclusion find how many solutions does

$$x_1 + x_2 + x_3 = 11$$

have, where x_1, x_2 and x_3 are non negative integers with $x_1 \leq 3, x_2 \leq 4$ and $x_3 \leq 6$?

7. A new employee checks the hats of n people at a restaurant, forgetting to put claim check numbers on the hats. When customers return for their hats, the checker gives them back hats chosen at random from the remaining hats. What is the probability that no one receives the correct hat?

8. What is the closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, ...$?

9. If the ordinary generating function of a sequence is:

$$\frac{1+z}{(1-z)^3}$$

then what is $a_3 - a_0$?

10. Consider the recurrence relation $a_1 = 8$, $a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = k * 10^4$. Then what is the value of k?