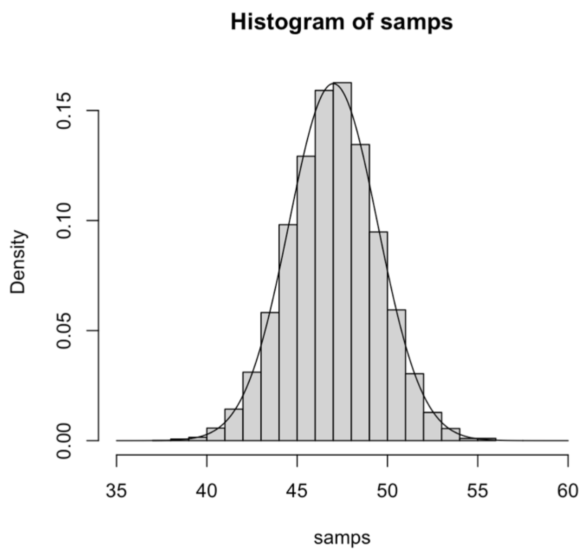
2.

f) i)



ii)

Because $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, we let $W = \frac{(n-1)S^2}{\sigma^2}$ and so we get $W \sim \chi^2(n-1)$ and $S^2 = \frac{W\cdot\sigma^2}{n-1}$ and we let $V = s^2 = \frac{W\cdot\sigma^2}{n-1}$.

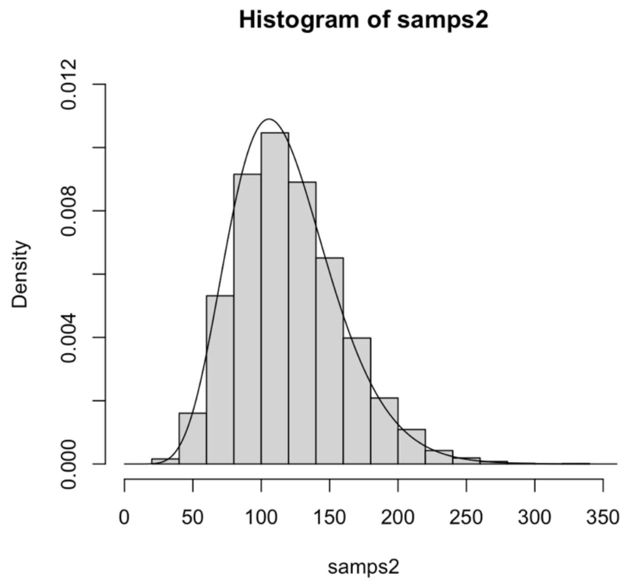
Thus, we have

$$
\begin{aligned}
F_V(v) &= P(V \le v) \\
&= P\left(\frac{W\cdot\sigma^2}{n-1} \le v\right) \\
&= P\left(W \le \frac{v\cdot(n-1)}{\sigma^2}\right) \\
&= F_W\left(\frac{v(n-1)}{\sigma^2}\right)
\end{aligned}
$$

Taking the derivative, we get

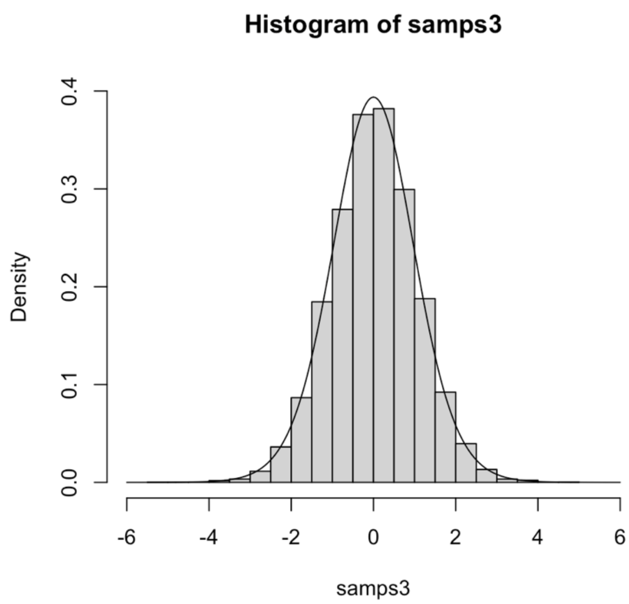
$$
\begin{aligned}
f_V(v) &= \frac{d}{dv} F_V(v) \\
&= \frac{d}{dv} F_W\left(\frac{v(n-1)}{\sigma^2}\right) \\
&= f_W\left(\frac{v(n-1)}{\sigma^2}\right)\frac{d}{dv}\left(\frac{v\cdot(n-1)}{\sigma^2}\right) \quad \text{by chain rule and } \frac{d}{dw}F_W(w) = f_W(w) \\
&= \frac{\left(\frac{v(n-1)}{\sigma^2}\right)^{\frac{n-1}{2}-1}\cdot e^{-\frac{v\cdot(n-1)}{2\sigma^2}}}{2^{\frac{n-1}{2}}\,\Gamma\left(\frac{n-1}{2}\right)} \cdot \frac{(n-1)}{\sigma^2} \quad \text{because } W \sim \chi^2(n-1) \\
&= \frac{v^{\frac{n-1}{2}-1}\cdot\left(\frac{n-1}{2\sigma^2}\right)^{\frac{n-1}{2}}\cdot e^{-\frac{v\cdot(n-1)}{2\sigma^2}}}{\Gamma\left(\frac{n-1}{2}\right)} \quad \text{for } v > 0
\end{aligned}
$$

Because $V = S^2$, the pdf of S^2 is the above one by replacing v with s^2.



Histogram of samps2
Density
0.000
0.004
0.008
0.012
0
50
100
150
200
250
300
350
samps2

iii)



Histogram of samps3
Density
0.0
0.1
0.2
0.3
0.4
-6
-4
-2
0
2
4
6
samps3