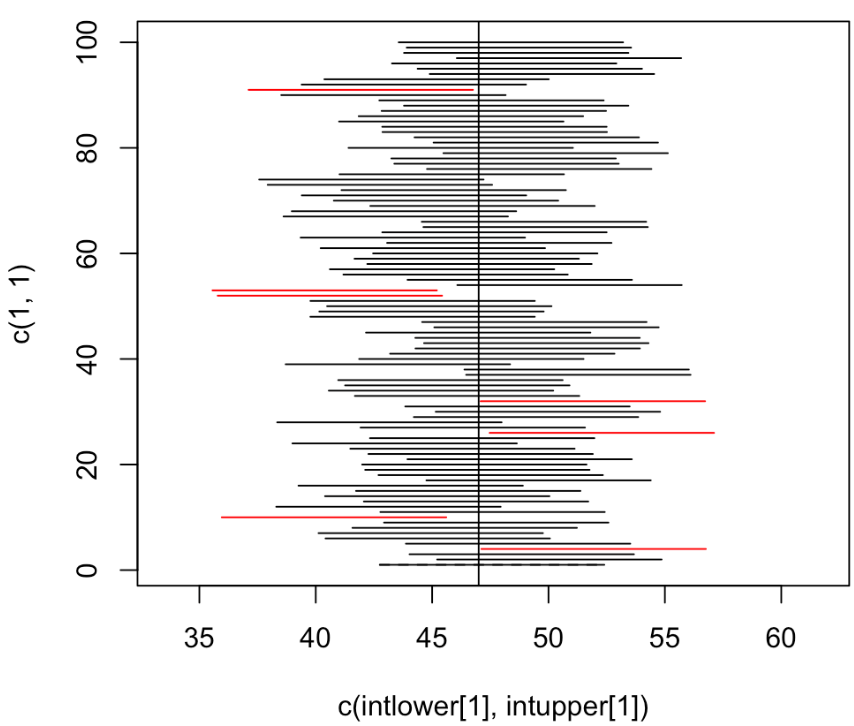
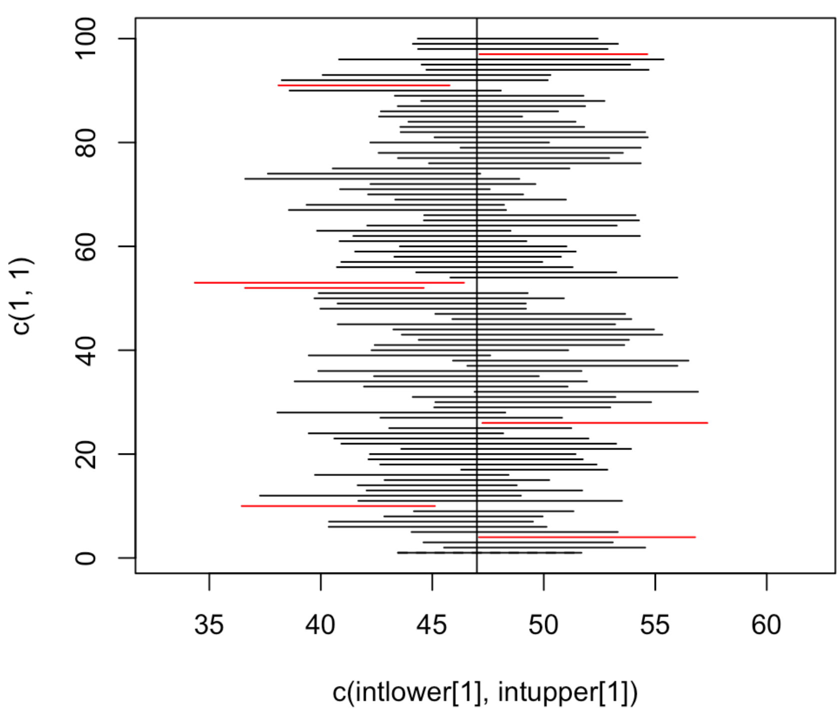
3.

g) i) The empirical coverage probability of a 95% confidence interval for μ when σ = 11 is known, based on 10,000 simulated datasets having μ = 47, is 0.9507. This means that if we conduct an experiment for a large number of times and construct the interval based on the observed data every time, we know that approximately 95.07% of these intervals would contain the true value of μ. Also, observing the plot of the confidence intervals for the first 100 simulated datasets, we find that there are 100-7=93 out of 100 intervals containing the true value of μ, which means that 93% of these 100 intervals contain the true value of μ. 93% and 95.07% are close and so the plot also shows that approximately 95.07% of these intervals would contain the true value of μ.



ii) The empirical coverage probability from the simulations is 0.9351. The coverage of the resulting intervals is lower than when we used the true σ. This is shown by we obtain that the coverage of the resulting intervals is 93.51%, which is lower than 95.07% obtained in part g)i). Also, the plot shows that there are 100-7=93 out of 100 intervals containing the true value of μ, which means that 93% of these 100 intervals contain the true value of μ. The reason that the coverage of the resulting intervals is lower than when we used the true σ is because degree freedom is not large enough to make the graphs of Gaussian distribution and t distribution similar and the value of **a** is taken from a G(0,1) distribution instead of t distribution. To be specific, when we use the true σ, we should use the formula $\bar{y} \pm \boldsymbol{a} * \frac{\sigma}{\sqrt{n}}$, where **a** is given by P($Z \leq \boldsymbol{a}$) = $\frac{1+p}{2}$, where $Z \sim G(0,1)$. However, when we use estimate σ for each simulated dataset using the sd function, we should use the formula $\bar{y} \pm \boldsymbol{a} * \frac{s}{\sqrt{n}}$, where **a** is given by P($T \leq \boldsymbol{a}$) = $\frac{1+p}{2}$, where $T \sim t(n-1)$. The value of **a** from a G(0,1) distribution is smaller than the value of a from the t distribution. Thus, it causes that the interval obtained is smaller. This further causes the coverage of the resulting intervals is lower than when we used the true σ. Also, we should notice that sample variance is an unbiased estimator, which means that E(S^2)=σ^2. Thus, replacing σ by s would not influence too much on the coverage.



iii) The empirical coverage probability from the simulations is 0.9512. Also, observing the plot, we get that there are 100-4 = 96 out of 100 intervals containing the true value of μ. The coverage of the resulting intervals should be the same as i) because we both use correct formulas and the results of i) and iii) are approximately 0.95. Furthermore, the coverage of the resulting intervals should be higher than ii) because we use the value of **a** from t distribution, which would be larger than the value obtained from G(0,1) distribution as explained in part ii). Thus, the intervals obtained in this question would be larger than the intervals obtained in part ii). This causes that the coverage of the resulting intervals is higher than ii).