

2.

e)

$$H_0: \mu_S - \mu_H = 0 \quad H_A: \mu_S - \mu_H \neq 0$$

$$\text{Test statistic is } d = \frac{|\hat{\mu}_S - \hat{\mu}_H - 0|}{\text{Se}(\hat{\mu}_S - \hat{\mu}_H)} = \frac{|\bar{y}_S - \bar{y}_H|}{s_p \sqrt{\frac{1}{n_S} + \frac{1}{n_H}}}, \text{ where}$$

$$\hat{\mu}_S = \bar{y}_S = \frac{\sum_{i=1}^{n_S} y_{Si}}{n_S}, \quad \hat{\mu}_H = \bar{y}_H = \frac{\sum_{i=1}^{n_H} y_{Hi}}{n_H},$$

$$s_p = \sqrt{\frac{(n_S-1)S_S^2 + (n_H-1)S_H^2}{n_S + n_H - 2}}, \quad S_S^2 = \frac{\sum_{i=1}^{n_S} (y_{Si} - \bar{y}_S)^2}{n_S - 1},$$

$$S_H^2 = \frac{\sum_{i=1}^{n_H} (y_{Hi} - \bar{y}_H)^2}{n_H - 1}.$$

Because $\bar{y}_S = 46.8599$, $\bar{y}_H = 46.70695$, $n_S = 20$, $n_H = 20$, we

have $s_p = 9.411988$ and so $d = 1.059341$.

Its distribution is $T \sim t(n_S + n_H - 2)$, which is

$$T \sim t(38)$$

$$p\text{-value} = P(|T| \geq d) = 2 \times P(T \geq d) = 2 \times [1 - P(T \leq d)], \text{ where}$$

$$T \sim t(38). \quad p\text{-value is } 0.296132.$$

f) Professor Stringer's cars use the same amount of gas on average. Based on part c and part d, both two intervals contain 0. This means that whether we assume the standard deviation of gas consumption of car S equals to the standard deviation of gas consumption of car H or not, both confidence intervals we get contain 0, which means

that the mean value of gas consumption of car S equals to the mean value of gas consumption of car H. Also, based on part e, we have p-value is 0.296132. Because $0.296132 > 0.1$, there is no evidence that against null hypothesis based on the data. Thus, Professor Stringer's cars use the same amount of gas on average.