

3.

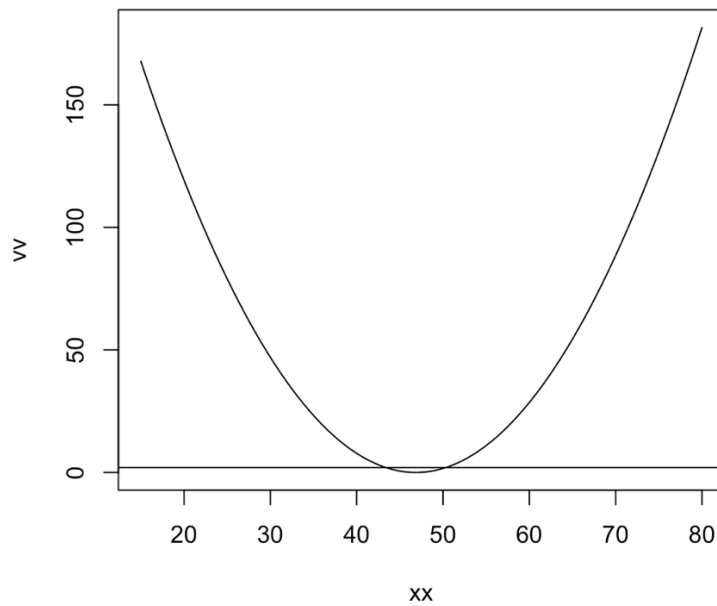
a) The formula for a 95% confidence interval for  $\mu$  when  $\sigma = 11$  is known is  $\bar{y} \pm a * \frac{\sigma}{\sqrt{n}}$ , where a is given by  $P(Z \leq a) = \frac{1+p}{2}$ , where  $Z \sim G(0,1)$ ,  $\bar{y}$  is the sample mean, and n is the number of observed data. By evaluating the interval for the gasS data, we obtain 42.03903 and 51.68077.

b) The formula for a 95% confidence interval for  $\mu$  when  $\sigma$  is unknown is  $\bar{y} \pm b * \frac{s}{\sqrt{n}}$ , where b is given by  $P(T \leq b) = \frac{1+p}{2}$ , where  $T \sim t(n-1)$ ,  $\bar{y}$  is the sample mean, n is the number of observed data, and s is the sample standard deviation. By evaluating the interval for the gasS data, we obtain 41.77343 and 51.94637.

c) The formula for a 95% confidence interval for  $\sigma^2$  when  $\mu$  is unknown and estimated by  $\bar{Y}$  is  $[\frac{(n-1)s^2}{d}, \frac{(n-1)s^2}{c}]$ , where c and d are given by  $P(W \leq c) = \frac{1-p}{2} = P(W > d)$ , where  $W \sim \chi^2(n-1)$ , s is the sample standard deviation, and n is the number of observed data. By evaluating the interval for the gasS data, we obtain 68.31286 and 251.9769.

d) The formula for a 95% confidence interval for  $\sigma$  when  $\mu$  is unknown and estimated by  $\bar{Y}$  is  $[\sqrt{\frac{(n-1)s^2}{d}}, \sqrt{\frac{(n-1)s^2}{c}}]$ , where c and d are given by  $P(W \leq c) = \frac{1-p}{2} = P(W > d)$ , where  $W \sim \chi^2(n-1)$ , s is the sample standard deviation, and n is the number of observed data. Thus, we obtain 8.265159 and 15.87378.

e)



f)

Using the relative likelihood from Assignment 1 to construct a 95% confidence interval for  $\mu$  when  $\sigma = 11$  is known, we get

$$\frac{1}{\sigma^2} \sum_{i=1}^n [(y_i - \mu)^2 - (y_i - \hat{\mu})^2] \leq m$$

, where  $m$  is given by  $P(W \leq m) = 0.95$ , where  $W \sim \chi^2(1)$ .

In this formula,  $\sigma = 11$  and  $n = 20$  and  $\hat{\mu} = 46.8599$ .

The 95% confidence interval for  $\mu$  is consisted of 42.03903 and 51.68076. This interval is approximately the same as the interval constructed in part a.