2.

a)

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

b) estimate of μ is 46.8599

estimate of σ^2 is 118.1177

estimate of σ is 10.8682

c) The sampling distribution of $\bar{Y}$ is $\bar{Y} \sim G(\mu, \frac{\sigma}{\sqrt{n}})$, where μ represents the average fuel consumption per fill-up, σ represents the standard deviation of the fuel consumption per fill-up, and n represents the number of gas data for car S. The formula for a pivotal quantity for μ based off $\bar{Y}$ is $\frac{\bar{Y}-\mu}{\sigma\,/\sqrt{n}}$.

d) The sampling distribution of S^2 is $S^2 \sim \frac{\chi^2(n-1)\sigma^2}{n-1}$, where σ^2 represents the variance of the fuel consumption per fill-up, and n represents the number of gas data for car S. The formula for a pivotal quantity for σ^2 based off S^2 is $\frac{(n-1)S^2}{\sigma^2}$ and S^2 represents the sample variance of the fuel consumption per fill-up.

e) The sampling distribution of T is T = $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim t(n-1)$, where n represents the number of gas data for car S. T is a pivotal quantity. This is because T is a function of the data Y due to the presence of $\bar{Y}$ and the unknown parameter μ such that the distribution of the random variable T is fully known, which is t distribution and degree freedom is n – 1.

That is, probability statements such as P($T \leq b$) and P($T \geq a$) depend on a and b but not on μ or any other unknown information.