

Question #1 (5 points):

A recent [POLITICO Magazine/Ipsos poll](#) asked 1,005 adults (i.e., ages 18 or older) Americans if they believe that Trump is guilty in the case related to retaining sensitive documents. As expected, results vary greatly depending on the political affiliation; see table below.

Is Trump guilty in the case related to retaining sensitive documents?

	Republicans	Democrats	Independents
Yes	25%	81%	36%
No	56%	11%	21%
Don't know	20%	8%	43%

Source: POLITICO Magazine/Ipsos poll
Catherine Kim/POLITICO

- (a) Using the fact that (as of Jan 2024) 26% of adult Americans are Republicans, 28% are Democrats and 46% are Independents, estimate the overall proportion of Americans who believe that Trump is guilty [†]
- (b) Provide a 95% CI for the estimate obtained in part (a)

$$(a) 26\% \times 25\% + 28\% \times 81\% + 46\% \times 36\% = 0.4574 = 45.74\% = \hat{p}_{pst}$$

$$(b) \text{ the 95\% CI for a proportion is } \hat{p}_{pst} \pm t_{1004; 0.975} \sqrt{\hat{V}_{SRS}(\hat{p}_{pst})}$$

By $qt(0.975, 1004)$ using R, we have 1.96233.

$$\hat{V}_{SRS}(\hat{p}_{pst}) = \left(\frac{1}{n} - \frac{1}{N}\right) \sum_{h=1}^H w_h S_{gh}^2 + \left(\frac{1}{n} - \frac{1}{N}\right) \frac{1}{n} \sum_{h=1}^H (1 - w_h) S_{gh}^2$$

$$= \frac{1}{1005} \sum_{h=1}^H w_h S_{gh}^2 + \left(\frac{1}{1005}\right) \left(\frac{1}{1005}\right) \sum_{h=1}^H (1 - w_h) S_{gh}^2 \quad \text{since } N \text{ is too large because there are too many adults Americans and so we can approximate } \frac{1}{N} \text{ by } 0.$$

$$= \frac{1}{1005} \sum_{h=1}^H w_h \frac{n_h}{n_h - 1} \hat{p}_{\pi_h} (1 - \hat{p}_{\pi_h}) + \frac{1}{1005^2} \sum_{h=1}^H (1 - w_h) \frac{n_h}{n_h - 1} \hat{p}_{\pi_h} (1 - \hat{p}_{\pi_h})$$

$$= \frac{1}{1005} \left[26\% \times 25\% (1 - 25\%) + 28\% \times 81\% (1 - 81\%) + 46\% \times 36\% (1 - 36\%) \right]$$

$$+ \frac{1}{1005^2} \left[(1 - 26\%) \times 25\% (1 - 25\%) + (1 - 28\%) \times 81\% (1 - 81\%) + (1 - 46\%) \times 36\% (1 - 36\%) \right]$$

$$\text{since } \frac{n_h}{n_h - 1} \approx 1$$

$$= 0.0001972121$$

$$45.74\% \pm 1.96233 \times \sqrt{0.0001972121}$$

$$= 45.74\% \pm 1.96233 \times 0.01404322$$

$$= 45.74\% \pm 0.02755743$$

$$= [42.98426\%, 48.49574\%]$$