4. a) Y_i represents the random variable of the fuel consumption on the i^{th} fill-up.

b) μ represents the average of the fuel consumption on every fill-up.

c) σ represents the standard deviation of the fuel consumption on every fill-up.

d) The parameter space for μ is $\mu > 0$.

e) $$L(\mu) = (2\pi)^{-\frac{n}{2}} 11^{-n} e^{-\frac{1}{2\times 11^2}\sum_{i=1}^{n}(y_i-\mu)^2}$$

$$= (2\pi)^{-\frac{n}{2}} 11^{-n} e^{-\frac{1}{242}\sum_{i=1}^{n}(y_i-\mu)^2}$$

$$= e^{-\frac{n}{242}(\bar{y}-\mu)^2} \quad \text{for } \mu > 0$$

because σ is known, which is $\sigma = 11$, then we get likelihood function of μ ignoring constants with respect to μ.

$$l(\mu) = -\frac{n}{242}(\bar{y}-\mu)^2 \quad \text{for } \mu > 0$$

$$\frac{dl(\mu)}{d\mu} = \frac{n}{121}(\bar{y}-\mu)$$

f) To obtain $\tilde{\mu}$, we have $\frac{d\ell(\mu)}{d\mu} = \frac{n}{121}(\bar{y} - \mu) = 0$,

and so $\hat{\mu} = \bar{y}$, and so $\tilde{\mu} = \bar{Y}$.

$$R(\mu) = \frac{L(\mu)}{L(\hat{\mu})} = \frac{e^{-\frac{n}{242}(\bar{y}-\mu)^2}}{e^{-\frac{n}{242}(\bar{y}-\hat{\mu})^2}}$$

$$= \frac{e^{-\frac{n}{242}(\bar{y}-\mu)^2}}{e^{-\frac{n}{242}(\bar{y}-\bar{y})^2}} \quad \text{because } \bar{y} = \hat{\mu}$$

$$= e^{-\frac{n}{242}(\bar{y}-\mu)^2}$$

$$= e^{-\frac{n}{242}(\hat{\mu}-\mu)^2} \quad \text{because } \bar{y} = \hat{\mu}$$

g) Because $\hat{\mu} = \bar{y}$ by part (f), we use mean(gas$gas) in

R to compute $\bar{y} = 46.8599$, and so $\hat{\mu} = 46.8599$