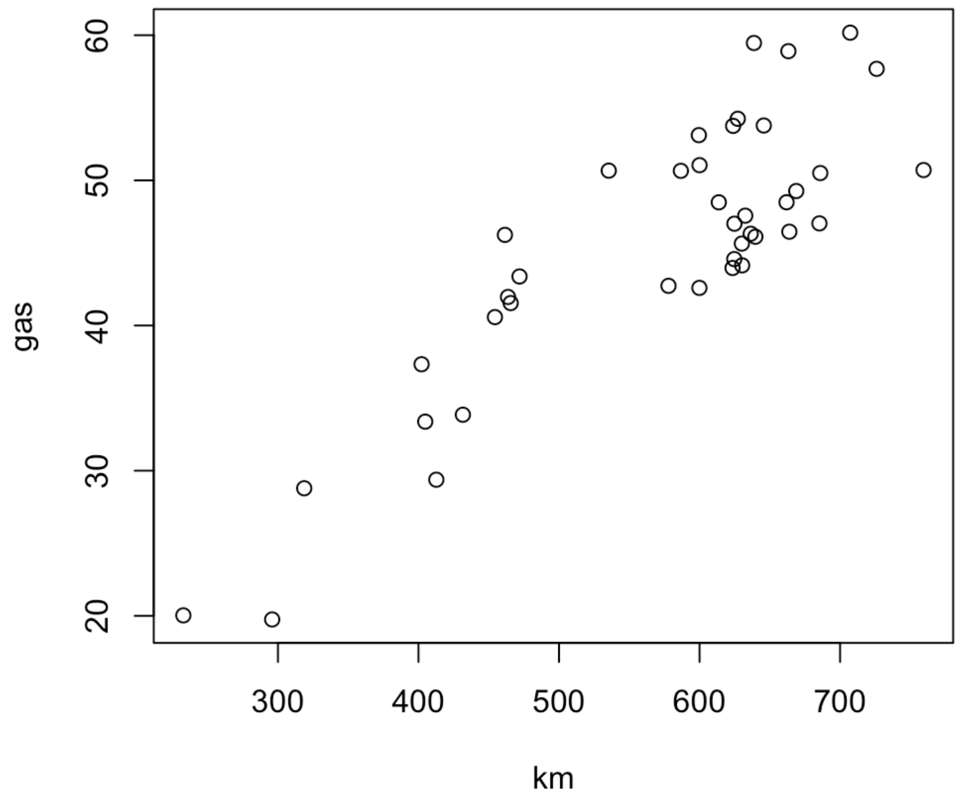
1.

a)



b) $Y_i = \alpha + \beta x_i + R_i$ where $R_i \sim G(0, \sigma)$ independently for i = 1, … , n. x_i represents the ith distance driven, and Y_i represents the ith fuel consumption.

c) The maximum likelihood estimates for α is 8.041825, and the maximum likelihood estimates for β is 0.06554341. The unbiased estimate of σ is 4.814252. The standard error of $\tilde{\beta}$ is 0.006206305. The standard error of $\tilde{\alpha}$ is 3.607627. 95% confidence interval for β is from 0.05297940 to 0.07810741. 95% confidence interval for α is from 0.7385669 to 15.3450835.

d) As the distance driven increases by one kilometer, the mean value of the fuel consumption increases by $\hat{\beta}$ = 0.06554341 liter. When the distance driven is 0, the mean value of the fuel consumption is $\hat{\alpha}$ = 8.041825 liters. The estimate of α is 8.041825 and the endpoints of its associated confidence interval are 0.7385669 and 15.3450835. These do not make sense to me. This is because when the distance driven is 0 kilometer, there should be almost no fuel consumption as a matter of common sense. However, these three numbers are larger than 0, which indicates that there is fuel consumption when the distance driven is 0 kilometer. This contradicts what we normally think. Thus, the estimate of α and the endpoints of its associated confidence interval do not make sense to me.

e)

$$H_0: \alpha = 0 \qquad H_A: \alpha \neq 0$$

In words, the null hypothesis is that when the distance driven is 0 kilometer, the fuel consumption is 0 liter, which means that there is no fuel consumption. The alternative hypothesis is that when the distance driven is 0 kilometer, the fuel consumption is not 0 liter, which means that there is fuel consumption. The formula for the test statistic is

$$\frac{|\tilde{\alpha} - \alpha_0|}{Se\sqrt{\frac{1}{n} + \frac{(\bar{x})^2}{Sxx}}}$$

, where $\alpha_0 = 0, \mathrm{Se} = 4.814252, \mathrm{n} = 40, \bar{x} = 568.1975, \mathrm{Sxx} = 601715.8$, $\tilde{\alpha}$ is a random variable so we use $\hat{\alpha}$ = 8.041825. Thus, the value of test statistic is 2.229118. Also, the formula for p-value is

$$2P\left(T \geq \frac{|\hat{\alpha} - \alpha_0|}{Se\sqrt{\frac{1}{n} + \frac{(\bar{x})^2}{Sxx}}}\right) \text{ where } T \sim t(n-2)$$

, where $\alpha_0 = 0, Se = 4.814252, \mathrm{n} = 40, \bar{x} = 568.1975, \mathrm{Sxx} = 601715.8$, $\hat{\alpha}$ = 8.041825. Thus, p-value is 0.03179553.

f) The $(1 - \alpha)100\%$ confidence interval for the mean gas consumption as a function of distance driven is

$$\left[\hat{\mu}(x) - a\, s_e\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}}\ ,\ \hat{\mu}(x) + a\, s_e\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}}\right]$$

, where x is the distance driven and a is obtained by P($T \le a$) = $\frac{1+p}{2}$ where T $\sim t(n-2)$ and p = 1 – α. The (1 − α)100% prediction interval for a new gas consumption amount as a function of distance driven is

$$\left[\hat{\mu}(x) - a\, s_e\sqrt{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}}\ ,\ \hat{\mu}(x) + a\, s_e\sqrt{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}}\right]$$

, where x is the distance driven and a is obtained by P($T \le a$) = $\frac{1+p}{2}$ where T $\sim t(n-2)$ and p = 1 – α.

g)