Artificial Neural Networks

Perceptron for AND gate

Question:

Perform the realization of AND function by using single layer perceptron algorithm. Where X_1 and X_2 are inputs and Y is the output, learning rate/threshold is 0.5. Output is 1, if weighted sum is greater than 0.5 otherwise 0 and weights W_1 and W_2 is equal to 0.9.

Given:

1. AND Gate

X_1	X_2	Y
0	0	0
0	1	0
1	0	0
1	1	1

2. Weights,
   $w_1 = 0.9$
   $w_2 = 0.9$
3. Learning rate (threshold),
   $\alpha = 0.5 \quad (0 \quad 1)$
4. Activation
   $$Output = \begin{cases} 0, if \sum \leq 0.5 \\ 1, if \sum > 0.5 \end{cases}$$
5. Bias,
   b = 0

Formulae:

1. $\sum ¿ b + \sum_{i=1}^{n} w_i x_i$
2. Error, ε = Actual – Predicted
3. Weights Calculations

$$w_i = w_{i-1} + \propto \times \varepsilon$$

Training:

Round 1:

We will apply 1st instance to the perceptron. x1 = 0 and x2 = 0.

Sum unit will be 0 as calculated below.

$\Sigma = b + x_1 * w_1 + x_2 * w_2$

$= 0 + 0 * 0.9 + 0 * 0.9$

$= 0$

Activation unit checks if the sum unit is greater than a threshold. If this rule is satisfied, then it is fired and the unit will return 1, otherwise it will return 0.

Activation threshold would be 0.5.

Sum unit was 0 for the 1st instance. So, activation unit would return 0 because it is less than 0.5. Similarly, its output should be 0 as well. We will not update weights because there is no error in this case.

Let's focus on the 2nd instance. x1 = 0 and x2 = 1.

Sum unit: $\Sigma = b + x1 * w1 + x2 * w2$

$= 0 + 0 * 0.9 + 1 * 0.9$

$= 0.9$

Activation unit will return 1 because sum unit is greater than 0.5. However, output of this instance should be 0. This instance is not predicted correctly. That's why, we will update weights based on the error.

Error, ε = actual – prediction = 0 – 1 = -1

We will add error times learning rate value to the weights. Learning rate would be 0.5. We mostly set learning rate value between 0 and 1.

w1 = w1 + α * ε = 0.9 + 0.5 * (-1) = 0.9 – 0.5 = 0.4

w2 = w2 + α * ε = 0.9 + 0.5 * (-1) = 0.9 – 0.5 = 0.4

Focus on the 3rd instance. x1 = 1 and x2 = 0.

Sum unit: $\Sigma = b + x1 * w1 + x2 * w2$

$= 0 + 1 * 0.4 + 0 * 0.4$

$= 0.4$

Activation unit will return 0 this time because output of the sum unit is 0.5 and it is less than 0.5. Output that we were expecting is also 0 so, we will not update weights.

Mention the 4th instance. x1 = 1 and x2 = 1.

Sum unit: Σ = x1 * w1 + x2 * w2 = 1 * 0.4 + 1 * 0.4 = 0.8

Activation unit will return 1 because output of the sum unit is 0.8 and it is greater than the threshold value 0.5. Its actual value should 1 as well. This means that 4th instance is predicted correctly. We will not update anything.

Round 2:

In previous round, we've used previous weight values for the 1st instance, and it was classified correctly. Let's apply feed forward for the new weight values.

Remember the 1st instance. x1 = 0 and x2 = 0.

Sum unit: Σ = b + x1 * w1 + x2 * w2

= 0 + 0 * 0.4 + 0 * 0.4

= 0.4

Activation unit will return 0 because sum unit is 0.4 and it is less than the threshold value 0.5. The output of the 1st instance should be 0 as well. This means that the instance is classified correctly. We will not update weights.

Feed forward for the 2nd instance. x1 = 0 and x2 = 1.

Sum unit: Σ = b + x1 * w1 + x2 * w2

= 0 + 0 * 0.4 + 1 * 0.4

= 0.4

Activation unit will return 0 because sum unit is less than the threshold 0.5. Its output should be 0 as well. This means that it is classified correctly, and we will not update weights.

We've applied feed forward calculation for 3rd and 4th instances already for the current weight values in the previous round. They were classified correctly.

So, the trained model is shown in Figure 1.

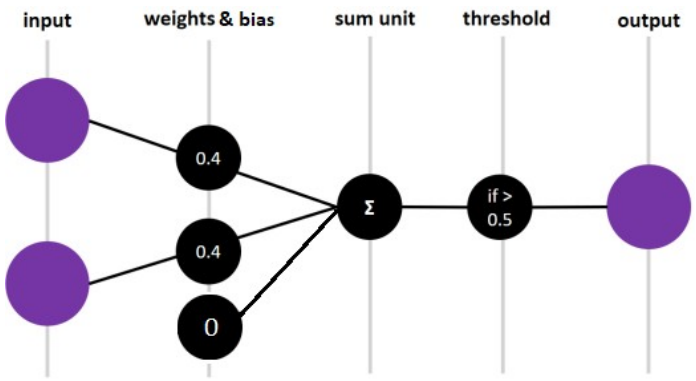

Figure 1: Trained perceptron on AND Gate.