Previous Lecture

* Applications of Gauss Law

i) Planar symmetry (Infinite sheet)
ii) Cylindrical symmetry (Infinitely long cylindrical rod)
iii) Spherical symmetry

i) Planar Symmetry

- → Non conducting sheet → $E = \frac{\sigma}{2\epsilon_0}$
- → Conducting sheet → $E = \frac{\sigma}{\epsilon_0}$

* We have also discussed, Electric field produced by two parallel conducting | non conducting plates at their central point.

II) Cylindrical Symmetry

Goal of study -: we have to find an expression for the electric field (E) at a radial distance (r) from the central axis of the rod.

Condition on charges and field

↳ Assuming that charge distribution and the field have cylindrical symmetry

Explanation:

For this purpose, we have a section of long cylindrical plastic rod with a uniform charge density (λ).

~~To find~~

E-field

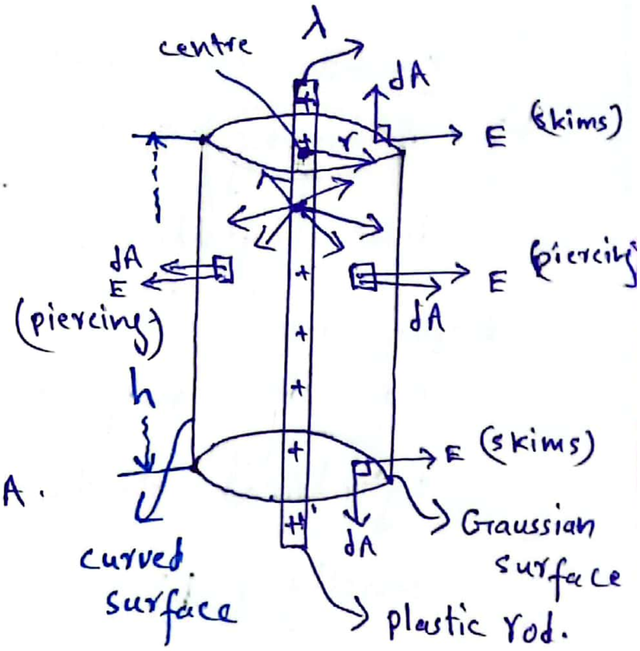
To find the field at radius 'r', we consider a cylindrical gaussian surface concentric with a rod. (see figure in Book)

Flux

Since the charges are +ive so for each patch element, the E-field must be radially outward.

we know that

$$\Phi = EA\cos\theta$$



For end caps (top | bottom).

At both top and bottom ends E-field skims the surface of the surface and makes an angle of 90° with area A. so, we have.

$$\Phi_{top} = EA\cos 90^\circ = 0$$

$$\Phi_{bottom} = EA\cos 90^\circ = 0$$

For curved surface

E-field and A are parallel, so

$$\Phi_{curved} = EA\cos 0^\circ = EA$$

where $A = 2\pi r \times h$

$$\Phi = E(2\pi r h)$$

using Gauss Law

$$\epsilon_0 \Phi = q_{enc} \rightarrow (a)$$

where $q_{enc} = \lambda h \rightarrow$ (i

$$\Phi = E(2\pi r h) \rightarrow \text{(ii}$$

using (i & (ii in eqn (a) we get.

$$\epsilon_0 \cdot E \cdot 2\pi r \cancel{h} = \lambda \cancel{h}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$ (magnitute of $\vec{E}$)

Direction of $\vec{E}$

The direction of $\vec{E}$ is radially outward for +ve charges and radially inward for -ve charges.

III) Spherical Symmetry

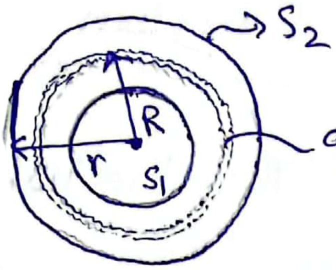
A shell of uniform charge attracts or replels a charged particle that is outside the shell as if all the shell's Charge were concentrated at the center of the shell.

To understand spherical symmetry, first we ~~x~~ check E-field inside/ out site the charged spherical shell. by drawing Grassian surfaces.

$S_1 \rightarrow$ surface S_1

$S_2 \rightarrow$ surface S_2

$r \rightarrow$ radius from center to outer surface S_2.



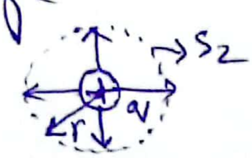
$R \rightarrow$ radius from center to total charge (Shell).

$q \rightarrow$ total charged enclosed by shell.

Gauss Law for S_2 $(r \geq R)$

For the assumption, $r \geq R$ the amount of total charge is enclosed so we can write electric field value equal to Electric field due to a point charge.

e.g.:



$$\boxed{E = \frac{kq}{r^2}}$$

Gauss Law for S_1 $(r<R)$

under, $r<R$ assumption, we can see, no charge enclosed by S_1, so

$$\boxed{E=0}$$

E-field due to any arbitrary sphere enclosing charge

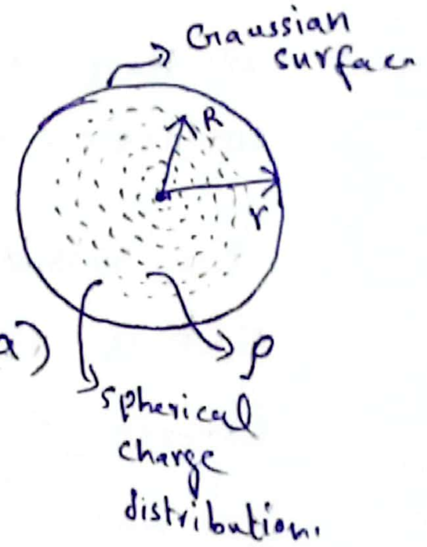
→ spherical Gaussian surface

→ Total charge enclosed $= q$

radius from center to charge distribution $= R$

radius from center to any point of Gaussian surface $= r$

ρ = ~~surface~~ volume charge density.

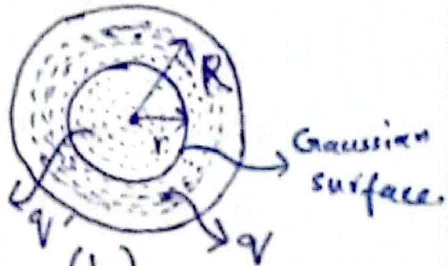


(a)

Assumption

→ ρ can vary with central radius but not vary from surface to surface.

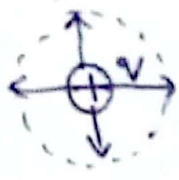
Gaussian surface (a) $(r>R)$



(b)

For this surface, entire charge lies inside the surface for, $r>R$.

we have $E = \dfrac{qk}{r^2}$ (behaves as a point charge)



Gaussian surface (b) ($r \leq R$)

For this Gaussian surface, total charge is divided into two regions, inside and outside the surface (b). So,

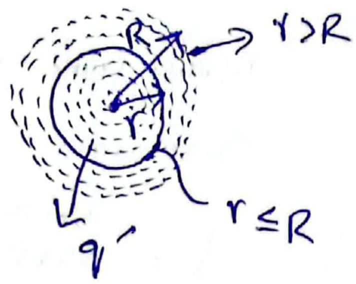
q = total charge enclosed by surface $r > R$.

q' = charge enclosed by surface $r \leq R$.

we have to find total E-field due to both surfaces.

For outside the surface

that is $r > R$, there is no Grassian surface so outside charge does not set up field.



For inside the surface, $r \leq R$ the charge enclosed is q' and the electric field due to this charge will be

$$E = \frac{k q'}{r^2} \longrightarrow (1)$$

If total charge is uniform, then q' enclosed is also uniform so we can write

$$\frac{\text{total charg}}{\text{total volume}} = \frac{\text{charge enclosed}}{\text{volum enclosed}}$$

$$\frac{q}{V} = \frac{q'}{V'}$$

$\because$ $V = \frac{4}{3}\pi R^3$

$V' = \frac{4}{3}\pi r^3$

and

$$\frac{q}{\cancel{\frac{4}{3}}\cancel{\pi} R^3} = \frac{q'}{\cancel{\frac{4}{3}}\cancel{\pi} r^3}$$

$$q r^3 = R^3 q'$$

$$q' = q\left(\frac{r}{R}\right)^3$$ putting in 1

$$E = \frac{k q'}{r^2} = \frac{k\, q \frac{r^3}{R^3}}{r^2}$$

$$\boxed{E = \frac{k q}{R^3} \cdot r}$$

$=$