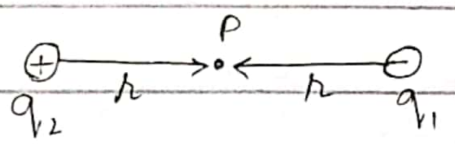
13th March, 2023

Lecture # 1

APPLIED PHYSICS :-

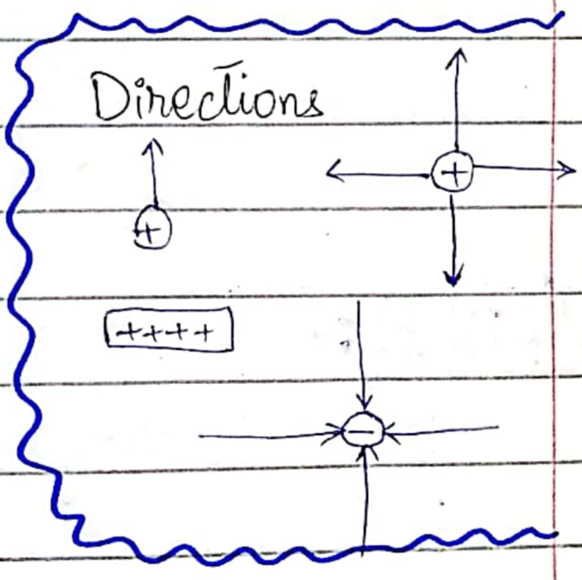


$$\boxed{\bar{F} = \frac{K q_1 q_2}{r^2}}$$ ∵ Columb's law

$$\boxed{\bar{E} = \frac{F}{q}}$$ ∵ Electric field.

$$\boxed{F = \frac{K q q_0}{r^2}}$$

$$\boxed{E = \frac{K q_0}{r^2}}$$



Q) If two point charges are given but both are positive charges. so what is electric feild?

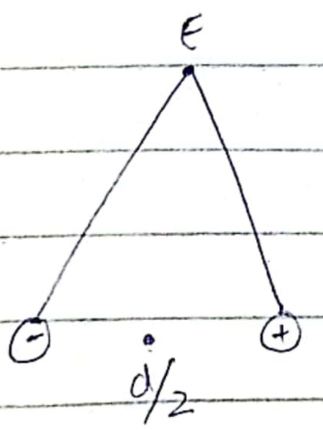
A) The electric feild is double.

Q) In this case 2 different charges are at distance. one is positive and the one is negative. So what is electric feild?

A) It makes a dipole.

DIPOLE :-

Different charges with same magnitude



$$E_+ = \frac{K q_0}{r^2} \qquad E_- = \frac{K(-q_0)}{r^2}$$

$E = E_+ - E_-$

$\oplus\!\!- \Longrightarrow$ dipole

$$\boxed{E = \frac{Kq}{r^3}}$$

In this case of dipole the electric feild is less.

charge – Q – coulomb
Field – E

∵ Force and electric field directions are same

#2

Properties of Electric feild lines:

i → Originate from +ve charge terminate on –ve charge.

ii → closer the EFL, greater is the E-Field.

iii → At any point E-feild is directed tangent in the E-feild.

→ E-field never cross each other due to their undirected properties.

Electric field due to point charge:

Find a charge object experiences a force defined by Coulomb's law is

$$F = \frac{k q_1 q_2}{r^2}$$

$$E = \frac{F}{q_0} \Rightarrow \frac{k \cancel{q_0} q}{r^2 \cancel{q_0}}$$

$$= \frac{kq}{r^2}$$

$$E = \frac{k|q|}{r^2} \hat{r}$$ → outward direction

$$E = \frac{kq}{r^2}$$

∵ Uni positive charge is test charge.

Q) If $q_1 = 2c$ & $q_2 = 3c$
then Total electric field = ?

A) Sum of electric feild ie:-

$$E_1 = \frac{K2q}{r^2}$$

$$E_2 = \frac{K3q}{r^2}$$

Total electric feild:-

$$= E_1 + E_2$$

$$= \frac{Kq(5)}{r^2}$$

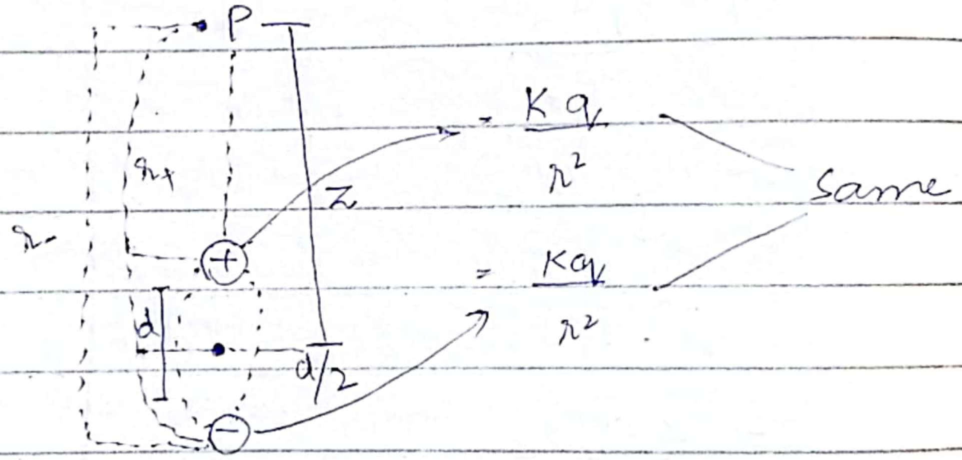
Q) Electric field lines never cross each other? why?

A) Electric field lines never cross each other due to their unidirectional properties.

#3

Electric field due to dipole:

Two equal and opposite charges at distance r is always consider as dipole.



∵ mid point of dipole is D/2

$$E_+ = \frac{Kq}{r_+^2}$$

$$E_- = \frac{Kq}{r_-^2}$$

$$E = E_+ - E_-$$

$$= \frac{Kq}{r_+^2} - \frac{Kq}{r_-^2}$$

$$= Kq\left[\frac{1}{r_+^2} - \frac{1}{r_-^2}\right]$$

$$E = Kq\left[\frac{1}{(z-d/2)^2} - \frac{1}{(z+d/2)^2}\right]$$

Simplify:-

$$E = Kq\left[\frac{1}{(z-d/2)^2} - \frac{1}{(z+d/2)^2}\right]$$

$$\because A = \frac{d}{2}$$

$$E = Kq\left[\frac{1}{(z-A)^2} - \frac{1}{(z+A)^2}\right]$$

$$E = Kq\left[\frac{(z+A)^2 - (z-A)^2}{(z-A)^2(z+A)^2}\right]$$

$$= Kq\left[\frac{4zA}{(z^2-A^2)^2}\right] \Rightarrow Kq\left(\frac{4\frac{d}{2}z}{\left(z^2-\left(\frac{d}{2}\right)^2\right)^2}\right)$$

$$= Kq\left(\frac{2dz}{\left(z^2-\frac{d^2}{4}\right)^2}\right)$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{q}{r^3} \qquad \because K = \frac{1}{2\pi\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon(r^3)}$$

Cases:- Why in dipole case electric field decays:-

→ ELF is decays by 1/3 by 1/r q factor in far field as compare to point charge case.

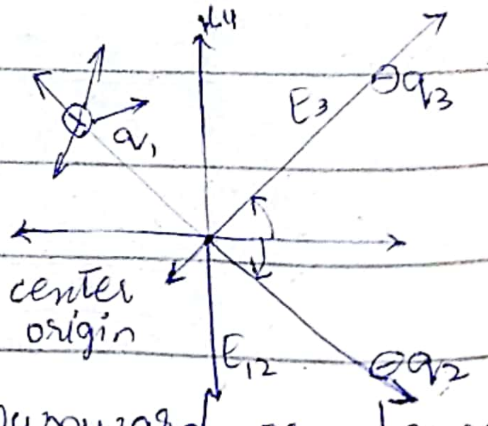
→ In electric field line decays by distance factor $1/r^3$ in far field as compare to point charge case.

Example:-

$q_1 = +2Q$

$q_2 = -2Q$

$q_3 = -4Q$



move downward or toward.

∵ magnitude and direction is also same

$$E_1 = \frac{K\,+2q}{r^2}$$

$$E_2 = \frac{K\,-2q}{r^2}$$

$$E_3 = \frac{K\,-4q}{r^2}$$

$\boxed{\begin{array}{l}\text{Q) In case of dipole, why it decreases?} \\ \text{A) Electric field decays.}\end{array}}$

$$E_1 = \frac{K(+2q)}{r^2} = \frac{K(2Q)}{r^2}\hat{r}$$

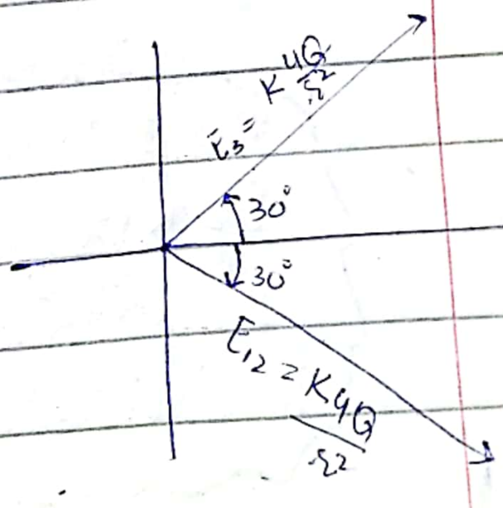
$$E_2 = \frac{K-2q}{r^2} = \frac{K(2Q)}{r^2}\hat{r}$$

$$E_3 = \frac{K-4q}{r^2} = \frac{K(4Q)}{r^2}\hat{r}$$

$$E_{12} = E_1 + E_2$$

$$= \frac{K(2q)}{r^2} + \frac{(2q)K}{r^2}$$

$$E_{12} = \frac{K(4Q)}{r^2}$$



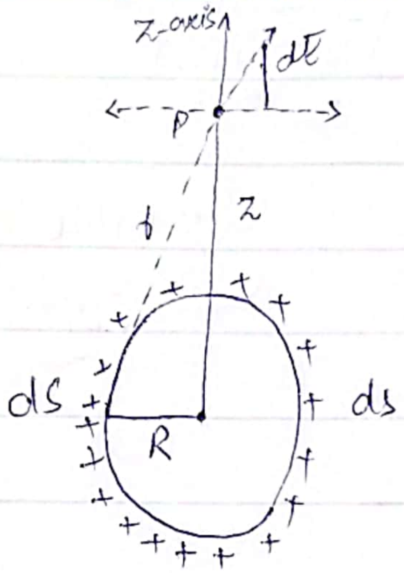
$$E = 2E_{3x}$$

$$= 2E_3 \cos\theta$$

$$= 2\left(\frac{K4Q}{r^2}\right)\cos 30^\circ$$

$$E = 0$$

Electric feild due to Charged ring:



→ A plastic charged ring divided into small pieces each of charges dq with circumference ds

→ λ = Line charge density

$$\lambda = \frac{dq}{ds}$$

$$dq = \lambda ds \longrightarrow (1)$$

→ Using pythagorus theorem:

$$(hyp)^2 = (base)^2 + (Prep)^2$$

$$r^2 = z^2 + R^2$$ ∵ value of extended object.

→ We know that,

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{kdq}{(z^2+R^2)}$$ ∵ $r^2 = z^2 + R^2$

$$dE = \frac{k\lambda ds}{(z^2+R^2)}$$ ∵ $dq = \lambda ds$

→ Resolving dE into horizantal & vertical components;

$$dE \cos\theta = \frac{kdq}{z^2+R^2} \cdot \cos\theta$$ ∵ $\cos\theta = \frac{z}{r}$

→ electric field of a ring

$$dE\cos\theta = \frac{k\,dq}{z^2+R^2}\cdot\cos\theta$$

→ integrate on b/s:

$$\int dE\cos\theta = \int \frac{k\lambda\,ds}{z^2+R^2}\cdot\cos\theta$$

$$\because \cos\theta = \frac{z}{r}$$

$$E = \int \frac{k\lambda\,ds}{z^2+R^2}\cdot\frac{z}{r}$$

$$E = \frac{k\lambda z}{(z^2+R^2)\cdot r}\int_0^{2\pi r} ds$$

$$\because s = 2\pi r$$

$$\because q = \lambda(2\pi r)$$

$$\because r = (z^2+R^2)^{1/2}$$

$$E = \frac{k\lambda z\,(2\pi r)}{(z^2+R^2)(z^2+R^2)^{1/2}}$$

$$E = \frac{kqz}{(z^2+R^2)^{3/2}}$$

→ Far feild

for $z \gg R$

$$E = \frac{k\lambda z\, 2\pi r}{(z^2+R^2)^{3/2}}$$

$$E = \frac{k\lambda z 2\pi r}{(z^2)^{3/2}}$$

$$E = \frac{k\lambda\, 2\pi r}{z^2}$$

$$E = \frac{k\lambda s}{z^2}$$

$\because s = 2\pi r$

$$\boxed{E = \frac{kq}{z^2}}$$

$\because \lambda s = q$

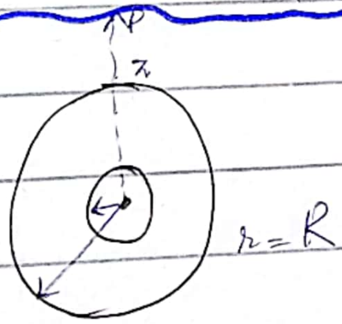
Electric field due to charged disk

$\because$ due to charge ring

$$E = \frac{Kqz}{(R^2+z^2)^{3/2}} \quad ; \quad dE = \frac{Kz\,dq}{(z^2+r^2)^{3/2}}$$

$$q = \lambda(2\pi r) \quad ; \quad \lambda = \frac{dq}{ds}$$

divide a ring into small ring and one ring is $dE = \frac{Kz\,dq}{(r^2+z^2)^{3/2}}$



$\sigma = \frac{dq}{dA}$ $\quad\therefore\ dA = (2\pi r)\cdot dr$

$\therefore\ dq = \sigma\, dA$

$$dE = \frac{Kz\,dq}{(r^2+z^2)^{3/2}}$$

$$= \frac{Kz\,\sigma\, dA}{(r^2+z^2)^{3/2}}$$

$\therefore\ dA = (2\pi r\cdot dr)$

$$dE = \frac{Kz\sigma\cdot 2\pi r\cdot dr}{(z^2+r^2)^{3/2}}$$

$$\int_0^R dE = \int_0^R \frac{Kz\sigma\cdot 2\pi r\cdot dr}{(z^2+r^2)^{3/2}}$$

$$E = Kz\sigma \cdot \pi \int_0^R (r^2+z^2)^{-3/2} \cdot 2r\,dr$$

By substituting:-

$$2r\,dr = dx$$

$$r^2+z^2 = x$$

$$\frac{-3}{2} = m$$

$$= \frac{z\sigma \cdot \cancel{\pi}}{4\cancel{\pi}\varepsilon_0} \int_0^R x^m\,dx \qquad \because K = \frac{1}{4\pi\varepsilon_0}$$

$$= \frac{z\sigma}{4\varepsilon_0} \cdot \frac{x^{m+1}}{m+1}\Big|_0^R$$

$$= \frac{z\sigma}{4\varepsilon_0} \left[\frac{(r^2+z^2)^{-\frac{3}{2}+1}}{-3/2+1} \Big|_0^R \right]$$

$$= \frac{z\sigma}{4\varepsilon_0} \left[\frac{(z^2+R^2)^{-1/2} - (z^2)^{-1/2}}{-1/2} \right]$$

$$= -\frac{\cancel{2}\,z\sigma}{\cancel{4}_{2}\varepsilon_0} \left[(z^2+R^2)^{-1/2} - (z^2)^{-1/2} \right]$$

$$= \frac{-z\sigma}{2\varepsilon_0}\left[(z^2+R^2)^{-1/2} - (z^2)^{-1/2}\right]$$

$$= \frac{-z\sigma}{2\varepsilon_0}\left[\frac{1}{(z^2+R^2)^{1/2}} - \frac{1}{(z^2)^{1/2}}\right]$$

$$= \frac{-z\sigma}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{-\sigma z}{2\varepsilon_0 z}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

→ Scalling factors that tells the results of far and near.

Q) from far disk infinite sheet results.

Special Cases:-

→ radius is Large in size.

→ If $R \to \infty$, z = infinite → near feild.

$$\therefore \frac{1}{\infty} = 0$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\infty}\right)$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\infty}\right)$$

$$E = \frac{\sigma}{2\varepsilon_0}(1)$$

$$= \frac{\sigma}{2\varepsilon_0}$$ → electric feild of infinite charge sheet

→ if $z \to 0$, $R =$ infinite
size of disk is same but in center axis.

$$E = \frac{\sigma}{2\varepsilon_0}$$

The result of infinite charge sheet is same either the distance is far or near.

:5

Coulomb's Law Limitations:-

→ Coulomb's law is applicable only for the point charges which are at rest.

→ This law can only be applied in the cases where the inverse square law is obeyed.

→ It is difficult to implement this law where the charges are in arbitrary shape because in those cases we cannot determine the distance between the charges.

→ The law cannot be used directly to calculate the charge on the big planets.

→ The separation between the charges must be greater than the nuclear size (10^{-15}m), bcz for distances $< 10^{-15}$m. The strong nuclear force dominates over the electrostatic force.

→ The formula is easy to use while dealing with charges of regular and smooth shape, and it becomes too complex to deal with charges having irregular shapes.

#6 GAUSS'S LAW:-

case I → only spherical surface

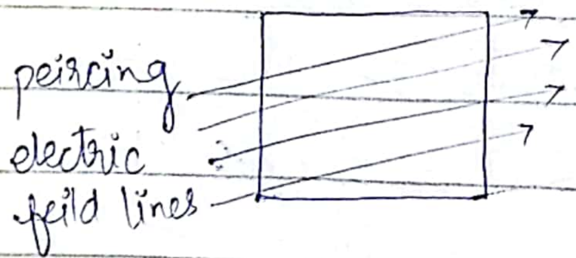
① → any Type of surface, surface should be obthirary.

② → surface should be closed and both +ve and -ve charges are must in it.

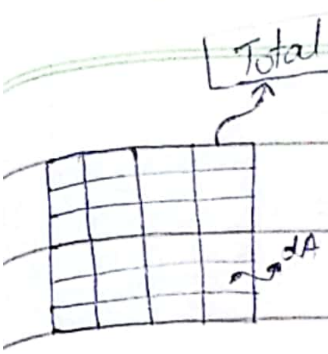
③ → enter from one place and leave from any one hole.

→ amount of electric feild / lines passing through a unit area, is called electric flux (ϕ)

$$\phi = \vec{E} \cdot \vec{A} \Rightarrow E\cos\theta A$$



in this same lines are enter and same are out

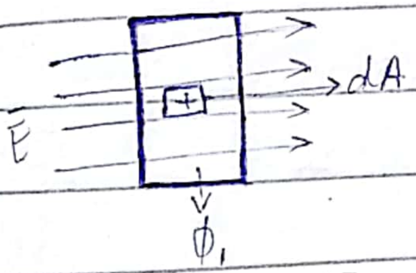


flat surface placed in electric feild lines.

(dA) tells the direction of area.

Cases:-

Case I:



$\phi_1 = EdA\cos\theta$

$\phi_1 = EdA\cos(0)$

$\phi_1 = EdA \Rightarrow$ flux is maximum and positive in outward direction

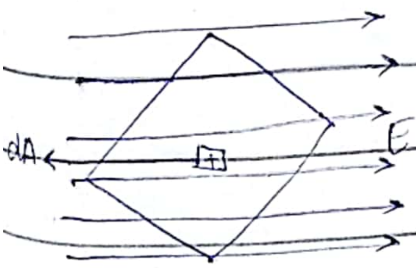
$(\phi > 0)$

$\because E \parallel dA$

$\because \theta = 0°$

$\because$ when both vectors are perpendicular then the dot product is used.

Case II:



$\phi_2 = EdA\cos\theta$

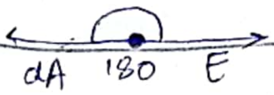
$= EdA\cos(180)$

$\phi_2 = -EdA$

$\because E \parallel dA$ but in opposite direction.

$\because \theta = 180°$

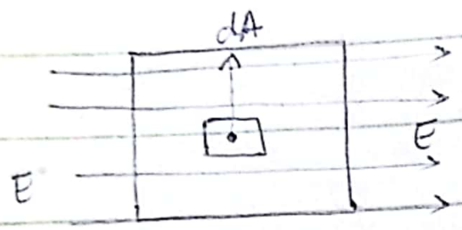
$\because \cos(180°) = -1$



flux is maximum and negative because in inward direction

$(\phi < 0)$

Case III



$\phi_3 = EdA\cos\theta$

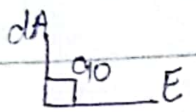
$\phi_3 = EdA\cos(90)$

$\phi_3 = 0$

flux is zero

$\because$ E is perpendicular to dA

$\because \cos 90^\circ = 0$



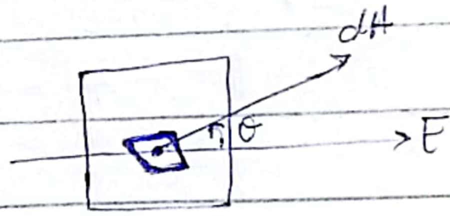
$\because$ imp Q) when flux is zero?

A) when electric feild and unit area make 90° angle then flux is zero in this case.

Case IV general case

$\phi_4 = EdA\cos\theta$

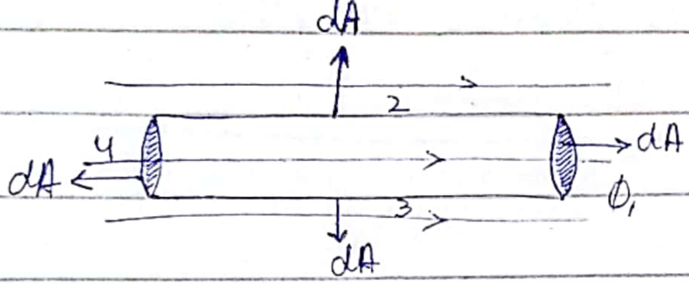
$\int \phi_4 = \int E\, dA\cos\theta$



$\phi = EA$

$d\phi = EdA$

7 Gausian Surface (cylindrical surface)



→ closed cylinder in free space
→ in electric feild lines
→ area will never be negative.

$\phi_1 = EdA\cos\theta$
$\because \theta = 0$ →dA →E
$\phi_1 = EdA$

$\phi_2 = EdA\cos\theta$
$\theta = 90^\circ$ dA E
$\phi_2 = 0$

$\phi_3 = EdA\cos\theta$
$\because \theta = 90^\circ$ E dA
$\phi_3 = 0$

$\phi_4 = EdA\cos\theta$
$\theta = 180^\circ = -1$ dA E
$\phi_4 = -EdA$

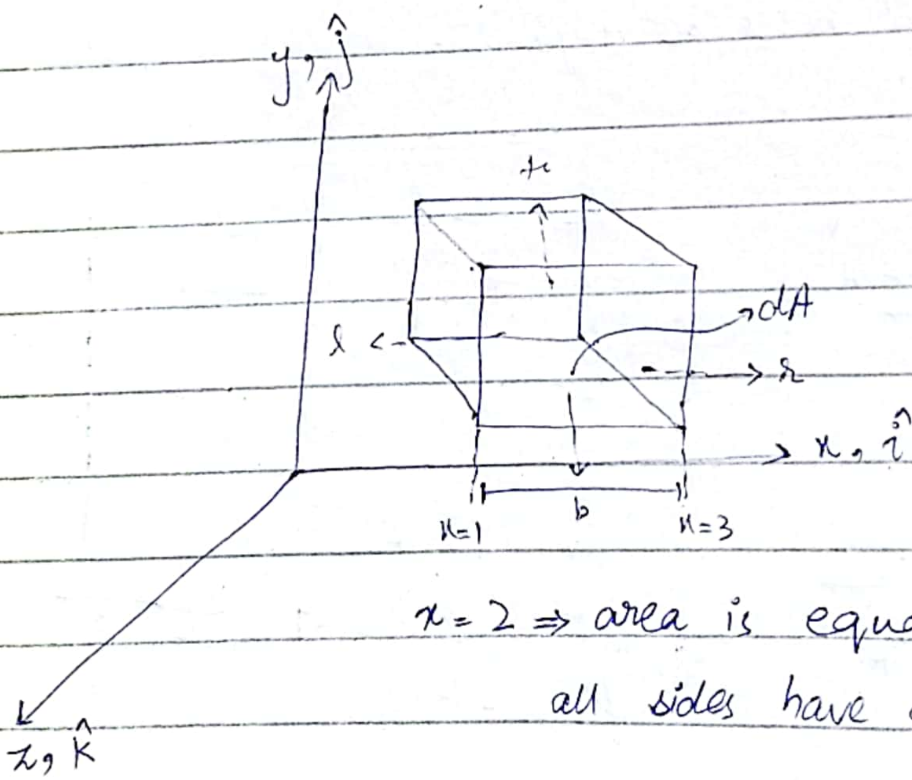
$$\phi_{total} = \phi_1 + \phi_2 + \phi_3 + \phi_4$$
$$= \cancel{EdA} + 0 + 0 - \cancel{EdA}$$

$$\boxed{\phi_{total} = 0}$$

Q) If one side of cylinder is closed and one is open
So, the area is find or not find?
A) The area is also find.

EXAMPLE:

→ Square gaussian Surface.
→ each side is same.



$x = 2 \Rightarrow$ area is equal to 2 so all sides have area 2.

$E = 3x\hat{i} + 4\hat{j}$

$\phi = \oint \bar{E} \cdot \overline{dA}$

$\because \overline{dA} = dA\hat{i} + dA\hat{j} + dA\hat{k}$

right:

$$\phi_r = \oint (3x\hat{i} + 4\hat{j}) \cdot dA\hat{i}$$

$$= 3x\oint (\hat{i}\cdot\hat{i})\, dA + 4\oint (\hat{i}\cdot\hat{j})\, dA$$

$$= 3x\int dA$$

$\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
because these are parallel vectors and the angle is 0 & $\cos(0) = (1)$
$\because \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

$$\phi_r = 3x(A)$$

$$\phi_r = 3x(4)$$

$$\phi_r = 3(3)(4)$$

$$\boxed{\phi_r = 36\, Nm^2/c}$$

left:

$$\phi_l = \oint (1x\hat{i} + 4\hat{j}) \cdot dA\hat{i}$$

$$\phi_l = -1x\oint (\hat{i}\cdot\hat{i})\, dA + 4\oint (\hat{i}\cdot\hat{j})\, dA$$

$$\phi_l = -1x\int dA$$

$$= -1xA$$

$$= -1(3)(4)$$

$$\boxed{\phi_l = -12\, Nm^2/c}$$

top: $\phi_t = \oint \vec{E}\cdot d\vec{A}$

$= \oint (3x\hat{i} + 4\hat{j})\cdot dA\hat{j}$

$= 3x\int(\hat{i}\cdot\hat{j})dA + 4\int(\hat{j}\cdot\hat{j})\,dA$

$= 4\int dA$

$= 4(A)$

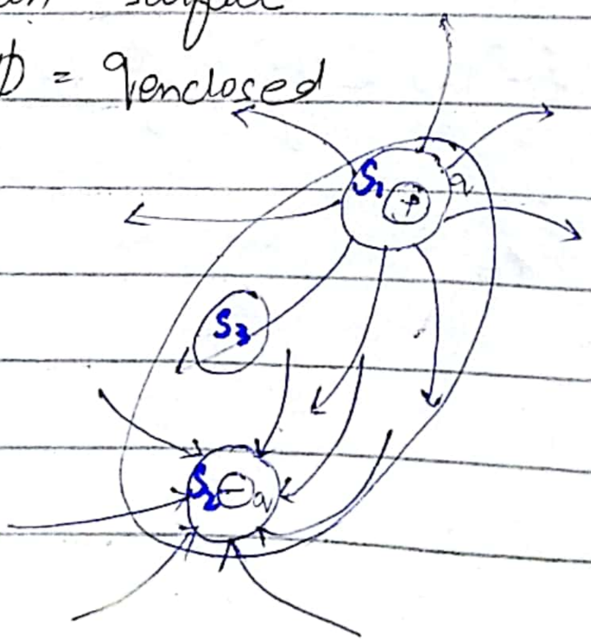
$= 4(4)$

$\boxed{\phi_t = 16\ Nm^2/C}$

Gauss's law:-

Total flux is enclosed in a total charge gaussian surface

$\varepsilon_0\phi = q_{enclosed}$



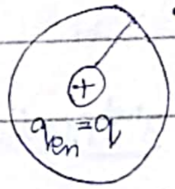
Q) How can we get E-feild of charged particle using gauss law?

A) Gauss's Law & Coulomb's Law:-

We Know that:

$$\varepsilon_0 \phi = q_{enclosed}$$

$$\phi = \oint \vec{E} \cdot \vec{dA}$$



∵ electric feild of ever path will be same-

$$\phi = E \oint dA$$

$$\phi = E \cdot A$$

$$\phi = E(4\pi r^2) \qquad \because A = 4\pi r^2$$

$$\boxed{\phi = E(4\pi r^2)}$$

$$\varepsilon_0 \left(E \cdot (4\pi r^2)\right) = q$$

$$E = \frac{q}{\varepsilon_0 4\pi r^2}$$

$$E = \frac{q}{\varepsilon_0 4\pi r^2}$$

$$E = \frac{q}{r^2} \cdot k \qquad \because k = \frac{1}{4\pi\varepsilon_0}$$

i- Electric feild over the surface should be same.
ii- surface should be spherical.

Last lecture: 3rd April, 2023
#8

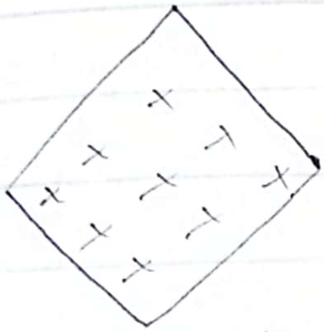
APPLICATIONS of GAUSS'S LAW:

→ due to planar symmetry
→ due to cylindrical symmetry
→ due to spherical symmetry

Planar Symmetry:-

→ Electric feild due to conducting/non-conducting
→ feild is uniform
→ sheets using Gauss Law:

$$\varepsilon_0 \phi = q_{enc}$$

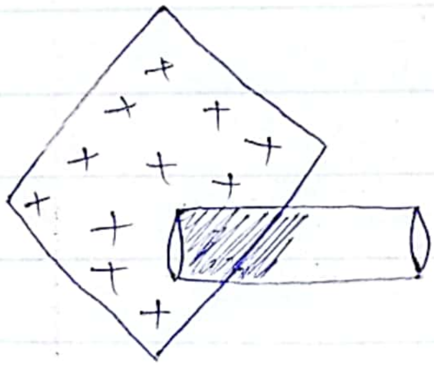


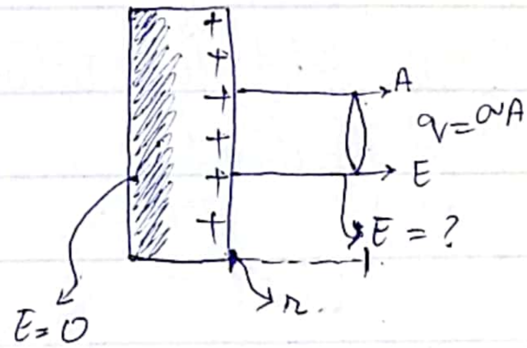
a peice of plane sheet

One side of sheet { electric feild = 0

Case I :-

→ Surface should be conducting
→ electric feild due to conducting





→ electric feild lines which are passes by it is flux.

→ gaussion surface with spherical surface.

Q) → at finite distance using infinite sheet find electric feild / charge using gauss law?

$$\varepsilon_0 \cdot \oint \bar{E} \cdot d\bar{A} = \sigma A$$

$$\varepsilon_0 E A = \sigma A.$$

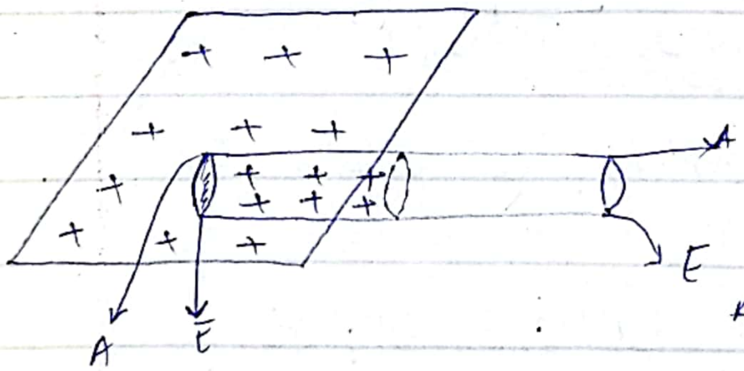
$$\varepsilon_0 E = \sigma$$

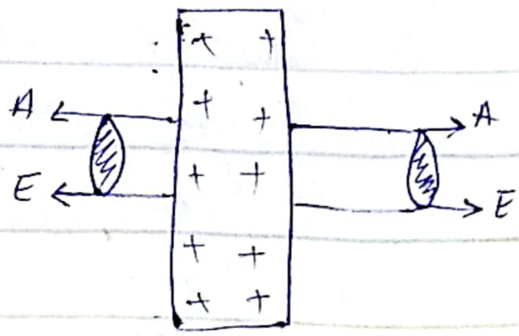
$$\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{r}}$$

$\hat{r}$ = direction outward due to +ve charge

Case II:-

→ electric feild due to non conducting sheet using gauss Law.





$$\varepsilon_0 \phi = q_{enc}$$

$$\varepsilon_0 (EA + EA) = \sigma A$$

$$\varepsilon_0 [(E+E)A] = \sigma A$$

$$\varepsilon_0 [2E] \cancel{A} = \sigma \cancel{A}$$

$$\varepsilon_0 2E = \sigma$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$