MALIK SULTAN Ameer (20-ARID-496)

LINEAR Algebra.

Topics :

(1) VECTOR SPACE
(2) SUB SPACE
(3) LINER DEPENDENT & INDEPENDENT
(4) LINER TRASfoomattion.
(5) SPANING
(6) BASE and DIAMENSIONS
(7) Application of linear System.

20-Arid-496
Malik SULTAN Ameer

Topic:- No: 01:-

Vector Space:-

Define:- Let F be a field and V be a non-empty set. Then V is said to be vector space over F if following axioms hold:

(A)

(i) Closure property: if u and v are elements of V then u+v is in V.

(ii) Commutative property:

$u+v = v+u$.

(iii) Associative property:

$u+(v+w) = (u+v)+w$.

(iv) Identity ~~m~~element:- There is an element 0 in V such that.

$0+u = u$.

(V) Inverse of matrix :-

for each u in V there is $-u$ in V
Such that $u+(-u)=0$.

(B) if u & v are element of V then $c \cdot u$ is in V.

(i) $c(u+v) = c \cdot u + c \cdot v$

(ii) $(c+d)u = cu + du$.

(iii) $c(d \cdot u) = (c \cdot d)u$.

(iv) $1 \cdot u = u$.

$\longleftrightarrow$

Example NO : 01

The Set $R^3 = \{(x, y, z)\}$ is a vector space.

Sol:- $u = \{x_1, y_1, z_1\}$ $v = \{x_2, y_2, z_2\}$

(A)

(i) $u + v = \{(x_1, y_1, z_1) + (x_2, y_2, z_2)\}$

$= \{(x_1 + x_2, y_1 + y_2, z_1 + z_2)\}$.

$= (x_1 + x_2, y_1 + y_2, z_1 + z_2) \in R^3$.

(ii). $u + v = v + u$.

So

$\{(x_1, y_1, z_1) + (x_2, y_2, z_2)\}$.

$= (x_1 + x_2, y_1 + y_2, z_1 + z_2)$.

$= (x_2 + x_1, y_2 + y_1, z_2 + z_1)$.

$= (x_2, y_2, z_2) + (x_1, y_1, z_1)$.

$= v + u$.

(iii) $u+(v+w) = (u+v)+w$.

$u+(v+w) = (x_1, y_1, z_1) + [(x_2, y_2, z_2) + (x_3,$

$= (x_1, y_1, z_1) + (x_2+x_3, y_2+y_3, z_2+$

$= (x_1+x_2+x_3, y_1+y_2+y_3, z_1+z_2+z_3)$

$= (x_1+x_2, y_1+y_2, z_1+z_2) + (x_3, y_3, z_3$

$= [(x_1, y_1, z_1) + (x_2, y_2, z_2)] + (x_3, y_3, z_3$

$= (u+v)+w$.

(iv) $0+u = u$.

$(0,0,0) + (x_1, y_1, z_1)$

$= (x_1, y_1, z_1)$

$= u$.

$z_3)]$.

(v) :- $u + (-u) = 0$.

$= (x_1, y_1, z_1) + (-x_1, -y_1, -z_1)$

$= (\cancel{x_1} - \cancel{x_1}, \cancel{y_1} - \cancel{y_1}, \cancel{z_1} - \cancel{z_1})$

$= (0, 0, 0)$

$= 0$.

(B) :-

(i). $c(u+v) = c \cdot u + c \cdot v$.

$c(u+v)$

$= c[(x_1, y_1, z_1) + (x_2, y_2, z_2)]$.

$= c[(x_1+x_2, y_1+y_2, z_1+z_2)]$

$= c(x_1+x_2), c(y_1+y_2), c(z_1+z_2)$

$= cx_1 + cx_2, cy_1 + cy_2, cz_1 + cz_2$

$= (cx_1, cy_1, cz_1) + (cx_2, cy_2, cz_2)$

$= c(x_1, y_1, z_1) + c(x_2, y_2, z_2)$

$= c.u + c.v.$

(ii) :- $(c+d)u = cu + du.$

$(c+d)u = (c+d)(x_1, y_1, z_1).$

$= ((c+d)x_1, (c+d)y_1, (c+d)z_1)$

$= cx_1 + dx_1, cy_1 + dy_1, cz_1 + dz_1$

$= (cx_1, cy_1, cz_1) + (dx_1, dy_1, dz_1).$

$= c(x_1, y_1, z_1) + d(x_1, y_1, z_1)$

$= c.u + d.u.$

(iii) $c(d \cdot u) = (c \cdot d)u$.

$c(d \cdot u) = c(d \cdot (x_1, y_1, z_1))$

$= c(dx_1, dy_1, dz_1)$

$= cdx_1, cdy_1, cdz_1$

$= (c \cdot d)(x_1, y_1, z_1)$

$= (c \cdot d)(u)$.

(iv) $1 \cdot u = u$.

$= (1)(x_1, y_1, z_1)$

$= 1 \cdot x_1, 1 \cdot y_1, 1 \cdot z_1$

$= (x_1, y_1, z_1)$

$= u$.

As all the conditions of vector space are satisfied. Given set R^3 is a vector space.

Example No: 02:-

If a set of all vectors in set of R^2 of the form $\begin{bmatrix} x \\ x \end{bmatrix}$ with the usual defination of vector addition and scalar multiplication a vectorspace?

Sol:-

$u = \begin{bmatrix} x \\ x \end{bmatrix}$ $\quad v = \begin{bmatrix} y \\ y \end{bmatrix}$

(i) $u+v = \begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} y \\ y \end{bmatrix}$

$= \begin{bmatrix} x+y \\ x+y \end{bmatrix} \in V.$

(ii) $u+v = v+u$.

$$u+v = \begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} y \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x+y \\ x+y \end{bmatrix}$$

$$= \begin{bmatrix} y+x \\ y+x \end{bmatrix}$$

$$= \begin{bmatrix} y \\ y \end{bmatrix} + \begin{bmatrix} x \\ x \end{bmatrix}$$

$$= v+u$$

(iii) $u+(v+w) = (u+v)+w$

$$u+(v+w) = \begin{bmatrix} x \\ x \end{bmatrix} + \left(\begin{bmatrix} y \\ y \end{bmatrix} + \begin{bmatrix} z \\ z \end{bmatrix} \right)$$

$$= \begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} y+z \\ y+z \end{bmatrix}$$

$$= \begin{bmatrix} x+y+z \\ x+y+z \end{bmatrix}$$

$$= \begin{bmatrix} x+y \\ x+y \end{bmatrix} + \begin{bmatrix} z \\ z \end{bmatrix}$$

$$= \left(\begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} y \\ y \end{bmatrix} \right) + \begin{bmatrix} z \\ z \end{bmatrix}$$

$$= (u+v) + w$$

iv) :- $0 + u = u$.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x \\ x \end{bmatrix}$$

$$\begin{bmatrix} 0+x \\ 0+x \end{bmatrix}$$

$$\begin{bmatrix} x \\ x \end{bmatrix} = u.$$

(v) :- $u + (-u) = 0$.

$$\begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} -x \\ -x \end{bmatrix}$$

$$\begin{bmatrix} x - x \\ x - x \end{bmatrix}$$

$$= 0.$$

<u>B</u> :-

(i) $C(u+v) = C \cdot u + C \cdot v$

$$C(u+v) = C\left(\begin{bmatrix} x \\ x \end{bmatrix} + \begin{bmatrix} y \\ y \end{bmatrix}\right).$$

$$= C\begin{pmatrix} x+y \\ x+y \end{pmatrix}.$$

$$= Cx + Cy$$

$$= Cx + Cy.$$

$$= \begin{bmatrix} Cx \\ Cx \end{bmatrix} + \begin{bmatrix} Cy \\ Cy \end{bmatrix}$$

$$= Cu + Cv.$$

ii) $(c+d)u = cu + du$.

$= (c+d)\begin{bmatrix} x \\ x \end{bmatrix}$

$= \begin{bmatrix} (c+d)x \\ (c+d)x \end{bmatrix}$

$= \begin{bmatrix} cx + dx \\ cx + dx \end{bmatrix}$

$= \begin{bmatrix} cx \\ cx \end{bmatrix} + \begin{bmatrix} dx \\ dx \end{bmatrix}$.

$= c\begin{bmatrix} x \\ x \end{bmatrix} + d\begin{bmatrix} x \\ x \end{bmatrix}$.

$= cu + dv$.

(iii) $1 \cdot u = u$.

$= 1 \cdot \begin{bmatrix} x \\ x \end{bmatrix}$

$= \begin{bmatrix} 1 \cdot x \\ 1 \cdot x \end{bmatrix} = \begin{bmatrix} x \\ x \end{bmatrix} = u$.

Scanned with CamScanner

(iv) $c(d \cdot u) = (c \cdot d)u$.

$= c\left(d \cdot \begin{bmatrix} x \\ x \end{bmatrix}\right)$

$= c\begin{bmatrix} dx \\ dx \end{bmatrix}$.

$= \begin{bmatrix} cdx \\ cdx \end{bmatrix}$

$= (c \cdot d)\begin{bmatrix} x \\ x \end{bmatrix}$

$= (c \cdot d)u$.

As all the conditions of the vector space are satisfied given set $\mathbb{R}^2$ is a vector space.

Example No :- 03 :-

Verify that R^2 is a vector space.

Where.

Sol :-

$u = (x, y)$ & $v = (x_1, y_1)$.

(i) $u+v = (x, y) + (x_1, y_1)$

$= (x+x_1), (y+y_1)$

$= (x+x_1, y+y_1) \in V.$

(ii) $u+v = v+u$

$u+v = (x, y) + (x_1, y_1)$

$= (x+x_1, y+y_1)$

$= (x_1+x, y_1+y)$

$= (x_1, y_1) + (x, y)$

$= v + u.$

(iii) $u + (v + w) = (u + v) + w$

$u + (v + w) = (x, y) + [(x_1, y_1) + (x_2, y_2)]$

$= (x, y) + (x_1 + x_2, y_1 + y_2)$

$= (x + x_1 + x_2, y + y_1 + y_2)$

$= (x + x_1, y + y_1) + (x_2, y_2)$

$= [(x, y) + (x_1, y_1)] + (x_2, y_2)$

$= (u + v) + w.$

(iv) $0 + u = u.$

$= (0, 0) + (x, y)$

$= (0 + x, 0 + y)$

$= (x, y) = u.$

(v) $u + (-u) = 0.$

$= (x, y) + (-x, -y)$

$= (\cancel{x} - \cancel{x}, \cancel{y} - \cancel{y})$

$= (0, 0)$

$= 0.$

B:- (i) $c(u+v) = c\cdot u + c\cdot v.$

$= c((x, y) + (x_1, y_1))$

$= c(x + x_1, y + y_1)$

$= c(x + x_1), c(y + y_1).$

$= cx + cx_1, cy + cy_1.$

$= (cx, cy) + (cx_1, cy_1)$

$= c(x, y) + c(x_1, y_1)$

$= cu + cv.$

(ii) $(c+d)u = cu + du.$

$= (c+d)(x, y)$

$= (c+d)x, (c+d)y.$

$= cx + dx, cy + dy.$

$= (cx, cy) + (dx, dy)$

$= c(x, y) + d(x, y)$

$= cu + du.$

iii). $c(d \cdot u) = (cd)u.$

$c(d \cdot u) = c(d(x, y))$

$= c(dx, dy)$

$= cdx, cdy.$

$= (cd)(x, y)$

$= (cd)u$

iv). $1 \cdot u = u.$

$1 \cdot (x, y)$

$(1 \cdot x, 1 \cdot y)$

$(x, y) = u.$

As all the conditions of the vector space are satisfied. Given set R^2 is a vector space.

Topic No: 02 :-

Subspace :-

Example:-

Q: The set of all functions $f \in C(a, b]$ such that $f(a) = f(b)$.

Sol:- As we know that $C(a,b)$ is a vector space of all real valued continous functions we show that the given set of the set is the subspace of $C(a,b)$,

Scanned with CamScanner

let $(f, g) \in C(a, b)$.

$\cancel{f(a), g(a) =}$

Such that $f(a) = f(b)$

$g(a) = g(b)$.

$(f, g)(a)$

$= f(a) \cdot g(a)$.

$= f(b) \cdot g(b)$.

$= (f, g)(b) \in C(a, b)$.

also.

Let $k \in R$ & $f \in$

$kf(a) = kf(b)$.

$= (kf)(b)$.

So the given set is the subset of C(a,b).

Topic No :03

Linear Dependent & Linear independent vectors.

Definations:-

Linear Dependent :-

Let V be a vectorspace over a field F. The vector $v_1, v_2, \ldots v_m \in V$ are said to be linearly dependent over F if

$$a_1 v_1 + a_2 v_2 + \ldots a_m v_m = 0.$$

and not all a_i are zero, $a_i \in F$, $i = 1, 2, \ldots m$

is known as linearly dependent.

Linearly independent :-

If a_i are all are zero then vector are called linearly independent over F.

Also set of those vectors is called linearly independent set of vectors.

Examples.

Example No: 01:- Show that vectors $(1,0,0)$ $(0,1,0)$ $(0,0,1)$ belong to R^3 are linearly independent over R.

Sol:-
$$a(1,0,0) + b(0,1,0) + c(0,0,1) = (0,0,0).$$

$$(a,b,c) = (0,0,0)$$

$a = 0$
$b = 0$
$c = 0$.

Thus the given vectors are linearly independent.

Example 2:-

Show that the vectors.
$(3,0,-3)$, $(-1,1,2)$, $(4,2,-2)$ & $(2,1,1)$
are linearly depend over $\mathbb{R}$.

$a(3,0,-3)+b(-1,1,2)$
$+c(4,2,-2)+d(2,1,1)=(0,0,0)$

$(3a-b+4c+2d,\ b+2c+d$
$,\ -3a+2b-2c+d)=(0,0,0)$.

$3a - b + 4c + 2d = 0.$

$b + 2c + d = 0.$

$-3a + 2b - 2c + d = 0.$

$$\begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & 1 & 2 & 1 \\ -3 & 2 & -2 & 1 \end{bmatrix} \quad \cancel{R_1 + R_1}.$$

$$\begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \quad R_3 + R_1$$

$$\begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad R_3 - R_2.$$

As $3 < 4$ than

$3a - b + 4c + 2d = 0 \longrightarrow (i)$

$b + 2c + d = 0 \longrightarrow (ii)$

$2d = 0 \longrightarrow (iii)$

$2d = 0$

$\boxed{d = 0}$

Let $c = t$. & $\boxed{t = 1}$

then

so $\boxed{c = 1}$

$b + 2t + 0 = 0$.

$\boxed{b = -2t}$

$b = -2(1)$ $\boxed{b = -2}$

$3a - (-2) + 4(1) + 2(0) = 0$.

$3a + 2 + 4 = 0$.

$3a = 6$.

$a = \frac{6}{3}$

$\boxed{a = 2}$

$a = 2$, $b = -2$

$c = 1$ $d = 0$.

then the Given vectors is linearly dependent

Example No: 03:-

Show that vector

$\{(2,4,-3), (0,1,1), (0,1,-1)\}$..

Sol:-

$a_1(2,4,-3) + a_2(0,1,1) + a_3(0,1,-1)$
$= (0,0,0)$.

~~$2a_1 = 0$~~ (i)

$(2a_1, 4a_1 + a_2 + a_3, -3a_1 + a_2 - a_3)$
$= (0,0,0)$

$2a_1 = 0$ —— (i)

$4a_1 + a_2 + a_3 = 0$ —— (ii)

$-3a_1 + a_2 - a_3 = 0$ —— (iii)

$2a_1 = 0$

$\boxed{a_1 = 0}$

Put a_1 in e.q ii & iii

$4(0) + a_2 + a_3 = 0.$

$a_2 + a_3 = 0$ —— iv

$-3(0) + a_2 - a_3 = 0.$

$a_2 - a_3 = 0$ —— (v).

Add e.q iv & v.

$$a_2 + \cancel{a_3} = 0$$
$$a_2 - \cancel{a_3} = 0$$

$$2a_2 = 0$$

$$\boxed{a_2 = 0}$$

Put the value of a_2 in e-q(v) to get the value of a_3.

$$a_2 - a_3 = 0.$$
$$0 - a_3 = 0$$

$$\boxed{a_3 = 0}$$

$a_1 = 0$, $a_2 = 0$, $a_3 = 0$.

So the Given vector are linear independent.

Example NO: 04 :-

Determined whether the following vectors are in R^4 are linearly independ or linearly dependent.

$(1, 3, -1, 4), (3, 8, -5, 7); (2, 9, 4, 23)$

$a(1, 3, -1, 4) + b(3, 8, -5, 7)$
$+ c(2, 9, 4, 23) = (0, 0, 0, 0)$.

$(a + 3b + 2c, 3a + 8b + 9c$
$, -a - 5b + 4c, 4a + 7b + 23c)$
$= (0, 0, 0, 0)$.

$a + 3b + 2c = 0$ ——— (i)
$3a + 8b + 9c = 0$ ——— (ii)
$-a - 5b + 4c = 0$ ——— (iii)
$4a + 7b + 23c = 0$ ——— (iv)

Here,

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 8 & 9 \\ -1 & -5 & 4 \\ 4 & 7 & 23 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ 0 & -2 & 6 \\ 0 & -5 & 15 \end{bmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 + R_1 \\ R_4 - 4R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ 0 & -1 & 3 \\ 0 & -1 & 3 \end{bmatrix} \begin{matrix} \\ \\ R_3/2 \\ R_4/5 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \\ R_4 - R_2 \end{matrix}$$

As. Rank $= 2 < 3$ So is non trival.

$a + 3b + 2c = 0.$

$-b + 3c = 0.$

$c = t \qquad \boxed{t = 1}$

$\boxed{c = 1}$

$-b + 3(1) = 0$

$-b = -3$

$\boxed{b = 3}$

$a + 3(3) + 2(1) = 0$

$a + 9 + 2 = 0$

$\boxed{a = -11}$

and a, b, c will not have all zero, Hence given vectors are linearly dependent.

Scanned with CamScanner

Example No: 05

Let $a, b, c \in \mathbb{R}$ and Suppose that

$(1,-2,4,1)$, $(2,1,0,-3)$ $(1,-6,1,4)$

Sol:-

$$a(1,-2,4,1)+b(2,1,0,-3)$$
$$+c(1,-6,1,4) = (0,0,0,0)$$

$$(a+2b+c, -2a+b-6c, 4a+c$$
$$, a-3b+4c) = (0,0,0,0).$$

$a+2b+c=0$ —— (i)

$-2a+b-6c=0$ —— (ii)

~~$a-3b+4c=0$ —— (iii)~~

$4a+c=0$ —— (iii)

$a-3b+4c=0$ —— (iv).

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & -6 \\ 4 & 0 & 1 \\ 1 & -3 & 4 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 5 & -4 \\ 0 & -8 & -3 \\ 0 & -5 & 3 \end{bmatrix} \begin{matrix} \\ R_2 + 2R_1. \\ R_3 - 4R_1 \\ R_4 - R_1. \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -4/5 \\ 0 & 0 & 17/5 \\ 0 & 0 & -1 \end{bmatrix} \begin{matrix} \\ \cancel{R_3/5}\ R_3/5 \\ R_3 + 8R_2 \\ R_4 + 5R_2. \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -4/5 \\ 0 & 0 & 17/5 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \frac{5}{17}R_4 + R_3. \end{matrix}$$

Rank = 3
as Rank = number of variable.
So the System will be trivial Solution

then.

$a + 2b + c = 0.$

$b + -4/5c = 0.$

$17/5c = 0.$

$\boxed{c = 0}$

$b + -4/5(0) = 0.$

$\boxed{b = 0}$

$a + 2(0) + (0) = 0.$

$a + 0 + 0 = 0$

$\boxed{a = 0}.$

Hence the given vector are linearly independent.

Scanned with CamScanner

Example # 06 :-

Let $V = P_3(x)$ be the vector space of all polynomials of degree ≤ 3 over R together with the zero polynomial. Degree weather $u, v, w \in V$ are linearly independent or linearly dependent.

$u = x^3 - 4x^2 + 2x + 3$.
$v = x^3 + 2x^2 + 4x - 1$
$w = 2x^3 - x^2 - 3x + 3$

$au + bv + cw = 0$.

$a(x^3 - 4x^2 + 2x + 3) + b(x^3 + 2x^2 + 4x - 1)$
$+ c(2x^3 - x^2 - 3x + 3) = (0, 0, 0, 0)$

$(ax^3 + bx^3 + 2cx^3 , -4ax^2$

$+ 2bx^2 \cancel{+} - cx^2 ,$

$2ax + 4bx - 3cx , 3a - b + 3c)$

$= (0, 0, 0, 0)$.

$(a+b+2c)x^3 , (-4a+2b-c)x^2$

$(2a+4b-3c)x , (3a-b+3c)$

$= (0, 0, 0, 0)$.

As x^3, x^2 & x are constants

$(a+b+2c , -4a+2b-c ,$

$2a+4b-3c , 3a-b+3c)$

$= (0, 0, 0, 0)$.

$a + b + 2c = 0$ —— (i)

$-4a + 2b - c = 0$ —— (ii)

$2a + 4b - 3c = 0$ —— (iii)

$3a - b + 3c = 0$ —— (iv)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -4 & 2 & -1 \\ 2 & 4 & -3 \\ 3 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 6 & 7 \\ 0 & 2 & -7 \\ 0 & -4 & -2 \end{bmatrix} \begin{matrix} \\ R_2 + 4R_1 \\ R_3 - 2R_1 \\ R_4 - 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 7/6 \\ 0 & 2 & -7 \\ 0 & -4 & -2 \end{bmatrix} \begin{matrix} \\ R_2/6 \\ \\ \\ \end{matrix}$$

-11

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 7/6 \\ 0 & 0 & -11 \\ 0 & 0 & 6 \end{bmatrix} \begin{matrix} \\ \\ R_3 - 2R_1 \\ R_4 + 4R_1 \end{matrix}$$

$-2+4(2)$
$-2+8$
$6.$

Rank = 3 = number of variable So System will have trivial $a=0$, $b=0$ & $c=0$.

So the given vectors are linearly independent.

Topic 4:- Linear Algebra! After Mid

LINEAR Transformation.

Def:- A linear Transformation L of R^n into R^m a function assigning a unique vector $L(u)$ in R^m to each u in R^n Such that.

1) $L(u+v) = L(u) + L(v)$
2) $L(ku) = k L(u)$.

Example:- # 01 :-

Q1:- $L : R^3 \longrightarrow R^2$ be defined by $L(x, y, z) = (x, y)$

Sol:- $u = (x_1, y_1, z_1)$. $v = (x_2, y_2, z_2)$.

(1) $L(u+v) = L(u) + L(v)$

$$L(u+v) = L\big((x_1, y_1, z_1) + (x_2, y_2, z_2)\big)$$
$$= L[(x_1, y_1) + (x_2, y_2)].$$
$$= (x_1 + x_2, y_1 + y_2).$$

$$L(u) + L(v) = L(x_1, y_1, z_1) + L(x_2, y_2, z_2)$$
$$= (x_1, y_1) + (x_2, y_2).$$
$$= (x_1 + x_2, y_1 + y_2).$$

(2) $L(ku) = kL(u)$

$= L(k(x_1, y_1, z_1))$

$L(ku) = L(kx_1, ky_1, kz_1)$

$= kx_1, ky_1,$

$kL(u) = kL(x_1, y_1, z_1)$

$= k(x_1, y_1)$

$= kx_1, ky_1$

Linear Transformed.

Example # 02 :-

Question :- $R^3 \rightarrow R^2$ be defined.

$$L\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} u_1 + 1 \\ u_2 - u_3 \end{bmatrix}.$$

Sol :- (1) $L(u+v) = L(u) + L(v)$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad v = \begin{bmatrix} u_1' \\ u_2' \\ u_3' \end{bmatrix}.$$

$L(u+v)$

$$= L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} u_1' \\ u_2' \\ u_3' \end{bmatrix}\right).$$

$$= L\left(\begin{bmatrix} u_1 + u_1' \\ u_2 + u_2' \\ u_3 + u_3' \end{bmatrix}\right)$$

$$L(u+v) = \begin{bmatrix} u_1 + u_1' + 1 \\ u_2 + u_2' - u_3 - u_3' \end{bmatrix}.$$

$$L(u) + L(v) = L\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + L\left(\begin{bmatrix} u_1' \\ u_2' \\ u_3' \end{bmatrix}\right).$$

$$= \begin{bmatrix} u_1 + 1 \\ u_2 - u_3 \end{bmatrix} + \begin{bmatrix} u_1' + 1 \\ u_2' - u_3' \end{bmatrix}.$$

$$= \begin{bmatrix} u_1 + u_1' + 2 \\ u_2 + u_2' - u_3 - u_3' \end{bmatrix}$$

$L(u+v) \neq L(u) + L(v)$.

Hense it is not linear Trans

Example # 03 :-

Q:- $R^2 \to R^2$ be given by.

$L(x_1, x_2) = (-x_2, x_1)$.

Sol:- ~~Let~~ $u = (x_1, x_2)$ $v = (y_1, y_2)$

(i) $L(u+v) = L(u) + L(v)$

$L(u+v) = L([(x_1, x_2) + (y_1, y_2)]$

$= L((x_1 + y_1), (x_2 + y_2))$.

$= [-(x_2 + y_2), (x_1 + y_1)]$

$L(u) + L(v) = L[x_1, x_2] + L[y_1, y_2]$

$= (-x_2, x_1) + (-y_2, y_1)$

$= -(x_2 + y_2), (x_1 + y_1)$

Scanned with CamScanner

(ii) $L(ku) = kL(u)$

$L(ku)$

$= L[k(x_1, x_2)]$

$= L[kx_1, kx_2]$.

$= -kx_2, kx_1$

$kL(u) = kL[x_1, x_2)$

$= k(-x_2, x_1)$

$= -kx_2, kx_1$

Hense proved

Linear Transformed.

Example No 4 :-

$R^2 \to R^2$

$T(x_1, x_2) = (x_1+1, x_2+2)$.

$u = (x_1, x_2)$ $v = (y_1, y_2)$

$L(u+v) = L(u) + L(v)$.

i)

$L(u+v) = L[(x_1, x_2) + (y_1, y_2)]$.

$= L[(x_1+y_1), (x_2, y_2)]$.

$= [(x_1+y_1+1), (x_2+y_2+2)]$

$L(u) + L(v) = L(x_1, x_2) + L(y_1, y_2)$

$= (x_1+1, x_2+2) + (y_1+1, y_2+2)$.

$= (x_1+y_1+2), (x_2+y_2+4)$.

As $L(u+v) \neq L(u) + L(v)$.

So it is not linear Trasformal.

Examples:-

$R^3 \longrightarrow R^2$

$L(x_1, x_2, x_3) = (x_1 - x_2, x_1 - x_3)$.

Sol:-

$u = (x_1, x_2, x_3) \quad v = (y_1, y_2, y_3)$.

1) $L(u+v) = L(u) + L(v)$.

$L(u+v) = L\big((x_1, x_2, x_3) + (y_1, y_2, y_3)\big)$

$= L(x_1 + y_1, x_2 + y_2, x_3 + y_3)$.

$= (x_1 + y_1 - x_2 - y_2, \; x_1 + y_1 - x_3 - y_3)$

$L(u) + L(v) = L(x_1, x_2, x_3) + L(y_1, y_2, y_3)$

$= (x_1 - x_2, x_1 - x_3) + (y_1 - y_2, y_1 - y_3)$

$= (x_1 + y_1 - x_2 - y_2 \; ; \; x_1 + y_1 - x_3 - y_3)$

(ii) $L(ku) = Lk(ku)$.

$$L(ku) = L\big(k(x_1, x_2, x_3)\big).$$

$$= L(kx_1, kx_2, kx_3).$$

$$= (kx_1 - kx_2, kx_1 - kx_3).$$

$$Lk(u) = Lk(x_1, x_2, x_3)$$

$$= L(kx_1, kx_2, kx_3).$$

$$= (kx_1 - kx_2, kx_1 - kx_3)$$

This Transformation is linear Transformation

Example NO 6 :-

$R^3 \longrightarrow R^2$

$L(x_1, x_2, x_3) = (|x_1|, x_2 - x_3)$

Sol:-

$u = (x_1, x_2, x_3)$

$v = (y_1, y_2, y_3)$

$L(u+v) = L(u) + L(v)$

$L(u+v) = L\left((x_1, x_2, x_3) + (y_1, y_2, y_3)\right)$

$= L(x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$= (|x_1 + y_1|, x_2 + y_2 - x_3 - y_3)$

Scanned with CamScanner

But $|x_1 + y_1| \leq |x_1| + |y_1|$.

and equality does not hold for all x_1 & y_1

$|x_1 + y_1| \neq |x_1| + |y_1|$ for some x_1, y_1

$L\phi(u+v) \neq L\phi(u) + L\phi(v)$

So Lin transform is not possible.

Example 7:-

$R^2 \longrightarrow R^3$.

$$L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x-y \\ x+y \\ 2x \end{bmatrix}.$$

Sol:-

$$u = \begin{bmatrix} x \\ y \end{bmatrix} \qquad v = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}.$$

$$L(u+v) = L(u) + L(v).$$

$$L(u+v) = L\left[\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right]$$

$$= L\begin{bmatrix} x+x_1 \\ y+y_1 \end{bmatrix}.$$

$$= \begin{bmatrix} x + x_1 - y - y_1 \\ x + x_1 + y + y_1 \\ 2x + 2x_1 \end{bmatrix}$$

$$L(u) + L(v) = L\begin{bmatrix} x \\ y \end{bmatrix} + L\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}.$$

$$= \begin{bmatrix} x - y \\ x + y \\ 2x \end{bmatrix} + \begin{bmatrix} x_1 - y_1 \\ x_1 + y_1 \\ 2x_1 \end{bmatrix}.$$

$$= \begin{bmatrix} x + x_1 - y - y_1 \\ x + x_1 + y - y_1 \\ 2x + 2x_1 \end{bmatrix}$$

(ii) $L(ku) = L k(u)$

$$L(ku) = L\left(k\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$= L\begin{pmatrix} kx \\ ky \end{pmatrix}.$$

$$= \begin{bmatrix} kx - ky \\ kx + ky \\ 2kx \end{bmatrix}$$

$$L\,k(u) = kL\begin{pmatrix} x \\ y \end{pmatrix}.$$

$$= kL\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} kx - ky \\ kx + ky \\ 2kx \end{bmatrix}$$

This transformation is Linear Transformation.

Scanned with CamScanner

Example No: 8

$R^2 \longrightarrow R^2$

$$L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}.$$

$$u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}.$$

Sol:-

$$L(u+v) = L(u) + L(v),$$

$$L(u+v) = L\left[\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right].$$

$$= L\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}.$$

$$= \begin{bmatrix} x_1 + x_2 \\ -y_1 - y_2 \end{bmatrix}$$

$L(u)+L(v)$

$$= L\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + L\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}.$$

$$= \begin{bmatrix} x_1 \\ -y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ -y_2 \end{bmatrix}.$$

$$= \begin{bmatrix} x_1 + x_2 \\ -y_1 - y_2 \end{bmatrix}$$

2) $L(ku) = kL(u).$

$$L\left[k\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right] = kL\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$L\begin{pmatrix} kx_1 \\ ky_1 \end{pmatrix} = k\begin{bmatrix} x_1 \\ -y_1 \end{bmatrix}$$

$$\begin{bmatrix} kx_1 \\ -ky_1 \end{bmatrix} = \begin{bmatrix} kx_1 \\ -ky_1 \end{bmatrix}$$

This transformation is Linear Transformation

Topic No: 05

SPANING

Defination:- vector $v_1, v_2 \ldots v_k$ in a vector space V are said to be span V if every vector is a linear combination of $v_1, v_2 \ldots v_k$.

$$c_1 v_1 + c_2 v_2 + \ldots c_k v_k = v.$$

Examples:

Example No: 01 :-

As $V_1(1,2,1)$, $V_2(1,0,2)$

$V_3(1,1,0)$ Do V_1, V_2, V_3

Span V ?

Solution :-

$$C_1V_1 + C_2V_2 + C_3V_3 = V$$

$$C_1(1,2,1) + C_2(1,0,2) + C_3(1,1,0)$$

$$= (a,b,c) \quad \text{—(i)}$$

$$(C_1 + C_2 + C_3, \; 2C_1 + C_3,$$

$$C_1 + 2C_2) = (a,b,c)$$

$$C_1 + C_2 + C_3 = a \quad \text{—— (i)}$$
$$2C_1 + C_3 = b \quad \text{—— (ii)}$$
$$C_1 + 2C_2 = C \quad \text{—— (iii)}.$$

$$\cancel{c_1 + c_2 + c_3} = a.$$
$$-2c_1 \quad \cancel{+c_3} = -b.$$

$$-c_1 + c_2 = a-b. \quad \text{——— (iii)}.$$

$$\cancel{-c_1} + c_2 = a-b$$
$$\cancel{c_1} + 2c_2 = c$$

$$3c_2 = a-b+c.$$

$$\boxed{c_2 = \frac{a-b+c}{3}.}$$

Put c_2 in e.q (3)

$$c_1 + 2c_2 = c$$

$$c_1 + 2\left(\frac{a-b+c}{3}\right) = c$$

$$\frac{3c_1 + 2a - 2b + 2c}{3} = c$$

$$3C_1 + 2a - 2b + 2C = 3C.$$

$$3C_1 = 8 - 2a + 2b - 2C + 3C.$$

$$3C_1 = 2b - 2a + C$$

$$\boxed{C_1 = \frac{2b - 2a + C}{3}}$$

$$\cancel{C_1 =}$$

$$2C_1 + C_3 = b.$$

$$2\left(\frac{2b - 2a + C}{3}\right) + C_3 = b.$$

$$4b - 4a + 2C + 3C_3 = 3b.$$

$$3C_3 = 3b - 4b + 4a - 2C$$

$$3C_3 = -b + 4a - 2C$$

$$\boxed{C_3 = \frac{4a - b - 2C}{3}}$$

$\cancel{\frac{a-b+c}{3}}$

Put the value of c_1, c_2 & c_3

in eq no (1)

$$\left(\frac{2b-2a+c}{3}\right)(1,2,1)+\left(\frac{a-b+c}{3}\right)(1,0,2)$$

$$+\left(\frac{4a-b-2c}{3}\right)(1,1,0)$$

$$\left(\frac{2b-2a+c}{3}+\frac{a-b+c}{3}+\frac{4a-b-2c}{3},\right.$$

$$\frac{4b-4a+2c}{3}+\frac{4a-b-2c}{3},$$

$$\left.\frac{2b-2a+c}{3}+\frac{2a-2b+2c}{3}\right)=(a,b,c)$$

$$\left(\frac{\cancel{2b}-2a+\cancel{c}+a-\cancel{b}+\cancel{c}+4a-\cancel{b}-\cancel{2c}}{3},\right.$$

$$\frac{4b-\cancel{4a}+\cancel{2c}+\cancel{4a}-b-\cancel{2c}}{3},$$

$$\left.\frac{\cancel{2b}-\cancel{2a}+c+\cancel{2a}-\cancel{2b}+2c}{3}\right)=(a,b,c)$$

$$\left(\frac{\cancel{3}a}{\cancel{3}},\frac{\cancel{3}b}{\cancel{3}},\frac{\cancel{3}c}{\cancel{3}}\right)=(a,b,c)$$

$$(a,b,c)=(a,b,c)$$

Answer.

Example No#02:-

Find an equation defining the subspace w of R^3 spanned by

$v_1 = (1,-3,2)$, $v_2 = (-2,1,2)$

$v_3 = (-3,-1,6)$.

Sol:-

$$c_1v_1 + c_2v_2 + c_3v_3 = v$$

$$c_1(1,-3,2) + c_2(-2,1,2)$$

$$+ c_3(-3,-1,6) = (x,y,z).$$

$$(c_1, -3c_1, 2c_1 + -2c_2, c_2, 2c_2$$

$$+ -3c_3, -1c_3, 6c_3) = (x,y,z).$$

$(c_1 - 2c_2 - 3c_3, -3c_1 + c_2 - c_3$
$, 2c_1 + 2c_2 + 6c_3) = (x, y, z)$.

$c_1 - 2c_2 - 3c_3 = x$ ——— (i).

$-3c_1 + c_2 - c_3 = y$ ——— (ii)

$2c_1 + 2c_2 + 6c_3 = z$ ——— (iii)

multiply eq (1) by (-2)

$-2x = -2c_1 + 4c_2 + 6c_3$. ——— (iv).

$\cancel{2c_1} + 2c_2 + 6c_3 = z$.
$\underline{-\cancel{2c_1} + 4c_2 + 6c_3 = -2x}$

$6c_2 + 12c_3 = z - 2x$

$6(c_2 + 2c_3) = z - 2x$ ——— (v).

Scanned with CamScanner

then

$$x+y+z = c_1 - 2c_2 - 3c_3 - 3c_1 + c_2 - c_3 + 2c_1 + 2c_2 + 6c_3.$$

$$\boxed{x+y+z = c_2 + 2c_3}$$

replace $c_2 + 2c_3$ in e.q (v).

$$6(x+y+z) = z - 2x.$$

$$6x + 6y + 6z = z - 2x.$$

$$6x + 6y + 6z - z + 2x = 0.$$

(iv). $$\boxed{8x + 6y + 5z = 0}$$

$W = (x, y, z) : \{ 8x + 6y + 5z)$

(v).

Example No: 03 :-

Determine whether the Set S,

$S = \{ V_1 = (1,-1,0), V_2 = (0,1,2)$
$V_3 = (2,0,1) \quad V_4 = (1,0,1) \}$

Let $X = (A, B, C)$

$X = V_1C_1 + C_2V_2 + C_3V_3 + C_4V_4.$

$C_1(1,-1,0) + C_2(0,1,2) + C_3(2,0,1)$

$C_4(1,0,1) = (a,b,c)$

$(C_1 + 2C_3 + C_4, \; -C_1 + C_2,$

$2C_2 + C_3 + C_4) = a, b, c.$

$$c_1 + 2c_3 + c_4 = a \text{ ------ (i)}$$

$$-c_1 + c_2 = b \text{ ------ (ii)}$$

$$2c_2 + c_3 + c_4 = c \text{ ------ (iii)}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & a \\ -1 & 1 & 0 & 0 & b \\ 0 & 2 & 1 & 1 & c \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & a \\ 0 & 1 & 2 & 1 & a+b \\ 0 & 2 & 1 & 1 & c \end{array}\right] R_2+R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & a \\ 0 & 1 & 2 & 1 & a+b \\ 0 & 0 & -3 & -1 & c-2a-2b \end{array}\right] R_3-2R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & a \\ 0 & 1 & 2 & 1 & a+b \\ 0 & 0 & 1 & 1/3 & \frac{c-2a-2b}{3} \end{array}\right]$$

The system has ∞ M.S

So S spans R^3.

Example No: 04

Show that the vector $(1,2)$ and $(3,5)$ span the vector space R^2.

Sol:-

$$v = (x, y)$$

$$c_1 v_1 + c_2 v_2 = v.$$

$$c_1(1,2) + c_2(3,5) = (x,y).$$

$$(c_1, 2c_1 + 3c_2, 5c_2) = (x,y)$$

$$c_1 + 3c_2 = x \quad \text{——— (i)}$$
$$2c_1 + 5c_2 = y \quad \text{——— (ii)}$$

$$A = \left[\begin{array}{cc|c} 1 & 3 & x \\ 2 & 5 & y \end{array}\right]$$

$$\left|\begin{array}{cc|c} 1 & 3 & x \\ 0 & -1 & y-2x \end{array}\right| \; R_2 - 2R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3y-5x \\ 0 & -1 & y-2x \end{array}\right] \quad R_1+3R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3y-5x \\ 0 & 1 & 2x-y \end{array}\right] \quad -R_2$$

$c_1 = 3y - 5x.$

$c_2 = 2x - y.$

Put the value of c_1 & c_2 in the given eq.

$$(3y-5x)(1,2) + (2x-y)(3,5) = (x,y)$$

Hence vector $(1,2)$ & $(3,5)$

Span the vector R^2.

Topic No. 6 :-

Base And Diamensions

Def :- The vector $v_1, v_2 \dots v_k$ in a vector space V are said to form basis for V if

(i) $v_1, v_2 \dots v_k$ span V.
(ii) $v_1, v_2 \dots v_k$ are linear independent

Examples :-

Example No: 02

Show that 2x2 matrix

$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $E_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$E_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

For the basic of vector Space V of all 2x2 Symmetrix matrix.

Sol:-

Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

then:

(i) $\begin{bmatrix} a & b \\ b & c \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$+ c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & b \\ b & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

So the E_1, E_2, E_3 Span V.

(ii) $a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Scanned with CamScanner

$$\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a & b \\ b & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a = 0$, $b = 0$, $c = 0$.

E_1, E_2 & E_3 are linealy independent.

So the E_1, E_2 & E_3 are the base of V.

Scanned with CamScanner

Example No: 02 :-

Determined whether or not the given set of vector is a basis of R^2.

Q: $\{(1,1),(3,1)\}$

Sol:-

(i) $a(1,1)+b(3,1) = (0,0)$.

~~a = a~~

$(a+3b, a+b) = (0,0)$

$a+3b=0$ —— (i)
$a+b=0$ —— (ii)

$\cancel{a}+3b=0$
$-\cancel{a}\ \underline{+b} = 0$

$2b=0$

Scanned with CamScanner

$\boxed{b = 0}$

Put b in eq (2).

$a + b = 0$
$a + 0 = 0$

$\boxed{a = 0}$.

So $a = 0$ & $b = 0$ the given set (1, 1) & (3, 1) are linearly independent.

(ii) $a(1, 1) + b(3, 1) = (x, y)$

$(a + 3b, a + b) = (x, y)$

$a + 3b = x$ —— (i)
$a + b = y$ —— (ii)

$\cancel{a} + 3b = x$
$\underline{\cancel{a} + b = y}$

$2b = x - y$

$$\boxed{b = \frac{x-y}{2}} \text{ — (i)}$$

Put in e.q (2) to get a.

$$a + b = y.$$

$$a + \frac{x-y}{2} = y.$$

$$\frac{2a + x - y}{2} = y$$

$$2a + x - y = 2y.$$

$$2a = 2y + y - x.$$

$$2a = 3y - x$$

$$\boxed{a = \frac{3y-x}{2}.} \text{ — (ii)}$$

So (1,1) & (3,1) Span R^2.

Hense, Set (1,1) & (3,1) form a basis for R^2.

Example No: 03.

Determined whether or not the given set of vector is a basic of R^2.

Q: $\{(2,1), (1,-1)\}$

Sol:

$$a(2,1) + b(1,-1) = (0,0)$$

$$(2a+b, a-b) = (0,0)$$

$$2a+b=0 \quad \text{——— (i)}$$
$$a-b=0 \quad \text{——— (ii)}$$

$$\begin{aligned} 2a+\cancel{b} &= 0 \\ a \cancel{-b} &= 0 \\ \hline 3a &= 0 \end{aligned}$$

$$\boxed{a=0}$$

$$a-b=0$$
$$0-b=0$$
$$\boxed{b=0}$$

So $a=b=0$ the given set $(2,1),(1,-1)$ are linearly independent.

then

$$a(2,1)+b(1,-1)=(x,y)$$

$$(2a+b,\ a-b)=(x,y).$$

$2a + b = x$ —— (i)

$a - b = y$ —— (ii)

$$
\begin{array}{r}
2a + \cancel{b} = x \\
a \cancel{- b} = y \\
\hline
3a = x + y
\end{array}
$$

$$\boxed{a = \frac{x+y}{3}}$$

$a - b = y$

$\frac{x+y}{3} - b = y$

$\frac{x+y-3b}{3} = y$

$x+y-3b = 3y$

$-3b = 3y - y - x$

$-3b = 2y - x$

$$\boxed{b = \frac{x-2y}{3}}$$

Scanned with CamScanner

So, $(2,1), (1,-1)$ span R^2.

Hense the set $\{(2,1)(1,-1)\}$ from the basic of $\mathbb{R}^2$.

Example NO: 04 :-

Determined whether or not the given set of vectors is a basic of R^3.

Q: $\{(1,2,-1),(0,3,1)(1,-5,3)\}$

Sol:-

$a(1,2,-1)+b(0,3,1)+c(1,-5,3)$

$=(0,0,0)$.

$(a+c,\quad 2a+3b-5c,\quad -a+b+3c)$

$=(0,0,0)$.

$a + c = 0$ ———— (i)

$2a + 3b - 5c = 0$ ———— (ii)

$-a + b + 3c = 0$ ———— (iii)

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & 3 & -5 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 3 & -7 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 + R_1 \end{array}$$

$$= \cancel{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -7/3 \\ 0 & 0 & 4 \end{bmatrix} \; R_2/3}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 3 & -7 & 0 \end{array}\right]$$

R_2 exchan by R_3.

$$\begin{bmatrix} 1 & 0 & 1 & \vdots & 0 \\ 0 & 1 & 4 & \vdots & 0 \\ 0 & 0 & -19 & \vdots & 0 \end{bmatrix} R_3 - 3R_2.$$

$$\begin{bmatrix} 1 & 0 & 1 & \vdots & 0 \\ 0 & 1 & 4 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix} R_3/_{-19}$$

$$\begin{bmatrix} 1 & 0 & 1 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix} R_2 - 4R_3.$$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix} R_1 - R_3.$$

$a = 0$, $b = 0$, $c = 0$.

So the given set is linearly independ.

(ii) $a(1,2,-1)+b(0,3,+1)$

$+c(1,-5,3)=(x,y,z)$

$(a+c,\ 2a+3b-5c,\ -a+b+3c)$

$=(x,y,z)$

$a+c=x$ ——— (i)
$2a+3b-5c=y$ ——— (ii)
$-a+b+3c=z$ ——— (iii)

$\boxed{c=x-a}$ ——— (iv)

$y=2a+3b-5(x-a)$
$y=2a+3b-5x+5a.$
$y=7a+3b-5x.$ ——— (v)

$z=-a+b+3(x-a)$
$=-a+b+3x-3a.$
$z=-4a+b+3x$ ——— (vi)

$$7a + \cancel{3b} - 5x = y$$
$$\mp 12a \cancel{\pm 3b} \pm 9x = \underset{-}{32}$$

$$19a - 14x = y - 32$$

$$19a = y - 32 + 14x$$

$$\boxed{a = \frac{y - 32 + 14x}{19}}$$

$$z = -4\left(\frac{y - 32 + 14x}{19}\right) + b + 3x$$

$$z - b = -4\left(\frac{y - 32 + 14x}{19}\right) + 3x$$

$$-b = -4\left(\frac{y - 32 + 14x}{19}\right) + 3x - z$$

$$b = 4\left(\frac{y - 32 + 14x}{19}\right) + z - 3x$$

$C = x - a$

$C = x - \left(\frac{y - 3z + 14x}{19} \right)$

So, given vectors span R^3.

$\{ (1,2,-1), (0,3,1) (1,-5,3) \}$

form basic of R^3.

Scanned with CamScanner

Example NO: 05:-

Q:- $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ R^3.

Sol:-

$$a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$$= \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$$

$$= \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

So the give vector Span R^3.

$$a\begin{bmatrix}1\\0\\0\end{bmatrix}+b\begin{bmatrix}0\\1\\0\end{bmatrix}+c\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}a\\0\\0\end{bmatrix}+\begin{bmatrix}0\\b\\0\end{bmatrix}+\begin{bmatrix}0\\0\\c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\boxed{a=0}$

$\boxed{b=0}$

$\boxed{c=0}$

So the give set is linear independent So it the basis of R^3

Topic No: 07

Application of linear System.

missing