subspaces of a vector space ... of V.

Definition :- let U & W be two subspaces of a vectorspace V. Define
$$U+W = \{u+w : u \in U, w \in W\}$$

Theorem :- (3.16) If U, W are subspaces of a vectorspace V then $U+W$ is a subspace of V containing both U & W. Further $U+W$ is the smallest subspace containing both U & W

Proof :- $U+W = \{u+w : u \in U, w \in W\}$

Definition :- Let U, W be subspaces of a vector space V. If $U \cap W = \{0\}$ then $U + W$ is called the direct sum of U & W & is written as $U \oplus W$.

Definition :- Let V be a vector space over a field F & $v \in V$. Then v is called linear combination of $v_1, v_2, \ldots\ldots v_n \in V$ if

$$v = a_1 v_1 + a_2 v_2 + \ldots\ldots + a_n v_n \quad \text{where } a_i \in F$$

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if $n=1$

$a_1 s_1$

if $n=2$

$a_1 s_1 + a_2 s_2$

Definition :- Let S be a non-empty subset of a vector space V. The set of all linear combinations of finite number of elements of S is called the linear span of S & is denoted by $\langle S \rangle$.

i.e $\langle S \rangle = \left\{ \sum_{i=1}^{n} a_i s_i \;:\; a_i \in F,\; s_i \in S,\; n \in N \right\}$

Theorem :- (3.20) Let S be a finite set of vectors in a vector space V over a field F. Then $\langle S \rangle$ is a subspace of V containing S & it is the smallest subspace containing S

Definition :- A vectorspace V is said to be finite dimensional if there is a finite subset S in V such that

$$\langle S \rangle = V$$

subspace of R^3.

(ii). $W = \{(x,y,z) : x \geq 0\}$

Sol:- Let $w_1 = (x_1, y_1, z_1) \in W$ & $\alpha \in R$

then $\alpha w_1 = \alpha(x_1, y_1, z_1)$

$= (\alpha x_1, \alpha y_1, \alpha z_1)$

As $w_1 = (x_1, y_1, z_1) \in W$ then $x_1 \geq 0$

$\alpha w_1 = (\alpha x_1, \alpha y_1, \alpha z_1) \notin W$

$\because \alpha = -1$, $w_1 = (1,2,3) \in W$

$\alpha w_1 = (-1)(1,2,3) = (-1,-2,-3) \notin W$

So W is not a subspace of R^3.

(iii). $W = \{(x,y,z) : x^2+y^2+z^2 \leq 1\}$

Sol:- Let $\alpha \in R$ & $w_1 = (x_1, y_1, z_1) \in W$

As $w_1 \in W$ then $x_1^2 + y_1^2 + z_1^2 \leq 1$

$\alpha w_1 = \alpha(x_1, y_1, z_1)$

$= (\alpha x_1, \alpha y_1, \alpha z_1) \notin W$

$\therefore \alpha w_1 \notin W$

$\because w_1 = (1,0,0) \in W$

$\alpha = 2 \in R$

$\alpha w_1 = 2(1,0,0)$

$= (2,0,0) \in W$

So W is not a subspace of R^3

Definition:- Let V be a vector space over a field F. The vectors $v_1, v_2, \ldots v_m \in V$ are said to be linearly depdt over F if

$a_1 v_1 + a_2 v_2 + \ldots + a_m v_m = 0$

& all $a_i \neq 0$, $a_i \in F$, $i = 1, 2, 3, \ldots m$

or if $a_1 v_1 + a_2 v_2 + \ldots + a_m v_m = 0$

then at least one $a_i \neq 0$.

Definition:- The vectors $v_1, v_2, \ldots v_m$ are said to be linearly independent over F if

$a_1 v_1 + a_2 v_2 + \ldots + a_m v_m = 0$ then all $a_i = 0$.

Note:-

(i) An empty set is defined to be linearly independent set

(ii) If $v \neq 0$ then $[v]$ is linearly indept set.

Definition :- A set of linearly independent vectors spanning a vector space V is a called a basis for V.

Exp 30 :- $V = R^3$ & $B = \{ e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1) \}$

Show that B is a basis for V.

Sol:- (i) Suppose $a_1 e_1 + a_2 e_2 + a_3 e_3 = 0$

$$a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1) = 0$$
$$(a_1, 0, 0) + (0, a_2, 0) + (0, 0, a_3) = 0$$
$$(a_1+0+0,\ 0+a_2+0,\ 0+0+a_3) = 0$$
$$(a_1, a_2, a_3) = (0,0,0)$$

$\therefore$ $a_1 = 0$, $a_2 = 0$, $a_3 = 0$

$\therefore$ B is linearly independent set.

(ii) Let $v = (x, y, z) \in R^3$

Suppose $v = a_1 e_1 + a_2 e_2 + a_3 e_3$

$$(x,y,z) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$$
$$(x,y,z) = (a_1, a_2, a_3)$$
$$a_1 = x,\ a_2 = y,\ a_3 = z$$

$\therefore$ $v = x e_1 + y e_2 + z e_3$

$\therefore$ $\langle B \rangle = R^3$

Hence B is a basis for R^3.

Q. (5). If u, v, w are linearly independent vectors Prove that

(i) $u+v-2w$, $u-v-w$, $u+w$ are linearly independt.

Sol:- Suppose $a(u+v-2w) + b(u-v-w) + c(u+w) = 0$

$\Rightarrow au + av - 2aw + bu - bv - bw + cu + cw = 0$

$\Rightarrow (a+b+c)u + (a-b)v + (-2a-b+c)w = 0$