Def Of Matrix

entries.

Order of a Matrix
(Size of a Matrix)

Equal Matrix A

General form of
a Matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Row Matrix

Column Matrix

$A_{m \times n} = [a_{ij}]_{m \times n}$

General form.

Square Matrix

Diagonal of a Matrix

diagonal elements - a_{ij} where i

Scalar Matrix

Identity Matrix / unit Matrix

Equal Matrices

Null Matrices

Transpose of a Matrix

Upper Triangular Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

upper elements are non zero

$a_{ij} = 0 \quad i > j$

Lower Triangular Matrix.

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

lower elements are non zero

$a_{ij} = 0 \quad i < j$

Scalar Multiple of a Matrix

$$c \cdot A$$

Symmetric matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \qquad A^t = A$$

Skew Symmetric Matrix.

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix} \qquad A^t = -A$$

Conjugagate of Matrix : $\bar{A}$ $\begin{bmatrix} 1 & 2+i \\ 3-2i & 5i \end{bmatrix}$

Hermition Matrix

square Matrices $\quad (\bar{A})^T = A$

$$A = \begin{bmatrix} 2 & 1-i \\ 1+i & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1-2i & 0 \\ 1+2i & 0 & -i \\ 0 & i & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & +i & 2+i \\ -i & 4 & 3-i \\ 2-i & 3+i & 6 \end{bmatrix}$$

Skew Hermitian Matrix

$$(\overline{A})^{T} = -A$$

(W) 1.

$$B = \begin{bmatrix} 0 & i & 2+i \\ -i & 0 & -3+i \\ -2+i & -3+i & 0 \end{bmatrix}$$

W

$$C = \begin{bmatrix} 0 & 2+i & 3i \\ 2-i & 0 & 1-3i \\ -3i & 1+3i & 0 \end{bmatrix}$$

W

$\longleftrightarrow$

W

~~Adjoint~~ of A

Trace of a Matrix

If A is a square matrix then the trace of A is denoted by trace (A), is defined to be the sum of elements of the main diagonal of A.

The Trace of A is undefined if Matrix is not square.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Tra}(A) = a_{11} + a_{22} + a_{33}$$

Addition of Matrices :—

$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

properties of Matrices
(i) Commulative
(ii) Associative properties
(iii) identity
(iv) Inverse of A.

(i) $A+B$ (ii) $2A+3B$
(iii) $3A-5B$

Multiplication of Matrices.

(i) $\begin{bmatrix} 1 & 2 & 1 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & 5 \\ -2 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} a & h & g \\ h & b & f \\ g & h & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}^2$ (iv) $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}^3$

Associative
left write distributive
Identity.

find B^2 $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^2$

find B^3

Periodic Matrix

$A = \begin{bmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{bmatrix}$ of period 2

A Square Matrix A is said to be periodic Matrix of period k, where k is the least positive integer such That

$$A^{k+1} = A$$

Idempotent

A SQUAR matrix A is Called idempotent if $\boxed{A^2 = A}$
or if $k=1$
$A^{1+1} = A$.

$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$A^2 = A$

Nilpotent Matrix.

A Square Matrix A is said to be nilpotent of ~~ord~~ index k where k is the least positive Integer such That

$$A^k = 0$$

$\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$

$A \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \Rightarrow A^3 = 0$

Involutory

A Square matrix A is called Involutory matrix if $A^2 = I$

$$\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & -4 \end{bmatrix}$$

Explain why

$$(A+B)^2 \neq A^2 + 2AB + B^2$$

$$A^2 - B^2 = (A-B)(A+B)$$

Q No: $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Show That

$$A^2 - 4A - 5I_3 = 0$$

Elementry row operations

The following ~~row~~ ~~row~~ operation ~~of~~ on a row are called elementary row operation.

(i) Interchanging ~~row~~ any two rows (R_{ij}) Interchanging i^{th} and j^{th} row

(ii) Multiplication of a row by any non zero Scalar. kR_i

(iii) Addition of any Multiple of one row to to other row.

$$R_i + kR_j$$

A matrix $B_{m\times n}$ is called row equivalent to a matrix $A_{m\times n}$ if B is obtained from A by performing a finite sequence of elementary row operation on A.

we write $B \overset{R}{\sim} A$ to denoted B is row equivalent.

Echelon form

$$\begin{bmatrix} 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduced Echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find rank

$$A = \begin{bmatrix} 2 & 6 & -2 \\ 5 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & -9 \\ 1 & 2 & 3 \end{bmatrix}$$

Rank of Matrix A

if we reduce a system to Achelon form No of Non zero rows is called rank of a matrix.

$$A = \begin{bmatrix} 5 & 9 & 3 \\ -3 & 5 & 6 \\ -1 & -5 & -3 \end{bmatrix} \quad A \overset{R}{\sim} \begin{bmatrix} -1 & -5 & -3 \\ -3 & 5 & 6 \\ 5 & 9 & 3 \end{bmatrix} \text{ by } R_{13}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 5 & 3 \\ -3 & 5 & 6 \\ 5 & 9 & 3 \end{bmatrix} -R_1 \quad \overset{R}{\sim} \begin{bmatrix} 1 & 5 & 3 \\ 0 & 20 & 15 \\ 0 & -16 & -12 \end{bmatrix} \begin{matrix} R_2+3R_1 \\ R_3+(-5)R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & 3/4 \\ 0 & -16 & -12 \end{bmatrix} \frac{1}{20}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank

Calculate Adjoint of a Matrix A.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ -2 & 1 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Adjoint of matrix is Transpose of cofactor Matrix:

let A_{ij} denote Cofactor of A

$$A_{ij} = (-1)^{i+j} \begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)^2 [(-1)(+1) - (1)(2)]$$

$$= (+1) [-1 - 2]$$

$$= -3$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix}$$

R_2

$$= (-1)^3 [(1)(1) - (-2)(2)]$$

$R_3 + kR_2$

$$= -1 (1 + 4)$$

$$= -5$$

$= 2 /$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix}$$

$$= +1 [(1)(1) - (-2)(-1)]$$

$$= +1 (1 - 2) = -1$$

Reduce Matrix to

System of linear equation

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots\; a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots\; a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots\; a_{mn}x_n &= b_m \end{aligned}\right\} \rightarrow (1)$$

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

Matrix from of these equat

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Matrix form of system of linear equation

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Homogeneous system of equation

if in eq 1 $b = 0$

then system is called Homogeneous system of equation

if $b \neq 0$ then system is called Non Homogeneous equation

* If we reduce a system by elementary row operations

$$x_1 + a_{12} x_2 + a_{13} x_3 + \cdots a_{1n} x_n = b_1$$

$$x_2 + a_{23} x_3 + \cdots a_{2n} x_n = b_2$$

$$\vdots$$

$$x_{n-1} + a_{(n-1),n} x_n = b_{n-1}$$

$$x_n = b_n$$

But if rank of matrix

Solve the system of eqs by Gauss elimination method

$$x_1 + 5x_2 + 2x_3 = 9$$
$$x_1 + x_2 + 7x_3 = 6$$
$$\cancel{x_1} - 3x_2 + 4x_3 = -2$$

$$Ab \begin{bmatrix} 1 & 5 & 2 & 9 \\ 1 & 1 & 7 & 6 \\ 0 & -3 & 4 & -2 \end{bmatrix}$$

$$R \sim \begin{bmatrix} 1 & 5 & 2 & 9 \\ 0 & -4 & 5 & -3 \\ 0 & -3 & 4 & -2 \end{bmatrix} \text{ by } R_2 - R_1$$

$$R \sim \begin{bmatrix} 1 & 5 & 2 & 9 \\ 0 & -1 & 1 & -1 \\ 0 & -3 & 4 & 2 \end{bmatrix} \text{ by } R_2 - R_3$$

$$R \sim \begin{bmatrix} 1 & 5 & 2 & 9 \\ 0 & +1 & -1 & -1 \\ 0 & -3 & 4 & -2 \end{bmatrix} \text{ by } (-1) R_2$$

$$R \sim \begin{bmatrix} 1 & 5 & 2 & 9 \\ 0 & 1 & -1 & +1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ by } R_3 + 3R_1$$

$$x_1 + 5x_2 + 2x_3 = 9 \quad \text{—— (1)}$$
$$x_2 - x_3 = +1 \quad \text{—— (2)}$$
$$x_3 = +1 \quad \text{—— (3)}$$

A

Put in ②

$x_2 - 1 = +1$

$x_2 = 1+1$

$x_2 = 2$

$x_1 + 5(2) + 2(1) = 9$

$x_1 + 10 + 2 = 9$

$x + 12 = 9$

$x = 9 - 12$

$\boxed{x = -3}$

$x_1 + 2x_2 + x_3 = 160$

$2x_1 + x_2 + x_3 = 200$

$2x_1 + \quad 2x_3 = 240$

Q2

$x_1 - x_2 + 2x_3 = 0$

$4x_1 + x_2 + 2x_3 = 1$

$x_1 + x_2 + x_3 = -1$

Q3

$\frac{1}{4}$ ~~xxxx~~

$\frac{1}{4}x_1 + \frac{2}{4}x_2 + \frac{1}{4}x_3 = 40$

$\frac{2}{4}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3 = 50$

$\frac{2}{4}x_1 + \quad + \frac{2}{4}x_3 = 60$

Equivalent system as

if Rank A = rank A_b < No of unknown
infinite many solution
Rank A = rank b = No of unknown
unique sol exist

$$x_1 - x_2 + x_3 - x_4 + x_5 = 1$$
$$2x_1 + x_2 + 3x_3 + 4x_5 = 3$$
$$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$$
$$x_2 + x_4 + x_5 = 0$$

Rank A ≠ Rank A_b
No solution exist

Sol:

$$A_b \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & : & 1 \\ 2 & 1 & 3 & 0 & 4 & : & 3 \\ 3 & -2 & 2 & 1 & 1 & : & 0 \\ 0 & 1 & 0 & 1 & 1 & : & 0 \end{bmatrix}$$

$$A_b \sim \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & : & 1 \\ 0 & 3 & 1 & 2 & 2 & : & 0 \\ 0 & 1 & -1 & 4 & -2 & : & -2 \\ 0 & 1 & 0 & 1 & 1 & : & 1 \end{bmatrix}$$ by $R_2 - 2R_1$, $R_3 - 3R_1$

$$R \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 4 & -2 & -2 \\ 0 & 3 & 1 & 2 & 2 & 1 \end{bmatrix}$$ by R_{24}

$$R \begin{bmatrix} 1 & 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 3 & -3 & -2 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$ by $R_1 + R_2$, $R_3 - R_2$, $R_4 - 3R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 & -4 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix} \quad \begin{matrix} R_1 + (-1)R_{34} \\ R_3 + R_4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 2 & -4 & -1 \end{bmatrix} \quad \text{by } R_{34}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -2 & -1/2 \end{bmatrix} \quad \tfrac{1}{2}R_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 5 & 1/2 \\ 0 & 1 & 0 & 0 & 3 & 1/2 \\ 0 & 0 & 1 & 0 & -3 & 1/2 \\ 0 & 0 & 0 & 1 & -2 & -1/2 \end{bmatrix} \quad \begin{matrix} \text{by } R_1 - R_4 \\ R_2 - R_4 \\ R_3 + R_4 \end{matrix}$$

rank A = Rank Ab = 4 < No of unknown infinite

$x_1 + 5x_5 = 1/2 \Rightarrow x_1 = 1/2 - 5x_5$

$x_2 + 3x_5 = 1/2 \qquad x_2 = 1/2 - 3x_5$

$x_3 - 3x_5 = 1/2 \Rightarrow$

$x_4 - 2x_5 = 1/2$

Q2

$$x_1 + x_2 - x_3 = 1$$
$$x_2 + x_3 - x_4 = 1$$
$$x_3 + x_4 - x_5 = 1$$
$$x_5 + x_4 - x_3 = 1$$
$$x_4 + x_3 - x_2 = 1$$

$$A_b \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$R \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 & 2 \end{bmatrix} \quad \begin{matrix} R_1 - R_2 \\ R_5 + R_2 \end{matrix}$$

$$R \begin{bmatrix} 1 & 0 & 0 & 3 & -2 & 2 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & -2 & 2 & 0 \end{bmatrix} \quad \text{by} \begin{matrix} R_1 + 2R_3 \\ R_2 - R_3 \\ R_4 + R_3 \\ R_5 - 2R_3 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 & 3 & -2 & 2 \\ 0 & 1 & 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} \text{by } \tfrac{1}{2} R_4 \\ \tfrac{1}{2} R_5 \end{array}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & -1 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 + 3R_4 \\ R_2 + 2R_4 \\ R_3 - R_4 \\ R_5 + R_4 \end{array}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 + 2R_5 \\ R_2 - R_5 \\ R_3 + R_5 \end{array}$$

clearly rank A = rank Ab = 5 (No of unknown

So system have unique solution

Q3

$$\cancel{x_1 - 2x_2 - 7x_3 + 7x_4 = 5}$$

$$\cancel{+x_1 + 2x_2 - 7x_3 + 7x_4 = +5}$$

$$x_1 - 2x_2 - 7x_3 + 7x_4 = 5$$
$$-x_1 + 2x_2 + 8x_3 - 5x_4 = -7$$
$$3x_1 - 4x_2 - 17x_3 + 13x_4 = 14$$
$$2x_1 - 2x_2 - 11x_3 + 8x_4 = 7$$

Q4

$$x_1 + 2x_2 + x_3 = -1$$
$$6x_1 + x_2 + x_3 = -4$$
$$2x_1 - 3x_2 - x_3 = 0$$
$$x_1 - x_2 = 1$$

Q

Q
$$2x_1 + x_2 + 5x_3 = 4$$
$$3x_1 - 2x_2 + 2x_3 = 2$$
$$5x_1 - 8x_2 - 4x_3 = 1$$

$$A_b = \begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 2 & 1 & 5 & 4 \\ 1 & -3 & -3 & -2 \\ 5 & -8 & -4 & 1 \end{bmatrix} \quad \text{by } R_2 - R_1$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -3 & -3 & -2 \\ 2 & 1 & 5 & 4 \\ 5 & -8 & -4 & 1 \end{bmatrix} \quad \text{by } R_{12}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -3 & -3 & -2 \\ 0 & 7 & 11 & 8 \\ 0 & 7 & 11 & 11 \end{bmatrix} \quad \text{by } \begin{matrix} R_2 - 2R_1 \\ R_3 - 5R_1 \end{matrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -3 & -3 & -2 \\ 0 & 7 & 11 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix} \quad \text{by } R_3 - R_2$$

$$\sim^{R} \begin{bmatrix} 1 & -3 & -3 & -2 \\ 0 & 1 & 11/7 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ by } \frac{1}{7}R_2, \frac{1}{3}R_3$$

thus rank $A = 2$

Rank $Ab = 3$

Rank $A \neq$ Rank Ab

Hence there is no solution

Q No.

$$x - 3y + z = 2$$
$$2x + y + 3z = 3$$
$$x + 5y + 5z = 0$$

Q

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y - 4z = 6$$

$2x_1 \quad 7x_3 = 1$

$2x_1 + 4x_2 - x_3 = -2$

$x_1 - 8x_2 - 3x_3 = 2$

Q no: 1

$x_1 + x_2 + x_3 = a$

$x_1 + (1+a)x_2 + x_3 = 2a$

$x_1 + x_2 + (1+a)x_3 = 3a$

Sol $A_b = \begin{bmatrix} 1 & 1 & 1 & a \\ 1 & 1+a & 1 & 2a \\ 1 & 1 & 1+a & 3a \end{bmatrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & a & 0 & a \\ 0 & 0 & a & 2a \end{bmatrix} \quad \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} \frac{1}{a} R_2 \\ \frac{1}{a} R_3 \end{matrix}$

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & a-3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_1 - R_2 \\ R_1 - R_3 \end{matrix}$

$x_3 = 2, \quad x_2 = 1, \quad x_1 = a-3$