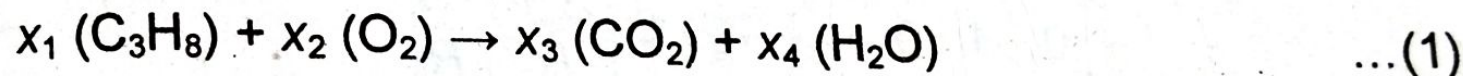


In Questions 68 – 72, write a balanced equation for the given chemical reaction.



Sol: Let x_1 , x_2 , x_3 , and x_4 be positive integers that balance the equation



Equating the number of atoms of each type on the two sides yields

$$3x_1 = 1x_3 \quad \text{Carbon (C)}$$

$$8x_1 = 2x_4 \quad \text{Hydrogen (H)}$$

$$2x_2 = 2x_3 + 1x_4 \quad \text{Oxygen (O)}$$

from which we obtain the homogeneous linear system

$$3x_1 - x_3 = 0 \quad \text{or} \quad 3x_1 + 0x_2 - x_3 + 0x_4 = 0$$

$$8x_1 - 2x_4 = 0 \quad \text{or} \quad 4x_1 + 0x_2 + 0x_3 - 1x_4 = 0$$

$$2x_2 - 2x_3 - x_4 = 0 \quad \text{or} \quad 0x_1 + 2x_2 - 2x_3 - x_4 = 0$$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}$$

Next we find the reduced echelon form of the augmented matrix as follows

$$A_b = \begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}, R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}, R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -4 & 3 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}, R_2 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 3 & 0 \end{bmatrix}, R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \end{bmatrix}, \frac{1}{2}R_2, -\frac{1}{4}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 + \frac{3}{4} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} - \frac{3}{4} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \end{bmatrix}, R_1 - R_3, R_2 + R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{4} & 0 \\ 0 & 1 & 0 & -\frac{5}{4} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \end{bmatrix}$$

from which we get

$$x_1 - \frac{1}{4}x_4 = 0$$

$$x_2 - \frac{5}{4}x_4 = 0$$

$$x_3 - \frac{3}{4}x_4 = 0$$

We can write above equations in terms of x_4 as follows:

$$x_1 = \frac{1}{4} x_4$$

$$x_2 = \frac{5}{4} x_4$$

$$x_3 = \frac{3}{4} x_4$$

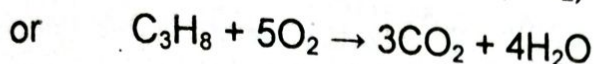
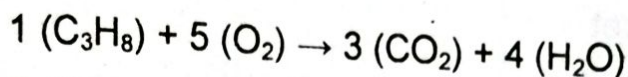
from which we conclude that the general solution of the system is

$$x_1 = \frac{t}{4}, x_2 = \frac{5t}{4}, x_3 = \frac{3t}{4}, x_4 = t$$

where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 4$, so the equation can be balanced by letting

$$x_1 = 1, x_2 = 5, x_3 = 3, x_4 = 4$$

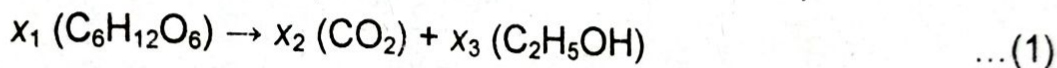
Making these substitutions in (1), we have



which is the required balanced equation.

69. $\text{C}_6\text{H}_{12}\text{O}_6 \rightarrow \text{CO}_2 + \text{C}_2\text{H}_5\text{OH}$ (fermentation of sugar)

Sol: Let x_1 , x_2 , and x_3 be positive integers that balance the equation



Equating the number of atoms of each type on the two sides yields

$$6x_1 = 1x_2 + 2x_3 \quad \text{Carbon (C)}$$

$$12x_1 = 6x_3 \quad \text{Hydrogen (H)}$$

$$6x_1 = 2x_2 + 1x_3 \quad \text{Oxygen (O)}$$

from which we obtain the homogeneous linear system

$$6x_1 - x_2 - 2x_3 = 0 \quad \text{or} \quad 6x_1 - x_2 - 2x_3 = 0$$

$$2x_1 - x_3 = 0 \quad \text{or} \quad 2x_1 + 0x_2 - 1x_3 = 0$$

$$6x_1 - 2x_2 - x_3 = 0 \quad \text{or} \quad 6x_1 - 2x_2 - x_3 = 0$$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 6 & -1 & -2 & 0 \\ 2 & 0 & -1 & 0 \\ 6 & -2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 6 & -1 & -2 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}, R_3 - R_1$$

$$\sim \begin{bmatrix} 0 & -1 & 1 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}, R_1 - 3R_2$$

$$\sim \begin{bmatrix} 0 & -1 & 1 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R_3 - R_1$$

$$\sim \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \frac{1}{2}R_2, -1R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R_2 \leftrightarrow R_1$$

from which we get

$$x_1 - \frac{1}{2}x_3 = 0$$

$$x_2 - x_3 = 0$$

We can write above equations in terms of x_3 as follows:

$$x_1 = \frac{1}{2}x_3$$

$$x_2 = x_3$$

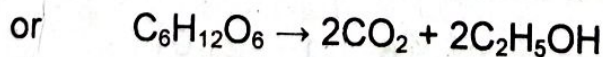
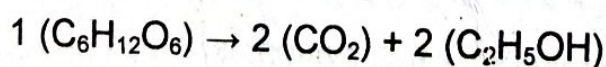
from which we conclude that the general solution of the system is

$$x_1 = \frac{t}{2}, x_2 = t, x_3 = t$$

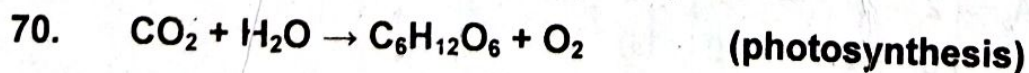
where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 2$, so the equation can be balanced by letting

$$x_1 = 1, x_2 = 2, x_3 = 2$$

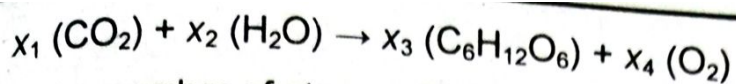
Making these substitutions in (1), we have



which is the required balanced equation.



Sol: Let x_1, x_2, x_3 , and x_4 be positive integers that balance the equation



...(1)

Equating the number of atoms of each type on the two sides yields

$$1x_1 = 6x_3$$

Carbon (C)

$$2x_2 = 12x_3$$

Hydrogen (H)

$$2x_1 + 1x_2 = 6x_3 + 2x_4$$

Oxygen (O)

from which we obtain the homogeneous linear system

$$1x_1 - 6x_3 = 0$$

$$\text{or } 1x_1 + 0x_2 - 6x_3 + 0x_4 = 0$$

$$1x_2 - 6x_3 = 0$$

$$\text{or } 0x_1 + 1x_2 - 6x_3 + 0x_4 = 0$$

$$2x_1 + x_2 - 6x_3 - 2x_4 = 0$$

$$\text{or } 2x_1 + 1x_2 - 6x_3 - 2x_4 = 0$$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 2 & 1 & -6 & -2 & 0 \end{bmatrix}$$

Next we find the reduced echelon form of the augmented matrix as follows:

$$A_b = \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 2 & 1 & -6 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 0 & 1 & 6 & -2 & 0 \end{bmatrix}, R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 12 & -2 & 0 \end{bmatrix}, R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{bmatrix}, \frac{1}{12}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{bmatrix}, \begin{matrix} R_1 + 6R_3, \\ R_2 + 6R_3 \end{matrix}$$

from which we get

$$x_1 - x_4 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 - \frac{1}{6}x_4 = 0$$

We can write above equations in terms of x_4 as follows:

$$x_1 = x_4$$

$$x_2 = x_4$$

$$x_3 = \frac{1}{6} x_4$$

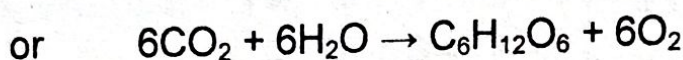
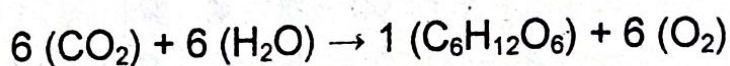
from which we conclude that the general solution of the system is

$$x_1 = t, x_2 = t, x_3 = \frac{t}{6}, x_4 = t$$

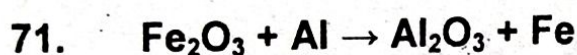
where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 6$, so the equation can be balanced by letting

$$x_1 = 6, x_2 = 6, x_3 = 1, x_4 = 6$$

Making these substitutions in (1), we have

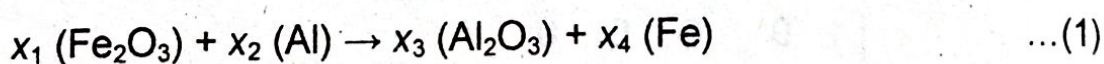


which is the required balanced equation.



(Fe = iron, Al = aluminium, O = oxygen)

Sol: Let x_1, x_2, x_3 , and x_4 be positive integers that balance the equation



Equating the number of atoms of each type on the two sides yields

$$2x_1 = 1x_4$$

Iron (Fe)

$$1x_2 = 2x_3$$

Aluminium (Al)

$$3x_1 = 3x_3$$

Oxygen (O)

from which we obtain the homogeneous linear system

$$2x_1 - 1x_4 = 0$$

$$\text{or } 2x_1 + 0x_2 + 0x_3 - 1x_4 = 0$$

$$1x_2 - 2x_3 = 0$$

$$\text{or } 0x_1 + 1x_2 - 2x_3 + 0x_4 = 0$$

$$1x_1 - 1x_3 = 0$$

$$\text{or } 1x_1 + 0x_2 - 1x_3 + 0x_4 = 0$$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

Next we find the reduced echelon form of the augmented matrix as follows:

$$A_b = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \end{bmatrix}, R_3 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}, R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}, R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}, \frac{1}{2}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}, R_1 + R_3$$

from which we get

$$x_1 - \frac{1}{2}x_4 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 - \frac{1}{2}x_4 = 0$$

We can write above equations in terms of x_4 as follows:

$$x_1 = \frac{1}{2}x_4$$

$$x_2 = x_4$$

$$x_3 = \frac{1}{2}x_4$$

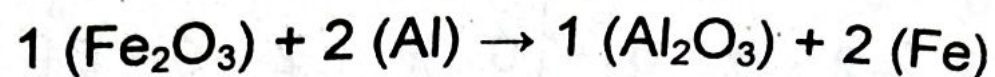
from which we conclude that the general solution of the system is

$$x_1 = \frac{t}{2}, x_2 = t, x_3 = \frac{t}{2}, x_4 = t$$

where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 2$, so the equation can be balanced by letting

$$x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 2$$

Making these substitutions in (1), we have



which is the required balanced equation.