In Questions 68 – 72, write a balanced equation for the given chemical reaction.

68. **$C_3H_8 + O_2 \rightarrow CO_2 + H_2O$ (propane combustion)**

Sol: Let x_1, x_2, x_3, and x_4 be positive integers that balance the equation

$$x_1\,(C_3H_8) + x_2\,(O_2) \rightarrow x_3\,(CO_2) + x_4\,(H_2O) \qquad \ldots(1)$$

Equating the number of atoms of each type on the two sides yields

$3x_1 = 1x_3$ **Carbon (C)**

$8x_1 = 2x_4$ **Hydrogen (H)**

$2x_2 = 2x_3 + 1x_4$ **Oxygen (O)**

from which we obtain the homogeneous linear system

$3x_1 - x_3 = 0$ or $3x_1 + 0x_2 - x_3 + 0x_4 = 0$

$8x_1 - 2x_4 = 0$ or $4x_1 + 0x_2 + 0x_3 - 1x_4 = 0$

$2x_2 - 2x_3 - x_4 = 0$ or $0x_1 + 2x_2 - 2x_3 - x_4 = 0$

The augmented matrix for this system is

ZR

$$A_b = \begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}$$

Next we find the reduced echelon form of the augmented matrix as follows

$$A_b = \begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}, \; R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}, \; R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -4 & 3 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}, \; R_2 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 3 & 0 \end{bmatrix}, \; R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \end{bmatrix}, \; \frac{1}{2}R_2, \; -\frac{1}{4}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1+\frac{3}{4} & 0 \\ 0 & 1 & 0 & -\frac{1}{2}-\frac{3}{4} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \end{bmatrix}, \; \begin{matrix} R_1 - R_3, \\ R_2 + R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{4} & 0 \\ 0 & 1 & 0 & -\frac{5}{4} & 0 \\ 0 & 0 & 1 & -\frac{3}{4} & 0 \end{bmatrix}$$

from which we get

$$x_1 - \frac{1}{4}x_4 = 0$$

$$x_2 - \frac{5}{4}x_4 = 0$$

$$x_3 - \frac{3}{4}x_4 = 0$$

We can write above equations in terms of x_4 as follows:

$$x_1 = \frac{1}{4}x_4$$

$$x_2 = \frac{5}{4}x_4$$

$$x_3 = \frac{3}{4}x_4$$

from which we conclude that the general solution of the system is

$$x_1 = \frac{t}{4},\ x_2 = \frac{5t}{4},\ x_3 = \frac{3t}{4},\ x_4 = t$$

where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 4$, so the equation can be balanced by letting

$$x_1 = 1,\ x_2 = 5,\ x_3 = 3,\ x_4 = 4$$

Making these substitutions in (1), we have

$$1\,(C_3H_8) + 5\,(O_2) \rightarrow 3\,(CO_2) + 4\,(H_2O)$$

or $\quad C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

which is the required balanced equation.

69. $\mathbf{C_6H_{12}O_6 \rightarrow CO_2 + C_2H_5OH}$ **(fermentation of sugar)**

Sol: Let x_1, x_2, and x_3 be positive integers that balance the equation

$$x_1\,(C_6H_{12}O_6) \rightarrow x_2\,(CO_2) + x_3\,(C_2H_5OH) \qquad \ldots(1)$$

Equating the number of atoms of each type on the two sides yields

$6x_1 = 1x_2 + 2x_3$ **Carbon (C)**

$12x_1 = 6x_3$ **Hydrogen (H)**

$6x_1 = 2x_2 + 1x_3$ **Oxygen (O)**

from which we obtain the homogeneous linear system

$6x_1 - x_2 - 2x_3 = 0$ or $6x_1 - x_2 - 2x_3 = 0$

$2x_1 - x_3 = 0$ or $2x_1 + 0x_2 - 1x_3 = 0$

$6x_1 - 2x_2 - x_3 = 0$ or $6x_1 - 2x_2 - x_3 = 0$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 6 & -1 & -2 & 0 \\ 2 & 0 & -1 & 0 \\ 6 & -2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 6 & -1 & -2 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}, R_3 - R_1$$

$$\sim\begin{bmatrix}0 & -1 & 1 & 0\\ 2 & 0 & -1 & 0\\ 0 & -1 & 1 & 0\end{bmatrix}, R_1-3R_2$$

$$\sim\begin{bmatrix}0 & -1 & 1 & 0\\ 2 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, R_3-R_1$$

$$\sim\begin{bmatrix}0 & 1 & -1 & 0\\ 1 & 0 & -\frac{1}{2} & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \frac{1}{2}R_2, -1R_1$$

$$\sim\begin{bmatrix}1 & 0 & -\frac{1}{2} & 0\\ 0 & 1 & -1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, R_2 \leftrightarrow R_1$$

from which we get

$$x_1 - \frac{1}{2}x_3 = 0$$
$$x_2 - x_3 = 0$$

We can write above equations in terms of x_3 as follows:

$$x_1 = \frac{1}{2}x_3$$
$$x_2 = x_3$$

from which we conclude that the general solution of the system is

$$x_1 = \frac{t}{2},\ x_2 = t,\ x_3 = t$$

where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 2$, so the equation can be balanced by letting

$$x_1 = 1,\ x_2 = 2,\ x_3 = 2$$

Making these substitutions in (1), we have

$$1\,(C_6H_{12}O_6) \rightarrow 2\,(CO_2) + 2\,(C_2H_5OH)$$

or $\quad C_6H_{12}O_6 \rightarrow 2CO_2 + 2C_2H_5OH$

which is the required balanced equation.

70. $\mathbf{CO_2 + H_2O \rightarrow C_6H_{12}O_6 + O_2}$ **(photosynthesis)**

<u>Sol</u>: Let x_1, x_2, x_3, and x_4 be positive integers that balance the equation

$$x_1\,(CO_2) + x_2\,(H_2O) \rightarrow x_3\,(C_6H_{12}O_6) + x_4\,(O_2) \qquad \ldots(1)$$

Equating the number of atoms of each type on the two sides yields

$1x_1 = 6x_3$ **Carbon (C)**

$2x_2 = 12x_3$ **Hydrogen (H)**

$2x_1 + 1x_2 = 6x_3 + 2x_4$ **Oxygen (O)**

from which we obtain the homogeneous linear system

$1x_1 - 6x_3 = 0$ or $1x_1 + 0x_2 - 6x_3 + 0x_4 = 0$

$1x_2 - 6x_3 = 0$ or $0x_1 + 1x_2 - 6x_3 + 0x_4 = 0$

$2x_1 + x_2 - 6x_3 - 2x_4 = 0$ or $2x_1 + 1x_2 - 6x_3 - 2x_4 = 0$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 2 & 1 & -6 & -2 & 0 \end{bmatrix}$$

Next we find the reduced echelon form of the augmented matrix as follows:

$$A_b = \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 2 & 1 & -6 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 0 & 1 & 6 & -2 & 0 \end{bmatrix},\ R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 12 & -2 & 0 \end{bmatrix},\ R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -6 & 0 & 0 \\ 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{bmatrix},\ \frac{1}{12}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{bmatrix},\ \begin{matrix} R_1 + 6R_3, \\ R_2 + 6R_3 \end{matrix}$$

from which we get

$$x_1 - x_4 = 0$$
$$x_2 - x_4 = 0$$
$$x_3 - \frac{1}{6}x_4 = 0$$

We can write above equations in terms of x_4 as follows:

Z.R. Bhatti

$$x_1 = x_4$$
$$x_2 = x_4$$
$$x_3 = \frac{1}{6}x_4$$

from which we conclude that the general solution of the system is

$$x_1 = t,\ x_2 = t,\ x_3 = \frac{t}{6},\ x_4 = t$$

where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 6$, so the equation can be balanced by letting

$$x_1 = 6,\ x_2 = 6,\ x_3 = 1,\ x_4 = 6$$

Making these substitutions in (1), we have

$$6\,(CO_2) + 6\,(H_2O) \rightarrow 1\,(C_6H_{12}O_6) + 6\,(O_2)$$

or $$6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2$$

which is the required balanced equation.

71. **$Fe_2O_3 + Al \rightarrow Al_2O_3 + Fe$**

(Fe = iron, Al = aluminium, O = oxygen)

Sol: Let x_1, x_2, x_3, and x_4 be positive integers that balance the equation

$$x_1\,(Fe_2O_3) + x_2\,(Al) \rightarrow x_3\,(Al_2O_3) + x_4\,(Fe) \quad \ldots(1)$$

Equating the number of atoms of each type on the two sides yields

$2x_1 = 1x_4$	**Iron (Fe)**
$1x_2 = 2x_3$	**Aluminium (Al)**
$3x_1 = 3x_3$	**Oxygen (O)**

from which we obtain the homogeneous linear system

$2x_1 - 1x_4 = 0$	or	$2x_1 + 0x_2 + 0x_3 - 1x_4 = 0$
$1x_2 - 2x_3 = 0$	or	$0x_1 + 1x_2 - 2x_3 + 0x_4 = 0$
$1x_1 - 1x_3 = 0$	or	$1x_1 + 0x_2 - 1x_3 + 0x_4 = 0$

The augmented matrix for this system is

$$A_b = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

Next we find the reduced echelon form of the augmented matrix as follows:

$$A_b = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \end{bmatrix}, \quad R_3 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}, \quad R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}, \quad R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}, \quad \frac{1}{2}R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}, \quad R_1 + R_3$$

from which we get

$$x_1 - \frac{1}{2}x_4 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 - \frac{1}{2}x_4 = 0$$

We can write above equations in terms of x_4 as follows:

$$x_1 = \frac{1}{2}x_4$$

$$x_2 = x_4$$

$$x_3 = \frac{1}{2}x_4$$

from which we conclude that the general solution of the system is

$$x_1 = \frac{t}{2}, \; x_2 = t, \; x_3 = \frac{t}{2}, \; x_4 = t$$

where t is arbitrary. The smallest positive integer values for the unknowns occur when we let $t = 2$, so the equation can be balanced by letting

$$x_1 = 1, \; x_2 = 2, \; x_3 = 1, \; x_4 = 2$$

Making these substitutions in (1), we have

$$1\,(Fe_2O_3) + 2\,(Al) \rightarrow 1\,(Al_2O_3) + 2\,(Fe)$$

or $$Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$$

which is the required balanced equation.