Probability

Chapter Outline

1. Dependent and Independent Events
2. Conditional Probability

Dependent and Independent Events

Independent Events: Two events A and B in the same sample space S, are defined to be independent if the Probability that one event occurs, is not affected by whether the other event has or has not occurred, that is

$$P(A/B) = P(A) \text{ and } P(B/A) = P(B).$$

It then follows that two events A and B are independent if and only if

$$P(A \cap B) = P(A).P(B)$$

Dependent Events

The events A and B are defined to be dependent If $P(A \cap B) \neq P(A).P(B)$.This means that occurrence of one of the events in some way affects the Probability of the occurrence of other event.

EXAMPLE

Two events A and B are such that $P(A)= 1/4$

$P(A/B)=1/2$ and $P(B/A)= 2/3$.

(i) Are A and B Independent events.

(ii) Are A and B mutually exclusive events.

(iii) Find $P(A \cap B)$ and $P(B)$.

Solution:

(i) If A and B are independent events , then
$P(A/B)=P(A)$

SOLUTION

Now $P(A)=1/4$ and $P(B)=1/2$ i.e $P(A/B)\neq P(A)$

Hence A and B are not independent.

(ii) If A and B are mutually exclusive events, then

$P(A/B)=0$ But it is given that $P(A/B)=1/2$.

Hence A and B are not mutually exclusive events.

(iii) Now $P(A\cap B)=P(A).P(B/A)$

$$=(1/4).(2/3)$$

$$= 1/6$$

Solution

By definition ,we have

$P(B)P(A/B)=P(A).P(B/A)$

$P(B)(1/2)=(1/4)(2/3)$

So that

$P(B)=(I/4)_x(2/3)_x(2/1)$

$= 1/3$

Conditional Probability

The sample space for an experiment must often be changed when some additional information pertaining to the outcome of the experiment is received .The effect of such information is to Reduce the sample space by excluding some outcomes as being impossible which before receiving the information were believed possible. The probabilities associated with such reduced Sample space are called conditional probabilities.

Example

Let us consider the die throwing experiment with sample space S =(1,2,3,4,5,6).Suppose we wish to know the probability of the outcome that the die shows 6 say event A . If before seeing the outcome, we are told that the die shows an even number of dots, say event B, then the information that the die shows the even number excludes the outcomes 1,3 and 5.Thus it reduces the original sample space that consist of 3 outcomes 2,4 and 6.

Conditional Probability

If A and B are two Events in a sample space S and if the P(B) is not equal to zero , then the Conditional probability of an event A given that Event B has occurred is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Similarly if P(A) is not equal to zero then B given A is

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Numerical Example

Two coins are tossed what is the conditional

Probability that two heads result given that

There is at least one head?

Solution:

S= (HH,HT,TH,TT)

Let A represent the event that two heads appear

And B the event that there is at least one head

Solution

Here

$A=(HH)$ $P(A)=1/4$

$B=(HH,HT,TH)$ $P(B)=3/4$

$A\cap B=(HH)$ $P(A\cap B)=1/4$

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

$$P(A/B) = \frac{1/4}{3/4} = 1/3$$

Numerical Example

A man tosses a fair dice .What is the conditional Probability that sum of two dice will be 7 given That the two dice had the same number.

Solution : Sample space of pair of dice is

S={ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

Solution

Let A ={ The sum is 7}={(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

P(A)=6/36

B={two dice had same outcome}={ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

P(B)=6/36

$A \cap B = O$

$P(A \cap B) = 0$

Solution

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) = \frac{O}{6/36} = O$$