

Probability

Chapter Outline

Problems solving related to Probability

Question 1

Find the Probability for each of following events:

- a) An odd number appears in a single toss of fair dice.

Solution:

$$\text{Probability} = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$$

$$S=\{1,2,3,4,5,6\}$$

Solution

Let A=Favourable outcomes=odd numbers in fair die={1,3,5}

Total outcomes in sample space=6

$$P(A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$$

$$\begin{aligned} P(A) &= 3/6 \\ &= 1/2 \end{aligned}$$

b)The sum 8 appears in a single toss of a pair of fair dice

Solution:

$$S=\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), \}$$

Solution

Let A=Sum 8 appears in two dice

$$A=\{(6,2), (5,3), (4,4), (3,5), (2,6)\}$$

Total outcomes in sample space=36

$$P(A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$$

$$P(A)=5/36$$

c)At least one head appears in three tosses of a fair coin.

Solution:

Three coins

$$S=\{\text{HHH}, \text{HTH}, \text{THH}, \text{TTH}, \text{HHT}, \text{HTT}, \text{THT}, \text{TTT}\}$$

Let A=At least one head in tosses of three coins

$$A=\{\text{HHH}, \text{HTH}, \text{THH}, \text{TTH}, \text{HHT}, \text{HTT}, \text{THT}\}$$

Total outcomes in sample space=8

Solution

$P(A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$

$$P(A) = 7/8$$

Question 2

In a single throw of two fair dice , find the Probability that the product of the numbers on The dice is

- (i) between 8 and 16 (both inclusive)
- (ii) Divisible by 4

Solution

i) between 8 and 16 (both inclusive)

$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
 $(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
 $(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
 $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6), \}$

A=Product on two dice is between 8 and 16 (both inclusive)

$= \{ (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (4,2), (4,3),$
 $(4,4), (5,2), (5,3), (6,2) \}$

Solution

$P(A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$

$$P(A) = 12/36$$

$$= 1/3$$

Solution (ii) Divisible by 4

Let B = Product is divisible by 4

$$= \{(1,4), (2,2), (2,4), (2,6), (3,4), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,4), (6,2), (6,4), (6,6)\}$$

$P(A) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$

$$P(A) = 15/36$$

$$= 5/12$$

Question 3

Of 12 eggs in a refrigerator, 2 are bad . From these 4 eggs are chosen at random to make a Cake .What are the Probabilities that

- (i) Exactly one is bad
- (ii) At least one is bad

Solution:

Good eggs	bad eggs	Total eggs
10	2	12

Solution

Chose or Select =4 eggs

(i) Exactly one is bad

$$= \frac{^2C_1 \cdot ^{10}C_3}{^{12}C_4}$$

$$= 0.4848$$

Solution

(ii) At least one is bad

$$= \frac{\underline{^2C_1} \underline{^{10}C_3}}{\underline{^{12}C_4}} + \frac{\underline{^2C_2} \underline{^{10}C_2}}{\underline{^{12}C_4}}$$

$$= 0.4848 + 0.0909$$

$$= 0.5758$$

Question 4

A bag contains 14 identical balls ,4 of which are Red,5 are black and 5 white . Six balls are drawn From the bag find the Probability that

- (i) 3 are red (ii) at least 2 are white

Solution:

Red	White	Black	Total
4	5	5	14

Solution

Balls drawn =6

(i) 3 are red

$$= \frac{{}^4C_3 \cdot {}^{10}C_3}{{}^{14}C_6}$$

$$= 160/1001$$

$$= 0.15984$$

Solution

(ii) At least 2 are white

$$= \frac{\underline{^5C_2} \underline{^9C_4}}{\underline{^{14}C_6}} + \frac{\underline{^5C_3} \underline{^9C_3}}{\underline{^{14}C_6}} + \frac{\underline{^5C_4} \underline{^9C_2}}{\underline{^{14}C_6}} + \frac{\underline{^5C_5} \underline{^9C_1}}{\underline{^{14}C_6}}$$
$$= \underline{1260+840+180+9}$$

$$\underline{^{14}C_6}$$

$$= 0.76223$$

Question 5

Three applicants are to be selected at random
Out of 4 boys and 6 girls. What is the Probability
of selecting (i) all girls (ii) all boys (iii) At least
One boy

Solution:

Boys	Girls	Total
4	6	10

Solution

Selected :3

(i) All girls

$$= \frac{{}^6C_3 \cdot {}^4C_0}{{}^{10}C_3}$$
$$= 20/120 = 1/6$$

(ii) All boys

$$= \frac{{}^6C_0 \cdot {}^4C_3}{{}^{10}C_3}$$
$$= 4/120 = 1/30$$

Solution

(iii) At least one is boy

$$= \frac{^4C_1}{^{10}C_3} \frac{^6C_2}{^{10}C_3} + \frac{^4C_2}{^{10}C_3} \frac{^6C_1}{^{10}C_3} + \frac{^4C_3}{^{10}C_3} \frac{^6C_0}{^{10}C_3}$$

$$= \frac{60+36+4}{120}$$

$$= 100/120$$

$$= 5/6$$

Question 6

An integer between 3 and 12 inclusive is chosen at random . What is the Probability that it is an even number ? That it is an even and divisible by 3?

Solution:

$$S=\{3,4,5,6,7,8,9,10,11,12\}$$

$$\text{Let } A=\{\text{Even number}\}=\{4,6,8,10,12\}$$

Total Outcomes:10

Solution

Probability= Favourable Outcomes
 Total Outcomes

$$P(A) = 5/10$$

$$P(A) = 1/2$$

Let $B = \{\text{Even number and divisible by 3}\}$

$$B = \{6, 12\}$$

$$P(B) = 2/10$$

$$P(B) = 1/5$$

Question 7

One integer is chosen at random from numbers 1,2,3.....50.What is the probability that Chosen number is divisible by 6 or 8.

Solution:

$$S=\{1,2,3,\dots,50\}$$

Let $A=\{\text{number divisible by } 6\}$

$$A=\{6,12,18,24,30,36,42,48\}$$

$B=\{\text{number divisible by } 8\}$

$$B=\{8,16,24,32,40,48\}$$

$$A \cap B = \{24,48\}$$

Solution

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 8/50 + 6/50 - 2/50$$

$$P(A \cup B) = 12/50$$

$$P(A \cup B) = 6/25$$

Question 8

In a group of 20 adults 4 out of 7 women and 2 out of 13 men wear glasses. What is the Probability that a person chosen at random is A woman or someone who wear glasses?

Solution:

	Women	Men	Total
	7	13	20
Wear glasses	4	2	6

Solution

Let $P(A) = \text{Woman} = 7/20$

$P(B) = \text{wear glasses} = 6/20$

$P(A \cap B) = \text{woman and wear glasses} = 4/20$

$P(\text{A or B}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 7/20 + 6/20 - 4/20$

$P(A \cup B) = 9/20$

Question 9

A pair of fair dice is thrown .If two numbers
Appearing are different find the Probability

- (i) The sum is 6 (ii)the sum is 4 or less

Solution : Sample space of pair of dice is

$$S=\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

Solution

Let $A=\{\text{two numbers appearing are different on two dice}\}$

$$A=\{ (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5) \}$$

$$P(A)=30/36$$

Solution

(i) The sum is 6

$$B = \{ \text{The sum is 6} \} = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$$

$$P(B) = 5/36$$

$$(A \cap B) = \{(1,5), (2,4), (4,2), (5,1)\}$$

$$P(A \cap B) = 4/36$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A/B) = \frac{4/36}{30/36} = 2/15$$

Solution

(ii) The sum is 4 or less

$C = \{\text{the sum is 4 or less}\} = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$

$$P(C) = 6/36$$

$A \cap C = \{(1,2), (1,3), (2,1), (3,1)\}$

$$P(A \cap C) = 4/36$$

$$P(C/A) = \frac{P(A \cap C)}{P(A)}$$

$$P(C/A) = \frac{4/36}{30/36} = 2/15$$

Question 10

The probability that A will be alive after 10 years
To come is $5/7$ and for B it is $7/9$. Find out the
Probability that:

- (i) both of them will die
- (ii) A will be alive and B will dead
- (iii) B will be alive and A dead
- (iv) both of them will be alive In 10 years to come.

Solution

Probability of alive for A and B is

$$P(A)=5/7 \quad P(B)=7/9$$

Probability for not alive is

$$P(\bar{A})= 1-P(A) \quad P(\bar{B})=1-P(B)$$

$$P(\bar{A})= 1-5/7 \quad P(\bar{B})=1-7/9$$

$$P(\bar{A}) =2/7 \quad P(\bar{B}) =2/9$$

Solution

(i) both of them will die

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= 2/7 \times 2/9 \\ &= 4/63 \end{aligned}$$

ii) A will be alive and B will dead

$$\begin{aligned} P(A \cap \bar{B}) &= 5/7 \times 2/9 \\ &= 10/63 \end{aligned}$$

Solution

(iii) B will be alive and A dead

$$\begin{aligned}P(B \cap \bar{A}) &= P(B) \times P(\bar{A}) \\&= 7/9 \times 2/7 \\&= 2/9\end{aligned}$$

(iv) both of them will be alive In 10 years to come.

$$\begin{aligned}P(A \cap B) &= P(A) \times P(B) \\P(A \cap B) &= 5/7 \times 7/9 \\P(A \cap B) &= 5/9\end{aligned}$$