Layout of Lecture 11

- ❑ **Mean(Average) Deviation**
  - ❑ Coefficient of M.D.
- ❑ Variance
- ❑ Standard Deviation
  - ❑ Coefficient of Variation

MEAN DEVIATION

The mean deviation (M.D.) of a set of data is defined as the arithmetic mean of the deviations measured either from mean or from the median, all deviations being counted as positive.

$$M.D. = \frac{\sum|x - \bar{x}|}{n} \quad (for\ sample\ ungrouped\ data)$$

$$M.D. = \frac{\sum f|x - \bar{x}|}{\sum f} \quad (for\ sample\ grouped\ data)$$

$$M.D. = \frac{\sum|x - \mu|}{n} \quad (for\ population\ data)$$

M.D.(Cont…)

MEAN DEVIATION		
	UNGROUPED DATA	GROUPED DATA
For Mean	$M.D. = \frac{\sum\lvert x-\bar{x}\rvert}{n}$	$M.D. = \frac{\sum f\lvert x-\bar{x}\rvert}{\sum f}$
For Median	$M.D. = \frac{\sum\lvert x-\tilde{x}\rvert}{n}$	$M.D. = \frac{\sum f\lvert x-\tilde{x}\rvert}{\sum f}$
For Mode	$M.D. = \frac{\sum\lvert x-\hat{x}\rvert}{n}$	$M.D. = \frac{\sum f\lvert x-\hat{x}\rvert}{\sum f}$

Ungrouped Data

EXAMPLE: Calculate the M.D. from (i) mean (ii) median and (iii) mode for the following data :
32,30,36,34,33,35,40,39,36.
Also compute their respective coefficients of M.D.
SOLUTION: Array 30,32,33,34,35,36,36,39,40
Here
Mode = Most frequented value or most repeated value
$\hat{x}$ = 36

Median= the $\left(\frac{n+1}{2}\right)^{th}$ value

$\tilde{x}$ = the $\left(\frac{9+1}{2}\right)^{th}$ value

$\tilde{x}$ = the $(5)^{th}$ value

$\tilde{x}$ = 35

Ungrouped Data

M.D.(Cont…)

Mean = $\frac{\sum X}{n}$

$\overline{x}$ = (315/9)=35

MEAN DEVIATION for Ungrouped data				
X	$X-\overline{X}$	$\lvert X-\overline{X}\rvert$	$\lvert X-\tilde{X}\rvert$	$\lvert X-\hat{X}\rvert$
32	-3	3	3	4
30	-5	5	5	6
36	1	1	1	0
34	-1	1	1	2
33	-2	2	2	3
35	0	0	0	1
40	5	5	5	4
39	4	4	4	3
36	1	1	1	0
315	0	22	22	23

Ungrouped Data

Now

$\sum|x-\bar{x}|= 22, \sum|x-\tilde{x}|= 22, \sum|x-\hat{x}|= 23.$

$$\text{M.D. from mean} = \frac{\sum|x-\bar{x}|}{n} = \frac{22}{9} = 2.44$$

$$\text{M.D. from median} = \frac{\sum|x-\tilde{x}|}{n} = \frac{22}{9} = 2.44$$

$$\text{M.D. from mode} = \frac{\sum|x-\hat{x}|}{n} = \frac{23}{9} = 2.56$$

COEFFICENT OF DISPERSION

Mean coefficient of dispersion $= \frac{\text{M.D.}}{\bar{x}} = \frac{2.44}{35} = 0.07$

Median coefficient of dispersion $= \frac{\text{M.D.}}{\tilde{x}} = \frac{2.44}{35} = 0.07$

Mode coefficient of dispersion $= \frac{\text{M.D.}}{\hat{x}} = \frac{2.56}{36} = 0.07$

Grouped Data

M.D.(Cont...)

EXAMPLE: Calculate the M.D. from (i) mean (ii) median and (iii) mode for the following data :

Class Interval	10-14	15-19	20-24	25-29	30-34	35-39	40-44
Freq.	01	04	08	11	15	09	02

Also compute their respective coefficients of M.D.

SOLUTION: First we have to find mean, median, mode.

MEAN DEVIATION for Grouped data					
C.I.	f	C.F.	C.B.	X=Mid Pt.	fX
10-14	01	01	09.5-14.5	12	12
15-19	04	05	14.5-19.5	17	68
20-24	08	13	19.5-24.5	22	176
25-29	f_1 = 11	c = 24	24.5-29.5	27	297
30-34	f_m=f =15	39	l=29.5-34.5	32	480
35-39	f_2 = 09	48	34.5-39.5	37	333
40-44	02	50	39.5-44.5	42	84
Total	$n=\sum f$=50	- -	- -	- -	$\sum fx$=1450

Median Class
Mode Class

Grouped Data

Now

$n = \sum f = 50, \sum fx = 1450$, c = 24,

Lower Class Boundary (l)= 29.5 ,

$h=(u-l)=(15-10)$ or $(20-15)= 5$,

$\left(\frac{n}{2}\right)^{th}$ class= $\left(\frac{50}{2}\right)^{th}$ class =25 th class

$f_m = f = 15$, $f_1 = 11$, $f_2 = 09$.

$$\text{Mean} = \bar{x} = \frac{\sum fx}{\sum f} = \frac{1450}{50} = 29$$

$$\text{Median} = \tilde{x} = l + \frac{h}{f}\left(\frac{n}{2} - c\right) = 29.5 + \frac{5}{15}\left(\frac{50}{2} - 24\right) = 29.83$$

$$\text{Mode} = \hat{x} = l + \frac{f_m - f_1}{(f_m - f_2) + (f_m - f_1)} \times h$$

$$= 29.5 + \frac{15-11}{(15-9)+(15-11)} \times 5 = 30.33$$

Grouped Data

M.D.(Cont…)

MEAN DEVIATION for Grouped data

C.I.	f	X	$\lvert X-\bar{X}\rvert$	$f\lvert X-\bar{X}\rvert$	$\lvert X-\tilde{X}\rvert$	$f\lvert X-\tilde{X}\rvert$	$\lvert X-\hat{X}\rvert$	$f\lvert X-\hat{X}\rvert$
10-14	01	12	17	17	17.83	17.83	18.33	18.33
15-19	04	17	12	48	12.83	51.32	13.33	53.32
20-24	08	22	7	56	7.83	62.64	8.33	66.64
25-29	11	27	2	22	2.83	31.13	3.33	36.63
30-34	15	32	3	45	2.17	32.55	1.67	25.05
35-39	09	37	8	72	7.17	64.53	6.67	60.03
40-44	02	42	13	26	12.17	24.34	11.67	23.34
Total	50	- -	- -	286	- -	284.34	- -	283.34

Grouped Data

M.D.(Cont…)

Now

$\sum f|x-\bar{x}|$= 286, $\sum f|x-\tilde{x}|$= 284.34, $\sum f|x-\hat{x}|$= 283.34

$$\text{M.D. from mean} = \frac{\sum|x-\bar{x}|}{n} = \frac{286}{50} = 5.72$$

$$\text{M.D. from median} = \frac{\sum|x-\tilde{x}|}{n} = \frac{284.34}{50} = 5.6868$$

$$\text{M.D. from mode} = \frac{\sum|x-\hat{x}|}{n} = \frac{283.34}{50} = 5.6668$$

COEFFICENT OF DISPERSION

$$\text{Mean coefficient of dispersion} = \frac{\text{M.D.}}{\bar{x}} = \frac{5.72}{29} = 0.1972$$

$$\text{Median coefficient of dispersion} = \frac{\text{M.D.}}{\tilde{x}} = \frac{5.6868}{29.83} = 0.1906$$

$$\text{Mode coefficient of dispersion} = \frac{\text{M.D.}}{\hat{x}} = \frac{5.6668}{30.33} = 0.1868$$

Layout of Lecture 11

- ❑ Mean(Average) Deviation
  - ❑ **Coefficient of M.D.**
- ❑ Variance
- ❑ Standard Deviation
  - ❑ Coefficient of Variation

Ungrouped Data

COEFFICENT OF DISPERSION

Mean coefficient of dispersion $= \frac{\text{M.D.}}{\bar{x}} = \frac{2.44}{35} = 0.07$

Median coefficient of dispersion $= \frac{\text{M.D.}}{\tilde{x}} = \frac{2.44}{35} = 0.07$

Mode coefficient of dispersion $= \frac{\text{M.D.}}{\hat{x}} = \frac{2.56}{36} = 0.07$

Grouped Data

COEFFICENT OF DISPERSION

Mean coefficient of dispersion $= \frac{\text{M.D.}}{\bar{x}} = \frac{5.72}{29} = 0.1972$

Median coefficient of dispersion $= \frac{\text{M.D.}}{\tilde{x}} = \frac{5.6868}{29.83} = 0.1906$

Mode coefficient of dispersion $= \frac{\text{M.D.}}{\hat{x}} = \frac{5.6668}{30.33} = 0.1868$

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VARIANCE

The variance of a set of observations is defined as the mea of the squares of deviations of all the observations from their mean. When it is calculated from the entire population, the variance is called the population variance, traditionally denoted by σ^2 (σ is the Greek lower-case "sigma").

If, instead , the data from the sample are used to calculate the variance, it is referred to as the sample variance and is denoted by S^2.

$$Var(x) = S^2 = \frac{\sum(x-\overline{x})^2}{n} \quad (for\ sample\ ungrouped\ data)$$

$$Var(x) = S^2 = \frac{\sum f(x-\overline{x})^2}{\sum f} \quad (for\ sample\ grouped\ data)$$

$$Var(x) = \sigma^2 = \frac{\sum(x-\mu)^2}{N} \quad (for\ population\ data)$$

Variance(Cont...)

VARIANCE	
UNGROUPED DATA	GROUPED DATA
$S^2 = \dfrac{\sum(x-\overline{x})^2}{n}$	$S^2 = \dfrac{\sum f(x-\overline{x})^2}{\sum f}$
$S^2 = \dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2$	$S^2 = \dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2$
$S^2 = \dfrac{\sum D^2}{n} - \left(\dfrac{\sum D}{n}\right)^2$	$S^2 = \dfrac{\sum fD^2}{\sum f} - \left(\dfrac{\sum fD}{\sum f}\right)^2$

Difference Between S^2 and s^2	
Biased Sample Variance= $S^2 = \dfrac{\sum(x-\overline{x})^2}{n}$	Unbiased Sample Variance= $s^2 = \dfrac{\sum(x-\overline{x})^2}{n-1}$

Ungrouped Data Variance(Cont...)

EXAMPLE: Calculate the Variance by (i) definition (ii) P.M.=32 and (iii) mean of square minus square of mean: 30,31,32,45,20,22,48,35,27,40.

SOLUTION: Here, Mean = $\overline{x} = \sum X/n$= $(330/10)$=33

VARIANCE for Ungrouped data

X	$X-\overline{X}$	$(X-\overline{X})^2$	D=$X-32$	D^2	X^2
30	-3	9	-2	4	900
31	-2	4	-1	1	961
32	-1	1	0	0	1024
45	12	144	13	169	2025
20	-13	169	-12	144	400
22	-11	121	-10	100	484
48	15	225	16	256	2304
35	2	4	3	9	1225
27	-6	36	-5	25	729
40	7	49	8	64	1600
330	0	762	10	772	11652

Ungrouped Data

Variance(Cont…)

Now

$\sum(x-\overline{x})^2 = 762$, $\sum x = 330$, $\sum x^2 = 11652$, $\sum D = 10$, $\sum D^2 = 772$.

$$S^2 = \frac{\sum(x-\overline{x})^2}{n} = \frac{762}{10} = 76.2$$

$$S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{11652}{10} - \left(\frac{330}{10}\right)^2 = 76.2$$

$$S^2 = \frac{\sum D^2}{n} - \left(\frac{\sum D}{n}\right)^2 = \frac{772}{10} - \left(\frac{10}{10}\right)^2 = 76.2$$

Grouped Data

Variance(Cont…)

EXAMPLE: Calculate variance for the following data :

Class Interval	10-14	15-19	20-24	25-29	30-34	35-39	40-44
Freq.	01	04	08	11	15	09	02

Also compute their respective coefficients of M.D.

SOLUTION: First we have to find mean, median, mode.

MEAN DEVIATION for Grouped data

C.I.	f	X=Mid Pt.	fX	fX^2	$(X-\bar{X})^2$=$(X-29)^2$	$f(X-\bar{X})^2$
10-14	01	12	12	144	$(-17)^2 = 289$	289
15-19	04	17	68	1156	$(-12)^2 = 144$	576
20-24	08	22	176	3872	$(-7)^2 = 49$	392
25-29	11	27	297	8019	$(-2)^2 = 4$	44
30-34	15	32	480	15360	$(3)^2 = 9$	135
35-39	09	37	333	12321	$(8)^2 = 64$	576
40-44	02	42	84	3528	$(13)^2 = 169$	338
Total	50	- -	$\sum fx = 1450$	$\sum fx^2 = 44400$	- -	$\sum f(X-\bar{X})^2 = 2350$

Grouped Data

Variance(Cont...)

Now

$\sum f(x-\overline{x})^2$ = 762, $\sum f$ = 50 , $\sum f x^2$ = 44400 , $\sum f x$ = 1450.

$$S^2 = \frac{\sum f(x-\overline{x})^2}{\sum f} = \frac{2350}{50} = 47$$

$$S^2 = \frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f}\right)^2 = \frac{44400}{50} - \left(\frac{1450}{50}\right)^2 = 47$$

Layout of Lecture 11

- Mean(Average) Deviation
  - Coefficient of M.D.
- Variance
- **Standard Deviation**
  - Coefficient of Variation

STANDARD DEVIATION

The positive square root of the variance is called standard deviation.

$$\boldsymbol{S.D.(x)} = S = \sqrt{\frac{\sum(\boldsymbol{x}-\overline{\boldsymbol{x}})^2}{\boldsymbol{n}}} \quad \boldsymbol{(for\ sample\ ungrouped\ data)}$$

$$\boldsymbol{S.D.(x)} = S = \sqrt{\frac{\sum \boldsymbol{f}(\boldsymbol{x}-\overline{\boldsymbol{x}})^2}{\sum \boldsymbol{f}}} \quad \boldsymbol{(for\ sample\ grouped\ data)}$$

$$\boldsymbol{S.D.(x)} = \sigma = \sqrt{\frac{\sum(\boldsymbol{x}-\boldsymbol{\mu})^2}{\boldsymbol{N}}} \quad \boldsymbol{(for\ population\ data)}$$

Ungrouped Data

Standard Deviation $= \sqrt{\text{Variance}} = \sqrt{76.2} = \mathbf{8.7293}$

Grouped Data

Standard Deviation $= \sqrt{\text{Variance}} = \sqrt{47} = \mathbf{6.8556}$

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COEFF. OF VARIATION(C.V.)

C.V. is the relative measure of dispersion and is defined as the percentage ratio between the S.D. and mean.

$$\text{C}.V.(x) = \frac{\text{S.D.}}{\bar{x}} \times 100$$

Ungrouped Data

COEFFICENT OF VARAITION

$$\text{Coefficient of Variation} = \frac{\text{S.D.}}{\bar{x}} \times 100$$

$$= \frac{8.7293}{33} \times 100 = 26.45\%$$

Grouped Data

COEFFICENT OF VARAITION

$$\text{Coefficient of Variation} = \frac{\text{S.D.}}{\bar{x}} \times 100$$

$$= \frac{6.8556}{29} \times 100 = 23.64\%$$

(Your Turn)

1. Why all mean deviations are counted as positive?
2. Why square root of variance is called standard deviation?
3. What is root mean square deviation?

Summary

- What did you learn in this lecture?
- What are some important facts to remember about a coefficient of variation?
- Is there something within the lecture that you need help on?