

# Chi-Square

## Exercise

# Steps in Test of Hypothesis

1. Determine the appropriate test
2. Establish the level of significance:  $\alpha$
3. Formulate the statistical hypothesis
4. Calculate the test statistic
5. Determine the degree of freedom
6. Compare computed test statistic against a tabled/critical value

# 1. Determine Appropriate Test

- ❑ Chi Square is used when both variables are measured on a nominal scale.
- ❑ It can be applied to interval or ratio data that have been categorized into a small number of groups.
- ❑ It assumes that the observations are randomly sampled from the population.
- ❑ All observations are independent (an individual can appear only once in a table and there are no overlapping categories).
- ❑ It does not make any assumptions about the shape of the distribution nor about the homogeneity of variances.

## 2. Establish Level of Significance

■  $\alpha$  is a predetermined value

■ The convention

- $\alpha = .05$
- $\alpha = .01$
- $\alpha = .001$

### 3. Determine The Hypothesis: Whether There is an Association or Not

- ❑  $H_o$  : The two variables are independent
- ❑  $H_a$  : The two variables are associated



## 4. Calculating Test Statistics

- Contrasts observed frequencies in each cell of a contingency table with expected frequencies.
- The expected frequencies represent the number of cases that would be found in each cell if the null hypothesis were true ( i.e. the nominal variables are unrelated).
- Expected frequency of two unrelated events is product of the row and column frequency divided by number of cases.

$$F_e = F_r F_c / N$$

## 4. Calculating Test Statistics

**Continued**

$$\chi^2 = \sum \left[ \frac{(F_o - F_e)^2}{F_e} \right]$$

## 4. Calculating Test Statistics

**Continued**

The diagram illustrates the calculation of the chi-square test statistic. It features the formula  $\chi^2 = \sum \left[ \frac{(F_o - F_e)^2}{F_e} \right]$  in the center. Three blue arrows point to the components of the formula: one from the top-left pointing to  $F_o$  (labeled 'Observed frequencies'), one from the bottom-left pointing to  $F_e$  (labeled 'Expected frequency'), and one from the bottom-right pointing to  $F_e$  (labeled 'Expected frequency'). The formula is enclosed in a light blue bracket on the left side.

$$\chi^2 = \sum \left[ \frac{(F_o - F_e)^2}{F_e} \right]$$

Observed frequencies

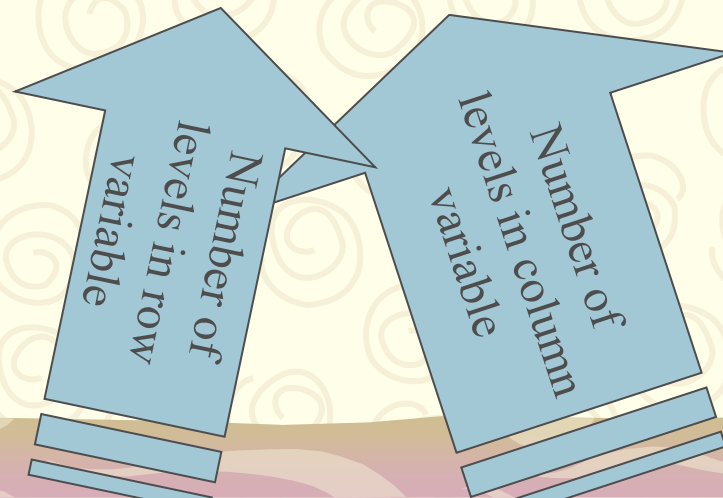
Expected frequency

Expected frequency



## 5. Determine Degrees of Freedom

$$df = (R-1)(C-1)$$



## 6. Compare computed test statistic against a tabled/critical value

- The computed value of the Pearson chi-square statistic is compared with the critical value to determine if the computed value is *improbable*
- The critical tabled values are based on sampling distributions of the Pearson chi-square statistic
- If calculated  $\chi^2$  is greater than  $\chi^2$  table value, reject  $H_0$

# Example

- Suppose a researcher is interested in voting preferences on gun control issues.
- A questionnaire was developed and sent to a random sample of 90 voters.
- The researcher also collects information about the political party membership of the sample of 90 respondents.

# Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	$f_{\text{row}}$
Democrat	10	10	30	50
Republican	15	15	10	40
$f_{\text{column}}$	25	25	40	$n = 90$

# Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	$f_{\text{row}}$
Democrat	10	10	30	50
Republican	15	15	10	40
$f_{\text{column}}$		25	40	$n = 90$

Observed  
frequencies



# Bivariate Frequency Table

## Contingency Table

Row frequency

	Favor	Neutral	Oppose	
Democrat	10	10	30	50
Republican	15	15	10	40
$f_{\text{column}}$	25	25	40	$n = 90$

# Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	$f_{\text{row}}$
Democrat	10	10	30	50
Republican	15	15	10	40
Column frequency	25	25	40	$n = 90$

# 1. Determine Appropriate Test

- 1. Party Membership ( 2 levels) and Nominal
- 2. Voting Preference ( 3 levels) and Nominal

## 2. Establish Level of Significance

Alpha of .05

### 3. Determine The Hypothesis

- $H_0$  : There is no difference between D & R in their opinion on gun control issue.
- $H_a$  : There is an association between responses to the gun control survey and the party membership in the population.



## 4. Calculating Test Statistics

	Favor	Neutral	Oppose	$f_{\text{row}}$
Democrat	$f_o = 10$ $f_e = 13.9$	$f_o = 10$ $f_e = 13.9$	$f_o = 30$ $f_e = 22.2$	50
Republican	$f_o = 15$ $f_e = 11.1$	$f_o = 15$ $f_e = 11.1$	$f_o = 10$ $f_e = 17.8$	40
$f_{\text{column}}$	25	25	40	$n = 90$

## 4. Calculating Test Statistics

**Continued**

	Favor	Neutral	Oppose	$f_{\text{row}}$
Democrat	$f_o = 10$ $f_e = 13.9$	$f_o = 10$ $f_e = 13.9$	$f_o = 30$ $f_e = 22.2$	50
Republican	$f_o = 15$ $f_e = 11.1$	$f_o = 15$ $f_e = 11.1$	$f_o = 10$ $f_e = 17.8$	40
$f_{\text{column}}$	25	25	40	$n = 90$

## 4. Calculating Test Statistics

Continued

	Favor	Neutral	Oppose	$f_{\text{row}}$
Democrat	$f_o = 10$ $f_e = 13.9$	$f_o = 10$ $f_e = 13.9$	$f_o = 30$ $f_e = 22.2$	50
Republican	$f_o = 15$ $f_e = 11.1$	$f_o = 10$ $f_e = 11.1$	$f_o = 30$ $f_e = 17.8$	40
$f_{\text{column}}$	25	25	40	$n = 90$

## 4. Calculating Test Statistics

**Continued**

$$\begin{aligned}\chi^2 &= \frac{(10-13.89)^2}{13.89} + \frac{(10-13.89)^2}{13.89} + \frac{(30-22.2)^2}{22.2} + \\ &\quad \frac{(15-11.11)^2}{11.11} + \frac{(15-11.11)^2}{11.11} + \frac{(10-17.8)^2}{17.8} \\ &= 11.03\end{aligned}$$

## 5. Determine Degrees of Freedom

$$\begin{aligned} df &= (R-1)(C-1) = \\ &= (2-1)(3-1) = 2 \end{aligned}$$



## 6. Compare computed test statistic against a tabled/critical value

- $\alpha = 0.05$
- $df = 2$
- Critical tabled value = 5.991
- Test statistic, 11.03, exceeds critical value
- Null hypothesis is rejected
- Democrats & Republicans differ significantly in their opinions on gun control issues