

WEEK 13 (2ND LEC)

lecture 24

Rule of combination

The classical or A Priori definition of probability

The relative frequency or a Posteriori definition of probability

RULE OF COMBINATION

A combination is any subset of r objects, selected without regard to their order, from a set of n distinct objects.

The total number of such combinations is denoted by the symbol nC_r , is given by .

The number of combinations of n objects taken r at a time is

$${}_nC_r = \frac{n(n-1)(n-2) \dots (n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$

where $r \leq n$. It should be noted that

Combinations and permutations are related according to the following formulas:

$${}_nP_r = {}_nC_r * r! \quad \text{and} \quad {}_nC_r = {}_nP_r / r!$$

EXAMPLE

Suppose we have a group of three persons, A, B, & C.

If we wish to select a group of two persons out of these three, the three possible groups are {A, B}, {A, C} and {B, C}.

In other words, the total number of combinations of size two out of this set of size three is 3.

Now, suppose that our interest lies in forming a committee of two persons, one of whom is to be the president and the other the secretary of a club.

The six possible committees are:

(A, B) , (B, A) , (A, C) , (C, A) , (B, C) & (C, B) .

In other words, the total number of permutations of two persons out of three is 6.

EXAMPLE

In how many ways can three student-council members be elected from five candidates?

$${}_5C_3 = \frac{5 \times 4 \times 3}{3!} = \frac{60}{6} = 10 \text{ ways}$$

A DJ will play three CD choices from the 5 requests.

The classical or A Priori definition of probability

There is an experiment with n mutually exclusive, equally likely, and exhaustive elementary events. m of the elementary events are favorable to the occurrence of an event. Then the probability of occurrence of the event (represented by $P(\text{Event})$) is given by the ratio of the Number of favorable choices for the event to the Total number of possible choices in the experiment.

Probability of Occurrence of an Event

$$= \frac{\text{Number of Favorable Choices for the Event}}{\text{Total Number of Possible Choices for the Experiment}}$$

$$\Rightarrow P(E) = \frac{m_E}{n}$$

E represents the event.

The relative frequency or a Posteriori definition of probability

If a random experiment is repeated a large number of times, say n times, under identical conditions and if an event A is observed to occur m times, then the **probability** of the event A is defined as the LIMIT of the relative frequency m/n as n tends to infinitely. Symbolically, we write

$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

As its name suggests, the relative frequency definition relates to the relative frequency with which an event occurs in the *long run*. In situations where we can say that an experiment has been repeated a very large number of times, the *relative frequency definition* can be applied.