Lecture 18

Rank Correlation

- Rank correlation is used in a situation when the variable under consideration is not measurable e.g. intelligence, knowledge, experience, beauty etc. Such types of variables are judged by two different people or by two procedures. Therefore it is necessary to find the correlation between judgment of two person or two procedures. For this purpose observations are ranked and the method is called rank correlation coefficient.
- The formula for calculating rank correlation is given as:

$$r = 1 - \frac{6\sum d^2}{n^3 - n}$$

Question

Problem: Following are the ranks assigned to the customer satisfaction by an organization and its customers. Find the rank correlation.

Characteristics	*Company Ranking*	*Customer Ranking*
Customer service	1	2
Product quality	2	3
Delivery	4.5	5
Technical support	7.5	5
Professionalism	6	9
Responsiveness	7.5	7.5
Special discounts	6	5

Solution

Characteristics	R_1	R_2	$d = R_1 - R_2$	d^2
Customer service	1	2	-1	1
Product quality	2	3	-1	1
Delivery	4.5	5	-0.5	0.25
Technical support	7.5	5	2.5	6.25
Professionalism	9	6	3	9
Responsiveness	7.5	7.5	0	0
Special discounts	6	5	1	1
				18.5

$$r = 1 - \frac{6(18.50)}{7^3 - 7}$$

$$r = 1 - \frac{111}{343 - 7}$$

$$r = 1 - \frac{111}{336}$$

$$r = 1 - 0.330357$$

$$r = 0.6696$$

Interpretation

The value of r shows that there is positive correlation between the company ranking and customer ranking. It also indicates that both company and customer consider these characteristics important for customer satisfaction due to which the association between two rankings is positive.

Example 15.15.

The following were the "performance under stress" rankings of 10 honor students before and after mid-semester:

Student	A	B	C	D	E	F	G	H	I	J
Rank before	1	2	3	4	5	6	7	8	9	10
Rank after	6	5	8	9	3	4	10	1	7	2

Compute the Spearman's rank correlation coefficient for this data set.

Solution:

Rank before (X)	Rank after (Y)	d = X – Y	d^2
1	6	–5	25
2	5	–3	9
3	8	–5	25
4	9	–5	25
5	3	+2	4
6	4	+2	4
7	10	–3	9
8	1	+7	49
9	7	+2	4
10	2	+8	64
			$\Sigma d^2 = 218$

Spearman's rank correlation coefficient, $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

$= 1 - \frac{6(218)}{10(100 - 1)} = 1 - 1.32 = -0.32$

$10(100-1)$

Example 15.16.

A Statistics instructor wants to know whether there is a correlation between students midterm averages and their final examination scores. The instructor takes a random sample of nine students from previous Statistics courses and obtains the following data:

Midterm average X	72	96	86	77	67	92	90	74	60
Final examination score Y	49	97	80	73	71	86	95	48	52

(i) Determine the rank correlation coefficient, r_s, of the data.

(ii) Interpret the value of r_s obtained in part (i).

Solution:

Midterm average (X)	Final examination score (Y)	Rank of X	Rank of Y	$d = X - Y$	d^2
72	49	3	2	1	1
96	97	9	9	0	0
86	80	6	6	0	0
77	73	5	5	0	0
67	71	2	4	−2	4
92	86	8	7	1	1
90	95	7	8	−1	1
74	48	4	1	3	9
60	52	1	3	−2	4
					$\Sigma d^2 = 20$

(i) $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)} = 1 - \frac{6(20)}{9(81 - 1)} = 1 - 0.17 = 0.83$

(ii) The rank correlation coefficient, $r_s = 0.83$ suggests that there is a strong positive correlation between midterm average and final-examination score in Statistics courses.

Example 15.17.

The number of hours of study for an examination and the grades received by a random sample of 10 students are:

Number of hours studied, X	8	5	11	13	10	5	18	15	2	8
Grade in examination, Y	56	44	79	72	70	54	94	85	33	65

Compute and interpret the Spearman's rank correlation coefficient.

Solution:

Number of hours studied (X)	Grade in examination (Y)	Rank of X	Rank of Y	d = X – Y	d^2
8	56	4.5	4	0.5	0.25
5	44	2.5	2	0.5	0.25
11	79	7	8	–1.0	1.00
13	72	8	7	1.0	1.00
10	70	6	6	0.0	0.00
5	54	2.5	3	–0.5	0.25
18	94	10	10	0.0	0.00
15	85	9	9	0.0	0.00
2	33	1	1	0.0	0.00
8	65	4.5	5	–0.5	0.25
					$\Sigma d^2 = 3$

Spearman's rank correlation coefficient, $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

$$= 1 - \frac{6(3)}{10(100 - 1)} = 1 - \frac{18}{990} = 1 - 0.02 = 0.98$$

$r_s = 0.98$, indicating strong positive correlation between the number of hours of study and the grade in examination.