HYPOTHESIS TESTING - POPULATION MEAN μ , σ KNOWN (SMALL SAMPLE $n < 30$)

When the standard deviation of the population is not known, it is estimated by The sample standard deviation 's' where $s = \sqrt{\frac{1}{n-1}\sum(X-\bar{X})^2}$.

Example 1:

A manufacturing company making automobile tires claims that the average life its product is 35000 miles. A random sample of 16 tires was selected; and it was found that the mean life was 34000 miles with a standard deviation $s = 2000$ miles. Test hypothesis $H_0 : \mu = 35000$ against the alternative $H_1 : \mu < 35000$ at $\alpha = 0.05$.

Solution:

Step 1: Null hypothesis: $H_0 : \mu = 35000$

Alternative hypothesis: $H_1 : \mu < 35000$

Step 2: Level of significance: $\alpha = 0.05$

Step 3: Test statistics:

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Step 4: Critical Region: t<-1.753

(From the t-table, we have $-t_{\alpha(n-1)} = -t_{0.05(15)} = -1.753$)

Step 5: Computation:

Here $n = 16, \bar{X} = 34000, s = 200$ and hence

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{34000 - 35000}{\frac{2000}{16}}$$

$$t = -\frac{1000}{2000}(4) = -2$$

Step 6: Conclusion:

Since the calculated value of "t = -2" falls in the critical region; so, we reject our null hypothesis $H_0 : \mu = 35000$ at 5 % level of significance.

Example 2:

A random sample of 8 cigarettes of a certain brand has an average nicotine content of 4.2 milligrams and a standard deviation of 1 4 milligrams. Is this in line with the manufacturer's claim that the average nicotine content does not exceed 3.5 milligrams? Use 1% level of significance and assume the distribution of nicotine contents to be normal.

Solution:

Step 1: Null hypothesis: $H_0 : \mu \leq 3.5$

Alternative hypothesis: $H_1 : \mu < 3.5$

Step 2: Level of significance: $\alpha = 0.01$

Step 3: Test statistics:

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Step 4: Critical Region: t > 2.998

(From the t-table, we have $t_{\alpha(n-1)} = t_{0.01(7)} = 2.998$)

Step 5: Computation:

Here $n = 8, \bar{X} = 4.2, s = 1.4$ and hence

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{4.2 - 3.5}{\frac{1.4}{\sqrt{8}}}$$

$$t = \frac{0.7}{1.4}\sqrt{8} = 1.414$$

Step 6: Conclusion:

Since the calculated value of t = 1.414 falls in the acceptance region, so _we accept our null hypothesis at 1 % level of significance.

HYPOTHESIS TESTING DIFFERENCE BETWEEN TWO POPULATION MEANS $\mu_1 - \mu_2$, σ_1^2 AND σ_2^2 NOT KNOWN, NORMAL POPULATION (SMALL SAMPLES)

Let $X_{11}, X_{12}, \ldots, X_{1n_1}$ and $X_{21}, X_{22}, \ldots, X_{2n_2}$ be two independent random samples from two normal populations with means μ_1 and μ_2, standard deviations σ_1 and σ_2 respectively. If $(\sigma_1^2 = \sigma_2^2 = \sigma)$ but unknown, then the unbiased pooled or combined means of the common variance σ^2.

The variances σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2 = \sigma^2$. The parameter σ^2 is estimated by the sample variances. The sample estimator of σ^2 is s_p^2, where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}$$

and $$s_p = \sqrt{\frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}}$$

s_p^2 is called pooled estimator of the common population variance σ^2. The difference $(\bar{X}_1 - \bar{X}_2)$ has the t – distribution with $(n_1 + n_2 - 2)$ degrees of freedom where

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The tabulated value of 't' for $n_1 + n_2 - 2$ degrees of freedom is seen from the t-table.

For $H_1 : \mu_1 \neq \mu_2$ the critical values are $-t_{\alpha/2\,(n_1+n_2-2)}$ and $t_{\alpha/2\,(n_1+n_2-2)}$

For $H_1 : \mu_1 > \mu_2$ the critical value is $t_{\alpha\,(n_1+n_2-2)}$

and For $H_1 : \mu_1 < \mu_2$ the critical value is $-t_{\alpha\,(n_1+n_2-2)}$

The null hypothesis H_0 is rejected when the calculated value of t lies in rejection region.

Example 2:

Two samples are randomly selected from two classes of students who have been taught by different methods. An examination is given, and the results are shown as follows:

	Class I	Class II
Sample Size	$n_1 = 8$	$n_2 = 10$
Mean	$\bar{X}_1 = 95$	$\bar{X}_2 = 97$
Variance	$s_1^2 = 47$	$s_2^2 = 30$

On the assumption that the test scores of the two classes of students have identical variances, determine whether the two different methods of teaching are equally effective at $\alpha = 0.01$.

Solution:

Step 1: Null hypothesis: $H_0 = \mu_1 = \mu_2$

Alternative hypothesis: $H_1 = \mu_1 \neq \mu_2$

Step 2: Level of significance: $\alpha = 0.01$

Step 3: Test statistics:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Step 4: Critical Region: $|t|$> 2.921 (t < -2.921 and t > 2.921)

(From the t-table, we have $t_{\frac{\alpha}{2}(n_1+n_2-2)} = t_{0.005(16)} = 2.921$)

Step 5: Computation:

Here $n_1 = 8, \bar{X}_1 = 95, n_2 = 10, \bar{X}_2 = 97, s_1^2 = 47, s_2^2 = 30$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$s_p^2 = \frac{(8-1)47 + (10-1)30}{8 + 10 - 2} = \frac{599}{16} = 37.4375$$

$$s_p = \sqrt{37.4375} = 6.12 \text{ and hence}$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(95 - 97) - (0)}{6.12\sqrt{\frac{1}{8} + \frac{1}{10}}} = -\frac{2}{2.9030} = -0.689$$

Step 6: Conclusion:

Since the calculated value of t = - 0.689 falls in the acceptance region, so we accept our null hypothesis $H_0 = \mu_1 = \mu_2$ at 1 % level of significance. On the basis of the evidence, we may conclude that the two different methods of teaching are equally effective.