Week 12
(LECT-23)

EQUALLY LIKELY EVENTS

Counting sample points

Rule of multiplication

Rule of permutation

EQUALLY LIKELY EVENTS

Two events A and B are said to be equally likely, when one event is as likely to occur as the other.

In other words, each event should occur in equal number in repeated trials.

EXAMPLE:

When a fair coin is tossed, the head is as likely to appear as the tail, and the proportion of times each side is expected to appear is 1/2.

Counting sample points

COUNTING RULES facilitate in the calculations of there probabilities are in certain situations. They are known as counting rules and include concepts of :

- Multiplication
- Permutations
- Combinations

Rule of multiplication

If a compound experiment consists of two experiments:

The first experiment has exactly m distinct outcomes and, If corresponding to each outcome of the first experiment there can be n distinct outcomes of the second experiment.

Then the compound experiment has exactly mxn outcomes.

Example 1

if A = {H, T} and B = {1, 2, 3, 4, 5, 6}, then the Cartesian product set is the collection of the following twelve (2 X 6) ordered pairs:

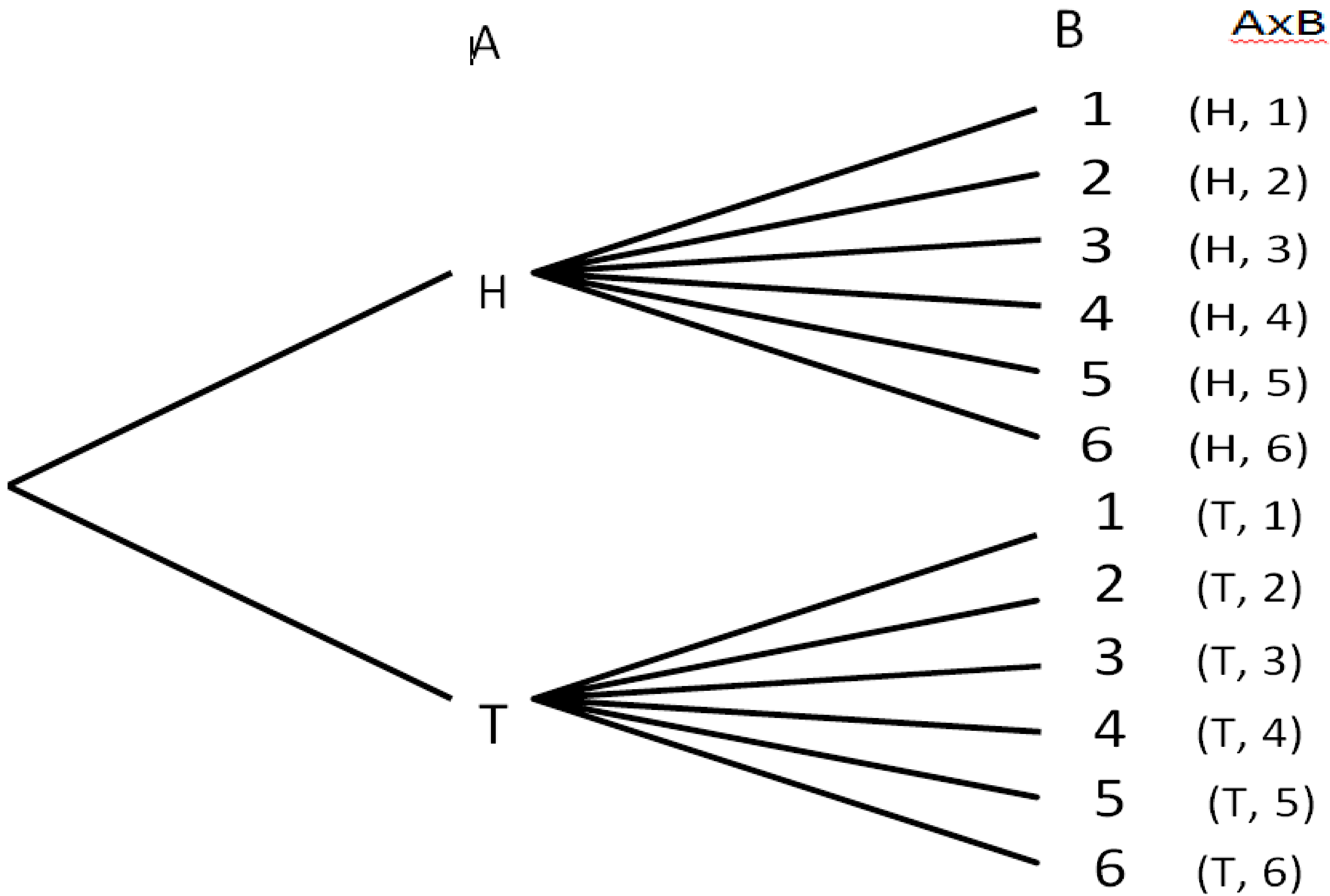
AXB = { (H, 1); (H, 2);(H, 3); (H, 4); (H, 5); (H, 6);(T, 1); (T, 2); (T, 3); (T, 4); (T, 5); (T, 6) }

EXAMPLE:2

- The compound experiment of tossing a coin and throwing a die together consists of two experiments:
- The coin-tossing experiment consists of two distinct outcomes (H, T), and the die-throwing experiment consists of six distinct outcomes (1, 2, 3, 4, 5, 6).

The total number of possible distinct outcomes of the compound experiment is therefore 2x6=12 as each of the two outcomes of the coin-tossing experiment can occur with each of the six outcomes of die throwing experiment.

Tree Diagram



EXAMPLES

- Suppose that a restaurant offers three types of soups, four types of sandwiches, and two types of desserts.

Then, a customer can order any one out of 3 X 4 X 2 = 24 different meals.

- Suppose that we have a combination lock on which there are eight rings. In how many ways can the lock be adjusted?

Solution:

The logical way to look at this problem is to see that there are eight rings on the lock, each of which can have any of the 10 figures 0 to 9:

A B C D E F G H

ring A can have any of the digits 0 to 9 and

ring B can have any of the digits 0 to 9 and

ring C can have any of the digits 0 to 9 and

. . .

ring H can have any of the digits 0 to 9

Hence the total No. of ways in which

these 8 rings can be filled is =10 X 10 X 10 X 10 X10 X 10 X10 x 10 X10

i.e. 100,000,000 — one hundred million.

RULE OF PERMUTATION

A permutation is any ordered subset from a set of n distinct objects.

For example, if we have the set {a, b}, then one permutation is ab, and the other permutation is ba.

The number of permutations of r objects, selected in a definite order from n distinct objects is denoted by the symbol ${}^nP_r$, and is given by n! ${}^nP_r = n(n-1)(n-2)\ldots(n-r+1) = (n\,P\,r)!$

FACTORIALS

7! = 7 X6 X 5 X 4 X 3 X 2 x 1

6! = 6 X 5 X 4 X 3 X 2 x 1

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1! = 1 Also, we define 0! = 1.

Example

A club consists of four members. How many ways are there of selecting three officers: president, secretary and treasurer?

SOL:

It is evident that the order in which 3 officers are to be chosen, is of significance.

Thus there are 4 choices for the first office, 3 choices for the second office, and 2 choices for the third office. Hence the total number of ways in which the three offices can be filled is

4 X 3 X 2 = 24.

- The same result is obtained by applying the rule of <u>permutations</u>:

$$_4P_3 = \frac{4!}{(4-3)!}$$

$$=4x3x2=24$$

EXAMPLE

Suppose that there are three persons A, B & D, and that they wish to have a photograph taken.

The total number of ways in which they can be seated on three chairs (placed side by side) is:

These are : $3P_3 = 3! = 6$

ABD, ADB, BAD, BDA, DAB, & DBA.