AutoData

November 20, 2025

1 Simple Linear Regression on the Auto Dataset

Chapter 3 – Question 8 (Applied)

This analysis investigates the relationship between **horsepower** (predictor) and **mpg** (response) using a simple linear regression model. We will perform regression using `sm.OLS()` from `statsmodels`, evaluate the model, create plots, and interpret the results.

[7]:
```
import pandas as pd
import statsmodels.api as sm
import matplotlib.pyplot as plt

auto = pd.read_csv("/home/mlahkim15/ve/Auto/Auto.csv")
auto.head()
```

[7]:
```
    mpg  cylinders  displacement horsepower  weight  acceleration  year  \
0  18.0          8         307.0        130    3504          12.0    70
1  15.0          8         350.0        165    3693          11.5    70
2  18.0          8         318.0        150    3436          11.0    70
3  16.0          8         304.0        150    3433          12.0    70
4  17.0          8         302.0        140    3449          10.5    70

   origin                       name
0       1  chevrolet chevelle malibu
1       1          buick skylark 320
2       1         plymouth satellite
3       1              amc rebel sst
4       1                ford torino
```

1.1 Part (a) — Fit the Linear Regression Model

We model `mpg` as the response variable and `horsepower` as the predictor variable.

[8]:
```
# Convert horsepower to numeric, coerce errors to NaN
auto['horsepower'] = pd.to_numeric(auto['horsepower'], errors='coerce')

# Drop rows with missing values in mpg or horsepower
auto = auto.dropna(subset=['horsepower', 'mpg'])
```

```python
# Check types and first rows
print(auto.dtypes)
auto.head

X = auto["horsepower"]
y = auto["mpg"]

X = sm.add_constant(X)
model = sm.OLS(y, X).fit()
model.summary()
```

```
mpg             float64
cylinders         int64
displacement    float64
horsepower      float64
weight            int64
acceleration    float64
year              int64
origin            int64
name             object
dtype: object
```

[8]:

Dep. Variable:	mpg	**R-squared:**	0.606
Model:	OLS	**Adj. R-squared:**	0.605
Method:	Least Squares	**F-statistic:**	599.7
Date:	Thu, 20 Nov 2025	**Prob (F-statistic):**	7.03e-81
Time:	13:36:18	**Log-Likelihood:**	-1178.7
No. Observations:	392	**AIC:**	2361.
Df Residuals:	390	**BIC:**	2369.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	39.9359	0.717	55.660	0.000	38.525	41.347
horsepower	-0.1578	0.006	-24.489	0.000	-0.171	-0.145

Omnibus:	16.432	**Durbin-Watson:**	0.920
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	17.305
Skew:	0.492	**Prob(JB):**	0.000175
Kurtosis:	3.299	**Cond. No.**	322.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

1.1.1 Interpretation of Regression Output

i. **Is there a relationship between horsepower and mpg?**
Yes — the p-value for horsepower is extremely small (much less than 0.05), which means the relationship is statistically significant.

ii. **How strong is the relationship?**
   The R-squared value is around **0.60**, meaning about **60% of the variation in mpg is explained by horsepower**.

iii. **Is the relationship positive or negative?**
   The coefficient for horsepower is **negative**, meaning **as horsepower increases, mpg decreases**.

iv. **Prediction for horsepower = 98**
   We will calculate the predicted mpg and 95% confidence and prediction intervals below.

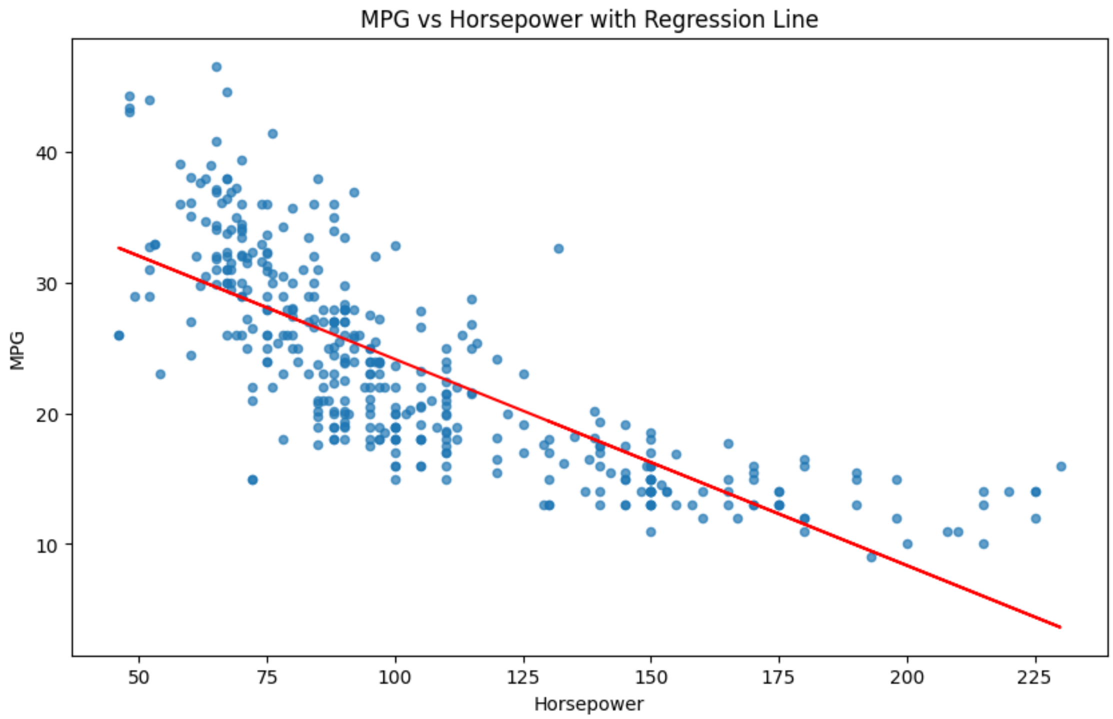
[9]:
```python
new_value = pd.DataFrame({"const":[1], "horsepower":[98]})
model.get_prediction(new_value).summary_frame(alpha=0.05)
```

[9]:
```
        mean   mean_se  mean_ci_lower  mean_ci_upper  obs_ci_lower  \
0  24.467077  0.251262      23.973079      24.961075     14.809396

   obs_ci_upper
0     34.124758
```

1.2 Part (b) — Plot mpg vs horsepower and the regression line

[14]:
```python
fig, ax = plt.subplots(figsize=(10,6))  # make figure wider and taller
ax.scatter(auto["horsepower"], auto["mpg"], s=20, alpha=0.7)  # smaller dots,␣
 ↪slightly transparent
ax.plot(auto["horsepower"], model.predict(sm.add_constant(auto["horsepower"])),␣
 ↪color='red')  # regression line
ax.set_xlabel("Horsepower")
ax.set_ylabel("MPG")
ax.set_title("MPG vs Horsepower with Regression Line")
plt.show()
```

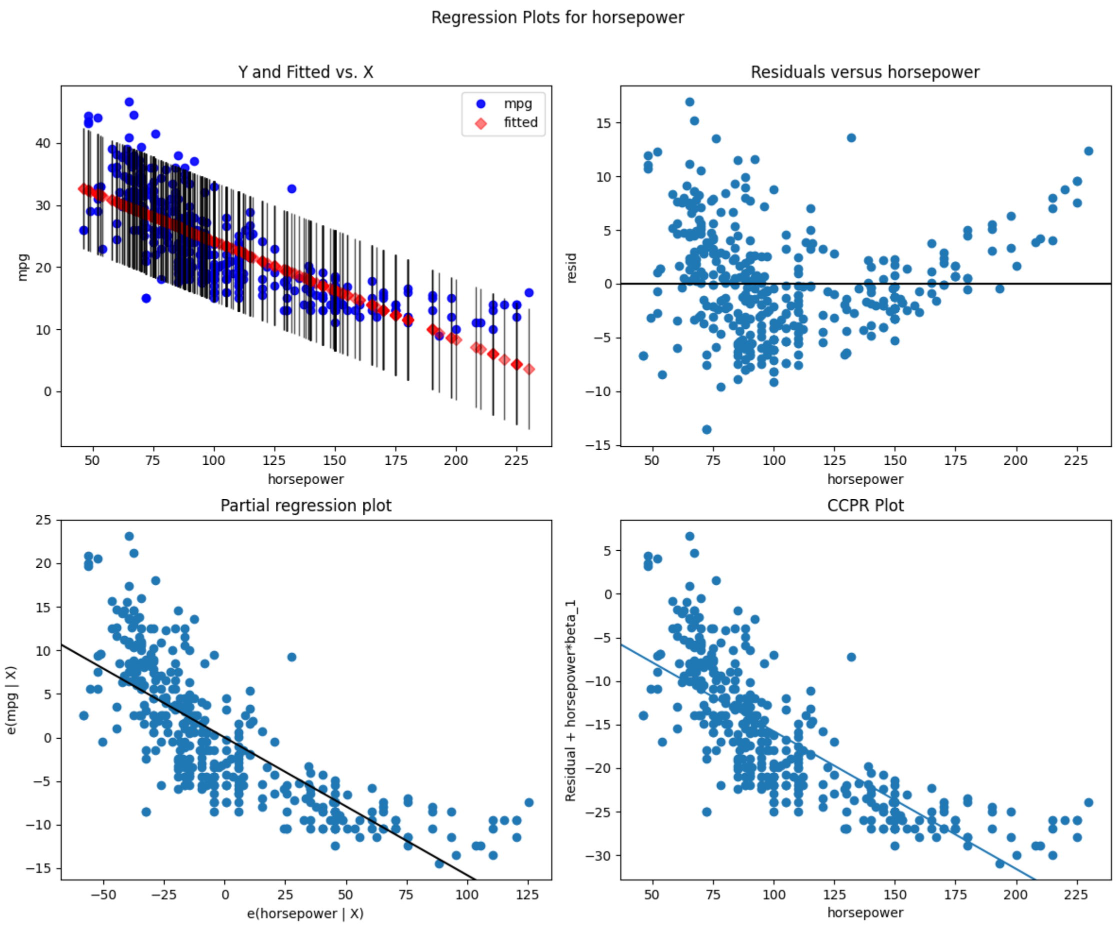


1.3 Part (c) — Diagnostic Plots

These plots help evaluate assumptions such as linearity, constant variance, and normality of residuals.

[16]:
```
import statsmodels.api as sm
import matplotlib.pyplot as plt

fig = plt.figure(figsize=(12, 10))  # make the figure larger
sm.graphics.plot_regress_exog(model, "horsepower", fig=fig)
plt.show()
```



1.3.1 Comments on Diagnostics

- There appears to be some curvature in the residuals, suggesting the relationship may not be perfectly linear.
- There is some evidence of non-constant variance (funnel shape), which means prediction accuracy varies across horsepower values.
- A few points may be potential outliers influencing the model.

Overall, the model shows a clear negative relationship, but improvements such as polynomial regression might yield a better fit.

[ ]:

```
```