3.

(Thm 2.1.13)

a) Since $\sqsubseteq_{D_1 \to D_2}$ is complete on $D_1 \to D_2$, it contains a smallest element $\perp_{D_1 \to D_2}$, where for any $g \in D_1 \to D_2$
$f(d) \sqsubseteq_{d_2} g(d)$ for all $d \in D_1$. (def 2.1.4)

Assume $f = \perp_{D_1 \to D_2}$, $f(d)$ and $f(d')$ are the value of f at d and d' respectively. where $d, d' \in D_1$

Define $g \in D_1 \to D_2$ and $g(d) = f(d')$ for all $d \in D_1$.

Because $f \sqsubseteq_{D_1 \to D_2} g$ as $f(d) \sqsubseteq_{D_2} g(d) = f(d')$ for all $d \in D_1$.

Define $h \in D_1 \to D_2$ s.t. $h(d') = f(d)$ for all $d' \in D_1$

$f \sqsubseteq_{D_1 \to D_2} h$, $f(d') \sqsubseteq_{D_2} h(d') = f(d)$, for all $d' \in D_1$

Then $f(d) = f(d')$ for all $d, d' \in D_1$ (Symmetry) and f is a constant function.

Assume C is a constant value of f. Since $f = \perp_{D_1 \to D_2}$, $f(d) \sqsubseteq_{D_2} g(d)$, then $C \sqsubseteq_{D_2} g(d)$ for all $d \in D_1$.

then C is the lower bound for the set of values of all the functions in $D_1 \to D_2$

$\Rightarrow C = \perp_{D_2}$

Therefore, $\perp_{D_2}$ exists.

b) (Def 2.1.9)

Let chain $S = \{S_1, S_2, \ldots\} \subseteq D_2$ and $S_1 \sqsubseteq S_2 \sqsubseteq \ldots$

for each $S_i \in S$, we can define a constant function f_i in $D_1 \to D_2$:

$f_i(d) = S_i$ for all $d \in D_1$.

because $S_i \sqsubseteq S_j$, $f_i \sqsubseteq f_j$, $F: \{f_1, f_2 \ldots\}$ is a chain in $D_1 \to D_2$

Since $\sqsubseteq_{D_1 \to D_2}$ is complete in $D_1 \to D_2$, for the Chain F there exists a lub $g \in D_1 \to D_2$ (def 2.1.12)

According to the definition of lub, $f_i \sqsubseteq_{D_1 \to D_2} g$, and $f_i(d) \sqsubseteq_{D_2} g(d)$ for all $d \in D_1$

Assume $g(d)$ is not constant, then $g(d) \neq g(d')$ for $d, d' \in D_1$.

then there exist $S_j \in S$ s.t. $S_j(d) \sqsubseteq_{D_2} g(d)$, $S_j(d) \sqsupseteq_{D_2} g(d')$, which conflicts the defination of lub.

Therefore g is constant.

Assume C_g is a constant value of g.

for all $S_i \in S$, $S_i \sqsubseteq C_g$. Then C_g is an upper bound of S.

Since g is lub for the chain F, C_g is lub of S in D_2.

$\Rightarrow \bigsqcup S$ of S exists in D_2