## CEOI'2012 Day1, Task: jobs

According to the task description, n ( $1 \le n \le 100000$ ) is the number of days the organization performs jobs, m ( $1 \le m \le 1000000$ ) is the number of job requests and d ( $0 \le d < n$ ) is the delay number.

It is not difficult to see that for given k  $(1 \le k \le m)$  we can decide whether the organization can process all jobs with at most d days of delay having k machines. In other words, whether k is an upper bound of the solution. If we precompute Cn[x], the number of requests submitted on day x, then the above decision can be done in  $\mathbf{O}(n)$  time by a greedy algorithm.

Therefore one can give an algorithm using sequential search of  $\mathbf{O}(m+k\cdot n)$  running time, where k is the minimum number of machines needed to process all jobs with at most d days of delays. Since in the worst case k=m, the running time of this naive algorithm is  $\mathbf{O}(m\cdot n)$ . Too slow.

We can speed up the above naive algorithm by using binary search. For the binary search the initial lower bound is 1 and the upper bound is m. We can give better initial lower and upper bound by a greedy method of  $\mathbf{O}(n)$  running time.

The worst case running time of this algorithm is  $\mathbf{O}(m + n \cdot \log m)$ .

## **Implementation**

```
#include <stdio.h>
 1
2
   #include <stdlib.h>
3
   #define
             \max N 100001
   using namespace std;
5
   struct Cell{
        int id; Cell* next;
6
7
   };
   Cell* Req[maxN];
8
9
   int Cn[maxN], m,n,d;
10
11
   bool Test(int k){
12
       int dd=1, r=0;
13
       for (int x=1;x \le n;x++)
14
          if (Cn[x]==0) continue;
15
          if (dd < x-d) \{ dd = x-d; r=0; \}
16
          dd+=(Cn[x]+r)/k;
          r = (Cn[x]+r)\%k;
17
          if (dd>x+1 \mid \mid dd=x+1 \&\& r>0) return false;
18
19
20
       return true;
21
   }
```

```
int main() {
22
23
        int a; int r=0, sol=0, left=1;
24
        Cell* p, *pp;
25
        scanf("%d_%d_%d",&n, &d, &m);
26
        for (int x=1; x <= n; x++) {
27
             Req[x]=NULL; Cn[x]=0;
28
        for (int i=1; i < m; i++) {
29
             scanf("%d",&a); a+=d;
30
31
             p = new Cell;
32
             p->id=i; p->next=Req[a];
33
             \operatorname{Req}[a]=p; \operatorname{Cn}[a]++;
34
35
       computing lower and upper bound for the binary search
36
       for (int x=1; x <= n; x++) {
37
           if (\operatorname{Cn}[x]==0){
38
              if (r<=d) r++;
39
          } else {
              if ((Cn[x]+d)/(d+1)>left) left = (Cn[x]+d)/(d+1);
40
              if (r*sol>=Cn[x]){
41
42
                  r = (Cn[x] + sol - 1)/sol;
                                                   r++;
43
              } else {
                  sol += (Cn[x]-r*sol+d)/(d+1); r=1;
44
              }
45
46
           }
       }//left=lower, sol=upper bound for solution
47
48
       int m;
49
       while (left < sol) {
50
          m=(left+sol)/2;
51
           if (Test (m))
52
              sol=m;
53
           else
54
              left=m+1;
55
       printf("%d \n", sol);
56
       int dc=1, dd=1, x=1; p=Req[1];
57
58
       while (dd \le n) {
59
           if (p=NULL){
60
              x++;
              while (x \le n \&\& Req[x] = = NULL) x++;
61
62
              if (x>n) break;
              p=Req[x];
63
64
           if (dd < x-d) {
65
              printf("0 \ n");
66
67
              dd++; dc=1;
68
          } else {
69
              printf("%d_{-}", p->id);
70
              p=p->next;
71
              if (++dc>sol)
72
                  dc=1; dd++;
73
                  printf("0 \ n");
              }
74
          }
75
76
77
       if (dc>1) { dd++; printf("0\n"); }
78
       while (dd++\leq n) printf("0\n");
79
       return 0;
80
  }
```