Домашнее задание к уроку № 4

① а) $f(x) = \ln(x+2)$

$x + 2 > 0$

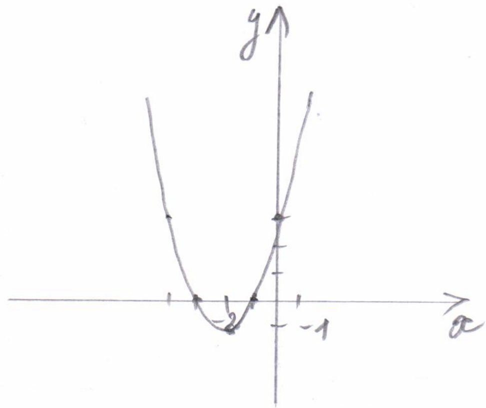
$x > -2$

$D(f) = (-2; +\infty)$

② а) $y = x^2 + 4x + 3$

$y = x^2 + 4x + 4 - 4 + 3$

$y = (x+2)^2 - 1$

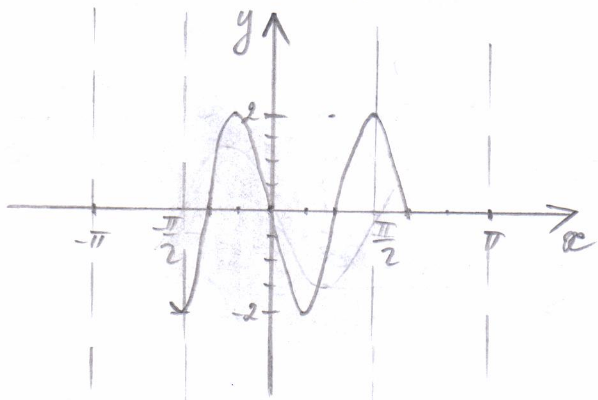


б) $y = -2\sin 3x$

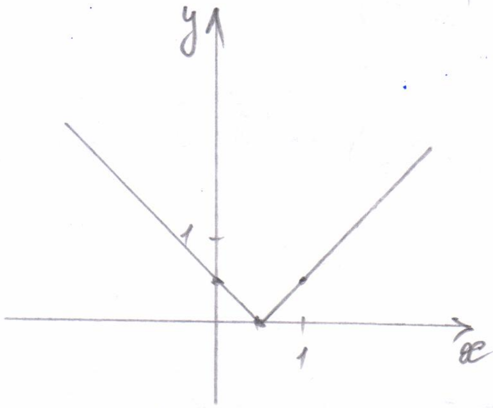
симм-но относ. Ox

растяжение вдоль Oy

сжатие вдоль Ox



в) $y = \left| \{x\} - \frac{1}{2} \right|$



③ 1) $\lim\limits_{x \to 5} \dfrac{x^2 - 6x + 5}{x^2 - 25} = \left[\dfrac{0}{0}\right] =$

$D = 36 - 4 \cdot 1 \cdot 5 = 16$

$x_1 = \dfrac{6+4}{2} = 5 \quad x_2 = \dfrac{6-4}{2} = 1$

$= \lim\limits_{x \to 5} \dfrac{(x-5)(x-1)}{(x+5)(x-5)} = \dfrac{x-1}{x+5} = \dfrac{4}{10} = \dfrac{1}{5}$

2) $\lim\limits_{x \to -1} \dfrac{x^3 + x + 2}{x^3 + 1} = \dfrac{(x+1)(x^2 - x + 2)}{(x+1)(x^2 - x + 1)} =$

$= \lim\limits_{x \to -1} \dfrac{x^2 - x + 2}{x^2 - x + 1} = \dfrac{(-1)^2 - (-1) + 2}{(-1)^2 - (-1) + 1} = \dfrac{4}{3}$

3) $$\lim_{x\to 3}\frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1}=\left[\frac{0}{0}\right]=\lim_{x\to 3}\frac{(\sqrt{2x+3}-3)(\sqrt{2x+3}+3)}{(\sqrt{x-2}-1)(\sqrt{2x+3}+3)}=$$

$$=\frac{(2x+3)-9}{(\sqrt{x-2}-1)(\sqrt{2x+3}+3)}=\frac{(2x-6)(\sqrt{x-2}+1)}{(\sqrt{x-2}-1)(\sqrt{x-2}+1)(\sqrt{2x+3}+3)}=$$

$$=\frac{(2x-6)(\sqrt{x-2}+1)}{((x-2)-1)(\sqrt{2x+3}+3)}=\frac{2(x-3)(\sqrt{x-2}+1)}{(x-3)(\sqrt{2x+3}+3)}=$$

$$=\frac{2(\sqrt{x-2}+1)}{\sqrt{2x+3}+3}=\frac{4}{6}=\frac{2}{3}$$

4) $$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{2\sin^2\left(\frac{x}{2}\right)}{x^2}=2\cdot\left(\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{x}\right)\right)^2=2\cdot\left(\frac{1}{2}\right)^2=\frac{1}{2}$$

5) $$\lim_{x\to 0} x\cdot \operatorname{ctg} x=x\cdot\frac{\cos x}{\sin x}=\frac{\cos x}{\frac{\sin x}{x}}=$$

$$=\lim_{x\to 0}\frac{\overset{=1}{\cos x}}{1}=1$$

6) $$\lim_{x\to 0}\sqrt[2x]{1+3x}=\left[1^{\infty}\right]$$

$$=\lim_{x\to 0}(1+3x)^{\frac{1}{2x}}=(1+3x)^{\frac{1}{2x}\cdot\frac{2}{3}\cdot\frac{3}{2}}=\underbrace{(1+3x)^{\frac{1}{3x}}}_{\Rightarrow e}{}^{\cdot\frac{3}{2}}=\sqrt{e^3}$$

$$=\lim_{x\to 0}$$

7) $$\lim_{x\to 0}\left(\frac{3+5x}{3+2x}\right)^{\frac{1}{x}}=\left[1^{\infty}\right]=\left(\frac{3+\frac{5}{y}}{3+\frac{2}{y}}\right)^{y}=$$

$$=\frac{\lim\limits_{x\to 0}\left(3+\frac{5}{y}\right)^{y}}{\lim\limits_{x\to 0}\left(3+\frac{2}{y}\right)^{y}}=\frac{\lim\limits_{x\to 0}\left(1+\frac{5}{3y}\right)^{y}}{\lim\limits_{x\to 0}\left(1+\frac{2}{3y}\right)^{y}}=$$

$$=\frac{e^{\frac{5}{3}}}{e^{\frac{2}{3}}}=e$$