Домашнее задание к уроку 10

① $\int \frac{x^4+x^2-6x}{x^3}dx = \int x - x^{-1} - 6x^{-2}dx = \frac{x^2}{2} + \ln(|x|) + 12\,\frac{x^{-1}}{-1} + C =$

$= \ln(|x|) + \frac{x^2}{2} + \frac{6}{x} + C = \ln(|x|) + \frac{x^3+12}{2x} + C$

② $\int \cos 2x\,dx = \left[f(ax+b) = \frac{1}{a}F(ax+b) + C \right] = \frac{1}{2}\sin 2x + C$

③ $\int \frac{5x-1}{\sqrt{4-x^2}}dx = \int \frac{5x}{\sqrt{4-x^2}}dx - \int \frac{1}{\sqrt{4-x^2}}dx = 5\int \frac{x}{\sqrt{4-x^2}}dx - \int \frac{1}{\sqrt{4-x^2}}dx$ ⊜

• $\int \frac{x}{\sqrt{4-x^2}}dx = \left| \begin{array}{l} u = 4-x^2 \\ \frac{du}{dt} = -2x \\ dx = -\frac{1}{2x}du \end{array} \right| = -\frac{1}{2}\int \frac{1}{\sqrt{u}}du = 2\sqrt{u}$

$-\frac{1}{2}\int \frac{1}{\sqrt{u}}du = -\sqrt{u} = -\sqrt{4-x^2} + C$

• $\int \frac{1}{\sqrt{4-x^2}}dx = \arcsin\frac{x}{2} + C$

⊜ $-5\sqrt{4-x^2} - \arcsin\left(\frac{x}{2}\right) + C$

④ $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = \left| \begin{array}{l} u = 1+\sqrt{x} \\ \frac{du}{dx} = \frac{1}{2\sqrt{x}} \\ dx = 2\sqrt{x}\,du \end{array} \right| = 2\int \frac{1}{u}du = 2\ln(u) + C =$

$= 2\ln(1+\sqrt{x}) + C$

⑤ $\int \underbrace{x^2}_{u}\,\underbrace{\cos x\,dx}_{d\upsilon} = \left| \begin{array}{l} u = x^2 \\ du = 2x\,dx \\ \cos x\,dx = d\upsilon \\ \sin x = \upsilon \end{array} \right| = \left[\int u\,d\upsilon = u\upsilon - \int \upsilon\,du \right] =$

$x^2 \cdot \sin x - \int \sin x \cdot 2x\,dx$ (=)

$$\int 2x \sin(x)\,dx = 2\int \underbrace{x}_{u} \underbrace{\sin x\,dx}_{d\upsilon} = \left| \begin{array}{ll} u = x & \sin x\,dx = d\upsilon \\ du = dx & \int \sin x\,dx = \int d\upsilon \\ & -\cos x = \upsilon \end{array} \right| =$$

$$= \left[\int u\,d\upsilon = u\upsilon - \int \upsilon\,du \right] = x \cdot (-\cos x) - \int (-\cos x)\,dx =$$

$$= 2(-x\cos x + \sin x) + c$$

(=) $x^2 \cdot \sin x - 2(\sin x - x\cos x) + c = x^2 \cdot \sin x - 2\sin x + 2x\cos x + C$

(6) $\int \frac{(x+6)\,dx}{x^2-2x+17}$ (=) $\qquad (x^2-2x+17)' = 2x-2 \qquad x+6 = \frac{2x-2}{2} + 7 \qquad x^2-2x+17 = (x-1)^2+16$

(=) $$\int \frac{\frac{2x-2}{2}+7}{x^2-2x+17}\,dx = \frac{1}{2} \cdot \int \frac{2x-2}{x^2-2x+17}\,dx + 7\int \frac{dx}{(x-1)^2+16} =$$

$$= \left| \begin{array}{ll} x^2-2x+17 = t & y = x-1 \\ dt = (2x-2)\,dx & dy = dx \end{array} \right| = \frac{1}{2}\int \frac{dt}{t} + 7\int \frac{dy}{y^2+16^2} =$$

$$= \frac{1}{2}\ln|t| + 7 \cdot \frac{1}{4}\,\mathrm{arctg}\,\frac{y}{4} + C = \frac{1}{2}\ln|x^2-2x+17| + \frac{7}{4}\,\mathrm{arctg}\,\frac{x+2}{3} + C$$

(7) $$\int \frac{x\,dx}{(x^2-1)(x^2+1)} = \left| \begin{array}{l} u = x^2+1 \\ \frac{du}{dx} = 2x \\ dx = \frac{1}{2x}\,du \end{array} \right|$$

$$\frac{1}{2}\int \frac{1}{(u-2)u}\,du = \int \frac{1}{(1-\frac{2}{u})u^2}\,du = \left| \begin{array}{l} \upsilon = 1-\frac{2}{u} \\ \frac{d\upsilon}{du} = \frac{2}{u^2} \\ du = \frac{u^2}{2}\,d\upsilon \end{array} \right| =$$

$$= \frac{1}{2}\int \frac{1}{\upsilon}\,d\upsilon = \frac{\ln(\upsilon)}{2} = \left| \upsilon = 1-\frac{2}{u} \right| = \frac{\ln(1-\frac{2}{u})}{2} \Rightarrow$$

$$\frac{1}{2}\int \frac{1}{(u-2)u}du = \frac{\ln\left(1-\frac{2}{u}\right)}{4} = \left| u = x^2+1 \right| = \frac{\ln\left(1-\frac{2}{x^2+1}\right)}{4} + C$$

(8) $$\int \frac{\sqrt[3]{x}\,dx}{\sqrt[3]{x^2}-\sqrt{x}} \qquad u = \frac{\sqrt[3]{x^2}-\sqrt[3]{x}}{\sqrt[3]{x}}$$

$$\frac{du}{dx} = \frac{\frac{2}{3\sqrt[3]{x}} - \frac{1}{3x\sqrt[3]{x^2}}}{\sqrt[3]{x}} - \frac{\sqrt[3]{x^2}-\sqrt[3]{x}}{3\sqrt[3]{x^4}}$$

$$dx = \frac{1}{\frac{\frac{2}{3\sqrt[3]{x}} - \frac{1}{3\sqrt[3]{x^2}}}{\sqrt[3]{x}} - \frac{\sqrt[3]{x^2}-\sqrt[3]{x}}{3\sqrt[3]{x^4}}}du \ (=)$$

- $\sqrt[3]{x^2} = (u+1)^2$
- $\frac{1}{\sqrt[3]{x^4}} = \frac{1}{(u+1)^4}$
- $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{(u+1)^2}$
- $\frac{1}{\sqrt[3]{x}} = \frac{1}{u+1}$

$$(=)\ 3\int \frac{(u+1)^2}{u}du = \int\left(u+\frac{1}{u}+2\right)du =$$

$$= \int u\,du + \int \frac{1}{u}du + 2\int 1\,du \ (=)$$

- $\int u\,du = \frac{u^2}{2}$
- $\int \frac{1}{u}du = \ln(u)$
- $\int 1\,du = u$

$$(=)\ \ln(u) + \frac{u^2}{2} + 2u$$

$$3\int \frac{(u+1)^2}{u}du = 3\ln(u) + \frac{3u^2}{2} + 6u = \left| u = \frac{\sqrt[3]{x^2}-\sqrt[3]{x}}{\sqrt[3]{x}} \right|$$

$$= \frac{6\left(\sqrt[3]{x^2}-\sqrt[3]{x}\right)}{\sqrt[3]{x}} + \frac{3\left(\sqrt[3]{x^2}-\sqrt[3]{x}\right)^2}{2\cdot\sqrt[3]{x^2}} + 3\ln\left(\frac{\sqrt[3]{x^2}-\sqrt[3]{x}}{\sqrt[3]{x}}\right) + C$$

(9) $$\int \frac{dx}{\sin x \sin 2x} = \left| \sin 2x = 2\cos x \sin x \right| = \int \frac{dx}{2\cos x \sin^2 x} = \frac{1}{2}\int \frac{dx}{\cos x \sin^2 x} \ (=)$$

- $$\int \frac{dx}{\cos x \sin^2 x} = \int \frac{dx}{\csc^2(x)\sec(x)} = \int (\cot^2(x)+1)\sec(x)\,dx =$$

$$= \int(\sec(x) + \cot(x)\csc(x))\,dx = \int \sec(x)\,dx + \int \cot(x)\csc(x)\,dx \ (=)$$

$\bullet \int \sec(x)dx = \ln(\tan(x) + \sec(x))$ $\quad \bullet \int \cot(x)\csc(x)dx = -\csc(x)$

$\ominus \ln(\tan(x) + \sec(x)) - \csc(x)$

$\Rightarrow \frac{1}{2}\int \frac{1}{\cos(x)\sin^2(x)}dx = \frac{\ln(\tan(x)+\sec(x))}{2} - \frac{\csc(x)}{2} + C$

(10) $\int_0^\pi (2x + \sin 2x)dx = \int_0^\pi 2x dx + \int_0^\pi \sin 2x dx \ominus$

$\bullet \int_0^\pi 2x dx = 2\int_0^\pi x dx = 2 \cdot \frac{x^2}{2}\Big|_0^\pi = \pi^2 - 0 = \pi^2$

$\bullet \int_0^\pi \sin 2x dx = \sin^2 x\Big|_0^\pi = 0 - 0 = 0$

$\ominus \pi^2 + 0 = \pi^2$

(11) $\int_{\frac{1}{2}}^1 \sqrt{4x-2} = \left| \begin{matrix} 4x-2=t & dx = \frac{1}{4}dt \\ x = \frac{t+2}{4} & \begin{matrix} x=1 & t=2 \\ x=\frac{1}{2} & t=0 \end{matrix} \end{matrix} \right| = \frac{1}{4}\int_0^2 \sqrt{t}dt = \frac{1}{4}\int_0^2 t^{\frac{1}{2}}dt =$

$= \frac{1}{4} \cdot \frac{t^{1,5}}{1,5}\Big|_0^2 = \frac{1}{4} \cdot \frac{2^{1,5}}{1,5} - \frac{0^{1,5}}{1,5} = \frac{1}{4} \cdot \frac{2\sqrt{2}\cdot 2}{3} = \frac{\sqrt{2}}{3}$

(12) $\int_0^{+\infty} e^{-4x}dx = \lim_{b\to+\infty} \int_0^b e^{-4x}dx = \lim_{b\to+\infty}\left(-\frac{e^{-4x}}{x}\right)\Big|_0^b = \lim_{b\to+\infty} -\frac{e^{-4b}}{b} + \lim_{b\to+\infty} -\frac{e^{0}}{0}$

$= 0 + \infty = \infty$

(13) $\int_0^1 \ln x dx = x\ln(x) - x\Big|_0^1 = (0-1) - 0 = \underline{-1}$