

Домашнее задание к уроку 3.

① 1)  $y = -10 \arctg x + 7e^x$

$$y'(x) = (-10 \arctg x)' + (7e^x)' = \left(-10 \cdot \frac{1}{1+x^2}\right) + (7 \cdot e^x) = -\frac{10}{1+x^2} + 7e^x$$

2)  $y = \frac{1}{\sqrt[3]{x^2}} - \frac{2}{x^3} + \sqrt{7} \cdot x$

$$y' = \left(x^{-\frac{3}{2}}\right)' - (2 \cdot x^{-3})' + (\sqrt{7} \cdot x)' = \left(-\frac{3}{2} \cdot x^{-\frac{3}{2}-1}\right) - (2 \cdot -3 \cdot x^{-4}) + \sqrt{7} =$$
$$= -\frac{3}{2} \cdot x^{-\frac{5}{2}} + 6x^{-4} + \sqrt{7}$$

3)  $y = \cos \frac{1-\sqrt{x}}{1+\sqrt{x}}$

$$y'(x) = \left(\cos \frac{1-\sqrt{x}}{1+\sqrt{x}}\right)' = -\sin\left(\frac{1-\sqrt{x}}{\sqrt{x}+1}\right) \cdot \left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)' =$$

$$= -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \cdot \frac{(1-\sqrt{x})' \cdot (\sqrt{x}+1) - (\sqrt{x}+1)' \cdot (1-\sqrt{x})}{(1+\sqrt{x})^2} =$$

$$= -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \cdot \frac{((1)' - (\sqrt{x})') \cdot (\sqrt{x}+1) - ((\sqrt{x})' + (1)') \cdot (1-\sqrt{x})}{(1+\sqrt{x})^2} =$$

$$= -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \cdot \frac{\left(-\frac{1}{2} \cdot \frac{1}{\sqrt{x}} + 0\right) \cdot (\sqrt{x}+1) - \left(\frac{1}{2} \cdot \frac{1}{\sqrt{x}} + 0\right) \cdot (1-\sqrt{x})}{(\sqrt{x}+1)^2} =$$

$$= - \frac{\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \left(\frac{-\sqrt{x}-1}{2\sqrt{x}} - \frac{1-\sqrt{x}}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2} =$$

$$= - \frac{\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \left(\frac{-2}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2} = - \frac{\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \cdot \left(-\frac{1}{\sqrt{x}}\right)}{(\sqrt{x}+1)^2}$$

$$\begin{aligned}
4) \quad y &= \ln \frac{(x+1)(x+3)^3}{(x+2)^3(x+4)} & y' &= \left( \ln \frac{(x+1)(x+3)^3}{(x+2)^3(x+4)} \right)' = \\
&= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \left( \frac{(x+1)(x+3)^3}{(x+2)^3(x+4)} \right)' = \\
&= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{((x+1)(x+3)^3)' \cdot (x+2)^3(x+4) - ((x+2)^3(x+4))' \cdot (x+1)(x+3)^3}{(x+2)^6(x+4)^2} = \\
&= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{((x+1)' \cdot (x+3)^3 + (x+3)^3)' \cdot (x+1) \cdot (x+2)^3(x+4) - ((x+2)^3)'(x+4) + (x+4)' \cdot (x+2)^3}{(x+2)^6(x+4)^2} \\
&\quad \cdot \frac{(x+1)(x+3)^3}{(x+1)(x+3)^3} = \\
&= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{((x+1)' + (1)') \cdot (x+3)^3 + 3 \cdot (x+3)^2 \cdot (x+3)' \cdot (x+1) \cdot (x+2)^3(x+4) -}{(x+2)^6(x+4)^2} \\
&\quad - (3 \cdot (x+2)^2 \cdot (x+2)' \cdot (x+4) + (x+4)' \cdot (x+2)^3) \cdot (x+1)(x+3)^3 = \\
&= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{((1+0) \cdot (x+3)^3 + 3(x+1)(x+3)^2 \cdot ((x+1)' + (3)')) \cdot (x+2)^3(x+4) -}{(x+2)^6(x+4)^2} \\
&\quad - (3(x+2)^2(x+4)(x'+2') + (1+0)(x+2)^3) \cdot (x+1)(x+3)^3 = \\
&= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{((x+3)^3 + 3(x+1)(x+3)^2 \cdot (1+0)) \cdot (x+2)^3(x+4) -}{(x+2)^6(x+4)^2} \\
&\quad - (3(x+2)^2(x+4)(1 \cdot 0) + (x+2)^3) \cdot (x+1)(x+3)^3 = \\
&= \frac{(x+2)^3(x+4) \left( (x+3)^3 + 3(x+1)(x+3)^2 \right) - (x+1)(x+3)^3 (x+2)^3 + 3(x+2)^2(x+4)}{(x+1)(x+2)^3(x+3)^3(x+4)}
\end{aligned}$$

$$(2) \quad y = \frac{\ln x}{x} \quad x_0 = e$$

$$y' = \frac{\ln x' \cdot x - \ln x \cdot x'}{x^2} = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2} = 0$$

$$(3) \quad y = x^{\ln x}$$

$$y' = ?$$

$$\ln y = \ln x^{\ln x}$$

$$(\ln y)' = (\ln x^{\ln x})'$$

$$\frac{y'}{y} = (\ln x)' \cdot \ln x + \ln x \cdot (\ln x)'$$

$$\frac{y'}{y} = \frac{\ln x}{x} + \frac{\ln x}{x} = \frac{2 \ln x}{x} \quad | \cdot y$$

$$y' = \frac{2 \ln x}{x} \cdot x^{\ln x} = 2$$

$$y' = \frac{2 \ln x}{x}$$

$$(4) \quad e^{xy} - \cos(x^2 + y^2) = 0$$

$$y' = ?$$

$$e^{xy} \cdot y' - (-\sin(x^2 + y^2) \cdot (x^2 + y^2)') = 0$$

$$e^{xy} \cdot y' - (-\sin(x^2 + y^2) \cdot (2x + 2y)) = 0$$

$$e^{xy} \cdot y' + \sin(x^2 + y^2) \cdot (2x + 2y) = 0$$

$$y' = \frac{-\sin(x^2 + y^2) \cdot (2x + 2y)}{e^{xy}}$$

$$(5) \quad x = t^3 + t$$

$$y = t^2 + t + 1$$

$$y'(x) = ?$$

$$y'(x) = \frac{(t^2 + t + 1)'}{(t^3 + t)'} = \frac{2t + 1}{3t^2 + 1}$$