Gliwice, 19.12.2022r.

Fundamentals and implementations of modeling and simulations.

Documentation of laboratory task no 5.

Title: Trajectory of a comet

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Field of studies: Informatics (sem.V)

Project Objective:

The goal of the project is to determine the trajectory of the comet moving around the Sun in dependence on the value of eccentricity. Since the mass of comet is very small, in comparision with the mass of a Sun, the situation may be considered as the trajectory of a point moving around the other fixed point.

Description:

Kepler's Laws of Planetary Motion

Kepler's three laws describe how planetary bodies orbit the Sun. They describe how (1) planets move in elliptical orbits with the Sun as a focus, (2) a planet covers the same area of space in the same amount of time no matter where it is in its orbit, and (3) a planet's orbital period is proportional to the size of its orbit (its semi-major axis).

Kepler's First Law: each planet's orbit about the Sun is an ellipse. The Sun's center is always located at one focus of the orbital ellipse. The Sun is at one focus. The planet follows the ellipse in its orbit, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit.

Since the comets are small bodies, the attraction between the comet and the Sun may cause that the comets have also the parabolic or slightly hyperbolic trajectories. A parameter that determines the deviation of the orbit of a comet, moving around the Sun, from a perfect circle is called the orbital eccentricity (denoted by e_c). The value of e_c equal to 0 means a circular orbit, values between 0 and 1 form an elliptical orbit, 1 means a parabolic escape orbit and greater than 1 forms a hyperbola.

Velocity of the comet is described by the following differential equation:

$$v'(t)\frac{(1+e_c)^2}{(1+\cos(v(t))\cdot e_c)^2} = 1, \quad v(0) = 0.$$

Coordinates of te comet position are defined by the following equations:

$$\begin{cases} x(t, e_c) = (1+e_c)\frac{\cos(v(t))}{1+e_c\cos(v(t))} \\ y(t, e_c) = (1+e_c)\frac{\sin(v(t))}{1+e_c\cos(v(t))} \end{cases}, \quad t \in \mathbb{R}$$

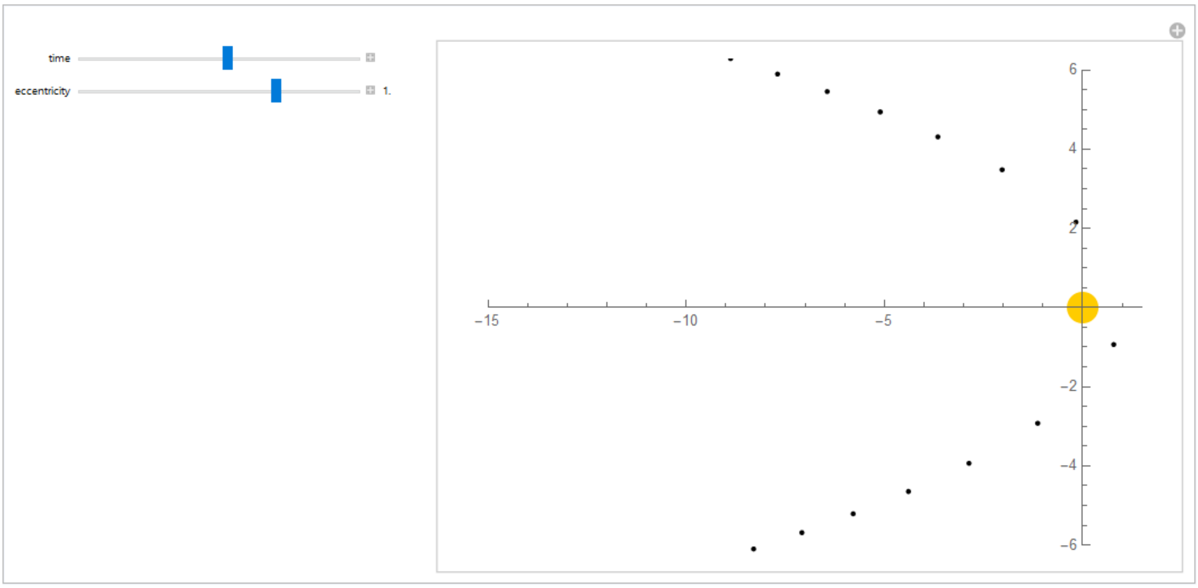
Solving the differentia equation for velocity:

```
abc = First[v /. NDSolve[{v'[t] * (1 + e) ^2 / (1 + Cos[v[t]] * e) ^2 == 1, v[0] == 0}, v, {t, -520, 520}]];
      pierwszy     rozwiąż numerycznie równanie różniczkowe cosinus
```

Equations of comet position:

```
x[nn_, ee_] := (1 + ee) Cos[abc[nn]] / (1 + ee Cos[abc[nn]]);
                        cosinus               cosinus
y[nn_, ee_] := (1 + ee) Sin[abc[nn]] / (1 + ee Cos[abc[nn]]);
                        sinus                 cosinus
```

Final result:



Enclosures:

File with the program(Łukaszek_Artur_proj5.nb)