Solution to analysis in Home Assignment 2

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Analysis

In this report I will present my independent analysis of the questions related to home assignment 2. I have discussed the solution with Qun Zhang, but I swear that the analysis written here are my own.

1 Scenario 1 – A first Kalman filter and its properties

a)

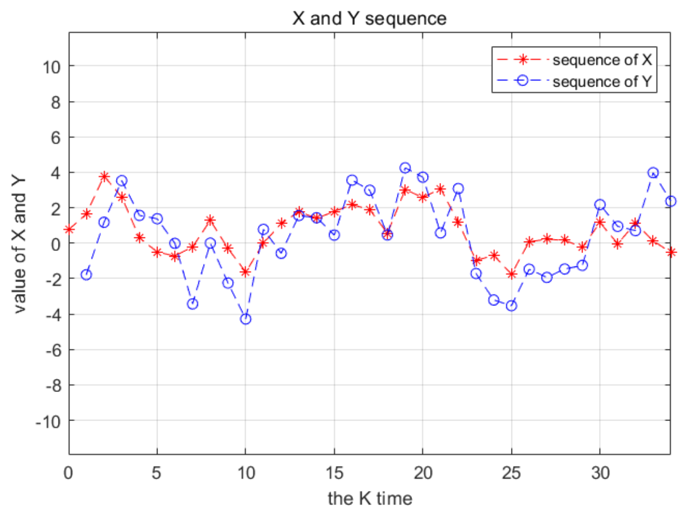

Figure 1

Yes. As we can see in figure, the general trend of the x and y series is almost the same, which means the measurement behave according to the model as the measurement model is $y_k = x_k + r_k$, but the specific values of the two are different due to measurement errors.

b)

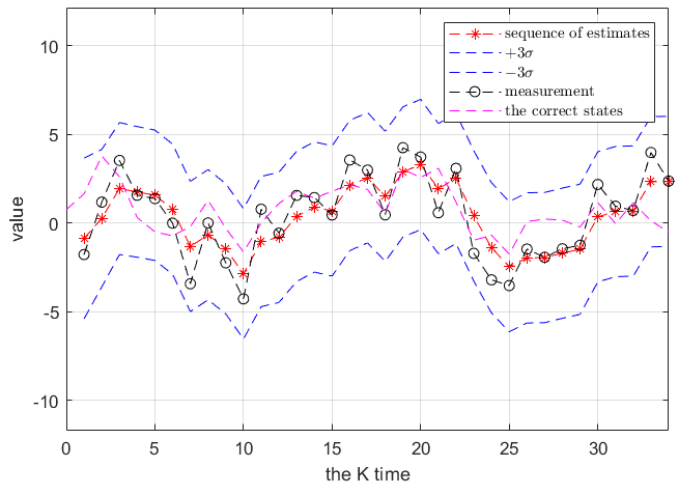

Figure 2

Yes, the estimates that the filter outputs are reasonable.Because we can see that the correct values are within 3 sigma range of the estimator and they share the same general trend.
Yes, the error covariance represent the uncertainty well. The correct values are strictly within the 3 sigma range of the estimator in the figure we get.

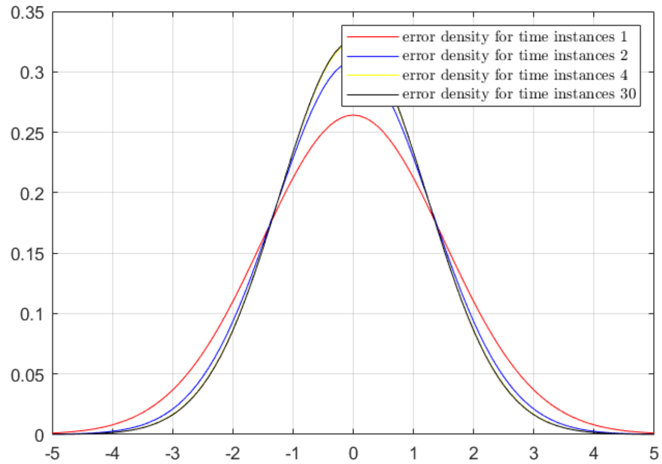

Figure 3

As we can see in the figure, the covariance of these densities are getting smaller from time instance 1 to 30 (and approaching a stationary P value).The value from time instance 4 is almost the same as 30, which means it's almost the stationary P.

c)

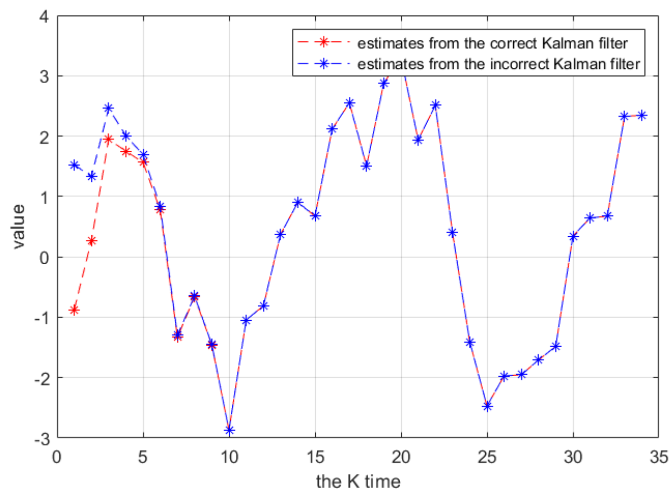

Figure 4

In figure 4, we have both estimates from correct Kalmen and Kalmen with incorrect prior at time 0. In the first, the two curves doesn't the same because we have different mean for the initial distribution, but they start to have the same performance over time because the Kalmen filter is affected by measurement, and we use the same measurement for two filter.

d)

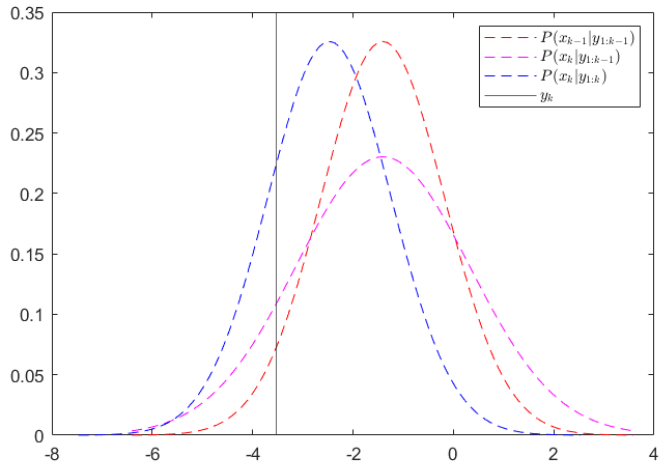

Figure 5

Yes, the behavior is reasonable. I use the time instance $k = 25$.
In the prediction step, the result is obtained from the result of last time instance with the function:

$$\begin{aligned}\hat{x}_{k|k-1} &= A * \hat{x}_{k-1|k-1} \\ P_{k|k-1} &= A * P_{k-1|k-1} * A^T + Q\end{aligned} \tag{1}$$

As the A is 1, the value of mean does not change after prediction but the covariance changed.
In the update step, the innovation makes the result affected by the measurement. so in the plot we can see the distribution of update result is moving towards the measurement value y.

e)

Firstly I generate a long true state sequence of length 100000. And I get the estimate values with the kalmen filter. Then we could have the histogram and compare to the normpdf with mean 0 and covariance $P_{k|k}$.

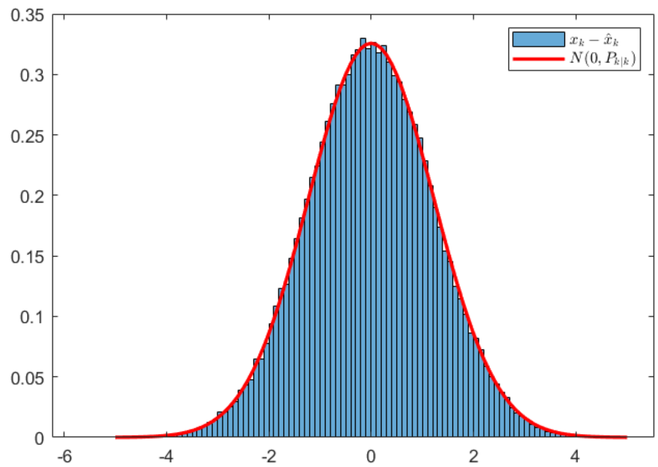

Figure 6

In the figure we can see that the two curve fit really well, they are the same distribution. After awhile, we get the constant $P_{k|k}$, and it represent the uncertainty of the estimates. So the $P_{k|k}$ is also the covariance of distribution of errors(a zero mean distribution).

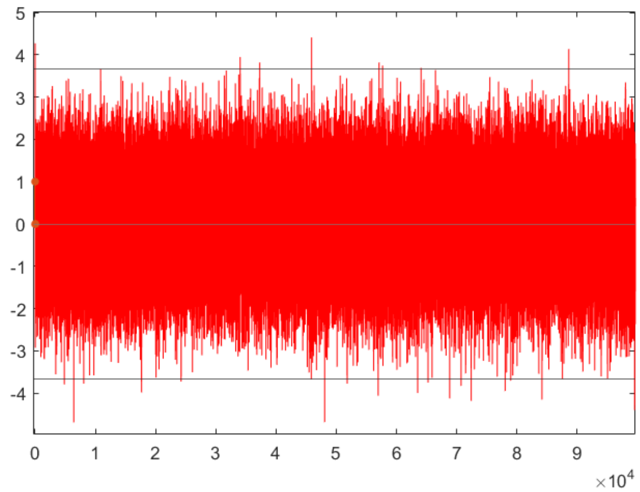

Figure 7

This figure shows the $S^{-\frac{1}{2}}V_k$ is within 3σ region. Shows consistency of innovation.

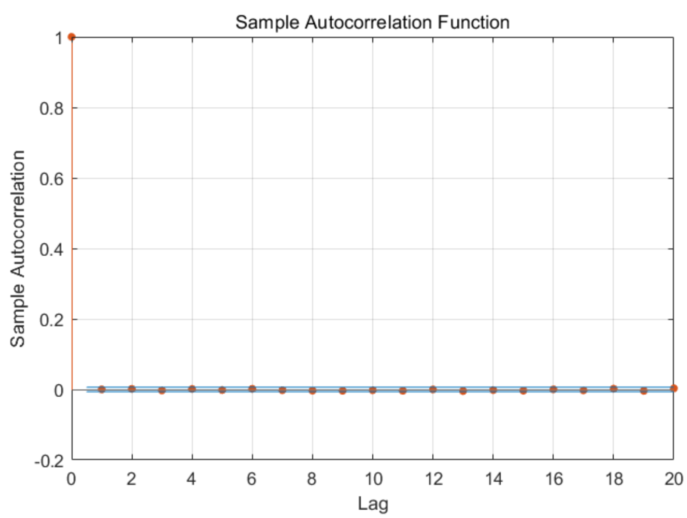

Figure 8

The auto correlation function of the innovation process shows that the correlation between sequence of innovations are really small which shows the whiteness of innovation.

2 Scenario 2 – Tuning a Kalman filter

a) Calibrate speed sensor model:

We get the mean from CalibrationSequenceVelocity_v10 and CalibrationSequenceVelocity_v20, and the C can be calculate by this, because the train was moving constantly with 10 or 20:

$$
\begin{aligned}
C &= mean_{20}/20 \\
C &= mean_{10}/10
\end{aligned} \tag{2}
$$

And we get $C = 1.1023$ for both equation. And we can get the variance σ of the three sequence, the variance of the velocity sensor noise can be calculate as this:

$$
R = \sigma/(C^2) \tag{3}
$$

As the following figures, we can see that the calibrated model is behaving appropriately:

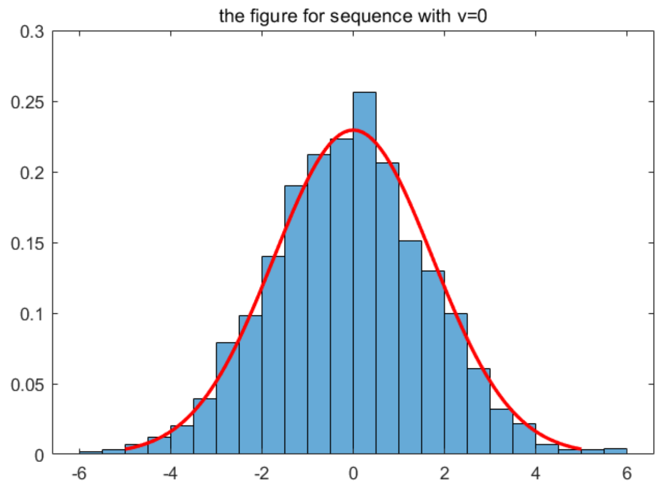

Figure 9

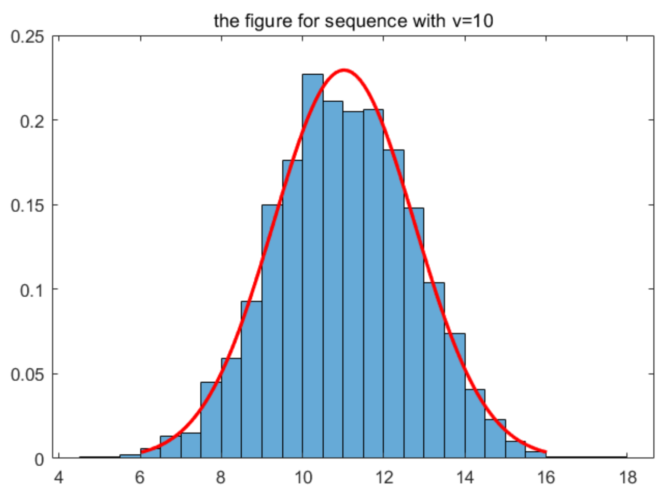

Figure 10

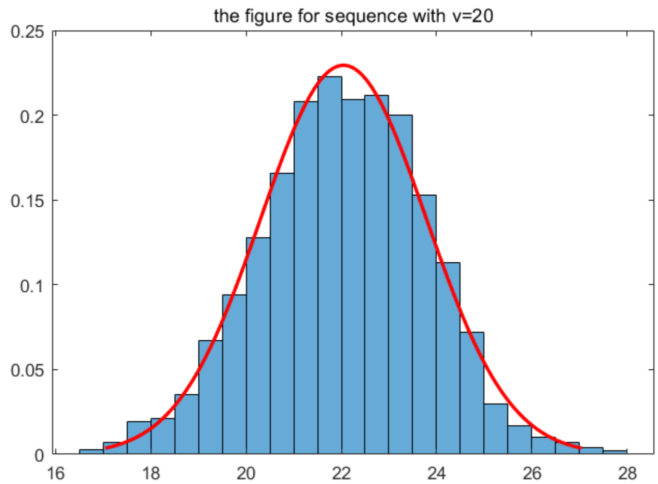

Figure 11

b)

In order to use the measurement in the Kalmen filter, we need to delete all the velocity values at the time when we have no information about position. Then we get a observation with $T = 0.2s$. And we can use the observation in the update step of Kalmen.

c) & d)

We need to tune the model by change the value of Q .
For CV and CA model, When we set$x_o p = 100, x_o v = 100, x_o a = 100$ and$Q_v = 3$,$Q_a = 3$ individually (I choose the value because they are similar to measurement noise):

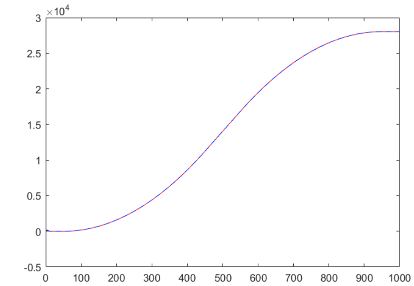

Figure 12: position

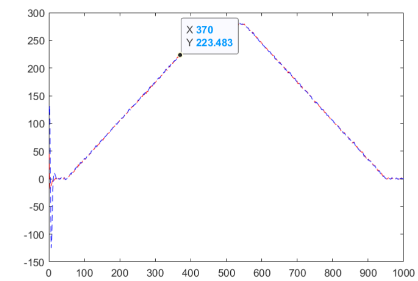

Figure 13: velocity

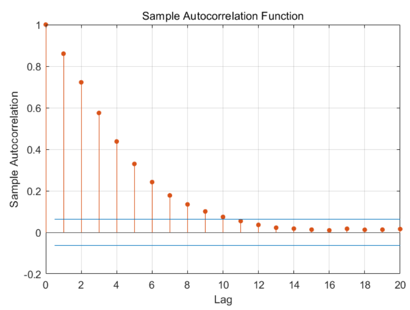

Figure 14: auto correlation for CV of position

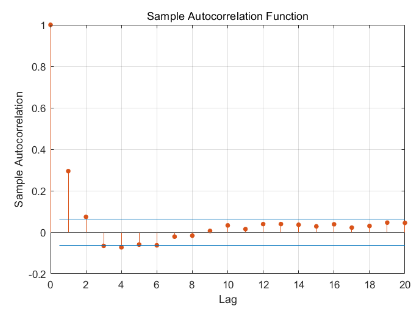

Figure 15: auto correlation for CV of velocity

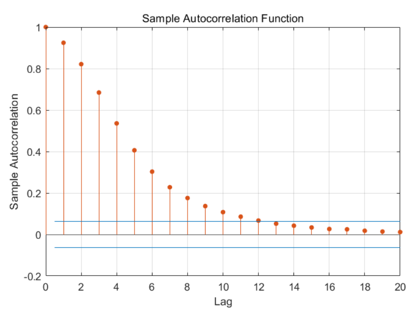

Figure 16: auto correlation for CA of position

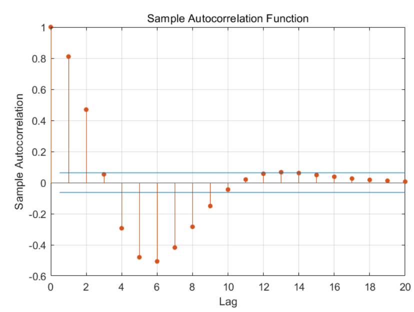

Figure 17: auto correlation for CA of velocity

We can see that the position and velocity changed from the big value to proper value in a few moments. And the correlations for both CA and CV model gradually get smaller, and now we can get the proper initial values.

For CV and CA model, When we set $x_op = 0, x_ov = 0, x_oa = 5$ and $Q_v = 3$, $Q_a = 3$ individually (I choose the value because they are similar to measurement noise):

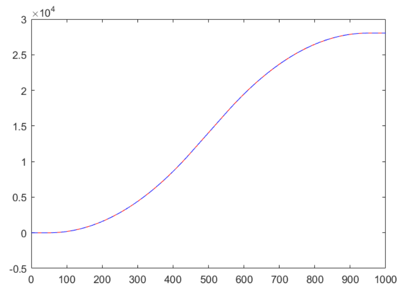

Figure 18: position

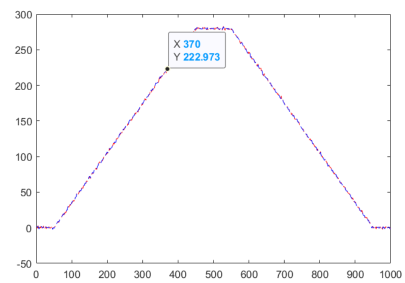

Figure 19: velocity

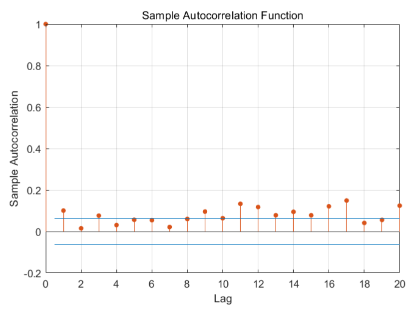

Figure 20: auto correlation for CV of position

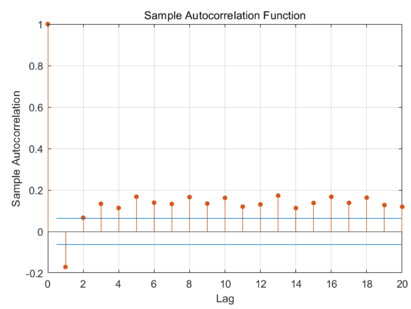

Figure 21: auto correlation for CV of velocity

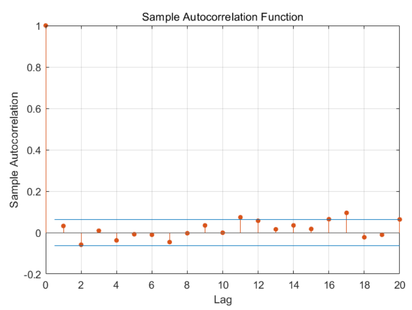

Figure 22: auto correlation for CA of position

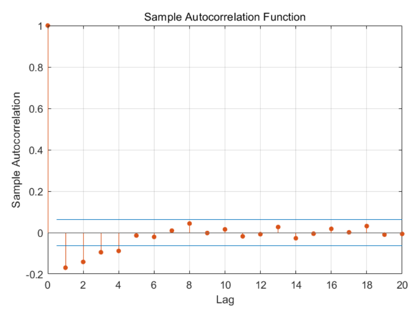

Figure 23: auto correlation for CA of velocity

The two model get almost the same figure and the correlations are small which means the two model are better.

For CV and CA model, When we set$x_o p = 0, x_o v = 0, x_o a = 5$ and$Q_v = 1000$,$Q_a = 1000$ individually (I choose the value because they are similar to measurement noise):

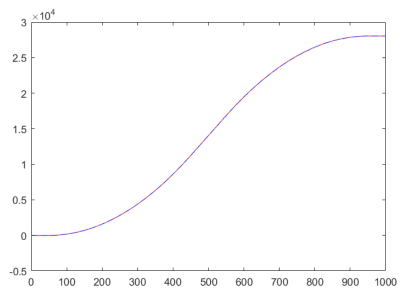

Figure 24: position

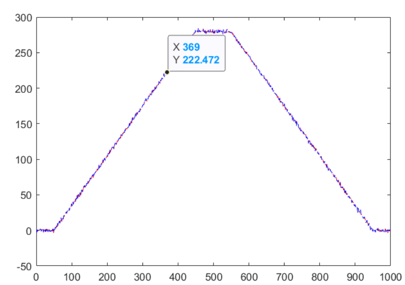

Figure 25: velocity

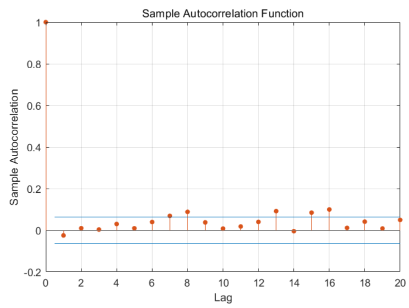

Figure 26: auto correlation for CV of position

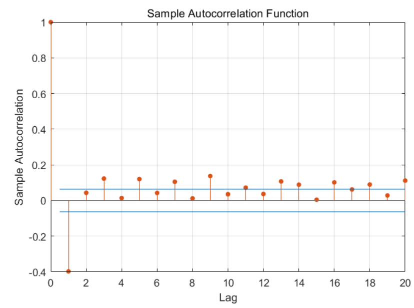

Figure 27: auto correlation for CV of velocity

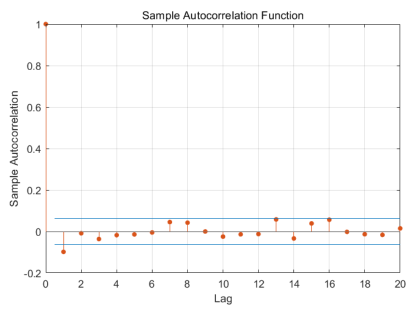

Figure 28: auto correlation for CA of position

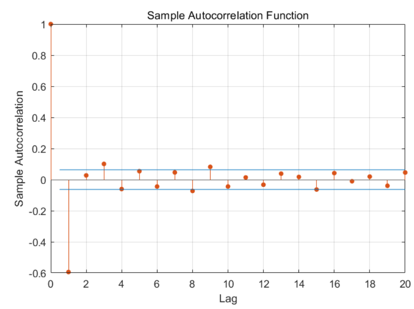

Figure 29: auto correlation for CA of velocity

In these model, the Q are too big, which means the result trust too much about the measurement and the model almost have no affection. Although the figures looks fine, the Q value doesn't make sense.
For CV and CA model, When we set$x_o p = 0, x_o v = 0, x_o a = 5$ and$Q_v =$ 0.001,$Q_a = 0.001$ individually (I choose the value because they are similar to measurement noise):

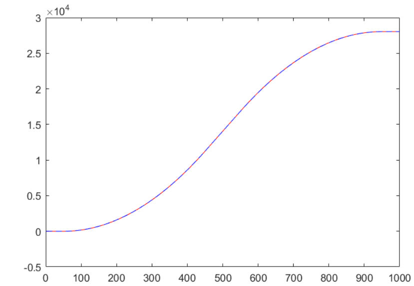

Figure 30: position

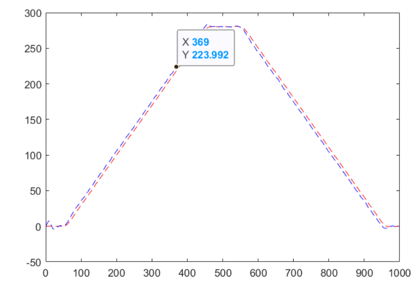

Figure 31: velocity

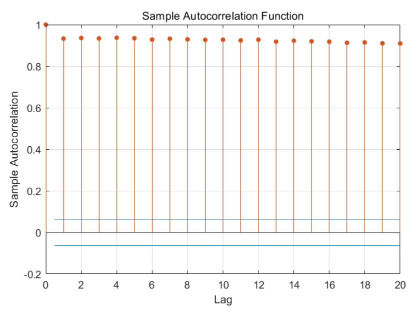

Figure 32: auto correlation for CV of position

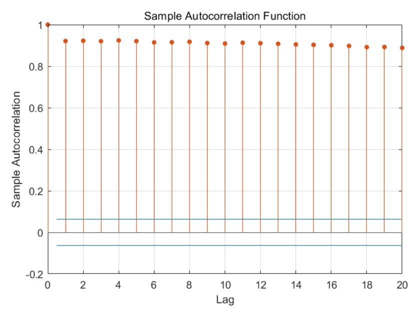

Figure 33: auto correlation for CV of velocity

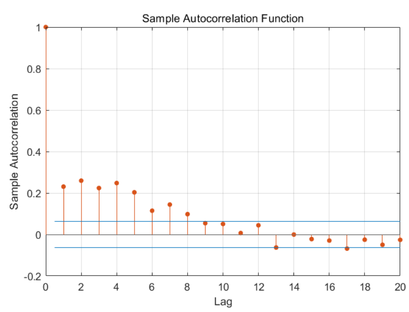

Figure 34: auto correlation for CA of position

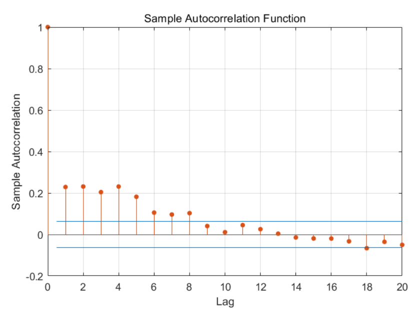

Figure 35: auto correlation for CA of velocity

In these models, the Q are too small, which means the result trust too much about the model and the measurement get smaller affection. There is different between the two model for velocity. And when we look at the correlations we can find that the two models are both not good but the CV model is a lot worse than the CA model. That is because the system is not a constant velocity system and the accelerate changes in the progress.

So for the tuning, we might choose $Q_v = 3$,$Q_a = 3$ for the system.
And we can see that the CA model is better than CV model. Because when we trust more about model, the result of CA model is a lot better than CV model.
The CA model can be used to describe the motion of objects that are experiencing constant acceleration, but sometimes it's difficult to apply the model.The CV model can not be used to describe the motion of objects that are experiencing constant acceleration very well, but sometimes it's more simply model and it's easier to be applied.