Orientation estimation using smartphone sensors

Lizi Teng + lizi(cid)
Luming Wang+luming(cid)

Task 1-Discuss pros and cons regarding the choice of input

Pros and cons using gyroscope measurements as inputs

- Pros of using gyroscope measurements as inputs:

1. Accuracy: Gyroscope measurements provided by smartphones are generally accurate. They can provide precise angular velocity information, which is crucial for orientation estimation.
2. Low latency: Gyroscopes have a high sampling rate and can provide real-time measurements with low latency. This enables faster updates and responsiveness in the orientation estimation system.
3. Direct measurement of angular velocity: Gyroscopes directly measure the rate of rotation, which is a fundamental parameter for orientation estimation. Using gyroscope measurements as inputs allows for a more direct and intuitive representation of the system dynamics.

- Cons of using gyroscope measurements as inputs:

1. Bias estimation: Gyroscopes often suffer from bias errors that can affect the accuracy of orientation estimation. Estimating and compensating for these biases is necessary to obtain accurate orientation estimates.

2. Drift over time: Gyroscopes are prone to drift, which means that even in the absence of rotation, small errors can accumulate over time and result in incorrect orientation estimates. Drift correction techniques are required to mitigate this issue.

3. Sensitivity to external disturbances: Gyroscopes can be sensitive to external factors such as vibrations and shocks, leading to inaccuracies in the orientation estimation. This sensitivity may require additional filtering or sensor fusion techniques to improve robustness.

Situation where specified input is inappropriate

One situation where using gyroscope measurements as the sole input may not be a good choice is when the gyroscope data is unreliable or contaminated with significant noise. In such cases, relying solely on gyroscope measurements can lead to inaccurate orientation estimation. Including additional sensor data, such as accelerometer or magnetometer measurements, can help compensate for the limitations of the gyroscope and improve the overall accuracy of the estimation.
Also it's also not a good choice to only use gyroscopes as input when we need long-term estimates since gyroscopes are prone to drift over long time. Small errors in gyroscope measurements can accumulate over time.

Include angular velocities in the state vector

It is better to include angular velocities in the state vector when there is a need for a more comprehensive representation of the system dynamics.
By incorporating angular velocities directly into the state vector, the estimation algorithm can take advantage of the additional information to model and track the motion more accurately. This can be particularly beneficial in scenarios with high dynamic movements, rapid changes in orientation, or complex motion patterns. Including angular velocities in the state vector allows for a more detailed and nuanced estimation of the orientation, leading to improved performance in such situations.

Task 2-collect a few seconds of data and compute mean and variance for the accelerometer, gyroscope, and magnetometer

Histograms of measurements and the signals overtime

The histograms and the the signal over time for the three axes of the accelerometer, gyroscope and magnetometer are shown respectively in Figure 1, Figure 2 and Figure 3.

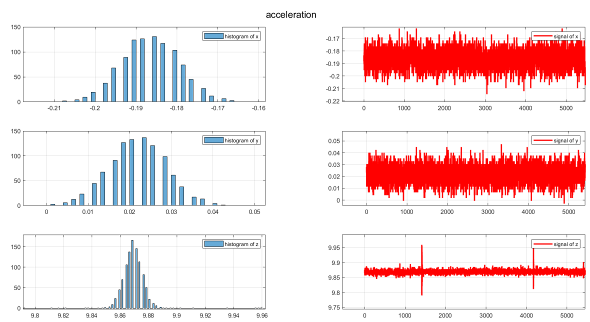

Figure 1: histogram and signal of accelerometer

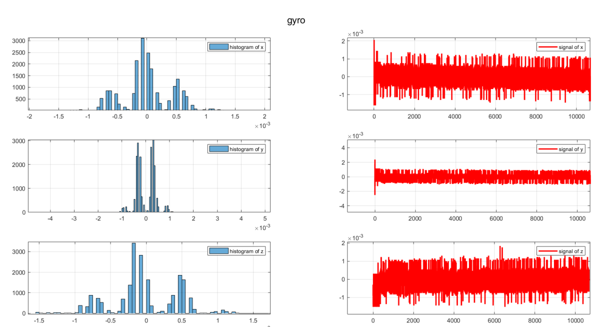

Figure 2: histogram and signal of gyroscope

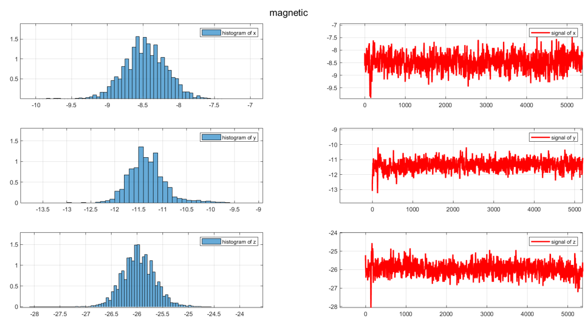

Figure 3: histogram and signal of magnetometer

Compute mean and covariances

The computed means and covariances for different sensors are shown below:

Table 1: Calculated mean values of calibration data

	Accelerometer	Gyroscope	Magnetometer
μ_x	-0.0698	$0.0382 \cdot 10^{-4}$	1.8617
μ_y	-0.0622	$0.0565 \cdot 10^{-4}$	-10.8390
μ_z	9.8720	$-0.1954 \cdot 10^{-4}$	-47.9589

Table 2: Calculated covariance of calibration data

	Accelerometer	Gyroscope	Magnetometer
σ_x^2	$5.0364 \cdot 10^{-5}$	$1.8082 \cdot 10^{-7}$	0.0843
σ_y^2	$4.9247 \cdot 10^{-5}$	$1.5847 \cdot 10^{-7}$	0.1062
σ_z^2	$4.6910 \cdot 10^{-5}$	$2.0807 \cdot 10^{-7}$	0.0974

Analysis of measurement

Our experiment device is Xiaomi 10T. The smartphone is placed faced up on the flat floor surface. The histograms and the signal over time are shown above as well their mean and variance.
As we can see from the histograms, the noises are all nearly Gaussian which means in the following EKF, the diagonal of the noise covariance can be used as the covariance matrix for measurement noise. Also the means of

accelerometer and magnetometer shows obvious trends because of the earth's magnetic field and gravity field.

- The accelerometer has a mean value of 9.872 in z-axes with the value of the gravity, the x and y axes have the mean value nearly to zero due to inevitable vibration.
- The gyroscope, at rest, the output of the gyroscope should be close to zero, and any small measurement errors or noise will be random and evenly distributed in the positive and negative directions
- The magnetometer has more noise compared with other two, considering the magnitude of the numbers. from the histogram, the magnetic field seems to follow the normal distribution. In practice, the magnetic field may be disturbed by weak magnetic fields from the environment, or by the phone's own magnetic or electronic components, which may cause the magnetometer's measurement data to deviate slightly from zero at rest.

Task 3-Derivation of a discretized model

From the given the equation:

$$\dot{q}(t) = \underbrace{\frac{1}{2}S(w_{k-1} + v_{k-1})}_{A} q(t) \tag{1}$$

Then we need to discrete it:

$$\begin{aligned} q_k &= e^{AT} q_{k-1} \\ &\text{Given that: } \exp A \approx I + A \\ &= (I + AT) q_{k-1} \\ &= (I + \frac{T}{2} S(w_{k-1} + v_{k-1})) q_{k-1} \end{aligned} \tag{2}$$

Then from the given equation:

$$S(w_{k-1} + v_{k-1}) = S(w_{k-1}) + S(v_{k-1}) \tag{3}$$

equation q_k can be rearranged as:

$$q_k = (I + \frac{T}{2}S(w_{k-1}))q_{k-1} + \frac{T}{2}S(v_{k-1})q_{k-1} \tag{4}$$

Then from the Given equation:

$$S(\omega)q = \bar{S}(q)\omega \tag{5}$$

equation q_k can be written as:

$$q_k = \underbrace{(I + \frac{T}{2}S(w_{k-1}))}_{F(\omega_{k-1})} q_{k-1} + \underbrace{\frac{T}{2}\bar{S}(q)}_{G(\hat{q}_{k-1})} v_{k-1} \tag{6}$$

Task 4- Matlab function for tu_qw

With the equation 6 from Task3, we can implement the predict step with input w:

$$\begin{aligned} x_k &= F(w_{k-1}) * x_{k-1} \\ P_k &= F(w_{k-1}) * P_{k-1} * F(w_{k-1})^T + G(x_{k-1}) * R_w * G(x_{k-1})^T \end{aligned} \tag{7}$$

The matlab function tu_qw is shown below:

```
function [x, P] = tu_qw(x, P, omega, T, Rw)
% TU_QW  EKF time update step
    % Update x and P when w is available
    % omega is the measured angular rate,
    % T the time since the last measurement,
    % Rw the process noise covariance matrix.
    % Calculate  F G Q
        F = eye(size(x,1))+(T/2)*Somega(omega);
        G = (T/2)*Sq(x);% process noise also has a
           motion model
        Q=G*Rw*G';
        x = F*x;%process noise is zero mean
        P = F*P*F' + Q;
end
```

If the no angular rate measurement available, we should implement the following equation:

$$\begin{aligned} x_k &= x_{k-1} \\ P_k &= P_{k-1} \end{aligned} \tag{8}$$

Task 5- Add the gyrscope update to the EKF

The smartphone is placed faced up on the flat floor surface, the filter performs almost as well as the google filter, however when we start estimate with the phone on the side. The estimate orientation of our own performs different as the google one, and is inconsistent with reality, as in figure 4.

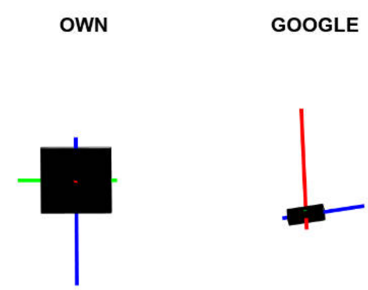

Figure 4

That is because the gyroscope could only give the information of angle velocity, so with wrong prior the filer can not estimate the actual orientation. When using only the gyroscope and lacking a magnetometer and accelerometer, the initial orientation of the device remains unknown. Without an absolute reference or prior position information, the gyroscope can only supply measurements of relative angular velocity.

Also if we shake the phone when we do the estimate, errors will occur between our filter and Google, and the orientation can not be back to initial orientation even if the phone returns to its original pose. This is because the vibrations and shocks may cause errors of gyroscope and the errors can accumulate to be a drift.

Task 6- EKF update with y_k^a and Matlab function mu_g

The measurement model is defined as:

$$y_k^a = Q\left(q_k\right)^T \left(g^0 + f_k^a\right) + e_k^a \tag{9}$$

where g^0 is the nominal gravity vector, f_k^a is a force which is non-zero if the sensor accelerates in the world frame (we assume that $f_k^a = 0$) and e_k^a is the measurement noise. And here is the update step with y_k^a.

$$\begin{aligned}
S_k &= h'\left(\hat{q}_{k|k-1}\right) P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T + R_a && \text{(innovation process)}\\
K_k &= P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T S_k^{-1} && \text{(kalman gain)}\\
\hat{q}_{k|k} &= \hat{q}_{k|k-1} + K_k\left(y_k^a - h\left(\hat{q}_{k|k-1}\right)\right)\\
P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T
\end{aligned}$$

The matlab function mu_g is shown below:

```
function [x,P] = mu_g(x,P,yacc,Ra,g0 )
            %yacc is shorthand for y ka
            %Ra is the measurement noise covariance
               matrix.
            %calculate Hx and hx
            [a, b ,c ,d]= dQqdq(x);
            Hx=[a'*g0 ,b'*g0, c'*g0, d'*g0];
            hx=Qq(x)'*g0;
            %implement EKF update
            S=Hx*P*Hx'+Ra;% innovation covariance
            k=P*Hx'/S;%Kalman Gain (K)
            %yacc-hx is the innovation
            x=x+k*(yacc-hx);
            P=P-k*S*k';
end
```

Task 7- Add the accelerometer update to the EKF

With the addition of the accelerometer update, the EKF can more accurately estimate the orientation up to a rotation in the horizontal plane. This is because the accelerometer provides information about the device's gravitational acceleration, allowing the filter to correct for tilt and provide accurate orientation estimation in the horizontal plane.
However, if performing experiments that introduces specific forces that are not purely due to gravity. f_k^a is large enough that can not be neglected.And our assumption $f_k^a = 0$ is no longer correct. These forces can affect the accuracy of the orientation estimation.

Task 8- Outlier detection and rejection for accelerometer

The process of outlier detection and rejection are shown in the following code:

```
if ~any(isnan(acc))  % Acc measurements are
   available.
if (norm(acc,2) < 10.8 && norm(acc,2) > 8.8)
[x, P] = mu_g(x,P,acc,Ra,g0);
[x, P] = mu_normalizeQ(x, P);
ownView.setAccDist(0)
else
ownView.setAccDist(1)
end
end
```

As the following figure 6 show, when specific force is introduced, the outlier accelerometer value would be rejected. And the errors that are caused by outlier accelerometer value will be prevented. So the error is smaller compared to the result of task 7 with specific forces.

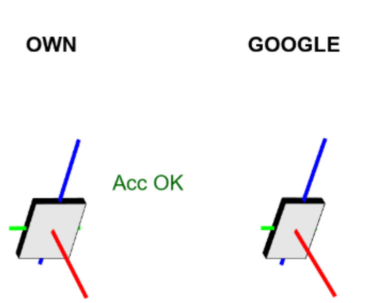

Figure 5: without acc disturbance

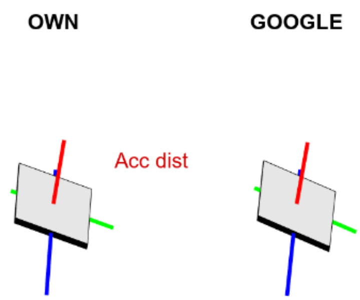

Figure 6: with acc disturbance

Task 9– EKF update with y_k^m and Matlab function mu_m

The measurement model is defined as:

$$y_k^m = Q^T(q_k)(m^0 + f_k^m) + e_k^m$$

where m^0 is the earth magnetic field in world coordinates, f_k^m represents other sources of magnetic fields (we assume that $f_k^m = 0$) and e_k^m is the measurement error.

$$\begin{aligned}
S_k &= h'\left(\hat{q}_{k|k-1}\right) P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T + R_a && \text{(innovation process)} \\
K_k &= P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T S_k^{-1} && \text{(kalman gain)} \\
\hat{q}_{k|k} &= \hat{q}_{k|k-1} + K_k\left(y_k^a - h\left(\hat{q}_{k|k-1}\right)\right) \\
P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T
\end{aligned}$$

Also the m_0 can be calculated by the equation using the means of magnetometer from task 2 when the phone lies horizontally:

$$m^0 = [0 \quad \sqrt{m_x^2 + m_y^2} \quad m_z]^T \tag{10}$$

And the matlab function mu_m is shown below:

```
function [x,P] = mu_m(x,P,mag,m0,Rm)
        %calculate Hx and hx
```

```
        [a, b ,c ,d]= dQqdq(x);
        Hx=[a'*m0,b'*m0, c'*m0, d'*m0];
        hx=Qq(x)'*m0;

        %implement EKF update
        S=Hx*P*Hx'+Rm;
        k=P*Hx'/S;

        x=x+k*(mag-hx);
        P=P-k*S*k';
end
```

Task 10- Add the magnetometer update to the EKF

The magnetometer can add an absolute reference to the sensor fusion algorithm, which can help in reducing drift errors that can accumulate over time in other sensors like accelerometers and gyroscopes. This drift can cause the orientation estimation to slowly deviate from the true orientation even when the device is stationary.

When the magneticometer update is added to the EKF, the magnetic disturbance can cause the estimated orientation to deviate from the true orientation, as in figure 7. The disturbance alters the magnetic field measurements, leading to inaccuracies in the estimation process. the EKF attempts to adjust the estimated orientation based on the perturbed magnetometer readings, which may introduce errors in the estimated orientation. This signifies that the presence of a magnetic disturbance can impact the accuracy and reliability of the orientation estimation obtained from the EKF with magnetometer update.

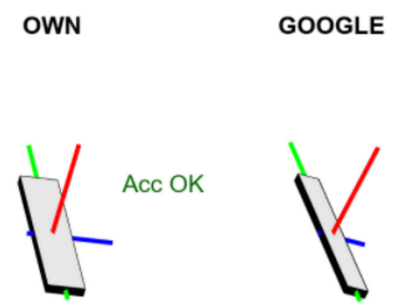

Figure 7: orientation estimate with magnetic disturbance

Task 11- Outlier detection and rejection for magnetometer

We use the following filter to estimates the magnitude:

$$\hat{L}_k = (1-\alpha)\hat{L}_{k-1} + \alpha \left\| m_k \right\| \tag{11}$$

In this equation we choose $\alpha = 0.02$. And then we can detect if the absence of disturbances cause the magnitude drift. The matlab code which detect and reject outlier is shown below:

```
if ~any(isnan(mag))  % Mag measurements are
   available
  L=0.98*L+0.02*norm(mag);
  if (L < 55 &&L > 45)
  % Do something
      [x,P] = mu_m(x,P,mag,m0,Rm);
      [x, P] = mu_normalizeQ(x, P);
      ownView.setMagDist(0)
  else
      ownView.setMagDist(1)
  end
end
```

Assumptions

The method presumes the magnetic field's expected magnitude remains mostly stable over time, with only slow drifts, free of rapid or significant disturbances.

Reasonableness of Assumptions:

The assumption is reasonable because we are in the environment where the magnetic field is relatively stable and changes are primarily due to slow drifts rather than sudden disturbances.

What happens now when you introduce a magnetic disturbance? Why?

When a magnetic disturbance is introduced, the implemented outlier rejection mechanism helps in mitigating its impact. Outliers caused by the disturbance can be identified and rejected based on the deviation of the measured magnetic field strength from the expected value. The rejection prevents the distorted measurements from significantly influencing the orientation estimation, leading to more accurate and reliable results.

The Euler angles with all filters

After adding all sensors into the filter, the Euler angles of both your orientation filter and the built in filter in the phone are shown below:

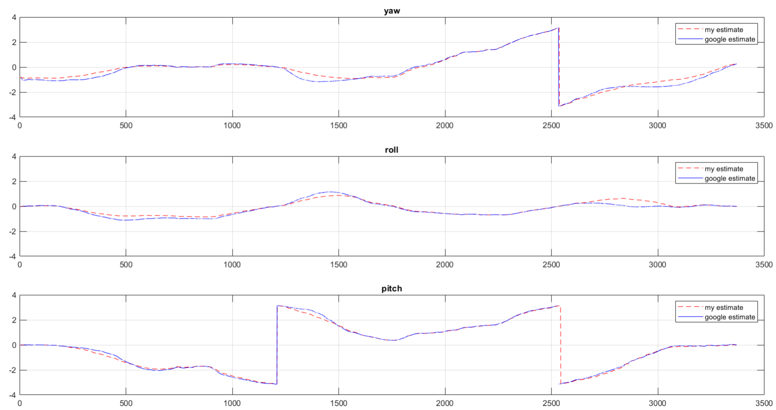

Figure 8: introduce a magnetic disturbance

Task 12- Evaluate

1. Only Accelerometer and Magnetometer (No Gyroscope):

   - Pros: The combination of accelerometer and magnetometer can provide orientation estimation in the absence of gyroscope data. The accelerometer helps estimate tilt and linear acceleration, while the magnetometer contributes to determining orientation relative to the Earth's magnetic field.
   - Cons: Without gyroscope data, the filter may be more prone to drift and unable to accurately estimate rotational motion and angular velocity. Dynamic movements and changes in orientation may not be accurately captured.

2. Only Magnetometer and Gyroscope (No Accelerometer):

   - Pros: The magnetometer and gyroscope combination can provide orientation estimation without accelerometer data. The magnetometer determines orientation relative to the Earth's magnetic field, while the gyroscope provides angular velocity measurements.

- Cons: Without accelerometer data, the filter may struggle to estimate linear acceleration and tilt. The absence of accelerometer information can limit the filter's ability to accurately capture linear motion and changes in tilt.

3. Only Accelerometer and Gyroscope (No Magnetometer):

   - Pros: The accelerometer and gyroscope combination can provide orientation estimation without magnetometer data. The accelerometer estimates linear acceleration and tilt, while the gyroscope provides angular velocity measurements.
   - Cons: Without magnetometer data, the filter may lack the ability to determine orientation relative to the Earth's magnetic field. The absence of magnetometer information can limit the filter's accuracy in determining orientation in relation to magnetic north.

4. Combination of All Three Sensors (Accelerometer, Gyroscope, and Magnetometer):

   - Pros: Sensor fusion using all three sensors offers the most comprehensive and accurate orientation estimation. It leverages the strengths of each sensor to provide robust estimation of tilt, angular velocity, and orientation in both static and dynamic scenarios.
   - Cons: The combination of all three sensors can be more computationally intensive and may require additional calibration and synchronization steps. However, the benefits in terms of accuracy and robustness generally outweigh these considerations.

Combining the accelerometer, gyroscope, and magnetometer sensors in sensor fusion enhances orientation estimation accuracy, reduces drift, improves responsiveness to dynamic movements, and increases robustness to environmental factors. By leveraging the strengths of each sensor and compensating for their limitations, the filter achieves more reliable and precise orientation estimation.