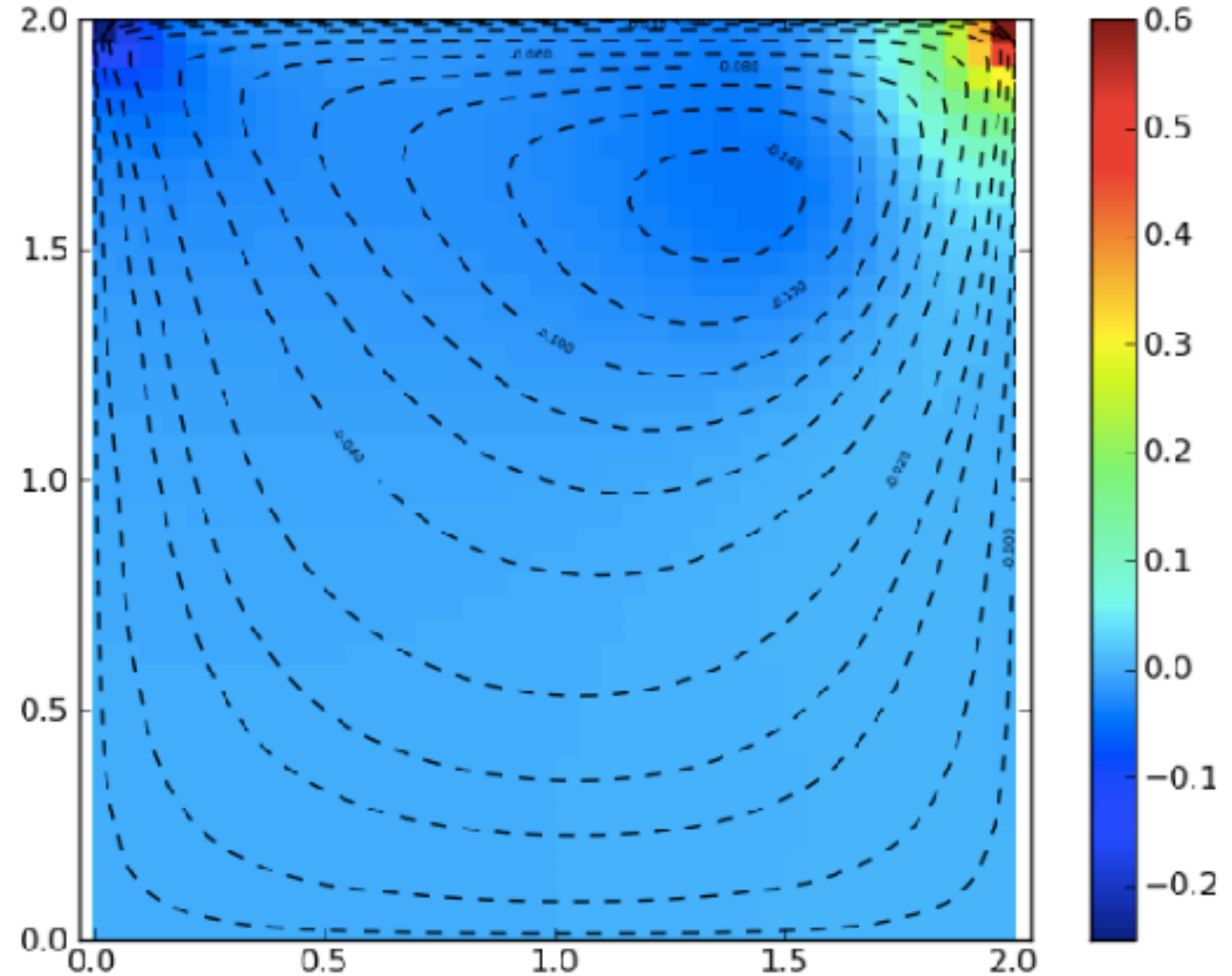


		授業計画	課題
04/06	第1回	微分方程式の離散化	前進・後退・中心差分，高次の差分を用いて微分方程式を離散化し，誤差を評価できる
04/10	第2回	有限差分法	時間積分の安定性や高次精度の積分を理解し移流・拡散・波動方程式を解析できる
04/13	第3回	有限要素法	Galerkin 法，テスト関数，isoparametric 要素の概念を理解し，弾性方程式を解析できる
04/17	第4回	スペクトル法	Fourier・Chebyshev・Legendre・Bessel などの直交基底関数による離散化の利点を説明できる
04/20	第5回	境界要素法	逆行列と δ 関数・Green 関数の関係を理解し境界積分方程式を用いた解析ができる
04/24	第6回	分子動力学法	時間積分の symplectic 性や熱浴の概念を理解し分子間に働く保存力の動力学を解析できる
04/27	第7回	Smooth particle hydrodynamics (SPH) 法	微分演算子の動径基底関数による離散化とその保存性・散逸性を評価できる
05/01	第8回	Particle mesh 法	粒子と格子の両方の離散化を組み合わせた場合の離散点からの補間法と高次モーメントの保存法

有限差分法

1. 1次元 線形移流方程式
2. 1次元 移流方程式
3. 1次元 拡散方程式
4. 1次元 移流拡散方程式
5. 2次元 線形移流方程式
6. 2次元 移流方程式
7. 2次元 拡散方程式
8. 2次元 移流拡散方程式
9. 2次元 Laplace方程式
10. 2次元 Poisson方程式
11. 2次元 Navier-Stokes方程式
12. 2次元 Navier-Stokes方程式2



$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} &= -\rho \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right)\end{aligned}$$

1次元 線形移流方程式

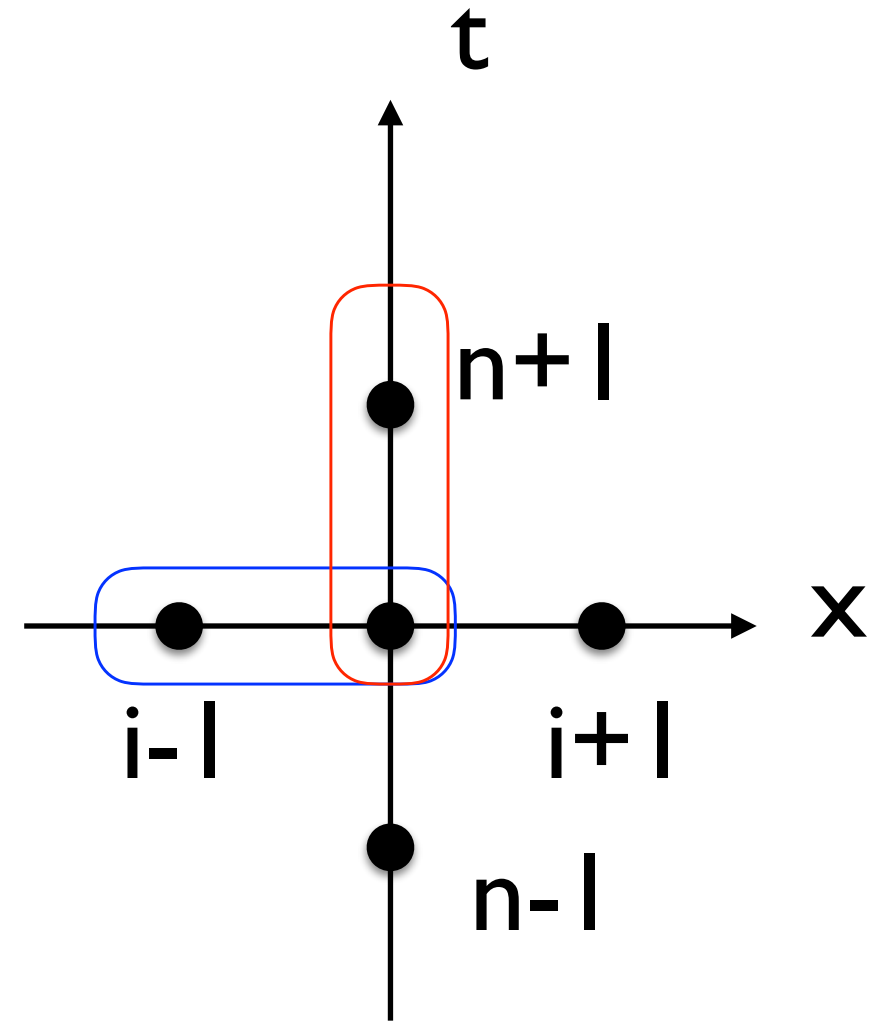
$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0$$

前進差分

後退差分

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + c \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

$$u_i^{n+1} = u_i^n - c \frac{\Delta t}{\Delta x} (u_i^n - u_{i-1}^n)$$



1次元 移流方程式

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0$$

cをuに変える

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

$$u_i^{n+1} = u_i^n - u_i^n \frac{\Delta t}{\Delta x} (u_i^n - u_{i-1}^n)$$

1次元 拡散方程式

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

2階微分

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$u_i^{n+1} = u_i^n + \frac{\nu \Delta t}{\Delta x^2} (u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

1次元 移流拡散方程式

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

移流 + 拡散

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \frac{u_i^n - u_{i-1}^n}{\Delta x} = \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$u_i^{n+1} = u_i^n - u_i^n \frac{\Delta t}{\Delta x} (u_i^n - u_{i-1}^n) + \nu \frac{\Delta t}{\Delta x^2} (u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

2次元 線形移流方程式

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} + c \frac{\partial u}{\partial y} = 0 \quad y \text{ 方向}$$

$$\frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} + c \frac{u_{i,j}^n - u_{i-1,j}^n}{\Delta x} + c \frac{u_{i,j}^n - u_{i,j-1}^n}{\Delta y} = 0$$

$$u_{i,j}^{n+1} = u_{i,j}^n - c \frac{\Delta t}{\Delta x} (u_{i,j}^n - u_{i-1,j}^n) - c \frac{\Delta t}{\Delta y} (u_{i,j}^n - u_{i,j-1}^n)$$

2次元 移流方程式

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = 0$$

v 成分

$$\frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} + u_{i,j}^n \frac{u_{i,j}^n - u_{i-1,j}^n}{\Delta x} + v_{i,j}^n \frac{u_{i,j}^n - u_{i,j-1}^n}{\Delta y} = 0$$

$$\frac{v_{i,j}^{n+1} - v_{i,j}^n}{\Delta t} + u_{i,j}^n \frac{v_{i,j}^n - v_{i-1,j}^n}{\Delta x} + v_{i,j}^n \frac{v_{i,j}^n - v_{i,j-1}^n}{\Delta y} = 0$$

$$u_{i,j}^{n+1} = u_{i,j}^n - u_{i,j}^n \frac{\Delta t}{\Delta x} (u_{i,j}^n - u_{i-1,j}^n) - v_{i,j}^n \frac{\Delta t}{\Delta y} (u_{i,j}^n - u_{i,j-1}^n)$$

$$v_{i,j}^{n+1} = v_{i,j}^n - u_{i,j}^n \frac{\Delta t}{\Delta x} (v_{i,j}^n - v_{i-1,j}^n) - v_{i,j}^n \frac{\Delta t}{\Delta y} (v_{i,j}^n - v_{i,j-1}^n)$$

2次元 拡散方程式

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} + \nu \frac{\partial^2 u}{\partial y^2}$$

y 方向

$$\frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} = \nu \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{\Delta x^2} + \nu \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{\Delta y^2}$$

$$u_{i,j}^{n+1} = u_{i,j}^n + \frac{\nu \Delta t}{\Delta x^2} (u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n) + \frac{\nu \Delta t}{\Delta y^2} (u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n)$$

2次元 移流拡散方程式

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)\end{aligned}$$

$$\begin{aligned}u_{i,j}^{n+1} &= u_{i,j}^n - \frac{\Delta t}{\Delta x} u_{i,j}^n (u_{i,j}^n - u_{i-1,j}^n) - \frac{\Delta t}{\Delta y} v_{i,j}^n (u_{i,j}^n - u_{i,j-1}^n) + \\ &\frac{\nu \Delta t}{\Delta x^2} (u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n) + \frac{\nu \Delta t}{\Delta y^2} (u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n)\end{aligned}$$

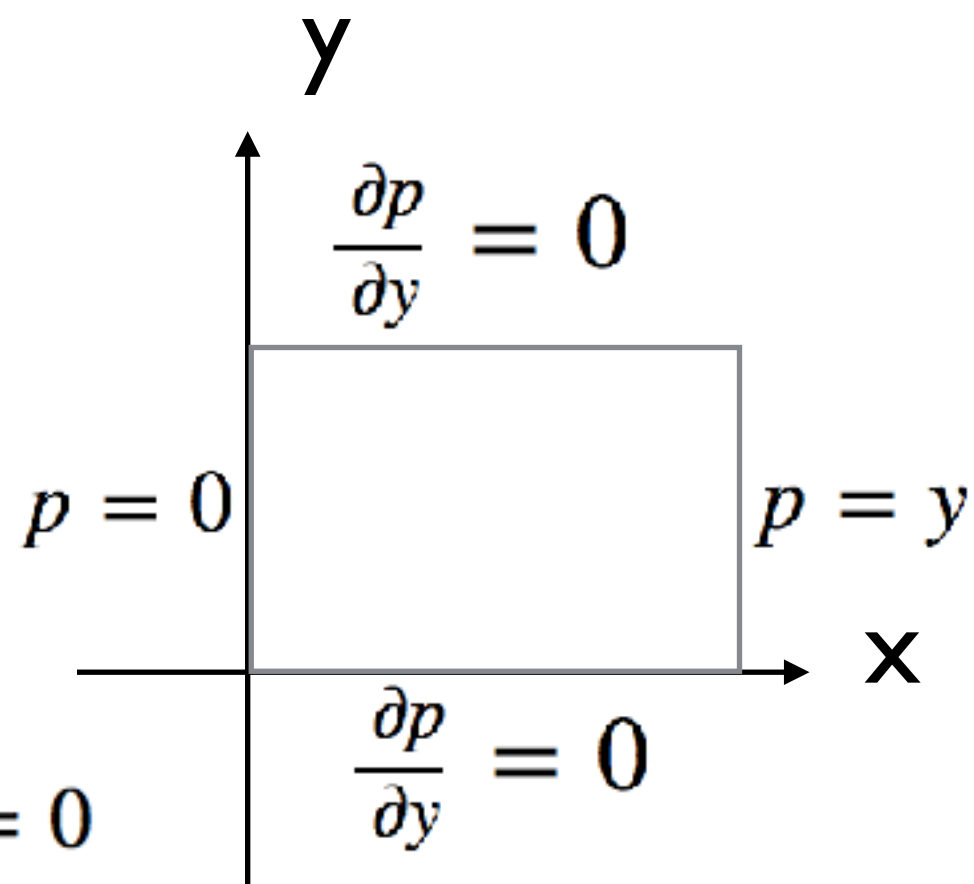
$$\begin{aligned}v_{i,j}^{n+1} &= v_{i,j}^n - \frac{\Delta t}{\Delta x} u_{i,j}^n (v_{i,j}^n - v_{i-1,j}^n) - \frac{\Delta t}{\Delta y} v_{i,j}^n (v_{i,j}^n - v_{i,j-1}^n) + \\ &\frac{\nu \Delta t}{\Delta x^2} (v_{i+1,j}^n - 2v_{i,j}^n + v_{i-1,j}^n) + \frac{\nu \Delta t}{\Delta y^2} (v_{i,j+1}^n - 2v_{i,j}^n + v_{i,j-1}^n)\end{aligned}$$

2次元 Laplace方程式

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

$$\frac{p_{i+1,j}^n - 2p_{i,j}^n + p_{i-1,j}^n}{\Delta x^2} + \frac{p_{i,j+1}^n - 2p_{i,j}^n + p_{i,j-1}^n}{\Delta y^2} = 0$$

$$p_{i,j}^n = \frac{\Delta y^2(p_{i+1,j}^n + p_{i-1,j}^n) + \Delta x^2(p_{i,j+1}^n + p_{i,j-1}^n)}{2(\Delta x^2 + \Delta y^2)}$$



2次元 Poisson方程式

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = b$$

非同次

$$\frac{p_{i+1,j}^n - 2p_{i,j}^n + p_{i-1,j}^n}{\Delta x^2} + \frac{p_{i,j+1}^n - 2p_{i,j}^n + p_{i,j-1}^n}{\Delta y^2} = b_{i,j}^n$$

$$p_{i,j}^n = \frac{(p_{i+1,j}^n + p_{i-1,j}^n)\Delta y^2 + (p_{i,j+1}^n + p_{i,j-1}^n)\Delta x^2 - b_{i,j}^n \Delta x^2 \Delta y^2}{2(\Delta x^2 + \Delta y^2)}$$

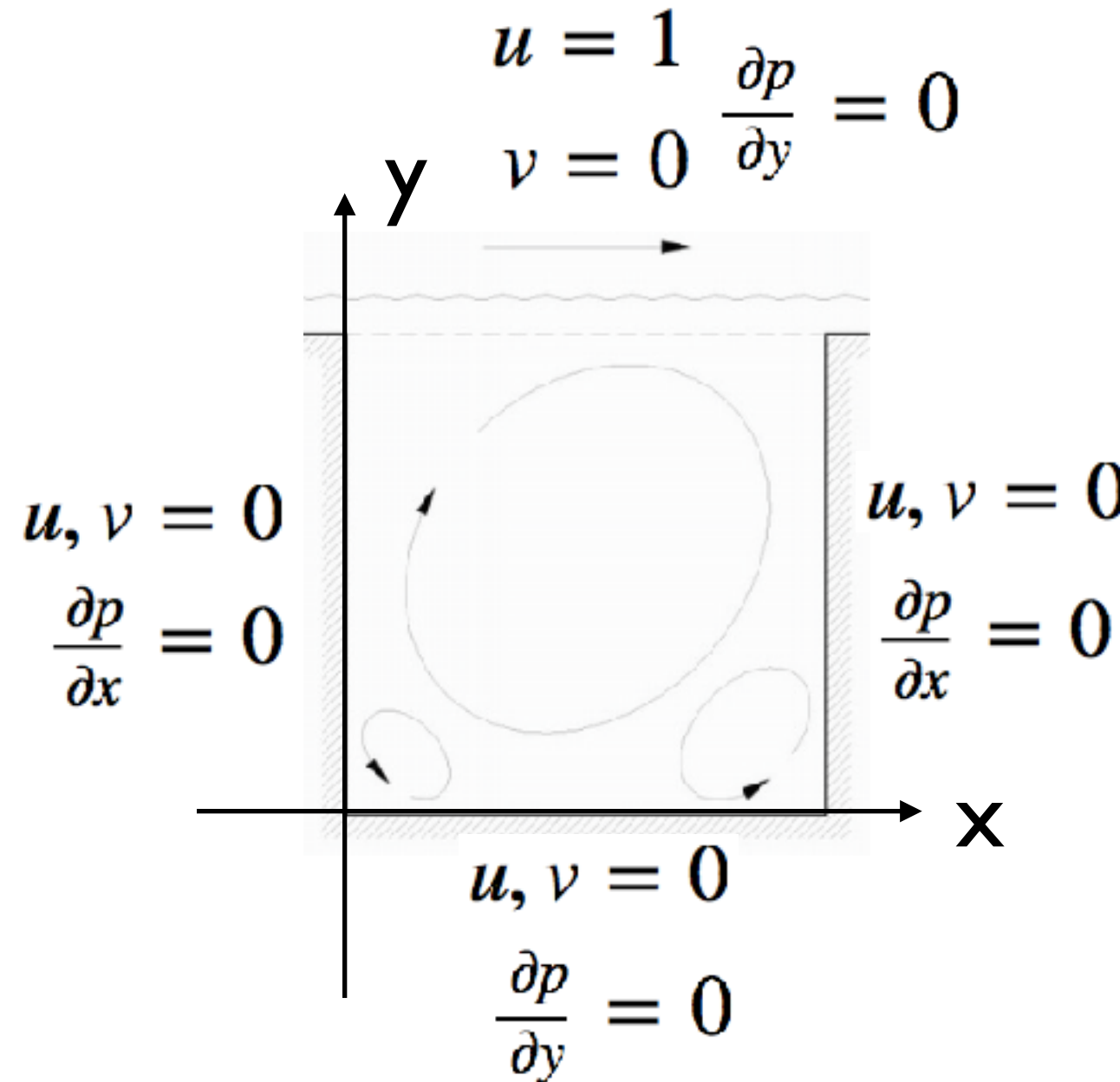
2次元 Navier-Stokes方程式 (Cavity)

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} &= -\rho \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right)\end{aligned}$$

$$\begin{aligned}u_{ij}^{n+1} &= u_{ij}^n - u_{ij}^n \frac{\Delta t}{\Delta x} (u_{ij}^n - u_{i-1,j}^n) - v_{ij}^n \frac{\Delta t}{\Delta y} (u_{ij}^n - u_{i,j-1}^n) \\ &\quad - \frac{\Delta t}{\rho 2 \Delta x} (p_{i+1,j}^n - p_{i-1,j}^n) + \nu \left(\frac{\Delta t}{\Delta x^2} (u_{i+1,j}^n - 2u_{ij}^n + u_{i-1,j}^n) \right. \\ &\quad \left. + \frac{\Delta t}{\Delta y^2} (u_{i,j+1}^n - 2u_{ij}^n + u_{i,j-1}^n) \right)\end{aligned}$$

$$\begin{aligned}v_{ij}^{n+1} &= v_{ij}^n - u_{ij}^n \frac{\Delta t}{\Delta x} (v_{ij}^n - v_{i-1,j}^n) - v_{ij}^n \frac{\Delta t}{\Delta y} (v_{ij}^n - v_{i,j-1}^n) \\ &\quad - \frac{\Delta t}{\rho 2 \Delta y} (p_{i,j+1}^n - p_{i,j-1}^n) + \nu \left(\frac{\Delta t}{\Delta x^2} (v_{i+1,j}^n - 2v_{ij}^n + v_{i-1,j}^n) \right. \\ &\quad \left. + \frac{\Delta t}{\Delta y^2} (v_{i,j+1}^n - 2v_{ij}^n + v_{i,j-1}^n) \right)\end{aligned}$$

$$\begin{aligned}p_{ij}^n &= \frac{(p_{i+1,j}^n + p_{i-1,j}^n) \Delta y^2 + (p_{i,j+1}^n + p_{i,j-1}^n) \Delta x^2}{2(\Delta x^2 + \Delta y^2)} - \frac{\rho \Delta x^2 \Delta y^2}{2(\Delta x^2 + \Delta y^2)} \times \\ &\quad \left[\frac{1}{\Delta t} \left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \right) - \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} \right. \\ &\quad \left. - 2 \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y} \frac{v_{i+1,j} - v_{i-1,j}}{2\Delta x} - \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \right]\end{aligned}$$



2次元 Navier-Stokes方程式 (Channel)

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} &= -\rho \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right)\end{aligned}$$

$$\begin{aligned}u_{ij}^{n+1} &= u_{ij}^n - u_{ij}^n \frac{\Delta t}{\Delta x} (u_{ij}^n - u_{i-1,j}^n) - v_{ij}^n \frac{\Delta t}{\Delta y} (u_{ij}^n - u_{i,j-1}^n) \\ &\quad - \frac{\Delta t}{\rho 2 \Delta x} (p_{i+1,j}^n - p_{i-1,j}^n) + \nu \left(\frac{\Delta t}{\Delta x^2} (u_{i+1,j}^n - 2u_{ij}^n + u_{i-1,j}^n) \right. \\ &\quad \left. + \frac{\Delta t}{\Delta y^2} (u_{i,j+1}^n - 2u_{ij}^n + u_{i,j-1}^n) \right)\end{aligned}$$

$$\begin{aligned}v_{ij}^{n+1} &= v_{ij}^n - u_{ij}^n \frac{\Delta t}{\Delta x} (v_{ij}^n - v_{i-1,j}^n) - v_{ij}^n \frac{\Delta t}{\Delta y} (v_{ij}^n - v_{i,j-1}^n) \\ &\quad - \frac{\Delta t}{\rho 2 \Delta y} (p_{i,j+1}^n - p_{i,j-1}^n) + \nu \left(\frac{\Delta t}{\Delta x^2} (v_{i+1,j}^n - 2v_{ij}^n + v_{i-1,j}^n) \right. \\ &\quad \left. + \frac{\Delta t}{\Delta y^2} (v_{i,j+1}^n - 2v_{ij}^n + v_{i,j-1}^n) \right)\end{aligned}$$

$$\begin{aligned}p_{ij}^n &= \frac{(p_{i+1,j}^n + p_{i-1,j}^n) \Delta y^2 + (p_{i,j+1}^n + p_{i,j-1}^n) \Delta x^2}{2(\Delta x^2 + \Delta y^2)} - \frac{\rho \Delta x^2 \Delta y^2}{2(\Delta x^2 + \Delta y^2)} \times \\ &\quad \left[\frac{1}{\Delta t} \left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \right) - \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} \right. \\ &\quad \left. - 2 \frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y} \frac{v_{i+1,j} - v_{i-1,j}}{2\Delta x} - \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \right]\end{aligned}$$

