

Chapter: *n Lives Inside N — The Real Connection Between n , N , and f_s*

Act 1 — The Big Block of Samples

Imagine you've just captured **N digital samples** from your IQ data stream.

This block is *exactly* what the FFT will chew on.

- N = **total number of samples in the block.**
- n = **which sample we're talking about inside that block.**

So if $N = 1024$:

- $n = 0 \rightarrow$ first sample in the block
- $n = 512 \rightarrow$ middle sample
- $n = 1023 \rightarrow$ last sample

Act 2 — n Has No Meaning Without N

In the DFT formula:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j \frac{2\pi}{N} nk}$$

- The **upper limit $N - 1$** tells us how many values n can take.
- **n exists only from 0 up to $N-1$** — because this **N** defines the FFT's “analysis window.”
- If you change N , you change the range of n .
- This means **n 's life span is tied to N** — it's part of N 's definition.

Act 3 — Where f_s Comes In

Outside the math world, your SDR is sampling at f_s samples/sec.

- Each **gap** between two n positions in the block is $\Delta t = 1/f_s$ seconds.
- The **time stamp** for a given n in the block is:

$$t_n = \frac{n}{f_s}$$

- f_s does **not** decide how many samples are in the block (that's N), but it **does** decide how far apart those N samples are in real time.

Act 4 — Why N and n Matter for Frequency

The FFT uses N and f_s together to set your frequency resolution:

$$\Delta f = \frac{f_s}{N}$$

- Bigger $N \rightarrow$ smaller Δf (better frequency resolution)
- Smaller $N \rightarrow$ larger Δf (coarser frequency resolution)

Here's the key chain:

1. **Choose N** \rightarrow defines how many n 's you have (0 to $N-1$)
2. n indexes the samples in that N -block
3. f_s tells you how those n 's map to actual time
4. Together, N and f_s give you your FFT frequency spacing

Brain Hook

Think of N as a *row of seats in a theater*, and n as *seat numbers*:

- If the theater has $N = 8$ seats, seat numbers go from $n = 0$ to $n = 7$.
- If you build a bigger theater ($N = 16$), you now have seat numbers $n = 0$ to $n = 15$.
- The **distance** between seats is set by f_s in time (seconds apart).

Recap Table

Symbol	Meaning	Tied To
N	Number of samples in FFT block	Your processing choice
n	Position of a sample in the block	Always 0 to N-1
f_s	Samples per second	Hardware/sample rate
Δt	Time between samples = $1/f_s$	From f_s
Δf	Frequency bin width = f_s/N	From f_s & N

Now the link is **explicit**:

- n lives **inside** N — it is meaningless without knowing N.
- N is the “container,” n is the “slot number” in that container.
- f_s turns those slot numbers into real-world time and frequency.