📖 Chapter: "How Hertz Gave Time and Frequency Their Deal"

🧑🔬 Act 1 — Meet Mr. Hertz

Back in the late 1800s, Heinrich Hertz was studying oscillating electrical waves. One day, he asks:

> "If I have a wave that repeats, how do I measure *how often* it happens?"

He decides: **Count the number of cycles in one second.**

Boom 💥 — that's **frequency**.

Mathematically:

$$f = \frac{\text{Number of cycles}}{\text{Time in seconds}}$$

If time = 1 second, the number you get **is** the frequency.

And the unit is named after him: **1 Hertz (Hz) = 1 cycle per second**.

🎯 Act 2 — Frequency is Just a Rate of Repetition

If something repeats regularly, its frequency tells you **how many repeats per second**.

Examples:

- A fan blade spinning 60 times per second → $60\ \text{Hz}$
- A light flickering 50 times per second → $50\ \text{Hz}$
- A sine wave going up–down 440 times per second → $440\ \text{Hz}$ (musical note A4)

⏳ Act 3 — The Inverse Magic: Period T

Hertz also realized:

If you know **how many times it repeats in a second**, you can figure out **how long each repeat lasts.**

He called that time **the period**, T.

Mathematically:

$$T = \frac{1}{f}$$

where T is in seconds and f is in Hz.

Example:

- $f = 10 \text{ Hz} \rightarrow T = 0.1 \text{ seconds}$ per cycle.
- $f = 1000 \text{ Hz} \rightarrow T = 0.001 \text{ seconds}$ per cycle.

This is where **time and frequency become reciprocals**:

$$f = \frac{1}{T}, \quad T = \frac{1}{f}$$

💻 Act 4 — The Sampling Twist (f_s)

Fast forward to the digital age. We still use Hertz's definition — but now instead of cycles of a wave, we can talk about **samples**.

If f_s is the **sampling rate** (samples per second):

- $f_s = 48,000$ means **48,000 samples in 1 second.**
- $1/f_s$ = time between two samples (the **sampling interval**).

Example:

- $f_s = 8,000 \text{ Hz} \rightarrow 1/f_s = 0.000125 \text{ s}$ = 125 μs between samples.
- $f_s = 1,000 \text{ Hz} \rightarrow$ 1 ms between samples.

So f_s is just the **frequency** of *your data collection process*.

🔄 Act 5 — Why Reciprocal is Everywhere

We can now write the “time–frequency handshake” in two contexts:

1. **Continuous signals (cycles)**:

$$f = \frac{1}{T}, \quad T = \frac{1}{f}$$

2. **Digital sampling (samples)**:

$$f_s = \frac{\text{Samples}}{\text{second}}, \quad \Delta t = \frac{1}{f_s}$$

where Δt is the time between samples.

Same idea — Hertz’s original definition just got re-applied to data points instead of sine waves.

🧠 Brain Hook

Think of Hertz as saying:

> “If you tell me how *fast* something repeats, I can tell you how *long* one repeat is. And vice versa.”

That’s all “reciprocal” really means here — swap **rate** ↔ **interval** and flip the number.

🍕 Pizza Analogy Upgrade (Sampling Edition)

- f_s = How many pizza slices (samples) you cut in **one second**.
- $1/f_s$ = How much time passes before you cut the *next* slice.

Cut slices faster → each gap in time is smaller.
Cut slices slower → each gap is bigger.