📖 Headfirst DSP: The Power of $e^{j2\pi f_m t}$ — with Time!

1. First, the idea

You already know that

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

is just a point moving around the **unit circle** in the complex plane.

When we say:

$$e^{j2\pi f_m \frac{n}{f_s}}$$

we're saying:

- **Spin around the circle** at a rate f_m (mixer frequency)
- **Take discrete steps** based on n (sample index)
- Each step is $\frac{1}{f_s}$ **seconds apart** (sampling frequency)
- So the **angle at sample** n is:

$$\theta_n = 2\pi f_m \cdot \frac{n}{f_s}$$

2. Timing element explained visually

Think of a **clock**:

- f_s = how many ticks per second you get (sampling rate)
- n = which tick we're at (sample number)
- $\frac{n}{f_s}$ = **time since start** (in seconds)

So:

- At $n = 0 \rightarrow t = 0$ sec → start at phase 0°
- At $n = 1 \rightarrow t = \frac{1}{f_s}$ sec → phase moves forward

- At $n = 2 \rightarrow t = \frac{2}{f_s}$ sec → phase moves further
- And so on.

3. Example: see the numbers

Let's take:

- Mixer frequency $f_m = 1000$ Hz
- Sampling rate $f_s = 8000$ Hz
- Samples $n = 0, 1, 2, 3$

Then:

$$t_n = \frac{n}{f_s}$$

and

$$\theta_n = 2\pi f_m t_n$$

n	t (sec)	θ (radians)	cos(θ)	sin(θ)	Complex value
0	0.0000	0.0000	1.000	0.000	$1.000 + 0.000j$
1	0.000125	0.7854 (45°)	0.707	0.707	$0.707 + 0.707j$
2	0.000250	1.5708 (90°)	0.000	1.000	$0.000 + 1.000j$
3	0.000375	2.3562 (135°)	-0.707	0.707	$-0.707 + 0.707j$

4. What's the "rotation" really?

- The **rotation per sample** in radians is:

$$\Delta\theta = \frac{2\pi f_m}{f_s}$$

- This is how much the phasor moves **each time you take a sample**.
- More $f_m \rightarrow$ spins faster

- More $f_s \rightarrow$ spins slower (because you're taking smaller time steps)

5. Big picture — why you care

- **Mixing**: Multiplying your signal by this complex exponential shifts its frequency up or down
- **Filtering**: Often done after mixing to isolate what you want
- **Signal scaling**: Controlled by amplitude, phase rotation rate controlled by f_m

💡 **Memory trick**:
Think of f_s as **"frames per second"** in a movie, n as the **frame number**, and f_m as **how fast the character spins**. The formula just tells you where the character is facing in each frame.