Headfirst - Two Rotations → One Spike

Imagine you're standing in front of a giant **complex plane** — like a spinning turntable, but floating in space.
We place one arrow (vector) spinning **counterclockwise** and another spinning **clockwise**, both at the **same constant speed**. Each one is tracing a perfect circle in time. No speeding up, no slowing down — just pure, **steady angular velocity**.

Time domain view (side view)

From the **side** (real axis view), something magical happens:

- The counterclockwise and clockwise spins combine so that all the up-and-down motions cancel.
- What's left is just the left-right "real" wiggle — a perfect **cosine wave**.

This is why a real cosine can be seen as the sum of two equal and opposite complex exponentials.

Frequency domain view (top view)

Now switch to the **top view** — you're looking straight down at the complex plane from above.
In this world, each steady rotation shows up as a **dot** on the frequency axis. Why a dot?

- Because nothing is changing in its speed or direction — it's a **pure tone**.
- In the frequency domain, "pure and unchanging" means **all the energy sits at one exact frequency**.

That dot is what we call a **spike** in the spectrum.
It's not a wave, it's not a fuzzy blob — it's a sharp pin stuck at exactly the frequency of rotation.

Why it's a spike (intuition)

Think of the frequency axis like a row of parking spots for rotating motions.

- If the rotation is **exactly** at 10 Hz, all the energy parks in the "10 Hz" slot.
- If it speeds up, slows down, or wobbles, it starts spilling into other slots — creating sidebands, spreads, or smears.

Since our two rotations are perfectly steady, they **never leave their parking spots**. One spike appears at $+f_0$ (counterclockwise), the other at $-f_0$ (clockwise).

Bottom line:
A spike in the spectrum means:

> "This is a pure, steady rotation at exactly this frequency — and it hasn't changed its mind."