Chapter 4.2 - FFT Twiddle Rotations and Frequency Bins — A Layman's Guide

1️⃣ Digital Samples and Sampling Interval

- FFT works on **N digital samples**.
- **ΔT** = time between consecutive digital samples.
- **Fs** = 1 / ΔT → sampling frequency.

Key:

- N → number of samples analyzed
- ΔT → spacing between samples, determines frequency resolution
- Total FFT window duration = N * ΔT

2️⃣ Rotating Arrows and Twiddle Factor

- Each FFT bin k has a **complex arrow** that rotates.
- Rotation per sample is determined by the **twiddle factor**:

$$W^{kn} = e^{-j2\pi kn/N}$$

- **Higher k** → **faster rotation** → **higher frequency**.
- Each sample x[n] scales the arrow; summing all arrows → X[k].

Intuition:

- If the arrow rotation matches the signal frequency → arrows align → strong magnitude.
- If not → arrows cancel → small magnitude.

3️⃣ Frequency Comes from Rotation

- Frequency represented by bin k:

$$f = k/(N \cdot \Delta T) = k \cdot Fs/N$$

- Phase progression at sample n:

$$phase = 2\pi f n \Delta T$$

Takeaway:

- k → controls rotation speed → higher k = faster rotation
- n → sample position in the FFT
- ΔT → determines how fast the phase rotates per sample

4️⃣ Complex Vector Sum

- X[k] = sum over n of x[n] * W^{kn}
- Magnitude → strength of that frequency
- Phase → timing offset

> Tiny imaginary parts in Python output are floating-point noise.

5️⃣ Nyquist Limit

- Maximum frequency measurable = Fs / 2 → **Nyquist frequency**
- Bin N/2 → Nyquist frequency
- Bins above N/2 → mirrored negative frequencies (aliasing if signal exceeds Nyquist)

6️⃣ Summary of Variables

Concept	Symbol	Meaning
Digital samples	N	Number of measurements in FFT
Sample interval	ΔT	Time between consecutive digital samples
Sampling frequency	Fs	1 / ΔT

Concept	Symbol	Meaning
Bin index	k	Frequency being analyzed
Sample index	n	Digital sample position
Frequency	f	Actual frequency represented by bin k
Twiddle factor	W^kn	Rotating arrow for that bin/sample
FFT output	X[k]	Complex sum → magnitude & phase

✅ **Head-First Insight:**

- Each bin = a **frequency tuner**
- Arrow rotation = how k, n, and ΔT interact to detect that frequency
- FFT bins sum contributions of all samples → magnitude and phase
- Maximum measurable frequency = Nyquist (Fs/2)
