Input signal:

Let's say we have 8 real samples:

$$x = [x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7]$$

◈ Step-by-Step Breakdown (Radix-2 FFT)

◈ Step 1: Divide the signal into even and odd indices:

- **Even samples**: x_0, x_2, x_4, x_6
- **Odd samples**: x_1, x_3, x_5, x_7

◈ Step 2: Recursively divide again:

Now recursively split even and odd:

- Evens of evens: x_0, x_4
- Odds of evens: x_2, x_6
- Evens of odds: x_1, x_5
- Odds of odds: x_3, x_7

Now each of these subgroups has **2 elements**, so you apply:

$$X[0] = a + b, \quad X[1] = a - b$$

You now have **DFTs of size 2** computed for each subgroup:

- $X_{\text{even even}}$: from x_0, x_4
- $X_{\text{even odd}}$: from x_2, x_6
- $X_{\text{odd even}}$: from x_1, x_5
- $X_{\text{odd odd}}$: from x_3, x_7

◈ Step 3: Combine results bottom-up using Twiddle Factors

Let's move upward one level:
Now combine the results of the two sub-DFTs (of size 2) into a DFT of size 4.

For the even half:

You combine:

- $X_{\text{even}}[k] = X_{\text{even even}}[k] + W_4^k \cdot X_{\text{even odd}}[k]$
- $X_{\text{even}}[k+2] = X_{\text{even even}}[k] - W_4^k \cdot X_{\text{even odd}}[k]$

Where $k = 0, 1$, and $W_4^k = e^{-j\frac{2\pi}{4}k}$

Similarly for the odd half:

- $X_{\text{odd}}[k] = X_{\text{odd even}}[k] + W_4^k \cdot X_{\text{odd odd}}[k]$
- $X_{\text{odd}}[k+2] = X_{\text{odd even}}[k] - W_4^k \cdot X_{\text{odd odd}}[k]$

◈ Step 4: Final recombination (size 8 DFT from two 4-point DFTs)

Now combine full even and odd results:

$$X[k] = X_{\text{even}}[k] + W_8^k \cdot X_{\text{odd}}[k]$$
$$X[k+4] = X_{\text{even}}[k] - W_8^k \cdot X_{\text{odd}}[k]$$

Where $k = 0, 1, 2, 3$, and $W_8^k = e^{-j\frac{2\pi}{8}k}$

🔁 Summary of What Happens After the Last Pair is Reached:

1. You **compute 2-point DFTs** at the bottom: $a + b$, $a - b$

2. Then **combine pairs of these 2-point results** into 4-point DFTs using twiddle factors
3. Then **combine those into 8-point DFTs** — again using twiddle factors
4. This **recursively assembles the full FFT output**: $X[0], X[1], \ldots, X[7]$

🧾 Analogy (Quick Intuition)

Think of FFT as:

- **Chopping a big problem into tiny problems**
- **Solving the tiny problems**
- **Merging them cleverly using twiddle factors** (which are like "phase shifts" that align results in frequency)
