📖 Chapter 2.1 - Your Signal's "Passport"

A signal is like a traveler — to let it into your DSP world, you've got to stamp its passport with some key details.

1️⃣ Duration (T) → *How long you watch it*

Question: How many seconds (or milliseconds, microseconds) are you recording or generating the signal?

- Think of it like the *length of the movie* you're shooting.
- If you stop recording early, you miss the story.
- If you record too long, you'll have a longer file, more samples, and better frequency resolution — but bigger processing costs.

💡 **Example:**

```
T = 1 second    →  short snapshot
T = 5 seconds   →  longer snapshot, more data to work with
```

2️⃣ Sampling Rate (Fs) → *How often you take pictures*

Question: How many times per second do you capture the signal's value?

- Units: **Hertz (Hz)** = samples per second.
- The faster you sample, the more detail you capture — *but* there's a law you must obey:

Nyquist theorem:
You can only capture frequencies up to

$$F_{\max} = \frac{F_s}{2}$$

💡 **Example:**
If Fs = 8 Hz → Max detectable frequency = 4 Hz.

3️⃣ Number of Samples (N) → *How many pictures you end up with*

Relation:

$$N = F_s \times T$$

- This is just *how many data points* you have.
- **FFT eats N for breakfast** — because it decides how many *frequency bins* you get.
- More samples = finer frequency resolution.

💡 **Example:**
If Fs = 8 Hz, T = 1 s → N = 8 samples.

4️⃣ Signal Frequency (f_0) → *The "beat" you put in*

- This is the actual "note" or tone in the signal you're generating.
- If you generate a sine wave at 2 Hz, it makes 2 complete cycles per second.

5️⃣ Amplitude (A) → *How tall the wave is*

- Bigger amplitude = higher peaks and deeper troughs.
- This doesn't change the frequency — only the size of the wave.

6️⃣ Phase (φ) → *Where the wave starts*

- Two sine waves with the same frequency and amplitude but different phase look like one is shifted sideways.
- Units: degrees (°) or radians.

💡 **Example:**
φ = 0° → starts at the center going up.
φ = 90° → starts at the top.

7️⃣ Frequency Resolution (Δf) → *How far apart your FFT "bin" notes are*

Relation:

$$\Delta f = \frac{F_s}{N} = \frac{1}{T}$$

- The longer you observe (bigger T), the **smaller** Δf — you can distinguish closer frequencies.
- Short observation → coarser frequency bins.

💡 **Example:**
N = 8, Fs = 8 → Δf = 1 Hz.
You can only get 0 Hz, 1 Hz, 2 Hz … up to 4 Hz.

8️⃣ Nyquist Limit → *The tallest note you can hear without distortion*

$$F_{\text{Nyquist}} = \frac{F_s}{2}$$

- If you put a 5 Hz signal into an 8 Hz sampler, it will "pretend" to be 3 Hz — this is aliasing.

🎯 Why this matters

When you see:

> "A 1-second signal sampled at 8 Hz containing a 2 Hz sine wave"

You can immediately decode:

- **Duration (T)** = 1 s
- **Sampling Rate (Fs)** = 8 Hz → Nyquist limit = 4 Hz
- **Samples (N)** = 8
- **Resolution (Δf)** = 1 Hz
- **Frequency of wave (f_0)** = 2 Hz
- You'll see a peak in FFT at bin index `2`.

❓ Head First Exercise

1. Take **Fs = 10 Hz**, **T = 2 seconds**
   - What's N?
   - What's Δf?
   - What's Nyquist limit?
2. Generate a sine wave of frequency 3 Hz with amplitude 2 and phase 90°.
3. Run FFT and identify the bin with the highest magnitude.