Compute the 4-point DFT of the time-domain sequence

$$x[n] = [\,1,\ 2,\ 3,\ 4\,], \quad n = 0, 1, 2, 3$$

—but **do it with the FFT method** (divide & conquer).

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2. Step 1: Split Signal into Even and Odd Samples

We introduce `n` here to index time samples.

- **Even indices** (`n = 0,2`):

$$x_{\text{even}}[m] = x[2m], \quad m = 0, 1 \quad \Longrightarrow \quad [\,x[0],\ x[2]\,] = [\,1,\ 3\,]$$

- **Odd indices** (`n = 1,3`):

$$x_{\text{odd}}[m] = x[2m+1], \quad m = 0, 1 \quad \Longrightarrow \quad [\,x[1],\ x[3]\,] = [\,2,\ 4\,]$$

> 🔍 Here, `n` has been re-parametrized into a **new index** `m` (runs 0→1) for each half.

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3. Step 2: Compute 2-Point DFTs of Each Half

Now we compute the **base-case DFTs** of length-2 directly, using `m`:

3.1 Even half DFT

$$E[k] = \sum_{m=0}^{1} x_{\text{even}}[m]\, e^{-j\frac{2\pi}{2}km} \quad \text{for } k = 0, 1$$

- For $k = 0$:
  $E[0] = x_{\text{even}}[0] + x_{\text{even}}[1] = 1 + 3 = 4$
- For $k = 1$:
  $E[1] = x_{\text{even}}[0] - x_{\text{even}}[1] = 1 - 3 = -2$

3.2 Odd half DFT

$$O[k] = \sum_{m=0}^{1} x_{\text{odd}}[m]\, e^{-j\frac{2\pi}{2}km} \quad \text{for } k = 0, 1$$

- For $k = 0$:
  $O[0] = 2 + 4 = 6$
- For $k = 1$:
  $O[1] = 2 - 4 = -2$

> 🔍 **Notice:** after this step, all sums over **time indices** (`m`) are done, and we have two small DFT results: **E[·]** and **O[·]**.

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4. Step 3: Combine via the FFT “Twiddle” Rule

Now we switch to the **frequency index** `k`—we no longer refer to `n` or `m`. We use:

$$W_4^k = e^{-j\frac{2\pi}{4}k}$$

$$X[k] = E[k] + W_4^k\, O[k], \quad X[k+2] = E[k] - W_4^k\, O[k]$$

for $k = 0, 1.$

- **Twiddle factors**:
  $W_4^0 = 1,\ W_4^1 = e^{-j\pi/2} = -j.$

4.1 For $k = 0$:

$$X[0] = E[0] + 1 \cdot O[0] = 4 + 6 = 10$$
$$X[2] = E[0] - 1 \cdot O[0] = 4 - 6 = -2$$

4.2 For $k = 1$:

$$X[1] = E[1] + (-j)\, O[1] = (-2) + (-j)(-2) = -2 + 2j$$
$$X[3] = E[1] - (-j)\, O[1] = (-2) - (-j)(-2) = -2 - 2j$$

> 🔍 **Key Point:** here **only** `k` appears—**no** `n` or `m`. They've done their work in Step 2.

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5. Final Result

$$\boxed{X = [\,10,\ -2+2j,\ -2,\ -2-2j\,]}$$

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6. The "Disappearance" of `n` (and `m`)

1. **DFT step (time sums)** uses `n` (or `m` after splitting).
2. **FFT combine step** uses only `k`, because you are now working **purely in the frequency domain**, merging **already-computed** partial results.
3. Once you've "summed over time" to get **E[k]** and **O[k]**, you **never need** to revisit `n` or `m`.

✅ Teaching Takeaway

- **Use** `n` when you're summing over time samples to get any DFT.
- **Once that's done**, you enter the FFT's **merge phase**—only the frequency bin index `k` matters.
- This clean separation is what makes FFT both **correct** (it computes the DFT) and **efficient** (it reuses partial results without re-summing time samples).