Here’s a **compiled set of concise yet complete notes** on:

- **Fundamental Frequency**
- **Normalized Frequency**
- **Normalization Factor** $\frac{k}{N}$
- **DFT Bin Interpretation**
- **Real-World Mapping**

📘 Understanding Frequency in DFT: A Practical Guide

🔹 1. Fundamental Frequency

Definition:
The **fundamental frequency** in the context of DFT is the **lowest non-zero frequency** that can be resolved by the transform.

$$f_{\text{fundamental}} = \frac{f_s}{N}$$

Where:

- f_s: Sampling frequency (Hz)
- N: Number of time-domain samples in DFT

Interpretation:

- It's the frequency corresponding to **1 full cycle** over **N samples**.
- All other DFT bin frequencies are **integer multiples (harmonics)** of this.

🔷 2. Normalized Frequency

Definition:
The frequency expressed as a **fraction of the sampling rate**, i.e., **cycles per sample**.

$$f_{\text{norm}} = \frac{f_{\text{actual}}}{f_s} = \frac{k}{N}$$

Where:

- k: DFT bin index
- N: DFT size
- f_{actual}: Actual frequency in Hz

Units: Cycles per sample (unitless)

Range:

- For real signals, normalized frequency typically spans $[0, 0.5]$ due to symmetry (Nyquist limit).
- For complex signals, full range is $[0, 1)$ or $[-0.5, 0.5)$

🔷 3. Normalization Factor $\frac{k}{N}$

This is the core frequency index of DFT — it determines:

- The rate of rotation of the complex exponential used in bin k
- The **harmonic component** DFT is measuring

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi \frac{k}{N} n}$$

- $\frac{k}{N}$: Normalized frequency
- k: Bin number = k-th harmonic

🔷 4. Harmonic Interpretation

Each DFT bin k measures how much of the k-th harmonic (i.e., sinusoid completing k cycles in N samples) is present in the signal.

$$f_k = k \cdot f_{\text{fundamental}} = \frac{k}{N} \cdot f_s$$

So:

- **Bin 0** → DC component (0 Hz)
- **Bin 1** → 1 cycle over N samples → fundamental
- **Bin 2** → 2nd harmonic
- ...
- **Bin $N/2$** → Nyquist frequency (if N is even)

🔷 5. Physical Interpretation of Frequencies

Type	Formula	Units	Meaning
Fundamental	$\frac{f_s}{N}$	Hz	Lowest non-zero freq. DFT can detect
Actual freq. of bin k	$\frac{k}{N} \cdot f_s$	Hz	Frequency component measured in bin k
Normalized freq. of bin k	$\frac{k}{N}$	cycles/sample	Fraction of sampling rate
Harmonic number	k	dimensionless	k-th sinusoid in the basis

🔷 6. Example:

Let's say:

- $f_s = 8000\,\text{Hz}$

- $N = 8$

  Then:
- $f_{\text{fundamental}} = \frac{8000}{8} = 1000\,\text{Hz}$
- Bin 2:
  - Normalized frequency = $\frac{2}{8} = 0.25$
  - Actual frequency = $0.25 \cdot 8000 = 2000\,\text{Hz}$
  - Harmonic = 2nd

🔷 7. Why It Matters

- **Normalized frequency** lets you work **independent of sampling rate** — useful for algorithms, plotting, etc.
- **Actual frequency** gives **physical meaning** — e.g., audio tone, RF carrier.
- **Fundamental frequency** shows your resolution: you **can't distinguish** between signals closer than $\frac{f_s}{N}$.
- Understanding $\frac{k}{N}$ is essential to interpreting DFT results correctly.

📝 Summary Table

Concept	Expression	Interpretation
Fundamental frequency	$\frac{f_s}{N}$	Lowest resolvable frequency
Normalized frequency	$\frac{k}{N}$	Cycles per sample
Actual frequency	$\frac{k}{N} \cdot f_s$	Hz
Harmonic number	k	k-th sinusoidal basis
Nyquist frequency	$\frac{f_s}{2}$	Maximum representable frequency for real signals