

Let  $A$  be the  $n \times n$  matrix whose  $(i, j)^{\text{th}}$  entry is  $i + j$  for all  $i, j = 1, 2, \dots, n$ . What is the rank of  $A$ ?

**Solution:**

The matrix  $A$  can be rewritten as

$$\begin{pmatrix} 2 & 3 & \dots & n+1 \\ 3 & 4 & \dots & n+2 \\ \vdots & \vdots & \ddots & \vdots \\ n+1 & n+2 & \dots & n+n \end{pmatrix}$$

Subtracting the first row from the rest, we obtain another matrix but with the same rank:

$$\begin{pmatrix} 2 & 3 & \dots & n+1 \\ 1 & 1 & \dots & 1 \\ 2 & 2 & \dots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ n-1 & n-1 & \dots & n-1 \end{pmatrix}$$

And this matrix has several rows that are proportional to the second row, so it is equivalent to the matrix

$$\begin{pmatrix} 2 & 3 & \dots & n+1 \\ 1 & 1 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

And this last matrix has rank equal to 2. Since it is equivalent to matrix  $A$ , then the rank of  $A$  is also 2.