Suppose that in a not necessarily commutative ring R the square of any element is 0. Prove that $abc + abc = 0$ for any three elements a, b, c.

Solution:

Initially it can be seen that $(a+b)^2 = 0$, then $(a+b)^2 = a^2+ab+ba+b^2 = ab+ba = 0$, where it follows that $ab = -(ba)$.

Once having this, then

$$\begin{aligned}
abc &= a(bc) \\
&= -((bc)a) \\
&= -(b(ca)) \\
&= (ca)b \\
&= c(ab) \\
&= -((ab)c) \\
&= -abc
\end{aligned}$$

And therefore, $abc + abc = 0$.