Determine whether there exist an odd positive integer n and $n \times n$ matrices A y B with integer entries, that satisfy the following conditions:

1. $\det(B) = 1$.
2. $AB = BA$.
3. $A^4 + 4A^2B^2 + 16B^4 = 2019I$.

Solution:

It is satisfied that

$$A^4 \equiv A^4 + 4A^2B^2 + 16B^4 = 2019I \pmod 4 \Longrightarrow A^4 \equiv 2019I \pmod 4$$

Therefore

$$\det(A^4) = (\det(A))^4 \equiv \det(2019I) = 2019^n \pmod 4$$

Since n is odd, $2019^n \equiv 3 \pmod 4$, then $(\det(A))^4 \equiv 2019^n \equiv 3 \pmod 4$, but 3 is not a quadratic residue modulo 4, so matrices A and B do not exist.