Let A be a real 4×2 matrix and B be a real 2×4 matrix such that

$$AB = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

Find BA.

Solution:

The previous product can be expressed as $AB = \begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$, with I being the 2×2 identity matrix. It can also be said that $A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$ and $B = \begin{pmatrix} B_1 & B_2 \end{pmatrix}$, with A_1, A_2, B_1, B_2 being 2×2 matrices.

Therefore,

$$AB = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \begin{pmatrix} B_1 & B_2 \end{pmatrix} = \begin{pmatrix} I & -I \\ -I & I \end{pmatrix} \iff \begin{cases} A_1B_1 & = & I \\ A_1B_2 & = & -I \\ A_2B_1 & = & -I \\ A_2B_2 & = & I \end{cases}$$

On the other hand, $BA = \begin{pmatrix} B_1 & B_2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = B_1A_1 + B_2A_2$. Since $A_1B_1 = I$, then $B_1A_1 = I$. And thus $A_2B_2 = I \iff B_2A_2 = I$. Having seen this, it is concluded that

$$BA = B_1A_1 + B_2A_2 = 2I = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$