Laboratory Manual EPHY108L
B.Tech, 1st Year, 2nd Semester
Department of Physics

BENNETT UNIVERSITY

Table of Contents

Experiment 1

Aim:
Estimation of Error in measurements of Vernier Calipers & Screw Gauge

Apparatus:
Vernier Calipers, Screw Gauge, metal ball, metal ring, weighing machine

Theory:
All experimental measurements even with the most sophisticated equipment are constrained by errors of measurement caused due to various factors. Errors can be reduced by using accurate instruments and performing the experiment carefully, but the final result will still have errors.
There are primarily two types of errors: systematic and random.

Systematic errors**:** These come primarily from the measurement instruments and could be due to limitations in accuracy of the instrument, incorrect calibration or due to errors in using the instruments. For example a defect in the thermometer that is used to measure the temperature will lead to systematic errors. Such errors will lead to results of multiple measurements which are close to each other but deviated from the true value.

Random errors**:** These are caused due to random variations in the conditions of the experiment. For example, while measuring the time period of a simple pendulum the randomness in starting and stopping of the stop watch will lead to random errors in the measured time period of the pendulum. In the presence of random errors, the results will be crowded around a mean value which is close to the true value of the measured quantity. The distribution of the values around the mean value is usually Gaussian in nature.

As an example, let us take the estimation of the acceleration due to gravity using a simple pendulum. It is well known that the time period T of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

where l is the length of the pendulum and g is the acceleration due to gravity. In order to estimate the value of g we take a simple pendulum of length l and measure the time period T of the pendulum. From these measurements, we can obtain the value of g.

The question is what is the error in the value of g obtained from the experiment? We first note that when the length l and the time period T are measured, both these quantities would have errors due to the limitations of measuring instruments. Thus, if we use a scale with a least count of 1 mm in measuring the length of the pendulum then the maximum possible error in the length of the pendulum is ± 1 mm. Similarly, to measure the time period of the pendulum we measure the time t taken by the pendulum for say 10 oscillations and estimate the time period from the formula

$$T = t/10$$

If the stop watch has a least count of 0.1 s then the measurement of t has a maximum error of 0.1 s and the maximum error in the time period (T) is 0.01 s.

These errors in the measurement of l and T will result in error in the estimation of g. In order to relate these, we write Eq. (1) as

$$g = \frac{4\pi^2 l}{T^2} \tag{2}$$

We now take natural logarithm of Eq. (2) to obtain

$$\ln(g) = \ln(4) + 2\ln(\pi) + \ln(l) - 2\ln(T)$$

We now differentiate the above equation to obtain

$$\frac{dg}{g} = 0 + 0 + \frac{dl}{l} - 2\frac{dT}{T} = \frac{dl}{l} - 2\frac{dT}{T} \tag{3}$$

where dg is the error in the value of g, dl is the error in the value of l and dT is the error in the value of T.

Since errors can be both positive or negative, we estimate the maximum probable error by adding the individual errors in l and T and obtain

$$\frac{dg}{g} = \frac{dl}{l} + 2\frac{dT}{T} \tag{4}$$

Since T is estimated by measuring the time taken for n oscillations, the error in T will be $dT = dt/n$ where dt is the least count of the stop watch used to measure the time taken for n oscillations. Thus

$$\frac{dg}{g} = \frac{dl}{l} + 2\frac{dt}{t} \tag{5}$$

As an example, we have dl = 1mm, dt = 0.1 s, n = 10, l = 20 cm, t = 9s and we have

$$\frac{dg}{g} = \frac{10^{-3}}{0.2} + 2\frac{0.1}{9} \approx 0.005 + 0.02222 \approx 0.027$$

Thus, the fractional error in the estimation of g is about 2.7%. The value of g, as given by the calculator, comes out to be 9.747757 m/s^2. Now since dg/g is 0.027, we obtain dg = 0.26318 m/s^2.

Although the calculator gives many significant digits for the value of g, since there is a fractional error of about 2.7 % in the experiment, it implies that the value of g that is calculated has fractional uncertainties of about 2.7 %. Hence, we need to quote the value of g appropriately keeping the uncertainty in account.

For this we first round off the error to one significant digit and obtain dg = 0.3 m/s^2. Since the value of error in the estimation of g goes up to the 1st decimal place, we need to round off the value of measured g to the 1st decimal place, i.e. 9.7 m/s^2. Thus, in this example the measured value of g is

$$g \approx (9.7 \pm 0.3)\ \text{m/s}^2$$

This implies that due to the errors in the measurement of the length and time period of the pendulum, the measured value of g lies between 9.4 m/s^2 and 10.0 m/s^2. The actual value of g is within the two limits.

Experiment:

Vernier calipers: A normal scale that we all use generally has a minimum measurable distance of 1 mm. Thus any measurement of length using a standard scale will have an uncertainty of ± 1 mm. In order to measure lengths more accurately we use an instrument called Vernier Calipers.

It consists of a fixed main scale and a movable Vernier scale as shown in Fig. 1. Using a Vernier caliper, we can measure lengths of objects, internal diameter of holes, or depth of a hole or height of a step.

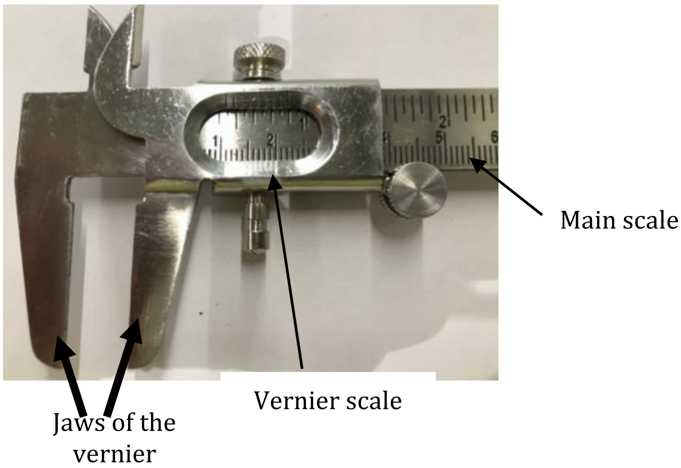


Figure 1: Vernier caliper

In a typical Vernier caliper nine main scale divisions (corresponding to 9 mm) are divided into ten Vernier scale divisions such that when the zero of the Vernier scale coincides with any main scale division the tenth Vernier division will coincide with another main scale division which is nine millimeters away. None of the other Vernier divisions will coincide with any main scale division.

We can estimate the least count of the Vernier calipers as follows:

- 1 main scale division = 1 mm
- Since 9 main scale divisions (MSD) are divided into 10 Vernier scale divisions (VSD), each Vernier scale division will correspond to 9/10 mm. Thus the least count of the Vernier calipers is

Least count = 1 main scale division – 1 Vernier scale division = 1.0 mm - 0.9 mm = 0.1 mm. Please note that this is a specific example, there are Vernier instruments which have a different least counts.

Thus, this Vernier caliper has a minimum measurable length of 0.1 mm. The maximum possible instrumental error is ± 0.1 mm, which is ten times better than a standard scale.

We can also write the least count as

$$\text{Least count} = 1\ \text{MSD}\ \times \left(1 - \frac{\text{Number of MSD}}{\text{Number of VSD}}\right)$$

- When the Vernier calipers is used to measure lengths, the total distance is estimated by first noting the position of the first VSD with regard to MSD. Thus if the zero of the Vernier scale lies between 11 and 12 mm divisions of the main scale, then the main scale value is taken as 11 mm.
- We also note down the particular number of the VSD that coincides with a division of the main scale. Thus, let us assume that the 6^{th} VSD coincides with some MSD.
- The total length then is the sum of the value in the main scale and the product of the least count and the number of the Vernier scale that coincides with the main scale.
- Thus, in this example the length is $11 + 6 \times 0.1 = 11.6$ mm
- Since the least count is 0.1 mm, we quote the measured length as 11.6 ± 0.1 mm.

Zero error: It is possible that when the Vernier scale is moved to the extreme left position, the zero of Vernier does not coincide with the zero of the main scale. In such a case the Vernier calipers has zero error. This is a systematic error and needs to be taken into account when estimating lengths using the Vernier. The zero error is subtracted from the length measured using the faulty Vernier calipers to get the actual length.

As an example, we first move the Vernier scale to the extreme left position so that the jaws of the main scale and Vernier scale touch each other. In this position for example if the zero of the Vernier does not coincide with zero of the main scale, then the Vernier has zero error. Let us assume that the zero of the Vernier is slightly to the right of the zero of the main scale. There will some particular division of the Vernier that will coincide with some main scale division. Let us assume that the 4^{th} Vernier division coincides with a main scale division. Then since the least count is 0.1 mm, in our example the zero error is $4 \times 0.1 = 0.4$ mm. So, even though the actual length is 0, the Vernier calipers is showing a reading of 0.4 mm. The zero error needs to be subtracted from all measured values of lengths using this Vernier calipers. Thus, if our measured length using this Vernier is 11.6 mm then the actual length will be $11.6 - 0.4 = 11.2$ mm.

If the zero of the Vernier lies to the left of the zero of the main scale when the jaws of the Vernier and main scale touch each other then the zero error is negative. As before if the 4^{th} Vernier division coincides with a main scale division, the zero error is -0.4 mm and the actual length in this case will be $11.6 - (-0.4) = 12.0$ mm

Screw Gauge: A screw gauge (see Fig. 2) is used to measure even smaller dimensions than Vernier calipers and is basically a fine screw with an accurate and constant pitch. An object such as a wire whose diameter is to be measured is placed between the anvil and spindle of the screw gauge and the thimble is rotated on the sleeve until the wire is held between the anvil and the thimble. The screw gauge contains a rachet knob which ensures that we always have the same amount of tightening of the given object between the anvil and the spindle.

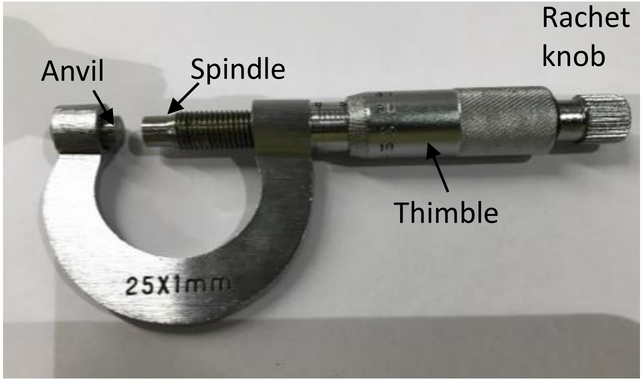


Figure 2: Screw gauge

The screw gauge contains a main scale on the spindle and a number of divisions on the rotating thimble. Typically, the main scale will have divisions every 0.5 mm and the circular scale will have 50 divisions. When the thimble is given one complete revolution, the thimble will move a certain distance along the main scale (also called the pitch of the screw gauge). In this example, when the thimble is rotated by one full revolution, the thimble will move 0.5 mm along the main scale. Thus, we find that 0.5 mm is divided into 50 divisions on the circular scale. Thus, the least count of the screw gauge will be 0.5/50 = 0.01 mm. Please note this is a specific example. Screw gauges with different least counts also exist.

It is possible that when the spindle is brought into contact with the anvil with no object placed in between, the zero of the circular scale does not coincide with the main scale line of the main scale. Thus, the screw gauge gives us a finite reading instead of zero. We need to find the zero error and subtract it from all subsequent readings.

For example, the zero of the circular scale is above the main scale line and the 5th division of the circular scale coincides with the main scale, then the zero error would be $5 \times 0.01 = 0.05$ mm. The zero error is positive in this case.

If the zero of the circular scale is below the main scale line and the 47th division of the circular scale coincides with the main scale line then zero error would be $-(50 - 47) \times 0.01 = -0.03$ mm. In such a case the zero error is negative.

For more details on Vernier calipers, screw gauge and spherometer, please visit the following site (IIT-PAL lecture by Prof Sanjeev Sanghi, IIT Delhi):

https://www.youtube.com/watch?v=DgMdre5q0tc&index=3&list=PLl9LSbHKSR63XPvso0DqKu5B029dS3MII

Measurement of volume of the given object and density of the material

In this experiment the objective is to use a Vernier caliper and a Screw gauge to estimate the density of the given metal balls and rings .

1. Calculate the least count of the Vernier calipers and the screw gauge provided to you.
2. Make at least five measurements of the diameter of the steel balls using the screw gauge. Since the steel ball may not be perfectly spherical, make measurements along different diameters.
3. Correct the values of diameter if the given screw gauge has zero error.
4. Calculate the average diameter of the given steel ball.
5. From the measured average diameter calculate the average volume of the steel ball.
6. Using the procedure mentioned above estimate the error in the measurement of the volume of the steel ball.
7. Measure the mass of the steel ball and from the measurement of mass and volume estimate the density of the steel ball. Also calculate the error in the estimated value of density and report the result in proper fashion.
8. Measure the inner and outer diameters of the given metal ring at least five times using a Vernier caliper. Both upper and lower jaws of the Vernier calipers should be used. Account for any zero error of the given Vernier.
9. Measure the thickness of the ring using a screw gauge.
10. From the measurements calculate the volume of the ring.
11. Calculate the error in the measured volume of the ring.
12. Measure the mass of the metal ring and from the mass and volume calculate the density of the material of the ring.
13. Calculate the error in the estimation of the density of the ring and report the result in proper fashion.

Example: Estimation of error in measuring volume of a spherical object measured using screw gauge.

To measure the volume of a spherical object we measure the diameter *D* of the object using a screw gauge. The volume *V* is then given by

$$V = \frac{4\pi}{3} R^3 = \frac{\pi}{6} D^3$$

In order to calculate the maximum probable error in the volume we take logarithm of both sides and obtain

$$\ln(V) = \ln\left(\frac{\pi}{6}\right) + 3\ln\,(D)$$

We now differentiate the above equation and get

$$\frac{dV}{V} = 0 + 3\,\frac{dD}{D} = 3\,\frac{dD}{D}$$

Now *dD* corresponds to the least count of the screw gauge and *D* is the average diameter of the spherical object. Thus, substituting for all values, we can calculate *dV* the error in the estimated value of volume *V*.

Write down the formulas for the volume of the ring and also for the density of the spherical ball and the ring.

Using these formulas obtain expressions for the errors, in a manner similar to the above example.

Show all calculations in your copy.

Observations:

Least count of Vernier calipers =
Least count of screw gauge =
Least count of weighing scale =

Table 1: Error in the volume and density of spherical metal ball.

S. No	Diameter (m)	Avg. Diameter (m)	Avg. Volume (m^3)	Error in Volume (%)	Error in Volume (m^3)	Avg. Density (kg/m^3)	Error in density (%)	Error in density (kg/m^3)
1.								
2.								
3.								
4.								
5.								

Table 2: Error in the volume and density of a metal ring.

S. No	Inner/Outer dia. (m)	Avg. Inner/Outer dia. (m)	Thickness (m)	Avg. Volume (m^3)	Error in Volume (%)	Error in Volume (m^3)	Avg. Density (kg/m^3)	Error in Density (%)	Error in density (kg/m^3)
1	/	/							
2	/								
3	/								
4	/								
5	/								

Results:

The measured volume of steel ball is $\dots . \pm \cdots$ m^3.

Value of density of the material of the steel ball is $\dots . \pm \cdots$ kg/m^3.

The measured volume of steel ring is $\dots . \pm \cdots$ m^3.

The measured value of density of the material of the steel ring is $\dots . \pm \cdots$ kg/m^3.

Experiment 2

Aim:
To determine the moment of inertia of a flywheel.

Apparatus:
Fly wheel, weight hanger, slotted weights, stop watch, a meter scale.

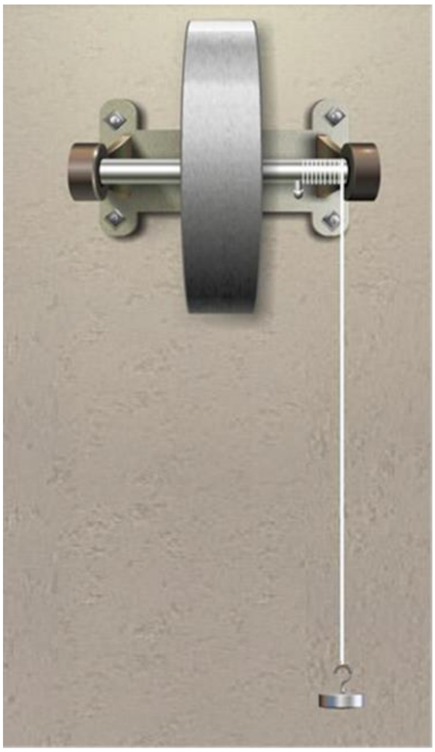


Figure 1: Experimental Setup

Theory:
The flywheel consists of a heavy circular disc/massive wheel fitted with a strong axle projecting on either side. The axle is mounted on ball bearings on two fixed supports. There is a small peg on the axle. One end of a cord is loosely looped around the peg and its other end carries the weight-hanger.

Let "m" be the mass of the weight hanger and hanging rings (weight assembly). When the mass "m" descends through a height "h", the loss in potential energy is

$$P_{loss} = mgh$$

The resulting gain of kinetic energy in the rotating flywheel assembly (flywheel and axle) is

$$K_{flywheel} = \frac{1}{2} I \omega^2,$$

where, I is the moment of inertia of the flywheel assembly, ω is angular velocity at the instant when weight assembly gets released from the fly wheel assembly. The gain in kinetic energy of the descending weight assembly is,

$$K_{weight} = \frac{1}{2} m v^2,$$

where v is the velocity at the instant when weight assembly gets released from the fly wheel assembly. The work done in overcoming the friction of the bearings supporting the flywheel assembly is

$$W_{friction} = nW_f$$

where, n is the number of times the cord is wrapped around the axle, W_f is work done to overcome the frictional torque in rotating the flywheel assembly completely once. Therefore, from the law of conservation of energy we get

$$P_{loss} = K_{flywheel} + K_{weight} + W_{friction}$$

On substitution, we get

$$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 + nW_f$$

After the falling mass gets detached from the fly wheel assembly, the fly wheel will come to a stop after completing say N rotations. This implies that the kinetic energy of the flywheel assembly is spent in rotating N times against the same frictional torque:

$$NW_f = \frac{1}{2} I\omega^2.$$

Therefore

$$W_f = \frac{1}{2N} I\omega^2,$$

If r is the radius of the axle, then velocity v of the weight assembly is given as

$$v = \omega r$$

Substituting the values of v and W_f we get,

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mr^2\omega^2 + \frac{n}{N}\left(\frac{1}{2} I\omega^2\right)$$

Now solving the above equation for I

$$I = \frac{Nm}{N+n}\left(\frac{2gh}{\omega^2} - r^2\right),$$

where, I = Moment of inertia of the flywheel assembly, N = Number of rotation of the flywheel before it stopped; m = mass of the rings, n = Number of windings of the string on the axle, g = Acceleration due to gravity, h =Height through which the weight assembly falls from its original position to the position when it gets detached from the fly wheel, r = Radius of the axle. Since the weight assembly detaches from the fly wheel or the axle after it had made n complete rotations

$$h = 2\pi rn$$

Now we begin to count the number of rotations, N until the flywheel stops and note the duration of time t for N rotations. Therefore, we can calculate the average angular velocity $\omega_{average}$ in radians per second.

$$\omega_{average} = \frac{2\pi N}{t}$$

Since we are assuming the torsional friction W_f is constant over time and final angular velocity = 0, the angular velocity at the moment when the weight assembly gets detached is simply twice the average angular velocity

$$\omega = \frac{4\pi N}{t}$$

Procedure:

1. Measure the radius of the axle of the flywheel (r) using a Vernier caliper.
2. Choose a chord of length less than the height of the fly wheel so that when it gets released from the fly wheel it does not touch the ground.

3. A suitable weight is placed in the weight hanger and the string is wound on the axle of the fly wheel without overlapping; let *n* be the number of loops of the winding of the cord on the axle of the fly wheel.
4. The height (h) of the weight hanger from the position where it will get released from the axle of the flywheel is calculated from *n* and *r*.
5. The flywheel is released from rest.
6. The weight hanger descends, and the flywheel starts to rotate.
7. The cord will slip off from the peg after it has made *n* rotations.
8. A stop clock is started just when the weight hanger gets released.
9. The time taken by the flywheel to come to a stop is measured; let it be *t* seconds.
10. The number of rotations (N) made by the flywheel during this interval is counted.
11. The experiment is repeated by changing the value of *n* and *m*.
12. From these values the moment of inertia of the flywheel is calculated using following equation:

$$I = \frac{Nm}{N+n}\left(\frac{2gh}{\omega^2} - r^2\right)$$

Observation:

Radius of the axle of the flywheel (r) = ……….

Table 1: Calculation of Moment of Inertia of flywheel

S. No.	Mass Suspended (m in kg)	No. of revolutions		Height of descend before detachment (h in m)	Time for N revolutions (t in s)	Angular velocity (ω in s^{-1})	Moment of Inertia of the flywheel (I in $kg - m^2$)
		n	N				
1							
2							
3							
4							
5							

Mean value of moment of inertia, I =.........kg-m^2

Results: The measured moment of inertia of the fly wheel =.........kg-m^2

Experiment 3

Aim: Determination of the co-efficient of viscosity of glycerin by falling sphere method.

Apparatus:
Measurement unit, Test Liquid: Glycerin,
Spherical steel balls, Stop watch, Long graduated cylinder,
Weighing scale.

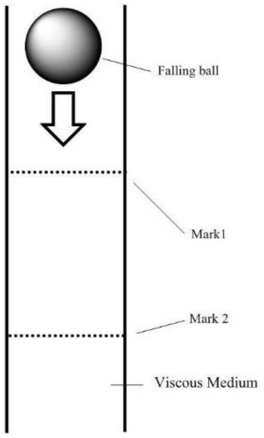


Figure 1: Schematic Diagram

Theory:

When a solid sphere is moving in a liquid, a viscous drag force will be exerted on the sphere. According to Stokes' law, the drag force is proportional to the viscosity of the fluid, the radius r of the sphere, and the velocity (or speed) of the sphere as:

$$f = 6\pi\eta r v. \qquad (1)$$

A steel ball is dropped into a fluid sample so that the gravitational force on the ball, mg, is larger than the buoyant force F_b. The net driving force on the ball is,

$$F = mg - F_b = \frac{4}{3}\pi r^3(\rho - \sigma)g, \qquad (2)$$

where ρ is the density of the ball and σ is the density of the liquid. Due to this driving force when the ball is dropped in the liquid it accelerates and its velocity increases. Simultaneously the viscous force, which opposes its motion also increases.

When $F = f$, the ball stops accelerating and falls with a constant speed v_f, which is known as terminal speed. Therefore, equating (1) and (2), we get the terminal velocity as

$$v_f = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}. \qquad (3)$$

Finally, equation (3) can be written as

$$\eta = \frac{2}{9}\frac{r^2 t(\rho - \sigma)g}{x}, \qquad (4)$$

where x is the distance travelled in time t, i.e. $v_f = x/t$

Figure 2: Experimental Set-up

Applications: Viscosity is an important concept that is taken into consideration in a variety of fields ranging from cooking to oil rigging. Understanding the applications of viscosity can help in both flow characterization and quality control.

Quality Control

- Since raw materials must be consistent from batch to batch, flow behavior can be used as an indirect measure of product consistency and quality. As mentioned earlier, similar viscosities are indicative of similar flows.

- Viscosity has a direct effect on the ability to be processed. When designing pumping and piping systems, it should be known that a high viscosity liquid requires more power to pump than a low viscosity one.

- The Viscosity Index of a liquid measures how variations in temperature directly affect the viscosity of a fluid. Liquids whose viscosity is greatly dependent on temperature have a high viscosity index. This is an important characteristic of a good lubricant.

Flow Characterization

- Rheology is the study of the flow of matter, primarily in the liquid state. The viscosity of a fluid helps predict whether the flow will be laminar or turbulent and it can be categorized accordingly.

Viscosity helps explain the behavior of fluids; thus, once the behaviors are understood, they can be manipulated according to specific needs.

<u>Procedure:</u>

1. Obtain three sets of metal spheres (with radii not more than 5 mm).
2. The sphere in each of three sets should be of same diameter.
3. Measure the diameter of each sphere three times with Screw Gauge and calculate the average diameter. Make a note of the least count and zero error of the given screw gauge.
4. Measure the mass of the sphere using weighing scale.
5. Calculate the density of the sphere.
   Density of sphere = ……. kg/m^3
6. Note down the density of liquid.

   Density of liquid = ………...kg/m^3
7. Set up the glass tube vertically in viscometer stand with the help of clamp.
8. Fill the glass tube with experimental liquid up-to upper marked line.
9. Carefully drop a sphere in the center of the tube.
10. Drop only one sphere at a time; which facilitates ease of timing measurement.

(Note: The removal of the sphere is not necessary for the completion of the experiment.)

11. Take measurement and observe the motion of sphere. Since the fall time of the sphere is very short so it is important to measure the time as accurately as possible. (Gradually the acceleration of the ball becomes zero and the

velocity of the ball becomes a constant to achieve its terminal velocity).

12. Press timer switch of measurement unit as soon as the bottom of the sphere passes the first marked line. (Timer starts counting).
13. When sphere just passes second marked line, press timer switch again. (For 1^{st} segment, time is noted and saved in a timer as t_1).
14. The sphere falls continuously, when it just passes third marked line, press timer switch again. (For 2^{nd} segment, time is noted and saved in a timer as t_2)
15. Sphere still falls downwards, and when it just passes last marked line, press timer switch again. (For 3^{rd} segment, time is noted and is saved in a timer as t_3).
16. Record t_1, t_2 and t_3 in the table.
17. Obtain the average time.
18. Now repeat this process for next two balls and note corresponding readings.
19. Tabulate the data in a table and calculate the coefficient of viscosity by using Eqn. (4).

Observation:

Table 1: Calculation of density of metal sphere

Balls	**Mass (gm)**	**Radii (mm)**			**Average radius of each ball (mm)**	**Average radius of three balls r (mm)**	**Average mass of three balls m (gm)**	**Density of sphere ρ (kg/m^3)**
		r_1	r_2	r_3	$\frac{r_1+r_2+r_3}{3}$			
A								
B								
C								

Table 2: Calculation of co-efficient of viscosity of glycerin

Balls	**Distance (mm)**	**Time of transit (sec)**			**Mean time (sec)** $t = \frac{t_1+t_2+t_3}{3}$	**Viscosity (η) (Pa.s)**	**Mean viscosity (η) (Pa.s)**
		t_1	t_2	t_3			
A							
B							
C							

Result: The viscosity of the given liquid was measured to be ……. Poise

Note: The SI unit of viscosity is (Pascal.second). The more common unit is Poise which is (dyne.second/cm^2). 1 Pa.s = 10 Poise.

Experiment 4

Aim:
Determination of Young's Modulus of the given sample by the method of bending

Apparatus:
Sample Stand, Weights of 500 gm, Given metal sample, DC Adaptor, Weight Holder, Spherometer, Stand with Buzzer.

Theory:
Young's modulus
Young's modulus (Y) is a mechanical property that measures the stiffness/elasticity of a solid material. It defines the relationship between stress (force per unit area) and strain (proportional deformation) in a material in the linear elasticity regime. So, in this regime stress is directly proportional to strain. When stress and strain are not directly proportional, Y may be represented as the slope of the tangent or the slope of the secant connecting two points on the stress-strain curve. The modulus is then designated as tangent modulus or secant modulus at stated values of stress. Young's modulus can be used to predict the elongation or compression of an object as long as the stress is less than the yield strength of the material.

Young's modulus (Y) = Applied load per unit area of cross section/increase in length per unit length.

According to Hooke's law, strain is proportional to stress, and therefore the ratio of the two is a constant that is commonly used to indicate the elasticity of the substance. Young's modulus is the elastic modulus for tension, or tensile stress, and is the force per unit cross section of the material divided by the fractional increase in length resulting from the stretching of a standard rod or wire of the material.
Young's modulus, Y, can be calculated by dividing the tensile stress by the tensile strain:
$Y = \frac{Tensile\ Stress}{Tensile\ strain} = \frac{\sigma}{\epsilon} = \frac{F/A_0}{\Delta L/L_0}$.
where,
Y is the Young's modulus (modulus of elasticity);
F is the force applied to the object;
Ao is the original cross-sectional area through which the force is applied;
ΔL is the amount by which the length of the object changes;
Lo is the original length of the object.

The SI unit of Y is the Pascal (kg $m^{-1}s^{-2}$). Some use an alternative unit form, kN/mm², which gives the same numeric value as gigapascals. The modulus of elasticity can also be measured in other units of pressure, for example pounds per square inch.

Cantilever
Beam supported at one end and carrying a load at the other end or distributed along the unsupported portion.

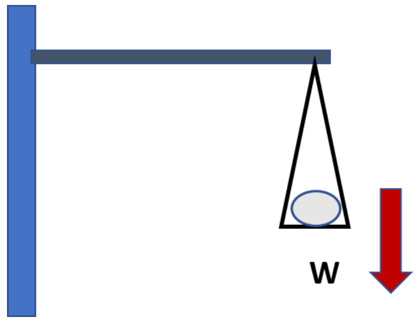


Figure 1: Cantilever

The upper portion of the thickness of such a beam is subjected to tensile stress, tending to elongate the fibers, the lower portion to compressive stress, tending to compress them. Cantilevers are employed extensively in building construction and in machines. In building, any beam built into a wall and with the free end projecting forms a cantilever. Longer cantilevers are incorporated in a building when clear space is required below, with the cantilevers carrying a gallery, roof, canopy, runway for an overhead traveling crane, or part of a building above.

A cantilever loaded at one end results in bending. In this process the top fibers of the beam will be subjected to tension and the bottom to compression. It is reasonable to suppose, therefore, **that somewhere between the two there are points at which the stress is zero. The locus of all such points is known as neutral axis.** The radius of curvature R is then measured to this axis.

The depression due to a load at one end of the cantilever can be determined by analyzing the bending due to the load.

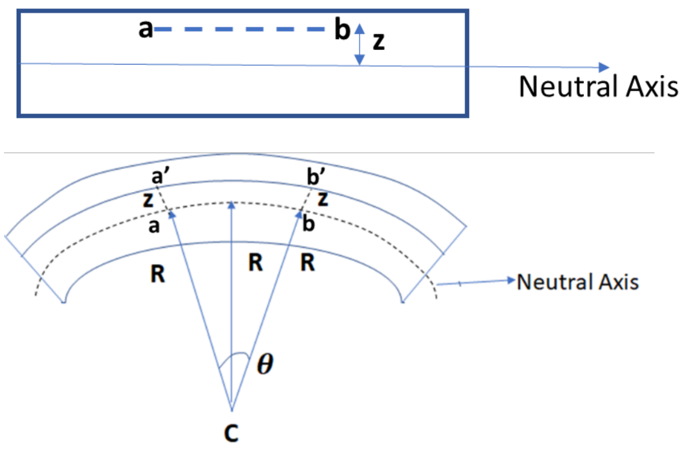


Figure 2: Bending of beam

Consider an arc formation due to the load subtending an angle θ at its center of curvature C. If the original length between two points is ab and modified length between the same two points is $a'b'$ then strain in the beam $=\frac{a'b'-ab}{ab}=\frac{(R+z)\theta-R\theta}{R\theta}=\frac{z}{R}$. Since stress or strain is zero on neutral axis therefore we have considered parallel points of ab on neutral axis as ab after bending.

Young modulus of the material is $Y=\frac{tensile\ stress}{tensile\ strain}=\frac{tensile\ stress}{z/R}$.

Tensile stress on a small area $\delta A=\frac{Yz}{R}$, so force on the area $\delta A=stress\times area=\left(\frac{Yz}{R}\right)\delta A$

Moment about the neutral axis will be $=\frac{Yz}{R}\ \delta Az=\frac{Yz^2}{R}\delta A$.

Total bending moment for the whole cross-section $=\sum\frac{Yz^2}{R}\delta A$.

Now, $\sum\delta Az^2$ is called geometrical moment of inertia of the cross section of the beam about the neutral axis and let us denote it as I.

<u>Determination of geometrical moment of inertia of rectangular cross section</u>

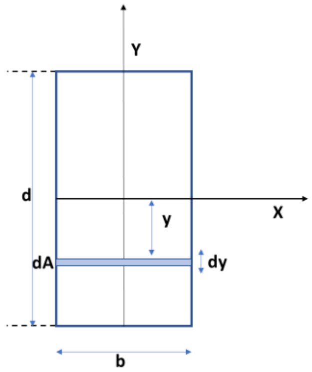


Figure 3: Geometric moment of inertia for rectangular cross-section

Let us calculate geometrical moment of inertia of a rectangular cross section of the beam about X axis (horizontal middle line or the neutral axis).

For a rectangular cross-section of height d and width b, the area is area $A=bd$.

So, the geometrical moment of inertia about the horizontal middle line, $I=b\int_{-\frac{d}{2}}^{\frac{d}{2}}y^2dy=\frac{bd^3}{12}$.

Hence bending moment of the beam $M=YI/R$.

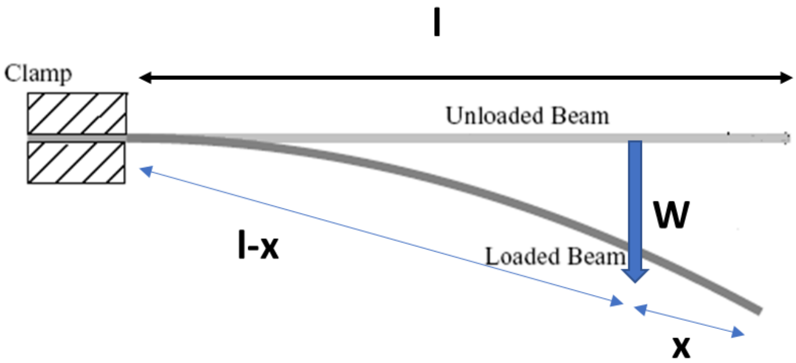


Figure 4: Bending of beam fixed at one end due to load

Bending moment due to load is $W(l-x)=\frac{YI}{R}$.

Now curvature $\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$. Since bending is small therefore we can neglect $\frac{dy}{dx}$. Hence, $\frac{d^2y}{dx^2} = \frac{W(l-x)}{YI} \Rightarrow \frac{dy}{dx} = \frac{W}{YI}\left(lx - \frac{x^2}{2}\right) + C_1$. Since one end ($x = 0$) is fixed therefore $\frac{dy}{dx} = 0$ at $x = 0$, thereby $C_1 = 0$.

So, $\frac{dy}{dx} = \frac{W}{YI}\left(lx - \frac{x^2}{2}\right) \Rightarrow y = \frac{W}{YI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$. However, at $y = 0$, $x = 0$, thus $C_2 = 0$.

This implies, $y = \frac{W}{YI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$. Now if the cantilever is loaded at one end i.e., $x = l$, the depression $y = d = \frac{Wl^3}{3YI}$

where, W is weight, I is moment of inertia, l is length and Y is young's modulus.

<u>Double Cantilever</u>

If a bar is supported at two knife edges A and B, l meter apart in a horizontal plane so that equal lengths of the bar project beyond the knife edges and a weight W is suspended at the middle point C, then it acts as a double cantilever.

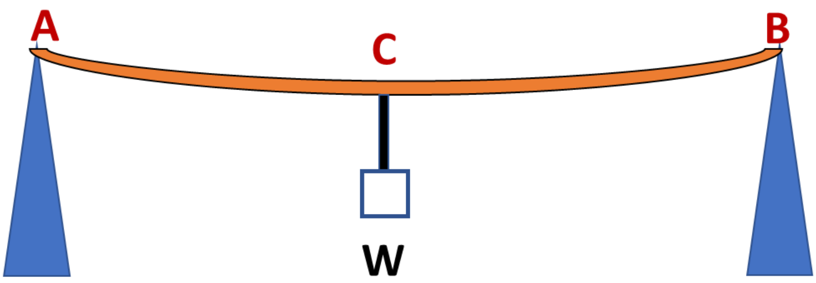


Figure 5: Double Cantilever beam

The middle part of the bar is practically horizontal. It is, therefore, equivalent to two inverted cantilevers fixed at the middle point C and loaded at A and B with load W/2 acting upward.

The depression (D) at C is given by $= \frac{\left(\frac{W}{2}\right)\left(\frac{l}{2}\right)^3}{3YI} = \frac{Wl^3}{48YI}$.

For a rectangular bar of breadth b and thickness d, $I = \frac{bd^3}{12}$.

Depression $D = \frac{Wl^3}{4Ybd^3}$

Young's modulus of elasticity can be determined by using this formula $Y = \frac{mgl^3}{4bd^3D}$

$W = mg$, where m is mass of load, g is gravity, l is length, b is breadth and d is depth of sample, and D is depression of bar.

<u>Trivia</u>

A good example of double cantilever is Howrah bridge which connects the twin cities of Kolkata and Howrah. Howrah is the 5th longest cantilever bridge in the world and one of the oldest.

Procedure

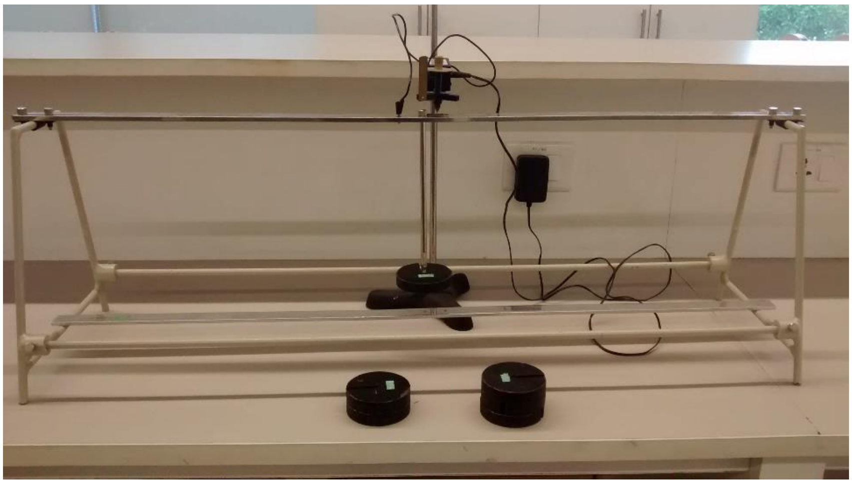


Figure 6: Experimental Setup

1. Determine the least count (L.C) of the spherometer.
2. Place the sample bar over sample stand and tighten the screws.
3. Tighten the weight holder at the center of the sample with the help of screw.
4. Place the spherometer stand, beyond the center of the sample.
5. Adjust the spherometer height with the help of screw according to the height of the sample.
6. Keep the spherometer leg in contact with the center of the sample by rotating the circular scale.
7. Connect buzzer with adaptor and connect patch cord with banana terminal, provided on the sample for buzzer connection.
8. Switch ‘On’ the supply for adaptor.
9. Buzzer blows because at this stage spherometer screw is in contact with the sample (because when spherometer touches the sample, circuit of buzzer becomes complete and it starts blowing).
10. Note the main scale reading (M.S) and circular scale reading (C.S) in observation Table 1 and find total reading (T = M.S +(C.S. x L.C.)) for 0 load.

Increment of the Loads:

11. Now place the 500 gm weight on the weight holder, at this stage buzzer stops blowing because spherometer screw is no longer in contact with sample bar owing to bending of the bar.
12. Rotate the circular scale or screw of the spherometer in clock wise direction till the buzzer blows.

13. Note down the total spherometer reading (T = M.S +(C.S. x L.C.)) in Table 1 for load of 0.5 kg.
14. Place one more 500 gm weight on the weight holder, total 1 kg weight is hanging, again buzzer will stop blowing because the spherometer leg is not in contact with the sample.
15. Rotate the circular scale of the spherometer till the buzzer blows.
16. Again, note the spherometer reading for load of 1 kg.
17. Repeat steps 14 to 16 till the total load is 3 kg.

<u>Decrement of the loads</u>

18. Now remove 500 gm weight from the weight holder. The buzzer will keep on blowing. Rotate the spherometer in anticlockwise direction until its leg does not touch the sample surface anymore.

19. Note spherometer reading for the new position, i.e. for load of 2.5 kg in the decrement mode.

20. Repeat steps 18 and 19, till the load on the bar is 0.

21. Take the mean of the total spherometer readings for load increment and decrement modes to find the mean position T_i corresponding to each load (m_i).

22. Determine the differences $T_3 - T_0$, $T_4 - T_1$, $T_5 - T_2$ and $T_6 - T_3$ to get D_1, D_2, D_3 and D_4, respectively. Here we are taking the difference between 2 readings that are 3 steps apart. In other words, we are taking the depressions caused by a load of 3 x 0.5 = 1.5 kg. Since the depressions by 0.5 kg loads are small numbers, taking depressions for 1.5 kg, which have larger values, helps in keeping the fractional errors in calculations low.

23. Take the mean of $D_1, D_2, D_3,$ and D_4 to get the mean depression (D) for a load (m) of 1.5 kg.

24. Note down the length, breadth and depth of the sample bar.

25. Put all the readings in the given formula to determine the elastic constant or Young's modulus of elasticity, Y (in Pascal or N/m^2),

$$Y = \frac{mgl^3}{4bd^3D}$$

where 'm' is mass (kg) for which depression (D) had been determined (here it is 1.5 kg), g = 9.8 m/s^2. Keep the units of all the quantities in SI system.

26. We will also use a second graphical method to determine Young's modulus. Note down the mean values of T_i corresponding to each load (m_i) in table 2.

27. Determine the depression (D_i) corresponding to each load, using $D_i = T_i - T_0$.

28. Plot depression (D_i) on Y axis and load (m_i) on X axis. Draw a best fit straight line and determine its slope (μ).

29. Since, $Y = \frac{mgl^3}{4bd^3D} \Rightarrow D = \frac{gl^3}{4bd^3Y}m.$

30. Hence, the slope, $\mu = \frac{gl^3}{4bd^3Y}$. Calculate Young Modulus from slope using: $Y = \frac{gl^3}{4bd^3\mu}$.

Observation:

Length (l), breadth (b), depth (d) of the bar.
l =..................
b =..................
d =..................

Table 1: Direct calculation of Young's modulus

Sl. No.	Load (m) (kg)	Load Increment			Load Decrement			Mean Value of T (mm)	Depression (D) (mm) for 1.5 kg load, $D_n = T_{n+2} - T_{n-1}$	Mean Depression (D) (mm) for 1.5 kg load
		$M.S$	$C.S$	$T = M.S + (C.S \times L.C)$ in mm	$M.S$	$C.S$	$T = M.S + (C.S \times L.C)$ in mm			
1	0.0							T_0=		
2	0.5							T_1=		
3	1.0							T_2=		
4	1.5							T_3=	D_1=	
5	2.0							T_4=	D_2=	
6	2.5							T_5=	D_3=	
7	3							T_6=	D_4=	

Table 2: Data for depression vs. load graph

Sl. No.	Load (m) (kg)	Mean Value of T (mm)	Depression (mm) $D_i = T_i - T_0$
1	0.0	T_0=	D_0=
2	0.5	T_1=	D_1=
3	1.0	T_2=	D_2=
4	1.5	T_3=	D_3=
5	2.0	T_4=	D_4=
6	2.5	T_5=	D_5=
7	3	T_6=	D_6=

Calculation:

1. Calculate Young's modulus (*Y)* from the mean Depression (*D*) for a load of 1.5 Kg using data from Table 1
2. Calculate Young's modulus (*Y)* from the slope (μ) of the best fit line obtained by plotting the data of Table 2.

Experiment 5

Aim:
To determine the coefficient of static friction between the surfaces of a steel slider and a wooden plane.

Apparatus:
Adjustable inclined plane, frictionless pulley, block, standard weights, and inextensible string.

Theory:
Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. It occurs because an object interacts with either the surface it lays upon, the medium it is contained in, or both. There are two types of friction: static and kinetic. Static friction is a force that keeps an object at rest. It must be overcome to start moving the object. Once an object is in motion, it experiences kinetic friction.

The maximum value of static frictional force is given by $f_{s,max} = \mu_s N$ and the kinetic frictional force is given by $f_k = \mu_k N$, where μ_s and μ_k are the coefficients of static and kinetic friction, respectively and N is the normal force. The coefficient of friction is a dimensionless constant.

In this experiment, the coefficient of static friction can be estimated using following methods:

Method 1 (Fixed angle of inclination): In this method, the angle of inclination (θ) of the wooden plane is kept fixed. The suspended mass (*M*) is increased until the slider just begins to slide or roll. Free-body diagram (figure 1) of the slider in equilibrium condition (just before it begins to move) gives

$$N = mg\ cos\theta \qquad (1)$$

$$T = Mg = mg\ sin\theta + \mu_s\ mg\ cos\theta \qquad (2)$$

$$Or,\ \mu_s = \frac{M - m\ sin\theta}{mcos\theta} \qquad (3)$$

The coefficient of static friction can be calculated using equation 3 by recording the mass of the slider (*m*), suspended mass due to the pan + weights (*M*) and the chosen angle of inclination (θ).

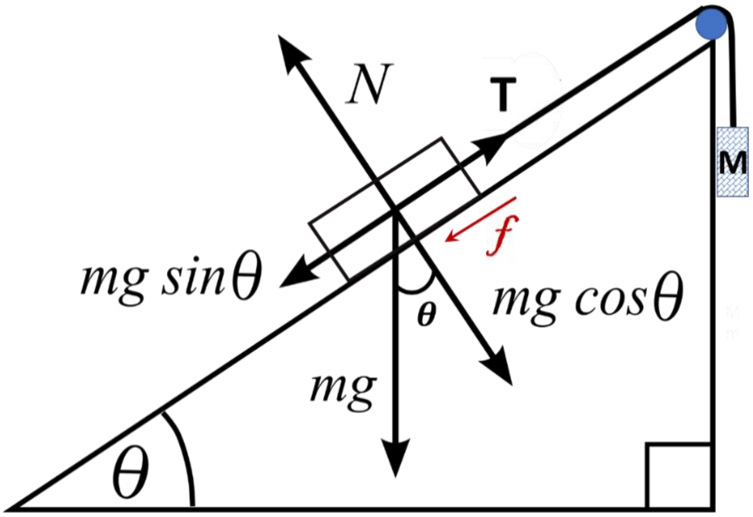


Figure 1: Free body diagram of a slider placed on an inclined wooden plane.

Method II (Angle of repose, $\boldsymbol{\theta_r}$): The angle of repose is defined as the angle at which an object just starts to slide down an inclined plane. In this method, there is no suspended mass (*M*) and the wooden plane is raised gradually until the steel slider just starts to slide or roll. In this case the friction force acts in direction opposite to that shown in figure 1 and there is no tension force (*T*). If θ_r is the angle of repose, then

$$\mu_s mgcos\theta_r = mgsin\theta_r \qquad (4)$$

$$Or,\ \mu_s = tan\theta_r \qquad (5)$$

<u>Procedure:</u>

Method I (Fixed Angle of Inclination Method)

1. Measure the mass of the slider and the pan using weighing scale provided in the laboratory.
2. Tie the slider and the pan using an inextensible string and place this assembly onto the wooden plane. Make sure that the string is passing through the frictionless pulley fixed at the end of the wooden plane.
3. Raise the wooden plane to a certain angle of inclination and clamp it.
4. Put a small weight on the scale pan and gradually increase it until the slider just starts to slide. Note that the slider should begin to move slowly and smoothly, it should not move abruptly.
5. As the slider starts to slide, note down the values of the mass of slider (*m*), suspended mass (*M*) and the angle of inclination, respectively in Table 1.
6. Repeat the experiment for five different values of angles of inclination and calculate the value of coefficient of static friction using the formula given in equation 3.

Observation:

Table 1: Coefficient of static friction by fixed angle of inclination method

S. No.	θ	m* (kg)	M** (kg)	$sin\theta$	$cos\theta$	μ_s	Mean μ_s
1.							
2.							
3.							
4.							
5.							

*m is the mass of the steel slider.

**M is the mass of the pan including its own mass and the mass of the weights added to it.

Method II (Angle of Repose Method)

After completing the above experiment, the coefficient of static friction must be calculated using the "angle of repose method" and has to be compared with the value obtained using "Fixed angle of inclination method".

1. In this experiment, adjust the angle of inclination to zero.
2. Untie the pan from the slider.
3. Place the slider on the wooden plane.
4. Increase the angle of inclination of the wooden plane gradually till the slider just begins to slide down the plane. Note: move the wooden plane very gently to increase the angle of inclination. The slider must begin to move slowly and smoothly, it should not move abruptly.
5. Note down the angle of repose in **Table 2**.
6. Repeat the steps from 2 to 4 four times. Each time place the slider at a different location on the inclined plane.
7. Calculate the coefficient of static friction using the formula given in equation 5.

Observation:

Table 2: Coefficient of static friction by Angle of Repose Method

S. No.	$Angle\ of$ Repose, θ_r	μ_s	Mean μ_s
1.			
2.			
3.			
4.			

Write down the final results.

Experiment 6

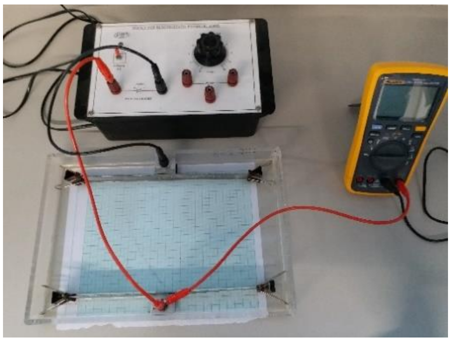
Aim:
To draw the equipotential lines of given electrodes

Apparatus:
Power source, Sets of metallic electrodes, Multimeter, Tray and H_2O

Theory:
The space surrounding an electric charge has a property called the electric field, which follows the superposition principle. The electric field exerts a force on other charges. The potential at a point in the electric field is the work done in bringing a unit positive charge from infinity to the given point. An equipotential line is a curve in 2D on which the electric potential is same everywhere. The displacement of a charge over such a curve would require no work. Since any two points on the curve have the same potential, there is zero potential difference along the curve. The electric field has no tangential component along the equipotential curve and it is always perpendicular to the curve. Thus, if we consider two point charges separated by some distance, then the potential is the same for a series of points, which when joined, form the equipotential curves for the dipole geometry. In three dimensions, the equipotential curves form the equipotential surfaces. For a point charge, the equipotential surfaces are concentric spheres and for a uniform electric field they are planes normal to the direction of the electric field.

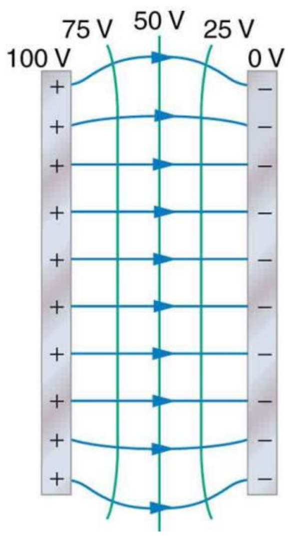
In this experiment to plot the equipotential lines, a tray half-filled with an electrolyte is used as an electrolytic tank. Metallic electrodes of desired shapes are kept in the tank. A potential difference is maintained between the electrodes, which causes a distribution of potential in the electrolyte. By measuring the potentials (using a digital multimeter) at different points, one can identify the coordinates of equipotential points and plot the equipotential lines on the graph sheet. By taking a combination of electrodes of desired shapes, one can plot the equipotential curves for various geometries.

Experimental Setup:

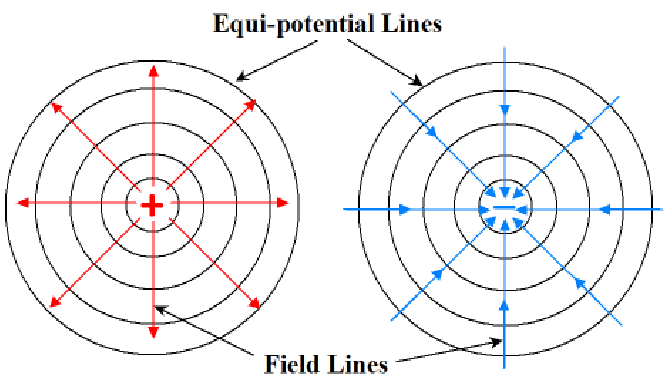


Procedure:

1. Fix the graph paper with tape below the tray with marked side facing the bottom of the tray. Keep another graph paper at the side on which you will mark the equipotential points.
2. Cover the tray with 1-2cm of the tap water.
3. Keep the electrodes E_1 and E_2 in the tray along the longer side of the tray. There should be some distance in between the electrodes.
4. Connect the electrodes with the power supply.
5. Switch on the multimeter to measure the voltage.
6. Dip the multimeter probe (red) at any point of the tray. The multimeter will show some reading. Now you should start from one end of the tray and follow the path along which the multimeter reading for the voltage would remain same.
7. To mark the equipotential points, start from one point. Note the point on the graph paper which is attached below the tray. Mark the same point on the other graph paper. Move the probe a little bit and find another point with same voltage reading. Note and mark the point on the graph paper. Keep doing this process for a *single* voltage value to note several points. Connect all these points. This will give you a *single* equipotential line.
8. Repeat the process for another value of voltage moving away from E_1 and getting near to E_2. Thus you will get different equipotential lines on the graph paper.
9. Obtain the equipotential lines for electrodes with other geometric shapes.



Equipotential lines for plate electrode.



Correct only for isolated charges and not if the two charges are kept close by

Experiment 7

Aim:
Measurement of band gap of a semiconductor using four probe method.

Apparatus:
Four probe experimental setup consisting of current source, voltmeter, temperature sensor, oven with controllable temperature, 4 probes for electrical connection, extrinsic semiconductor sample and an insulated sample holder.

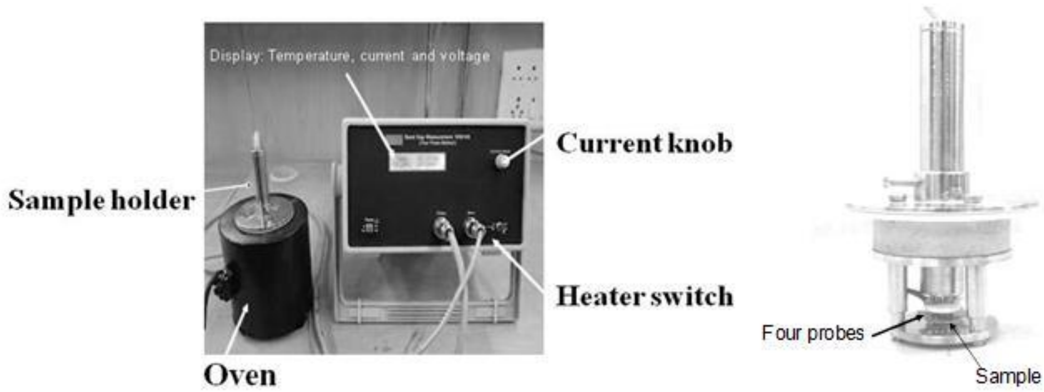


Fig. 1: Four probe experimental setup

Semiconductor

Semiconductor is a very important class of materials because of its applications. Some of the properties of semiconductors relevant to the experiment are:

1. The electrical conductivity of a semiconductor is generally intermediate in magnitude between that of a conductor and an insulator. That means conductivity is roughly in the range of 10^3 to 10^{-8} siemens (ohm^{-1}) per cm.
2. The electrical conductivity of a semiconductor varies widely with doping concentration, temperature and charge carrier injection.
3. Semiconductors have two types of charge carriers, electrons and holes.
4. Unlike metals, the number of charge carriers in semiconductors varies with temperature.
5. Generally, in case of semiconductor, increase of temperature enhances conductivity while in case of metals increase of temperature reduces conductivity.
6. The semiconductor can be best understood in the light of energy band model of solid.

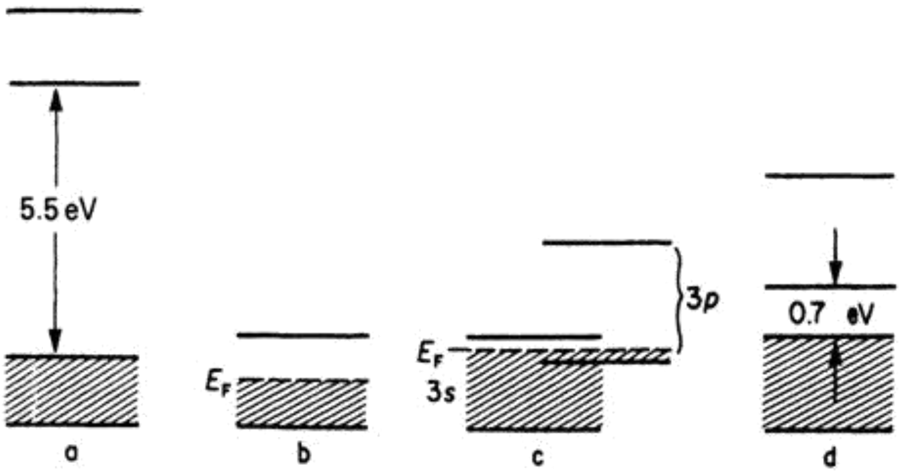


Fig. 2: Band gaps shown for (a) Insulator (b) Alkali Metal (c) Other Metal (d) Ge-semiconductor

Energy band structure of solid

Atom has specific/discrete energy levels/states. When atoms are put in a periodic arrangement in a solid the outer shell electrons no longer remain in discrete energy levels. Rather they occupy sets of continuous energy levels, each such set is called an energy band. In case of semiconductor and insulator, at temperature $0K$ all the energy levels up to a certain energy band, called valence band (VB), are completely filled with electrons, while next higher energy band (called conduction band (CB)) remains completely empty. The gap between bottom of the conduction band and top of the valence band is called *fundamental energy band gap (E_g)*. It is a forbidden gap, i.e. the energy states in the gap cannot be occupied by the electrons. In case of metals, valence band is either partially occupied by electrons or valence band has an overlap with conduction band, as shown in Fig. 2(b and c).

In case of semiconductor, the band gap $(0 - 4eV)$ is such that a significant number of electrons can move from VB to CB by absorbing thermal energy. When electron moves from VB to CB it leaves behind a vacant state in VB, called hole, which can be considered as a positively charged particle. When electric field is applied, movement of electrons in the VB can be considered equivalent to the movement of hole in opposite direction. The E_g is a very important characteristic property of semiconductor which dictates its electrical, optical and optoelectrical properties. There are two main types of semiconductor materials: intrinsic and extrinsic. Intrinsic semiconductor doesn't contain any impurity, while extrinsic semiconductor does. The process of adding impurities is called doping. Energy levels occupied by the impurity atoms are discrete and lie in the forbidden gap. The impurities are of two types: acceptor, which can accept an electron from the semiconductor and donor, which can donate an electron to the semiconductor. Extrinsic semiconductors with acceptor impurity are known as p-type semiconductors and they contain a discrete acceptor energy level just above the VB in the band gap.

Extrinsic semiconductors with donor impurity are known as n-type semiconductors and they contain a discrete donor energy level just below the CB in the band gap.

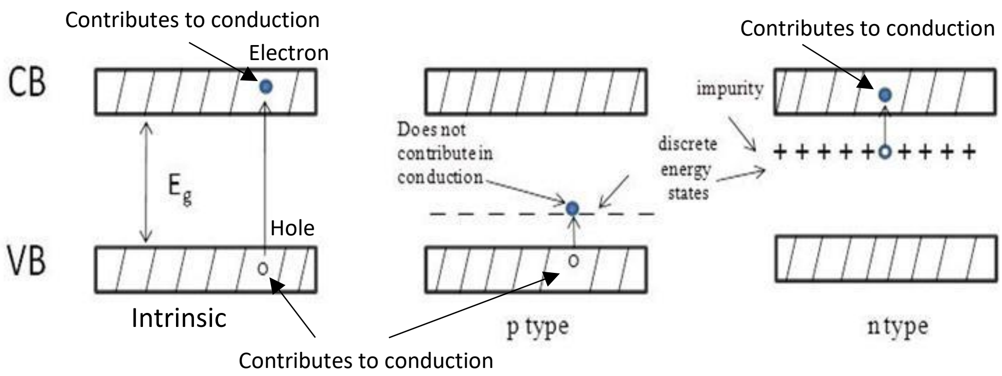


Fig. 3: Energy band diagram of a semiconductor

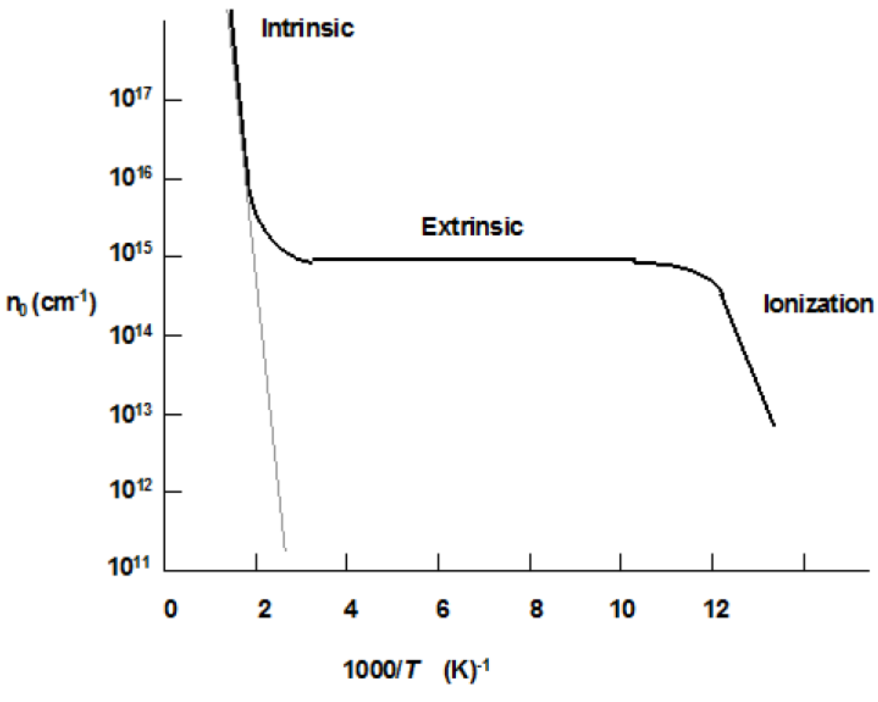


Fig. 4: Semi-log plot showing Temperature variation of charge carrier concentration

Fig. 4 shows the variation of carrier concentration (concentration of holes) in a p-type semiconductor with respect to 1000/T, where T is the temperature. Initially as temperature increases from 0K (i.e. ionization region), the vacant impurity energy level starts getting filled up with electrons from VB, which creates holes in VB. Beyond a certain temperature all the impurity states will be filled up with electrons, which will lead to the saturation region.

As temperature increases further, electrons, in the VB, get sufficient energy to occupy empty states of CB . In this temperature range the extrinsic semiconductor behaves as an intrinsic semiconductor and the threshold temperature above which this happens is called the "Intrinsic temperature".

Conductivity of a semiconductor

The conductivity of a semiconductor is given by

$$\sigma = e(\mu_n n + \mu_p p) \tag{1}$$

Where e is the electronic charge, μ_n and μ_p refer to the mobilities of the electrons and holes, and n and p refer to the density of electrons and holes, respectively. The mobility is drift velocity per unit electric field applied across the material, $\mu = V_d/E$.

Effects of temperature on conductivity of a semiconductor

In the semiconductor, both mobility and carrier concentration are temperature dependent. So conductivity as a function of temperature can be expressed by

$$\sigma(T) = e\,(\mu_n(T)n(T) + \mu_p(T)p(T)) \tag{2}$$

One interesting special case is when temperature is above intrinsic temperature where mobility is dominated by lattice scattering, i.e. scattering of the carriers due to the thermal vibration of atoms in the solid. In this temperature range mobility decreases with increase of temperature ($\mu \propto T^{-3/2}$).

In the intrinsic region, $n \approx p \approx n_i$, where n_i is the intrinsic carrier concentration. The intrinsic carrier concentration is given by

$$n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \left(m_n^* m_p^*\right)^{3/4} exp\left(\frac{-E_g}{2k_B T}\right) \tag{3}$$

where, m_n^* and m_p^* are effective mass of electron and hole, k_B is the Boltzmann constant and h is Planck's constant. Comparing all the temperature dependent terms in μ and n_i we find that the exponential term in n_i is the most dominant term.. Hence, the conductivity (which depends on μ and n_i) can easily be shown to vary with temperature as

$$\sigma = \sigma_0 exp\left(\frac{-E_g}{2k_B T}\right) \tag{4}$$

where σ_o is a constant. In this temperature range, conductivity depends on the semiconductor band gap and temperature. A plot of ln σ vs 1000/T is a straight line. From the slope of the straight line, the band gap (E_g) can be determined. The procedure of measurement of conductivity is given below.

Four probe technique

Four probe technique is generally used for the measurement of resistivity/conductivity of semiconductor sample. Before we introduce four probe technique, it is important to know two probe technique, where two probes (i.e. wires) are connected to a sample to supply constant current and measure voltage. The contact between metal sample and metal probe does not create appreciable contact resistance. But in the case of semiconductor, the metal – semiconductor contact gives rise to high contact resistance. If two probe configuration is followed for semiconductor sample, voltmeter measures the potential drop across the resistance of the sample as well as the contact resistances.

The potential drop across high contact resistance can be avoided by using four probe technique. In the four probe configuration, two outer probes are used to supply current and two inner probes are used to measure potential difference. When a voltmeter with very high impedance is connected to the inner two probes, almost no current goes through the voltmeter and the two contacts. So, the potential drop it measures, is only the potential drop across the sample resistance. From the measurement of current supplied and voltage drop across the sample, the resistance of the sample can be found out. Resistivity is given by $\rho = cV/I$, where c is a constant whose value depends on how the probes are connected. In our experiment, the probes are equispaced with the distance between two successive probes (a), being much smaller than the thickness of the sample (h). The four probes are far from the edge of the sample and the sample is placed on an insulating material to avoid leakage current. For such an arrangement the resistivity is given by

$$\rho = 2\pi a \frac{V}{I} \tag{5}$$

Since $\sigma = 1/\rho$, combining eqns. 4 and 5 gives us

$$\sigma = \sigma_0 exp\left(\frac{-E_g}{2k_BT}\right) = \frac{1}{2\pi a}\frac{I}{V}$$

$$ln\sigma_0 - \left(\frac{E_g}{2k_BT}\right) = ln\left(\frac{1}{2\pi a}\right) + ln\left(\frac{I}{V}\right)$$

$$ln\left(\frac{I}{V}\right) = -\left(\frac{E_g}{2k_BT}\right) + const. = -\left(\frac{E_g}{2000k_B}\right).\left(\frac{1000}{T}\right) + const. \tag{6}$$

Thus a plot of ln(I/V) versus $1000/T$ will be a straight line, with slope (m) given by

$$m = -\left(\frac{E_g}{2000k_B}\right)$$

The band gap (E_g) of the semiconductor can be calculated from the slope by

$$E_g = -m.2000k_B \tag{7}$$

Procedure

1. Switch ON the band gap setup.
2. Supply current to the semiconductor crystal and keep it constant (say, 7 mA) throughout the experiment. Periodically check the current, if it changes adjust it back to the set value. Turn the knobs very gently.
3. Initially the temperature of the oven must be at room temperature ($\sim 27^0$ C).
4. Switch on the oven to start increasing the temperature.

5. Note the voltage and temperature at intervals of 5^0 *C* starting from 40^0 C temperature.
6. Switch off the oven when temperature reaches 140^0 *C* or voltage becomes 0, whichever is earlier.
7. Now the oven starts to cool down. Note the voltage and temperature for decreasing temperature (5^0 *C* intervals) till it reaches 40^0 C.
8. Find the mean of the two voltages, for increasing and decreasing temperatures.
9. Calculate ln*(I/V)* for each temperature.
10. Convert the temperature scale from ^0C to Kelvin (K) by adding 273.15 and calculate 1000/*T*.

Observations

Constant Current during the experiment (*I*) = ……… mA

S. No.	Temperature (*T*) (^{0}C)	1000/*T* (K^{-1})	*V* during increasing *T* (mV)	*V* during decreasing *T* (mV)	Mean *V* (mV)	ln (*I/V*)

Calculations and Results:

Plot ln(*I/V*) (on Y axis) vs. 1000/*T* (on X axis), identify the region in graph where ln(*I/V*) decreases linearly with 1000/*T*, draw a best fit straight line and calculate slope (*m*).

Using eqn. 7 calculate the band gap (E_g) from the obtained slope. Use Boltzmann constant (k_B) = 1.38 x 10^{-23} $m^2kgs^{-2}K^{-1}$

Convert E_g from SI units, Joule (J) to units of electron Volt (eV) using the relation
1 eV = 1.6 x 10^{-19} J

Experiment 8

Aim:
Determination of the value of specific charge (e/m) of an electron by Thomson Method

Apparatus
Deflection magnetometer, two bar magnets, cathode ray tube with stand arrangement, power supply

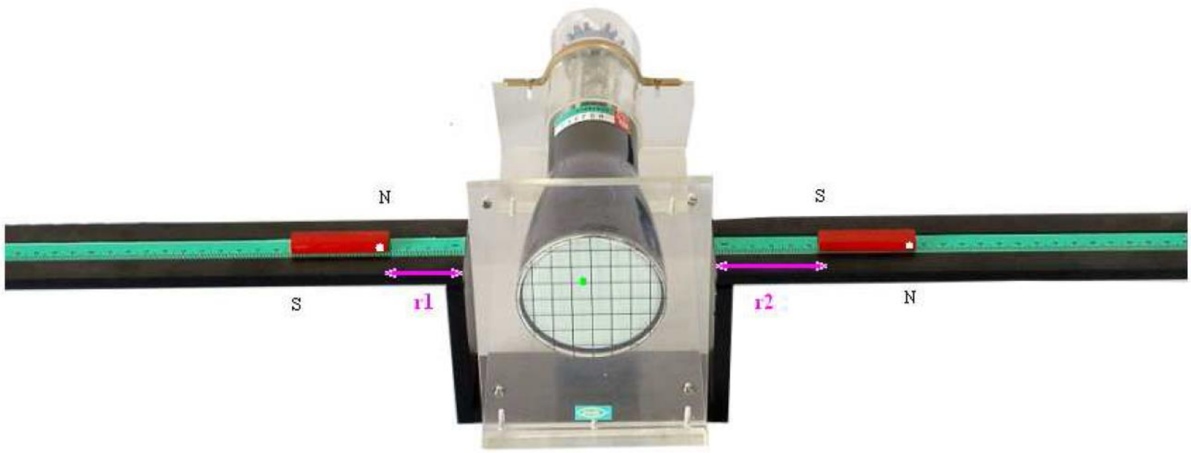


Fig. 1: Set-up used for determination of e/m by Thomson Method

Theory

The experiment is conducted inside a cathode ray tube (CRT). Electrons are emitted by a cathode, accelerated by a system of anode and concentrated into a fine beam (see fig. 2). Then they are passed between two parallel plates, which can deflect the beam in a vertical plane by an electric field E applied between both the plates. The beam of electrons can also be deflected in the same plane by applying a magnetic field B perpendicular to the plane of plates. This narrowed collimated beam of accelerated electrons then strikes the fluorescent screen to produce a glowing spot.

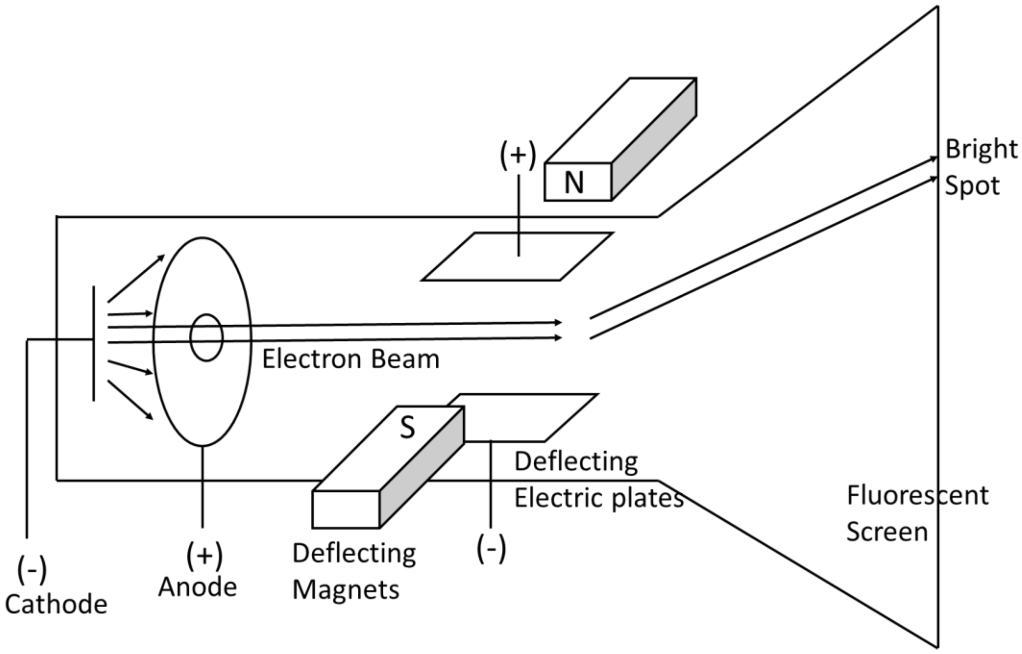


Fig. 2: Schematic diagram of the CRT used in Thomson method

1. If an electric field E is applied by a potential difference of V volts between the plates, the electrons experience a force F_e in a direction perpendicular to the direction of motion of the beam.

$$F_e = Ee \tag{1}$$

2. If B is the uniform magnetic field applied in an horizontal direction perpendicular to the direction of the electron beam, the magnetic force F_{mag} experienced by the electrons is

$$F_{mag} = e\left|\vec{v} \times \vec{B}\right| = Bev \sin 90^0 = Bev \tag{2}$$

where e is the electron charge and v is the velocity of the electrons. This force F_{mag} acts perpendicular to the direction of B as well as in the original direction of electron motion (in accordance with Fleming's left rule). The speed of electrons remains unchanged, but their path becomes circular, F_{mag} providing the required centripetal force

$$F_{mag} = Bev = mv^2/r \tag{3}$$

where m is the mass of an electron and r is the radius of the circular path. Thus

$$e/m = v/Br \tag{4}$$

3. If an electric field E is applied to deflect the beam in OO' direction (see Fig. 3), then a magnetic field B can be applied to bring the beam back to O. It means that the force of the electrostatic field is equal and opposite to applied magnetic field, so $F_e = F_{mag}$, and two forces null each other to bring the beam back to its original position. Thus

$$Ee = Bev$$

$$\text{or, } v = E/B \tag{5}$$

Substituting value of v from Eq.5 into Eq. 4,

$$\frac{e}{m} = \frac{E}{B^2 r} \tag{6}$$

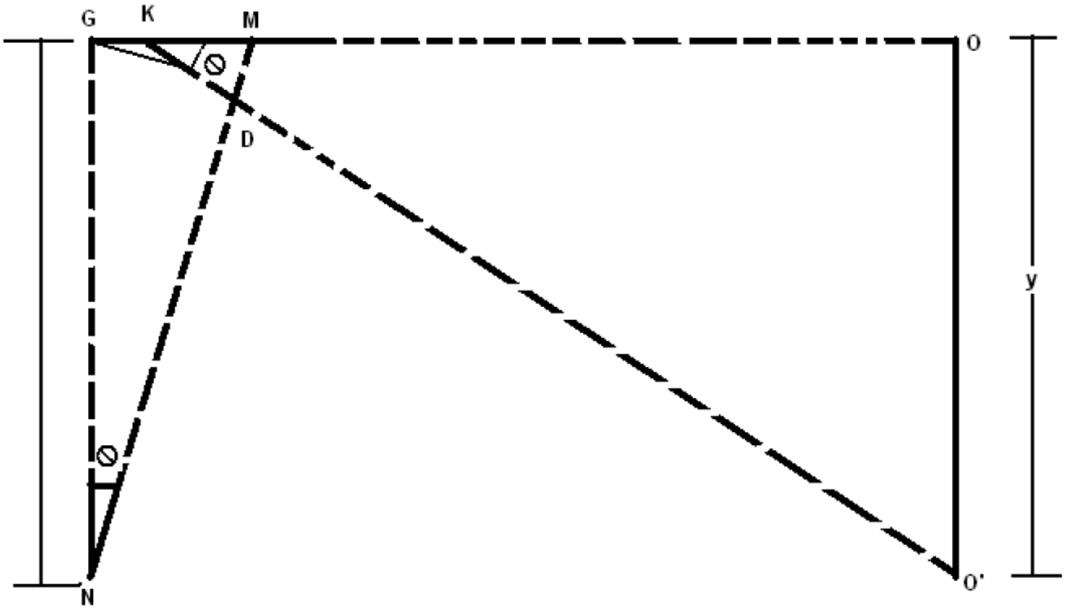


Fig. 3: Geometry of the path of the electron beam in presence of magnetic field

According to Fig.3, the original electron beam proceeds on the straight path G, M, O and impresses upon the screen at a point O. In the presence of magnetic field, the beam travels along a circular arc G, D whose radius is *r*. Beyond point D, the beam leaves the magnetic field and proceeds straight in the direction along the tangent KDO' (drawn on the circular arc at point D). GN is normal to GKO and MDN is normal to KDO'. Let these normals meet at point N. Then GN = ND = *r* = the radius of the circular arc. Let the angles GND = OKO' = θ. Then in the triangle KOO',

$$\tan\theta = OO'/KO \quad (7)$$

and if the angle θ is small enough,

$$\theta = \tan\theta = y/L \quad (8)$$

where L is distance of the screen from the mid point of the magnetic field region (generally mid point of electric field too). Again,

$$\theta = \tan\theta = \text{arc } GD/r = GM/r, \quad (9)$$

since GD is nearly equal to GM, or

$$\theta = l/r \quad (10)$$

where *l* is the length of the region of magnetic field, equal to the region of electric field. By comparing both values of θ

$$l/r = y/L \quad (11)$$

So

$$r = lL/y \quad (12)$$

Substituting the value of *r* into Eq.3.6,

$$\frac{e}{m} = \frac{Ey}{B^2 lL} \quad (13)$$

If a potential difference of *V* volts is applied between the plates P-P, and *d* is the gap between both plates, then the electric field is given by $E = V/d$. Therefore,

$$\frac{e}{m} = \frac{Vy}{B^2 lLd} \quad (14)$$

where y is the distance between spot positions displayed on the screen of CRT , l is the length of the deflection plates, L is the distance between screen and plates, d is the distance between plates, V is applied DC voltage across plates and B is magnetic field strength determined by $B = H\tan\theta$, where H is the horizontal component of earth's magnetic field at that place.

Procedure

1. Using compass needle, find and note North-South and East-West directions. Place CRT in between the stand in such a way that its screen faces towards North and both arms stand to East-West direction.
2. Adjust the Intensity and Focus potentiometers or knobs on the CRT to their mid positions.
3. Set the polarity selector switch at '0' position.
4. Set the deflection voltage potentiometer/knob in anti-clockwise direction.
5. Switch on the power supply and wait for some time (3-5 minutes) to warm up the CRT. A bright spot appears on the screen. Adjust intensity and focus knobs to obtain a sharp spot.
6. Bring the spot to the middle position of the CRT screen (in horizontal direction) with the help of X-plate deflection voltage knob given at the back-side of the instrument.
8. Note the zero error of the spot (in vertical position) if any. This has to be subtracted from the y-readings.
9. After rotating the voltage potentiometer/knob in anti-clockwise direction, the voltage should be zero. However, if any finite value appears, note it down as zero error in voltage reading. Subtract the same from all the future voltage readings.
10. Set polarity selector to '+' position, and adjust deflection voltage to deflect the spot in the upward direction (deflection should be ≤ 1 cm). Note the deflection voltage from the meter (V_1) and the deflection of the spot (y_1).
11. Now place the bar magnets on the stand arm, on both sides of CRT.
12. Adjust position of magnets to get the spot back down to its original position.
13. Note the positions of the bar magnets (i.e. positions of poles facing the screen) as r_1 and r_2 from the scale. Also note the polarity of the poles facing the CRT.
14. Now remove magnets from the arms of stand.
15. Select '−' position of polarity switch without changing the deflection voltage. This will deflect the spot in the downward direction. Adjust the voltage so that the magnitude of deflection is same as before. Note deflection voltage from display (V_1') and deflection of the spot (y_1).
16. Place the bar magnets again and adjust the position of magnets to bring spot back to its original central position. Note the positions of the magnets (positions of poles facing the screen) as r_1' and r_2'. Also note the polarity of the poles facing the CRT
17. Remove CRT and magnets. Place magnetometer or the compass arrangement in between the stand such that its centre lies on the centre of the stand arm. Note: Position of stand should not be disturbed.
18. Rotate the magnetometer so that the needle reads $0^o - 0^o$.
19. Now place the bar magnets at a distance equal to r_1 and r_2 with the same polarity as done previously. The magnetometer pointer deflects along the scale. Note the deflections of the two ends of the pointer as θ_1 and θ_2.
20. Repeat step 17 by placing the magnets at r_1' and r_2' distances. Note the deflection of the magnetometer needle as θ_3 and θ_4.
21. Now we know that magnetic field at the position of the magnetometer is

$$B = H\tan\theta, \tag{15}$$

where, $$\theta = \frac{\theta_1+\theta_2+\theta_3+\theta_4}{4} \tag{16}$$

and H is the horizontal component of earth's magnetic field (0.37×10^{-4} Tesla)

22. Calculate e/m using the following formula

$$\frac{e}{m} = \frac{Vy}{B^2 lLd} \tag{17}$$

where, d = distance between plates = 1.4 cm
l = length of plates = 3.23 cm
L = distance between screen and plates = 14.5 cm
V = deflection voltage = $(V_1 + V_1')/2$
y = magnitude of deflection in cm

23. Take more readings by repeating the experiment for a different value of deflection (y_2) (deflection should be ≤ 1 cm).

Precautions and sources of error

• The cathode ray tube should be handled carefully. There should not be any magnetic substance nearby the place of experiment. Other electronics equipment should be kept away from the setup.
• Rotate magnet(s) on their axes if spot does not come back to its original position.
• The magnets and CRT should be positioned as shown in figure 4 otherwise magnetic field due to the bar magnets will not be in the same location as the electrical deflection plates.
• The electric field between plates cannot be uniform due to short distance between them.
• The given constants of the instrument are generally taken from data provided by the manufacturer, there may be slight variations that can produce error.

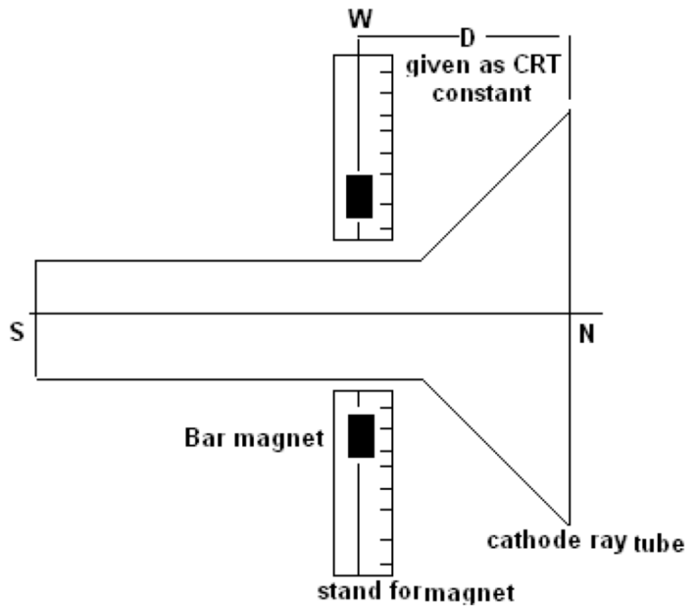


Fig. 4: Direction of orientation of the cathode ray tube and the magnets.

Observations:

Zero error in spot position along the vertical direction (y_0) =

Zero error in voltage reading (V_0) =

Table I: Data for deflection voltage and bar magnet positions

Electric field polarity	Deflection y (in cm)	Deflection voltage V (in V)	Average deflection voltage V (in V)	Bar magnet positions (in cm) and poles facing the CRT	
				Magnet 1	Magnet 2
+	y_1 =	V_1 =		r_1 =	r_2 =
−		V_1' =		r_1' =	r_2' =
+	y_2 =	V_2 =		r_1 =	r_2 =
−		V_2' =		r_1' =	r_2' =

Table II: Calculation of B from the magnetometer reading for $y = y_1$

Bar magnet positions (cm)	Magnetometer deflection (in degree)	Average deflection $\theta=(\theta_1+\theta_2+\theta_3+\theta_4)/4$ (in degree)	$B=H \tan\theta$ (Tesla), H = 0.37×10^{-4} Tesla
r_1 =	θ_1 =		
r_2 =	θ_2 =		
r_1' =	θ_3 =		
r_2' =	θ_4 =		

Table III: Calculation of B from the magnetometer reading for $y = y_2$

Bar magnet position (cm)	Magnetometer deflection (in degree)	Average deflection $\theta=(\theta_1+\theta_2+\theta_3+\theta_4)/4$ (in degree)	$B=H \tan\theta$ (Tesla), H = 0.37×10^{-4} Tesla
r_1 =	θ_1 =		
r_2 =	θ_2 =		
r_1' =	θ_3 =		
r_2' =	θ_4 =		

Calculations and Result:

Calculate *e/m* for electrons using eqn. 17 given in step no. 22 of the procedure.
Calculate the percentage error in *e/m* value as:
(Standard value - Calculated value)/Standard value × 100
Standard value of e = 1.6×10^{-19} C, standard value of m = 9.1×10^{-31} Kg

Experiment 9

Aim:
To study conservation of momentum and kinetic energy during collisions

Apparatus:
Linear air track, triangular gliders, air blower, digital data logger, gates with IR sensor and LED, accessories that can be plugged into gliders for different measurements, weights, balance.

Theory:
The law of conservation of linear momentum states that the total linear momentum of all constituent units or objects within an isolated system remains constant, unless the system is acted upon by external forces. This means in the event of collision of two objects, if the net external force on them is 0, then the total momentum before collision must be same as the total momentum after collision.

$$m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} \quad (1)$$

where m_i, $\overrightarrow{u_i}$, $\overrightarrow{v_i}$ are the mass, initial velocity (before collision) and final velocity (after collision) of the i^{th} object. For motion in one dimension there are two possible directions, which can be indicated by "+" and "–" signs. Kinetic energy of a system is not always conserved, even if the net external force on them is 0. Those collisions where kinetic energy is conserved are termed as elastic collisions, while those in which kinetic energy is not conserved are called inelastic collisions. One example of inelastic collision is when the colliding objects stick to each other after collision. In such a case the momentum conservation implies:

$$m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2} = (m_1 + m_2)\vec{v}_{1,2} \quad (2)$$

where $\vec{v}_{1,2}$ is the final velocity of the composite of the two objects after collision.

kinetic energy in elastic collisions: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2$ (3)

kinetic energy in inelastic collisions: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}(m_1 + m_2)v_{12}^2$ (4)

In this experiment we study the collisions between gliders on a linear air track. Air is forced out of the perforations in the track using a blower pump, so that the gliders float on a thin carpet of air above the surface of the air track. This make the friction acting on the gliders negligibly small. Hence, the net force acting on a glider (friction + weight + normal reaction) while moving on the air tack is ≈ 0. Observe how the glider keeps moving once set into motion. If we regard the two colliding gliders to be part of the same system, then the forces exerted by the gliders on each other during a collision are internal forces and the net external force on the system stays 0.

The air track consists of two gates fitted with IR sensors. Data from the sensors can be processed by a digital data logger or timer connected to them. There are different accessories that can be plugged into the gliders. Among these, the small u-shaped strips are used for measuring velocity of the gliders. The IR sensors on the gates record the time gap between the passing of the two arms of the u-shaped strip through the gate. It automatically computes the velocity of the glider by dividing the known separation between the two arms of the u-shaped strip by the time gap.

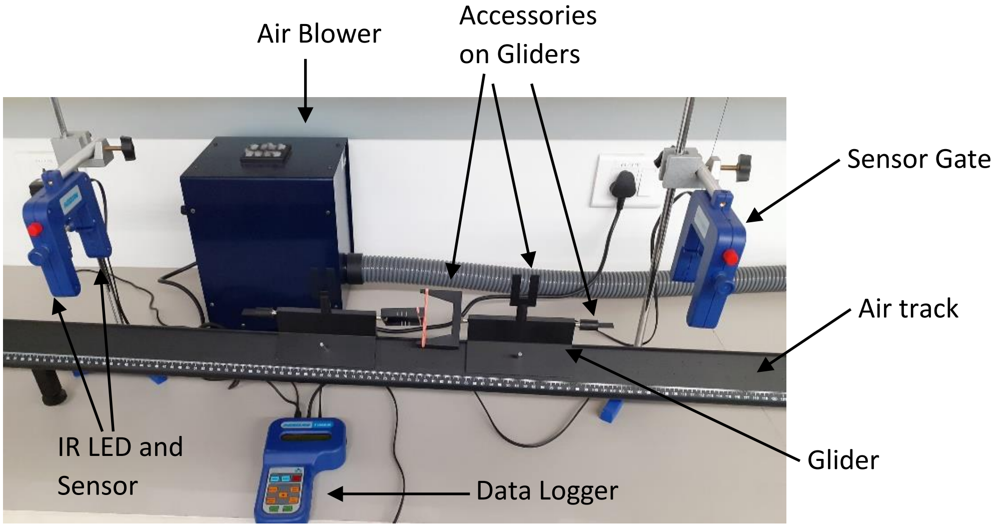


Figure 1: Linear air track set-up

Procedure:

1. Set up the linear air track in the manner shown above in figure 1. Make sure the u-shaped strip plugged to the top of the gliders passes perpendicularly between the arms of the sensor gates.
2. Attach rubber bands to the large U-shaped accessories and plug them in the front of the gliders, such that the rubber bands on the two gliders are roughly perpendicular to each other. This set up is for **elastic collisions**.
3. Confirm that the air track is levelled horizontally. One way to check levelling is to place the glider at the centre of the air track and ensure that it does not move along a particular direction, without the application of any force. If it does, then adjust the screws at the bottom of the air track. <u>Consult the instructor for this step</u>. The gliders oscillating about a point is normal.
4. Make sure that you identify the gate connected to port 1 of the data logger as gate 1. The glider that is closest to this gate should be called as glider 1. Accordingly, the mass, initial velocity (before collision) and final velocity (after collision) of glider 1 becomes m_1, u_1 and v_1, respectively. Similarly, the other glider becomes glider 2. This is to avoid confusion while noting data.
5. Press the "MODE" key on the data logger repeatedly till "speed" mode is displayed in the screen. Press the "SELECT" key repeatedly till "collision 2" is displayed. In this mode speeds of the gliders are measured twice at the two gates.
6. Press the "START/STOP" key on the data logger. The screen will display "…. …. * …. ….", indicating that the sensors are ready for measurement. Make sure nothing crosses the gates, except the u-shaped strips on the gliders, once the measurements are initiated.
7. Put one of the gliders at rest at the centre of the track between the two gates and impart a velocity towards the centre to the other glider by gently pressing it against the flexible lever attached to the end of the track. Let us call this <u>case 1</u>. While holding the glider against the

lever try to apply minimal force in vertically downward direction. The glider should move smoothly, without bumping on the track.

8. Closely observe how the gliders move after collision. After each gate has recorded two velocities "…. …. * …. …." will disappear from the screen of the data logger and it will display the results. Press the "REVIEW" key repeatedly to display the four velocities recorded.
9. The data logger will display the speeds as "initial 1", "final 1", "initial 2" and "final 2". Note these down. These are the 1st and 2nd speeds recorded at gate 1 and 1st and 2nd speeds recorded at gate 2, respectively. Assign the velocities of the two gliders accordingly. For example, if glider 1 was the glider that was given an initial velocity and glider 2 was at rest initially, then the speed "initial 1" is u_1 and $u_2 = 0$. In this case, after collision the glider 1 will come to rest and glider 2 will start moving (expected behaviour). So, the speed "initial 2" is v_2, while $v_1 = 0$. We can neglect the speeds "final 1" and "final 2" in this example.
10. Notice how there is a transfer of momentum from the glider that was moving initially to the glider that was at rest initially.
11. Weigh the two gliders and note down the masses as m_1 and m_2.
12. Press the "START/STOP" key on the data logger to restart the measurements.
13. Now press both gliders against the flexible levers at the two ends of the air track and release both simultaneously, so that they collide at the central region of the track. Let us call this <u>case 2</u>. Motion of the gliders should always be smooth and not bumpy.
14. Repeat steps 8 and 9. In this case the speed "initial 1" is u_1, "initial 2" is $-u_2$, "final 1" is $-v_1$ and "final 2" is v_2. Record data only when direction of motion of both gliders reverse after collision. If this does not happen then neglect the recorded speeds and repeat the collision.
15. Repeat step 7, but this time load two weights on the glider that is at rest initially. Put one weight on each side of the glider. Let us call this <u>case 3</u>. Weigh the glider with the weights and note down its mass. Record the speeds "initial 1", "final 1", "initial 2" and "final 2" and carefully assign $\overrightarrow{u_1}$, $\overrightarrow{u_2}$, $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$.
16. For each of the 3 cases calculate the total momentum before collision $(m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2})$, total momentum after collision $(m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2})$ and the percentage change in momentum due to collision $\left(\frac{\text{change}}{\text{initial value}}\text{ x } 100\right)$. Note that this change should be 0, as expected from the law of conservation of momentum. However, in this experiment it always turns out to be non-zero. There are many sources of error in the experiment. The smaller the change in momentum is, the better is the experiment.
17. Similarly, calculate the total kinetic energy before collision $(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2)$, total kinetic energy after collision $(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2)$ and the percentage change in kinetic energy due to collision. Again, for elastic collision this change should be ideally 0. Smaller values indicate better experiment.
18. Repeat the 3 cases above, but for **inelastic collision**. For this purpose, replace the accessories having rubber bands with accessories having a rubber pad and a sharp needle. <u>Be careful not to hurt yourself while handling the accessory with needle</u>. Now after collision the two gliders should stick to each other. If this does not happen then repeat the collision. Again, note down the speeds from the data logger and carefully assign values to the velocities $\overrightarrow{u_1}$, $\overrightarrow{u_2}$ and $\vec{v}_{1,2}$.

19. Again, compute the changes in total momentum and kinetic energy due to collision. For inelastic collisions change in momentum is expected to be 0 as before. Generally small non-zero values are obtained due to the limitations of the experiment. However, the change in kinetic energy is expected to be non-zero.

Observations:

Data for elastic collisions:

Case	Mass m_1 in g	Mass m_2 in g	Speeds from data logger in cm/s				Velocities * before collision in cm/s		Velocities * after collision in cm/s		% change in momentum	% change in kinetic energy
			"initial 1"	"final 1"	"initial 2"	"final 2"	$\vec{u_1}$	$\vec{u_2}$	$\vec{v_1}$	$\vec{v_2}$		
1												
2												
3												

* Direction should be indicated by sign

Mean value of % change in momentum = ……..
Mean value of % change in kinetic energy = ……..

Data for inelastic collisions:

Case	Mass m_1 in g	Mass m_2 in g	Speeds from data logger in cm/s				Velocities * before collision in cm/s		Velocity * after collision in cm/s	% change in momentum	% change in kinetic energy
			"initial 1"	"final 1"	"initial 2"	"final 2"	$\vec{u_1}$	$\vec{u_2}$	$\vec{v}_{1,2}$		
1											
2											
3											

* Direction should be indicated by sign

Mean value of % change in momentum = ……..
Mean value of % change in kinetic energy = ……..

Calculations:

Show calculations for momentum and kinetic energy before and after collision for each of the 6 cases (3 elastic and 3 inelastic collisions). Finally calculate the % changes in momentum and kinetic energy.

Results and discussion:

Write down the mean values of the % changes in momentum and kinetic energy for elastic and inelastic collisions. Discuss the factors responsible for the observed departure from the law of conservation of momentum. You should be able to enumerate at-least three major factors.

Experiment 10

Aim: To determine the normal mode frequencies of a coupled oscillator and verify beat frequency is a linear combination of the normal mode frequencies.

Apparatus:
1) Coupled pendulum
2) Two springs
3) Hanger
4) Weights 50 gms each
5) Stop watch

Theory: Let us consider two identical pendulums, each having a mass m suspended on a light rigid rod of length l. The masses are connected by a light spring of stiffness k whose natural length equals the distance between the masses when neither is displaced from equilibrium (Fig. 1).

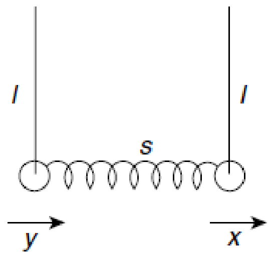


Fig. 1 Coupled pendulum

If x and y are the respective displacement of the masses, then the equations of motion are

$$m\ddot{x} = -mg\frac{x}{l} - k\,(x-y) \tag{1}$$

$$m\ddot{y} = -mg\frac{y}{l} + k(x-y) \tag{2}$$

These represent the simple harmonic motion terms of each pendulum plus a coupling term $k(x-y)$ from the spring.

Writing $\omega_0^2 = \frac{g}{l}$, where ω_0 is the natural vibration frequency of each pendulum, gives

$$\ddot{x} + \omega_0^2 x = -\frac{k}{m}(x-y) \tag{3}$$

$$\ddot{y} + \omega_0^2 y = -\frac{k}{m}(y-x) \tag{4}$$

Instead of solving these equations directly for x and y we are going to choose two new coordinates

$$X = x + y$$
$$Y = x - y$$

Adding (3) and (4) gives

$$\ddot{x} + \ddot{y} + \omega_0^2(x+y) = 0$$

$$\Longrightarrow\ \ddot{X} + \omega_0^2\, X = 0 \tag{5}$$

And subtracting (4) from (3) gives

$$\ddot{Y} + \left(\omega_0^2 + \frac{2k}{m}\right)Y = 0 \tag{6}$$

The motion of the coupled system is thus described in terms of the two coordinates X and Y, each of which has an equation of motion which is simple harmonic.

Normal modes of vibration: A vibration involving only one dependent variable X (or Y) is called a normal mode of vibration and has its own normal frequency. In such a normal mode all components of the system oscillate with the same normal frequency.

(i) In phase mode of vibration:

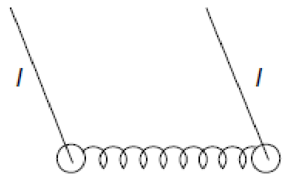


Fig. 2 The 'in phase' mode of vibration

If $Y = 0$, $x = y$ at all times, so that the motion is completely described by the equation

$$\ddot{X} + \omega_0^2 X = 0,$$

then the frequency of oscillation is the same as that of either pendulum in isolation and the stiffness of the coupling has no effect. This is because both the pendulums are always swinging in phase (Fig. 2) and spring is always at its natural length.

(ii) Out of phase mode of vibration:

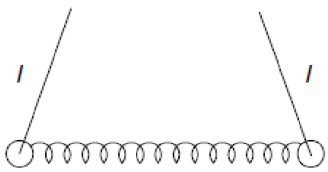


Fig. 3 The 'out of phase' mode of vibration

If $X = 0$, $x = -y$ at all times, so that the motion is completely described by

$$\ddot{Y} + \left(\omega_0^2 + \frac{2k}{m}\right) Y = 0.$$

The frequency of oscillation is greater because the pendulums are always out of phase (Fig. 3) so that the spring is either extended or compressed and the coupling is effective.

The importance of the normal modes of vibration is that they are entirely independent of each other. The energy associated with a normal mode is never exchanged with another mode; we can add the energies of the separate modes to get the total energy. If only one mode vibrates the second mode will always be at rest, acquiring no energy from the vibrating mode.

Any configuration of the coupled system can be represented by the superposition of the two normal modes.

$$X = x + y = X_0 \cos(\omega_1 t + \phi_1)\,; \quad Y = x - y = Y_0 \cos(\omega_2 t + \phi_2),$$

where X_0 and Y_0 are normal mode amplitudes, and $\omega_1^2 = \frac{g}{l}$ and $\omega_2^2 = \frac{g}{l} + \frac{2k}{m}$.

Beats: Now let us set the system in motion by displacing the right hand mass a distance $x = 2a$ and releasing both the masses from rest so that $\dot{x} = \dot{y} = 0$ at time $t = 0$.

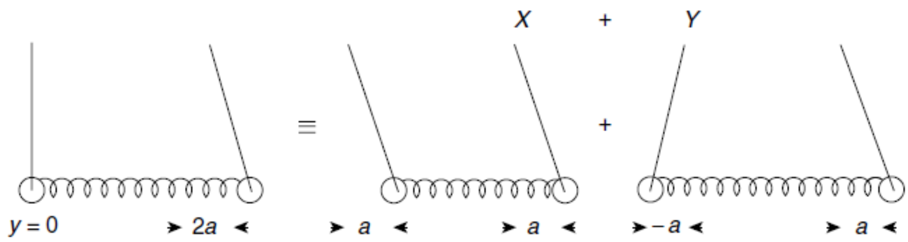


Fig. 4 The displacement of one pendulum is shown as equivalent to the combination of normal mode coordinates

As shown in Fig. 4, the initial displacement $x = 2a$, $y = 0$ at $t = 0$ can be seen as a combination of the 'in phase' mode ($x = y = a$ so that $x + y = X_0 = 2a$) and the 'out of phase' mode ($x = -y = a$ so that $Y_0 = 2a$).
After release, the motion of the right hand pendulum is given by

$$x = a\cos(\omega_1 t) + a\cos(\omega_2 t)$$

$$= 2a\cos\frac{(\omega_2-\omega_1)t}{2}\cos\frac{(\omega_1+\omega_2)t}{2} \tag{7}$$

and that of the left hand pendulum is given by

$$y = a\cos(\omega_1 t) - a\cos(\omega_2 t)$$

$$= 2a\sin\frac{(\omega_2-\omega_1)t}{2}\sin\frac{(\omega_1+\omega_2)t}{2}\ . \tag{8}$$

The behavior of the individual pendulums can be seen in Fig. 5. 'x' and 'y' oscillate cosinusoidally and sinusoidally respectively with a frequency that is the average of the two normal mode frequencies, and their amplitudes oscillate with a frequency that is half the difference between the normal frequencies (beats).

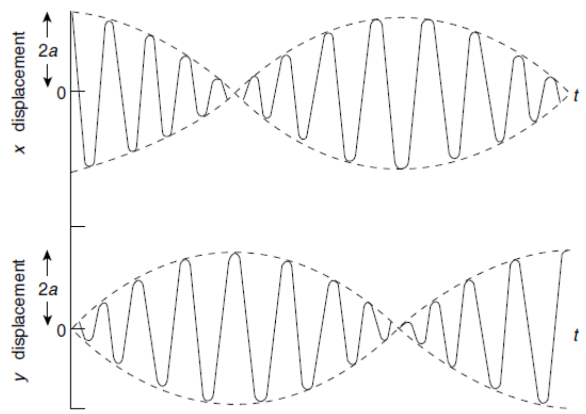


Figure 5 Behavior with time of individual pendulums, showing complete energy exchange between them

Procedure:

Fig. 6 Coupled pendulum setup

By considering damping due to air resistance and friction in the bearings at the top of each pendulum, the angular frequencies of the 'in phase' and 'out of phase' modes can be written as

$$\omega_1 = \sqrt{\frac{g}{l}} \tag{9}$$

$$\omega_2 = \sqrt{\omega_1^2 + \frac{2kd^2}{ml^2}}, \tag{10}$$

where $l = 72.5$ cm is the length of the rod, $m = 783.2$ gm is the mass of the pendulum, and $d = 36.25$ cm is the length of the rod from the coupling spring to the bearings as shown in Fig. 6.

Procedure:

Exp. 1: Measurement of the spring constant of helical spring

1) Set the apparatus as shown in Fig. 7.
2) Measure the length of the spring without any weight.
3) Add a weight (50 gm) to the spring and measure the extension (x) of the spring due to the weight.
4) Increase the weight to 100, 150, and 200 gm and measure the corresponding extensions (x).
5) Plot x (x-axis) vs mg (y-axis) and determine the slope.
Spring constant $k = slope$

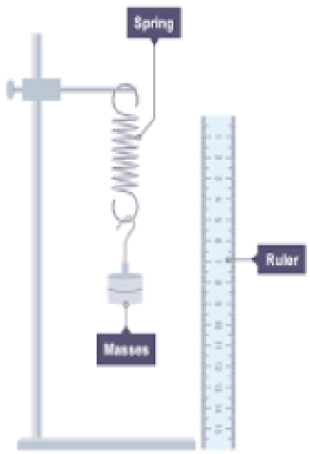


Fig. 7 Spring constant measurement

Exp. 2: In phase oscillation

1) Couple the two pendula with the spring provided and adjust the distance between the rods so that the spring is neither extended nor compressed.
2) Displace both the pendula equally in the same direction as shown in Fig. 8 and measure the time period T_1 by measuring the time duration t_1 for 10 cycles.
$T_1 = \frac{t_1}{10}$ and Normal mode frequency $\omega_1 = \frac{2\pi}{T_1}$

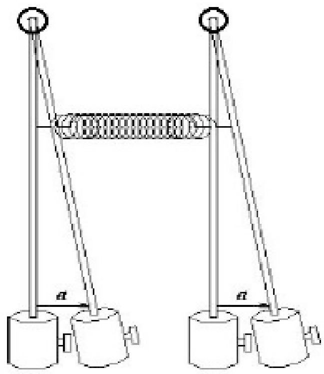


Fig. 8 In phase oscillation

Exp. 3: Out of phase oscillation

1) Displace both the pendula equally in opposite directions as shown in Fig. 9 and measure the time period T_2 by measuring the time duration t_2 for 10 cycles.
$T_2 = \frac{t_2}{10}$ and Normal mode frequency $\omega_2 = \frac{2\pi}{T_2}$

Now, compare the experimental values of $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_2}$ with the theoretically calculated ones from equations (9) and (10).

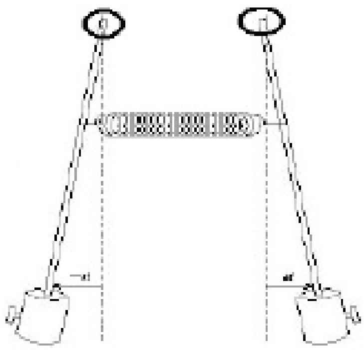


Fig. 9 Out of phase oscillation

Exp. 4: Beats

1) Keeping one pendulum at rest, displace the other and measure the time period T_C of coupled oscillations by noting the time duration t_C for 6 oscillations.
$T_C = \frac{t_C}{6}$ and $\omega_C = \frac{2\pi}{T_C}$.
2) Measure the 'beat' time period T_B by observing one of the pendula becoming stand still 3 times and noting the corresponding time duration t_B.
$T_B = \frac{t_B}{3}$ and $\omega_B = \frac{2\pi}{T_B}$

Now, calculate $\boldsymbol{\omega_C}$ and $\boldsymbol{\omega_B}$ (angular beat frequency) from $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_2}$ measured in Exp. 2 and Exp. 3 by using the following expressions

$$\boldsymbol{\omega_C = \frac{\omega_1 + \omega_2}{2}}$$

$$\boldsymbol{\omega_B = \frac{\omega_2 - \omega_1}{2}},$$

and compare them with the values measured in Exp. 4.
Also, calculate $\boldsymbol{\omega_C}$ and $\boldsymbol{\omega_B}$ by using theoretical $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_2}$ values (from equations (9) and (10)) and compare them with the above values.

Observation:

Spring constant measurement:

Length of the unextended spring (x_0) = ……………

S. No.	Weight (mg)	Spring length (x_1)	Spring extension $x = x_1 - x_0$
1			
2			
3			
4			
5			

Time period measurement:

Time period	Angular frequency
T_1=………..	ω_1=………..
T_2=………..	ω_2=………..
T_C=………..	ω_C=………..
T_B=………..	ω_B=………..

Result:

1) Spring constant k = ……………..

2) Theoretical normal mode frequencies: $\omega_1 = \sqrt{\frac{g}{l}}$ = ……………….

$\omega_2 = \sqrt{\omega_1^2 + \frac{2kd^2}{ml^2}}$ = ……………………………..

3) Experimental normal mode frequencies: ω_1 = …………….

ω_2 = …………………

4) Experimental carrier and beat frequencies: ω_C = …………….

ω_B = ………………

5) Calculated carrier and beat frequencies from the experimental normal mode frequencies:

$\omega_C = \frac{\omega_1+\omega_2}{2}$ = …………………….

$\omega_B = \frac{\omega_2-\omega_1}{2}$ = ……………………..

6) Calculated carrier and beat frequencies from the theoretical normal mode frequencies:

$\omega_C = \frac{\omega_1+\omega_2}{2}$ = …………………….

$\omega_B = \frac{\omega_2-\omega_1}{2}$ = ……………………..

Precautions:

1) The angular amplitude should be kept small.
2) While using the spring, keep its length natural, i.e., neither extended nor compressed.
3) The oscillations should be in plane.