Applied Linear Algebra
MAT3004

Due Date: 22/10/2022

1. Let $V = P_3(\mathbb{R})$ be a vector space of all polynomials of degree at most 3. If W_1 and W_2 are the subspaces of P_3 with corresponding bases $\alpha = \{1+t, 2t+t^2, 2+t^3\}$ and $\beta = \{t-2t^2, t+t^3\}$, then find the basis for
   (a) $W_1 + W_2$
   (b) $W_1 \cap W_2$.

2. (a) Let $T : M_{2\times 2} \to \mathbb{R}^2$, where $M_{2\times 2}$ is the real vector space of all 2×2 matrices and
$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{pmatrix} \max(a,b) \\ \min(a,b) \end{pmatrix}.$$
Verify, if T is a linear transform or not.

   (b) Let $T : P_2(\mathbb{R}) \to \mathbb{R}^3$, such that $T(a_0 + a_1 t + a_2 t^2) = (a_0 + 2a_1, a_2 - a_0, a_1 + 3a_2)$, $\forall (x,y) \in \mathbb{R}^2$.
   (i) Verify that if T is a linear transformation or not.
   (ii) Find the inverse map of T, if T is invertible.
   (iii) Find the kernel of T.

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transform, such that
$$T(x,y,z) = (2x+y, y-z, 3x+y-z),\ \forall (x,y,z) \in \mathbb{R}^3.$$
   (a) Then find the matrix of transformation of $[T]_\alpha$, where alpha is the standard basis of $\mathbb{R}^3$.
   (b) Find $[T]_\beta$, by using similarity transformation, where $\beta = \{(-1,1,2), (0,2,1), (1,1,4)\}$.

4. (a) Let $(\mathbb{R}^3, \langle\rangle)$ be a inner product space such that
$$\langle (x_1,x_2,x_3), (y_1,y_2,y_3)\rangle = 2x_1y_1 + 3x_2y_2 + 5x_3y_3.$$
   (i) Then find the matrix representation of this inner product w.r.t. standard basis of $\mathbb{R}^3$.
   (ii) $v_1 = (1,3,2)$, $v_2 = (-2,2,2)$, find norm of v_1, v_2 and angle between v_1, v_2.

   (b) Let $(P_3(\mathbb{R}), \langle\rangle)$ is an inner product space of all real polynomials of degree at most 3 with inner product as
$$\langle p(t), q(t)\rangle = \int_0^1 p(t).q(t)dt,\ \forall p(t), q(t) \in P_3.$$
Then find the ortho-normal basis of P_3 corresponding to basis $\left\{1-t, t+2t^2, 3t^2, 4t-t^3\right\}$.

5. Find QR decomposition of the matrix $\begin{bmatrix} 2 & 1 & -1 & 4 \\ 1 & 2 & 2 & -3 \\ 2 & 1 & -1 & 0 \\ -3 & 0 & 0 & 1 \\ 4 & -1 & 1 & 2 \end{bmatrix}$.