

Applied Linear Algebra(MAT3004)

DIGITAL ASSIGNMENT 2

1.

Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3 , where

$$v_1 = (1, 1, 1), \quad v_2 = (1, 1, 0), \quad v_3 = (1, 0, 0)$$

Let $T : R^3 \rightarrow R^2$ be the linear transformation for which

$$T(v_1) = (1, 0), \quad T(v_2) = (2, -1), \quad T(v_3) = (4, 3)$$

Find a formula for $T(x_1, x_2, x_3)$, and then use that formula to compute $T(2, -3, 5)$.

2.

$$\text{Let } M = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}.$$

(1) Find the unique linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ so that M is the matrix of T with respect to the bases

$$\alpha_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \quad \alpha_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

(2) Find $T(x, y, z)$.

3.

Let $T : R^2 \rightarrow R^3$ be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & 13 \\ -7 & 16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find the matrix for the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3 , where

$$u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}; \quad v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

4.

If $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is a linear transformation with matrix $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & -2 \end{bmatrix}$ relative to the standard bases of both spaces, find the matrix of T relative to the bases

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad B_2 = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}.$$

5.

Let the vector space P_2 have the inner product

$$\langle p, q \rangle = \int_{-1}^1 p(x)q(x) dx$$

Apply the Gram-Schmidt process to transform the standard basis $\{1, x, x^2\}$ for P_2 into an orthogonal basis $\{\phi_1(x), \phi_2(x), \phi_3(x)\}$.

6.

Use the Gram-Schmidt orthonormalization process to transform the given basis for \mathbb{R}^n into an orthonormal basis. Use Euclidean inner product for \mathbb{R}^n .

$$B = \{(3, 4, 0, 0), (-1, 1, 0, 0), (2, 1, 0, -1), (0, 1, 1, 0)\}$$

7.

Let $V = \mathbb{R}^3$ with the inner product defined by

$$\langle \mathbf{x}, \mathbf{y} \rangle = \left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = x_1 y_1 + 2x_2 y_2 + \frac{1}{3}x_3 y_3.$$

Convert the basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ into an orthonormal basis using this inner product.

8.

Suppose

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

is the matrix for $T: R^3 \rightarrow R^3$ relative to the standard basis. Find the matrix for T relative to the basis

$$B' = \{(1, 1, 0), (1, -1, 0), (0, 0, 1)\}.$$