Fundamental subspaces Let A be a $m \times n$ matrix. Then

$R(A)$ = Row space of A (Subspace of $\mathbb{R}^n$ spanned by rows)

$C(A)$ = Column space of A (Subspace of $\mathbb{R}^m$ spanned by columns)

$N(A)$ = Null space of A $(\{x \in \mathbb{R}^n \mid Ax = 0\})$

$N(A^T)$ = Null space of A^T $(\{y \in \mathbb{R}^m \mid A^T y = 0\})$

The above subspaces are called as Fundamental subspaces.

Relationship

(i) $N(A) = R(A)^\perp$ and $R(A) = N(A)^\perp$

(ii) $N(A^T) = C(A)^\perp$ and $C(A) = N(A^T)^\perp$

(iii) $\mathbb{R}^n = N(A) \oplus R(A) = N(A) \oplus N(A)^\perp$

(iv) $\mathbb{R}^m = N(A^T) \oplus C(A) = N(A^T) \oplus N(A^T)^\perp$