

Module-1

System of linear equations

Linear equation We call the equation $a_1x_1 + a_2x_2 + \dots + a_nx_n = b$ is a linear equation. Here a_1, a_2, \dots, a_n are real numbers. a_1 is known as leading coefficient and x_1 is known as leading variable.

Example:- $2x_1 - \frac{1}{2}x_2 + 5\frac{5}{3}x_3 - 0.0007x_4 + \pi x_5 = e$

Note:- 1) If the variables are present in product form (or) root form then the equation is not linear.

Example:- $xy + z = k$; $\sqrt{xy} + z^{1/3} = 5$

2) If the variable is involved in trigonometric, exponential (or) logarithmic expressions is known as non linear equation.

Example:- $e^x + \sin y = -7$; $\log y + z^{1/2} = 5/2$

3) We call a equation is linear if the degree of each variable in the equation must be one.

System of linear equations We call the following group of linear equations as system of linear equation

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

where a_{ij} 's are real numbers and b_i 's are real number.

Matrix form:- The above system can be written as $AX=B$ where

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} ; B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

Homogeneous and non homogeneous system Consider the system of equation

$AX=B$. We say this system is homogeneous if $B=0$ (that is) $b_1=b_2=\dots=0$

We say this system is non homogeneous if $B \neq 0$ (that is) there exists

$$b_i \neq 0.$$

Examples:- 1) The system

$$x + y + z + w = 0$$

$$4y - 5x + 5w - 7z = 0$$

$$w - 10y + \frac{51}{27}x - 12z = 0 \quad \text{is Homogeneous}$$

2) The system

$$2x + 7y - 5z + 11t = 1$$

$$5y - 7z + 10(x + 12t) = 0 \quad \text{is non Homogeneous.}$$

Solution Consider the system $AX=B$. we say this system has a

solution if there exists $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$ such that $Ay=B$.

Question:- Does a system $AX=B$ have solution?

Ans:- No, For example $x+y=1$ has no solution
 $2x+2y=4$

We classify system of equations into two parts.

1) Consistent System 2) Inconsistent System.

If a system $AX=B$ has a solution then, it is known as Consistent System. If the system $AX=B$ has no solution then it is known as Inconsistent

Example:-

$$\begin{aligned} x-y &= 1 \\ x+y &= 2 \\ x+y &= 3 \end{aligned}$$

↑

Inconsistent

$$\begin{aligned} x+y &= 2 \\ 2x+2y &= 4 \end{aligned}$$

↑

consistent

$$\begin{aligned} x+y &= 2 \\ x-y &= 1 \end{aligned}$$

↑

consistent.