Reg. No. :

Name :

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

CONTINUOUS ASSESSMENT TEST II – OCTOBER 2022

Programme	:	**B.Tech**	**Semester**	:	**Fall 2022**
Course Name	:	**Applied Linear Algebra**	**Course Code**	:	**MAT3004**
Faculty	:	**Dr.M.Kaliyappan, Dr.Hannah Grace, Dr.David Raj Micheal, Dr.S. Dhanasekar, Dr. Om Namha Shivay**	**Slot/ Class No:**	:	**C2+TC2+TCC2/CH2022231000390/391/392/ 393/394**
Time	:	**90 mins**	**Max. Marks**	:	**50**

Answer all questions (5 X 10 = 50 Marks)

1. (a) Determine the dimensions of the sum and of the intersection of the vector spaces V_1 and V_2 defined by the columns of these matrices $\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -2 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & -1 & 2 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -1 & 3 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ — 6+4

(b) Dim(v1)=3, dim(v2)=3, dim(v1+v2)=4. Dim(v1intsection v2)=2

1	1	1	4	0	0	-1	2
0	1	1	3	0	0	-1	2
0	0	1	2	0	1	-1	3
0	0	0	0	1	2	0	3

Find a second degree polynomial passing through the points (1,1),(2,3) and (4,8)

$f(x) = 0.1667x^2 + 1.5x - 0.6667$

2. Let $T_1 : P_2 \rightarrow R^3$ and $T_2 : P_2 \rightarrow R^3$ are the linear transformations defined as $T_1(a + bx + cx^2) = (a + c, a + b, 0)$, $T_2(a + bx + cx^2) = (0, a + b, a - c)$ where P_2 is the vector space of all polynomials of degree at most 2. — 4+6

(i) Find Ker(T_1) and Ker(T_2) and hence check whether of T_1 and T_2 are onto

Ker(T1)= a=-c, a=-b , b,c=-a

For example T1(-1+x+x^2) =(0,0,0) , ker(T2)= a=-b, a=c

T2(1-x+x^2) =(0,0,0)

Not onto

(ii) Let ($T_1 + T_2$): $P_2 \rightarrow R^3$ be another linear transformation defined as $(T_1 + T_2)(p(x)) = T_1(p(x)) + T_2(p(x))$, for every $p(x) = a + bx + cx^2$ in P_2 and a,b,c in R. Then find the inverse map of ($T_1 + T_2$)

(T1+T2)(p(x))=(a+c, 2a+2b, a-c)

Matrix of T1+T2

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

	B_1	B_2	B_3
1	1/2	0	1/2
2	-1/2	1/2	-1/2
3	1/2	0	-1/2

3. Let $T : R^3 \rightarrow R^3$ be the linear transformations given by $T_1(x, y, z) = (-2x + y, -y - z, x + 3z)$. — 6+4

(i) Find $[T]_\alpha^\beta$ where $\alpha = \{(1, -3, 1), (0,3, -1), (2, -2,1)$, $\beta = \{(2,0,1), (3, -1,1), (15, -6, 4)\}$ and (ii) Find the transition matrix $[id]_\alpha^\beta$

(i) $$\begin{bmatrix} -16 & 9 & -24 \\ 6 & -40 & 89 \\ -11 & 7 & -15 \end{bmatrix}$$

		ii[-10 5 -2 27 -33 14 -4 12 -5]	
4.	(a) (b)	Find $\lVert f \rVert, \lVert g \rVert$ and d(f,g) where $f = 1 + x,\ g = 1 + x + x^2$ for the inner product space V relative to the inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$. Sqrt (7/3) , sqrt(37/10), 1/sqrt(5) Find the value of K so that the following is an inner product space on R^2 $< u, v > = x_1y_1 - 2x_1y_2 - 2x_2y_1 + Kx_2y_2$ where $u = (x_1, x_2), v = (y_1, y_2) \in R^2$ k>4	5+5
5.		Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & -2 \\ 1 & 3 & 2 \\ 1 & -1 & 0 \end{bmatrix}$. Also find a vector orthogonal to first two columns of A. $Q \times R = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 \\ 0 & 4 & -2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & -2 \\ 1 & 3 & 2 \\ 1 & -1 & 0 \end{bmatrix}$	10