Applied Linear Algebra
MAT3004

Due Date: 10/09/2022

1. For the given matrix
$$A = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{bmatrix}.$$

   (a) Find A^{-1} using Gauss- Jordan elimination method and hence, find the solution of the system of equation $AX = b$, where $X = [x_1, x_2, x_3, x_4]$ and $b = [1, 0, 1, -1]$.

   (b) Determine the lower upper factorization for the matrix A, and hence, solve the system od equation $Ax = 0$,where $X = [x_1, x_2, x_3, x_4]^T$ and $b = [1, 1, -3, 4]^T$.

2. For what values of 'a' does the following system of equations have no solution, unique solution, or infinitely many solutions.

   (a) $$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$
   (b) $$\begin{aligned} x - y + z &= 1 \\ x + 3y + az &= 2 \\ 2x + ay + 3z &= 3 \end{aligned}$$

3. Show that the set $V = \{f(x) : a\cos(2x) + b\sin(2x),\ a, b \in \mathbb{R}\}$ is a real vector space. Where, for any vectors $f_1(x), f_2(x) \in V$, $f_1(x) = a_1\cos(2x) + b_1\sin(2x)$, $f_2(x) = a_2\cos(2x) + b_2\sin(2x)$ and scalar $\alpha \in \mathbb{R}$, the vector addition and scalar multiplication is defined as
$$f_1(x) + f_2(x) = (a_1 + a_2)\cos(2x) + (b_1 + b_2)\sin(2x)$$
$$\alpha f(x) = \alpha a\cos(2x) + \alpha b\sin(2x).$$

   Also, find the basis and dimension of the vector space.

4. Let V and W be two subspaces of $\mathbb{R}^5$ with bases $v_1 = (1, 3, -2, 2, 3), v_2 = (1, 4, -3, 4, 2), v_3 = (1, 3, 0, 2, 3)$ and $w_1 = (2, 3, -1, -2, 9), w_2 = (1, 5, -6, 6, 1), w_3 = (2, 4, 4, 2, 8)$,show that $\dim(V+W)+\dim(V\bigcap W)=\dim V+\dim W$.

5. Find bases for the row space, the column space, and the null space for of the given matrices

   (a) $\begin{bmatrix} 1 & 2 & 1 & 5 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & -1 & -2 & 1 \\ 1 & 1 & -1 & 3 & 1 \\ 2 & 1 & -1 & 8 & 3 \\ 0 & 0 & -2 & 2 & 1 \\ 3 & 5 & -5 & 5 & 10 \end{bmatrix}$.