MAT3004 C2 Slot Answer Key

1. (i) $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

   (ii) Set $L = E_1^{-1}E_2^{-1}E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$ and verification of $A = LU$.

2. (a) (i) $A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$

   (ii) $x = \begin{bmatrix} 4 \\ 21 \\ 15 \end{bmatrix}$

   (b) (i) Consistant if $b_1 = b_2 + b_3$

   (ii) Inconsistant if $b_1 \neq b_2 + b_3$

3. **To prove**: H_n is a subspace of $M_{n\times n}(\mathbb{R})$

   (i) $0 \in H_n$ (ii) $cA + B \in H_n, \quad A, B \in H_n$ and $c \in \mathbb{R}$

   To verify: $S = \left\{ \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \right\}$ forms basis for H_2

   (i) S is linearly independent.

   (ii) $\text{span}(S) = H_2$

   $$c_1 \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} a & -a \\ b & -b \end{bmatrix} \quad \Rightarrow a = c_1 \text{ and } b = c_2$$

4. (a) Basis $= \{(1,1,0), (0,1,1), (1,0,1)\}$

   (b) Basis $= \{1 - x^2, x - x^3\}$ and Dimension $= 2$

5. $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$

   (i) $\text{Rank}(A) = 4, \qquad \text{nullity}(A) = 1$

   (ii) $\{(1,0,0,0,2), (0,1,0,0,0), (0,0,1,0,-2), (0,0,0,1,-2)\}$

   (iii) $\{(1,2,3,4), (2,4,7,9), (1,1,2,3), (0,1,2,-1)\}$

   (iv) $\{(-2,0,2,2,1)\}$

   (v) Yes, the rows of A are linearly independent by $\text{rref}(A)$