Practice Problems

Module- 1

1. Given matrix $A = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 2 & 2 & 3 & -1 \\ 4 & 2 & 0 & 1 \end{bmatrix}$, find
   i. Echelon form of A.
   ii. Reduced row echelon for of A.
2. Solve the following system of equation using Gauss-elimination method

$$\begin{gathered} x - y + 2z + w = 1 \\ 3x + y - w = 3 \\ x + y + z = -1 \end{gathered}$$

3. Given system of equation

$$\begin{gathered} x + y - z + w = 1 \\ 2x + y + z - w = 2 \\ 3x + 3y + cz + 4w = 4 \\ 2x + 2y - z + dw = e \end{gathered}$$

   Find the values of parameters c,d and e, such the system of equations
   (i) Has infinitely many solutions
   (ii) Has unique solutions
   (iii) Has no solution.
4. Use the Gauss-Jordon method to find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}.$$

5. Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ and hence, us LU decomposition to find the solution of the following system of equations

$$\begin{gathered} x + 2y + 3z = 1 \\ x + 2y + 2z = -1 \\ 2x + y + z = 1 \end{gathered}$$

Module 2:

1. Are the following sets vector spaces with the indicated operations? If not, why not?
   i) The set V of nonnegative real numbers; ordinary addition and scalar multiplication.
   ii) he set V of all polynomials of degree ≥ 3, together with 0; operations of Polynomials.
   iii) The set V of 2 × 2 matrices with equal column sums; operations of $M_{2\times 2}$.
   iv) The set V of all ordered pairs (x, y) with the addition of $\mathbb{R}_2$, but using scalar multiplication $a(x, y) = (ax, -ay)$.
   v) The set V of all 2 × 2 matrices with the addition of $M_{2\times 2}$ but scalar multiplication $*$ defined by $a * X = aX^T$.
   vi) The set V of complex numbers; usual addition and multiplication by a real number.

2. Which of the following are subspaces of P_3 (Set of all polynomials of degree$\leq$ 3)? Support your answer
   a. $U = \{ f(x) \mid f(x) \in P_3, f(2) = 1\}$
   b. $U = \{xg(x) + (1-x)h(x) \mid g(x) \; and \; h(x) \in P_2\}$
3. Which of the following are subspaces of $M_{2\times 2}$(Set of all 2 × 2 matrices)? Support your answer.
   a. $U = \{A \mid A \in M_{2\times 2}, A^2 = A\}$
   b. $U = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a+b = c+d; a,b,c,d \in \mathbb{R}\right\}$
   c. $U = \{A \mid A \in M_{2\times 2}, BAC = CAB\}, B$ and C fixed 2×2 matrices.
4. Write each of the following vector as linear combination of $x+1,\ x^2 + x$, and x^2+2,
   I. x^2+3x+2, II. $2x^2-3x+1$, III. x^2+1.

5. Consider the vectors $p_1 = 1+x+4x^2 \; and \; p_2 = 1+5x+x^2$ in P_2. Determine whether p_1 and p_1 lie in span $\{1+2x-x^2, 3+5x+2x^2\}$.

6. Find the value of a such that the following subsets are linearly independent in $\mathbb{R}^3$

a. {(1, −1, 0), (a, 1, 0), (0, 2, 3)}

b. {(2, a, 1), (1, 0, 1), (0, 1, 3)}

7. Find the basis and dimension of the following

a. $w = \{a(1+x) + b(x+x^2) \mid a$ and $b \; in \; \mathbb{R}\}$, subspace of P_2 (set of polynomials of degree atmost 2)

b. w={p(x) | p(x) = p(−x)}, subspace of P_2 (set of polynomials of degree atmost 2)

c. $w = \{A|A^T = -A\}$, subspace of $M_{2\times 2}$ (set of all 2×2 matrices).

d. $w = \left\{A|A\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}A\right\}$, subspace of $M_{2\times 2}$ (set of all 2×2 matrices).

Module-3

1. Find the rank, basis and dimension of the row space, column space and null space of the following matrix

$$A = \begin{bmatrix} 1 & -2 & 2 & 1 & 3 \\ 1 & 1 & -1 & 7 & 5 \\ 2 & -4 & 4 & 2 & 6 \\ -1 & 2 & -2 & 3 & 4 \end{bmatrix}.$$