MAT3004- Applied Linear Algebra

Digital Assignment -1

1. Determine which matrices are in reduced echelon form and which others are only in echelon form

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

2. Solve the equation Ax = b by using the LU factorization given for A

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

3. Solve the equation Ax = b by using Gauss elimination method

$$A = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

4. Solve the equation Ax = b by using Gauss –Jordan elimination method

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

5. Test the consistency of the system of equations Ax= b

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$$

6, Find the inverse of A using Gauss –Jordan elimination method

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

7. Let S be the collection of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in R^2 that satisfy the given property. In each case, either prove that S forms a subspace of R^2 or give a counter example to show that it does not.

(a) $x = 0$ (b) $x \geq 0, y \geq 0$ (c) $y = 3x$ (d) $xy \geq 0$

8. Answer the following questions

Do $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$ form a basis for $\mathbb{R}^3$?

Do $\begin{bmatrix}1\\-1\\3\end{bmatrix}, \begin{bmatrix}-1\\5\\1\end{bmatrix}, \begin{bmatrix}1\\-3\\1\end{bmatrix}$ form a basis for $\mathbb{R}^3$?

Do $\begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\1\end{bmatrix}$ form a basis for $\mathbb{R}^4$?

Do $\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\-1\\1\end{bmatrix}, \begin{bmatrix}-1\\0\\1\\0\end{bmatrix}$ form a basis for $\mathbb{R}^4$?

9. For each subspace in Exercises a-f, (a) find a basis, and (b) state the dimension.

$$\left\{\begin{bmatrix}s-2t\\s+t\\3t\end{bmatrix} : s, t \text{ in } \mathbb{R}\right\} \qquad \left\{\begin{bmatrix}4s\\-3s\\-t\end{bmatrix} : s, t \text{ in } \mathbb{R}\right\}$$

$$\left\{\begin{bmatrix}2c\\a-b\\b-3c\\a+2b\end{bmatrix} : a, b, c \text{ in } \mathbb{R}\right\} \qquad \left\{\begin{bmatrix}a+b\\2a\\3a-b\\-b\end{bmatrix} : a, b \text{ in } \mathbb{R}\right\}$$

$$\left\{\begin{bmatrix}a-4b-2c\\2a+5b-4c\\-a+2c\\-3a+7b+6c\end{bmatrix} : a, b, c \text{ in } \mathbb{R}\right\}$$

$$\left\{\begin{bmatrix}3a+6b-c\\6a-2b-2c\\-9a+5b+3c\\-3a+b+c\end{bmatrix} : a, b, c \text{ in } \mathbb{R}\right\}$$

10(a) The first four Hermite polynomials are $1, 2t, -2+4t^2$, and $-12t + 8t^3$, these polynomials arise naturally in the study of certain important differential equations in mathematical physics. Show that the first four Hermite polynomials form a basis of P_3

10(b) The first four Laguerre polynomials are $1, 1-t, 2-4t+t^2$, and $6 - 18t + 9t^2 - t^3$, Show that these polynomials form a basis of P_3

11. For each of the following matrices find (a) the rank of the matrix, (b) a basis for the row space, (c) a basis for the column space and, (d) a basis for Nullspace

(a)

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & -3 & 1 \\ 5 & 10 & 6 \\ 8 & -7 & 5 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 4 & -3 & -6 \\ 7 & 14 & -6 & -3 \\ -2 & -4 & 1 & -2 \\ 2 & 4 & -2 & -2 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 2 & 4 & -2 & 1 & 1 \\ 2 & 5 & 4 & -2 & 2 \\ 4 & 3 & 1 & 1 & 2 \\ 2 & -4 & 2 & -1 & 1 \\ 0 & 1 & 4 & 2 & -1 \end{bmatrix}$$