Consider the following systems

System 1:

$x - 2y + z = 0$
$2y - 8z = 8$
$5x - 5z = 10$

The above system has solution

$x = 1; y = 0; z = -1$

System 2:-

$x - 2y + z = 0$
$6y - 24z = 24$
$5x - 5z = 10$

In this case we multiply the 2^{nd} equation of system ① by 3. So this System has solution $x = 1; y = 0; z = -1$

System 3:-

$x - 2y + z = 0$
$2y - 8z = 8$
$5x + 2y - 13z = 18$

Here we formed 3^{rd} equation by just adding 2^{nd} and 3^{rd} equation of system ①. So solution is $x = 1; y = 0; z = -1$

System 4:-

$5x - 5z = 10$
$2y - 8z = 8$
$x - 2y + z = 0$

We interchanged 1^{st} and 3^{rd} equation of system ①. So solution is $x = 1; y = 0; z = -1$

Row elementary operations:- Consider the system $AX = B$ and $[A\ B]$ is the augmented matrix corresponds to the system. We denote the i^{th} row of $[A\ B]$ by r_i. We perform the following operations on $[A\ B]$ to find the solution to the given system.

1) Addition of a multiple of one row to another row

$$r_j \longrightarrow r_j + \alpha r_k \quad (\alpha \neq 0)$$

2) Multiplication of a row by a non zero constant

$$r_j \longrightarrow \alpha r_j \quad (\alpha \neq 0)$$

3) Interchange of two rows

$$r_i \longleftrightarrow r_j$$

Echelon form:- A rectangular matrix is in echelon form (or row echelon form) if it has the following three properties

1) All nonzero rows are above any rows of all zeros.
2) Each leading entry of a row is in a column to the right of the leading entry of the row above it.
3) All entries in a column below a leading entry are zeros.

Example:-

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 & 29 & -3 \\ 0 & 1 & 0 & 16 & 4 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot element

It is an element in the augmented matrix such that we want elements below to be zero.

Gauss elimination method:- To solve the system $AX=B$ by Gauss elimination we proceed by the following steps.

1) Form the augmented matrix $[A \quad B]$

2) Apply row elementary operations to $[A \quad B]$ successively until we get the echelon form of $[A, B]$

3) Using the echelon form find the solution to the system $AX = B$

<u>Note:-</u> Suppose $[A \quad B]$ is the augmented matrix to the system $AX=B$ and $[C \quad D]$ is the echelon form of $[A \quad B]$ then the system $CX=D$ and $AX=B$ has same solution set.