Problem 1.

(a) R_n is indeed an equivalence relation since it is

- Reflexive because x equiv x (mod n).
- Symmetric because $x \equiv y(mod\, n)$ implies $y \equiv x(mod\, n)$.
- Transtive because $x \equiv y(mod\, n)$ and $y \equiv z(mod\, n)$ implies that $x \equiv z(mod\, n)$

(b) R is not equivlance relation because
if exists x such that x is taller than y, that doesn't imply that y is taller than x.

(c) R is not equivlance relation consider the case for example with $gcd(2,5)=1$ and $gcd(5,8)=1$ that doesn't imply that $gcd(2,8)$ is equal to 1.

(d) R_G is equivlance relation since

- Reflexive because every vertex is in itself connected.
- Symmetric because if x is connected to y through a path than implies that y is connected through the same path.
- Trasitive because if x is connected to y in a path and y is connected to z in a path that implies that x is connected to z through the sum of the two paths.

Problem 2.

(a) $f(x)=xsin(x)$ is

- Surjective since every number in R is assigned to atleast one element of A.
- NOT Injective because it maps different values to same value. Ex: $f(0)=f(\pi)$
  Hence it's not bijective.

(b) $f(x)=99x^{99}$ is

- Surjective since there exists an element y in R such that $f^{-1}(y)=x$.
- Injuctive because there exists no two values a, b in R such that $f(a)=f(b)$.
- Since every element of a is assinged to some element b in R (e.i. total), this implies that this function is bijection.

(c) $f(x)=\tan^{-1}(x)$ is

- NOT surjective since it ranges is $[\frac{-\pi}{2},\frac{\pi}{2}]$
- Injuctive because every number in R there exists $f^{-1}(y)$.
- since its NOT surjective and Injuctive, the function is NOT bijective.

(d) $f(x)= \textit{is the number of divisors}.$ is

- Surjective since every element in N is mapped atleast once.
- NOT Injective because there exists numbers B in N that is mapped more than once like the prime numbers all prime numbers has 2 divisors it self and 1.
- hence the function is NOT bijection.

Problem 3.

(a) to show that the relation is a weak poset, we need to proof the three properties of the poset, that is reflexive, anti-symmetric and transitive.

First with Reflexive, we know that if $i \geq i$.
second with anti- symm, we know that if $i \geq j \wedge j \geq y$, this implies that $i=j$.
Last with transitive, if $i \geq j \wedge j \geq k$ this for sure implies that $i \geq k$.

(b)let’s consider the ❑≼❑ relation in the set defined, length of the longest non-decreasing subsequance of the given integers is the length of the longest chain in the sequence. If the longest chain in the sequence have atleast n then, the length of the non-decreasing is equal to that.

Then from the theorem we know that if the sequence longest chain is n, then we can partition the seqence into n disjoint antichain. It’s long is $c \leq n-1$.
And since it can be partitioned into c parts with $\frac{(n-1)(m-1)+1}{c}$ from lemma 7.9.3 in the book.
resulting that this is aleast M.

Problem 4.

Diameter	Switch size	# switches	Congestion	LMC	CML
$\log N+4$	$2x2$	$N((\log n)+1)+2n\log n+4n$	1		

The diameter of the network is equal to the smaller (e.i. butterfly) + two switches at the the input and output.

Switch size at most at the two networks is $2x2$.

number of switches is equal to the number in butterfly + benes + 4N switch at input and output.
$N((\log n)+1)+2n\log n+4n$
congestion is forsure 1.

Problem 5.

(a) at any node v at any level i the number of inputs that can reach this node is 2^i and the number of outputs that can reach the same node is 2^{n-i} , so the congestion happen when the number of the inputs equals the number of outputs.and since n is even the two sides are equal when i = n/2, then $\sqrt{(N)}$.

(b) the congestion achieves when at vertex whose level is equal to i = n/2.