Problem 1.

a)

for the two separated component we have two cases to consider (WLOG):

1. Either of the resulting trees for the result tree is of degree 0, then from the definition of the SBTree a lone vertex is a SBTree.
2. Either of them is of degree 3, then by removing the root node the degree of the node will decrease by one, matching the properties of the root node and since the internal nodes and it's leaves never change, then the resulting tree is SBTree. #

b)

Lemma 1.

let induction hypothsis be N(l) = l-1

Proof by induction.

Basecase:

l = 1, then the number of nodes = 0, trivialy true.

Induction case:

assume N(l) is true, to proof N(l+1), lets consider a SBTree with n nodes, so adding a new node can't be done since every one must have degree of 3 or one so we can add two nodes this will result in the following:

1. The number of nodes will increase by one.
2. The number of leaves will increase be two. #

using Lemma1 concluding that they have the same number of nodes.

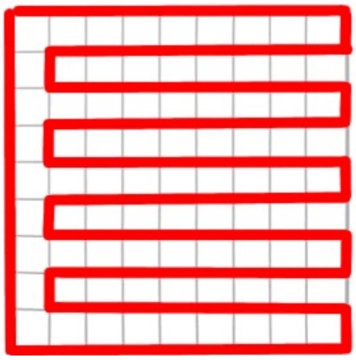
Problem 2.

a) a grid of N x M has a property that is it's highest degree 4 and any node of degree 4, it's 4 adjacent nodes are not adjacent to each other (the property of the grid) so it's 2 colorable. Since it's 2 colorable then we can rearrange the graph to L and R nodes according to the color to be a bipartite graph.

Using the fact that this graph is bipartite graph, we can notice that if N and M are both odd, then the total number vertices is odd, and rearranging the graph will lead to imbalance in the number of nodes in L side and R side. From the definition of the bipartite graph we know it can't contain odd cycles.

b)

1. we know that ODD x EVEN = EVEN, there exists even number of vertices(WLOG), and from the rearraning of the graph, we know there exists an even cycle in bipartite graphs.
   And the solution can be the in the following strategy:



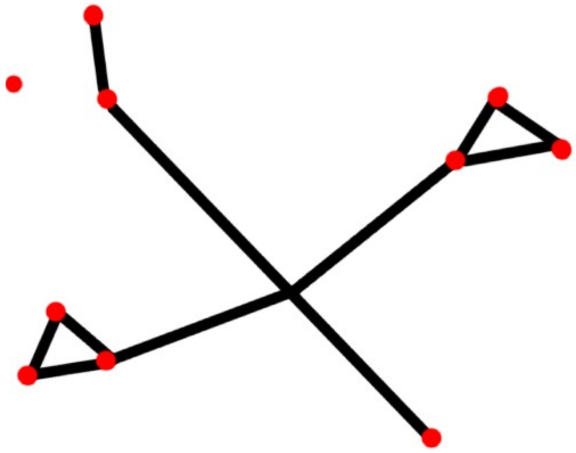
2. In bipartite graph to visit a node it takes two moves (entering node and leaving it) so the over all sum is even, And this contradict the case with odd numbers.
3. YES, they will survive, NO IT doesn’t depend.

Problem 3.

a)

the error in the prove where we assumed that the $\lceil \frac{n}{3} \rceil$ and every piece of the graph is connected in the first place.

b)



c) Assume there exists a set of node(subgraph) which is not connected.

Problem 4.

a)
First implication:
if G is a tree with n nodes then G has exactly n-1 edges.

PF: by induction.
Base Case: P(1) is true since a tree with one node has 0 vertices.

Inductive Case:
consider tree with (n+1) nodes and choose a leaf of the tree, removing this vertex will not affect the rest of the tree, so subgraph with n nodes has n−1 edges, reattatching the disconnected leaf will result a tree with n+1 nodes and n edges. #

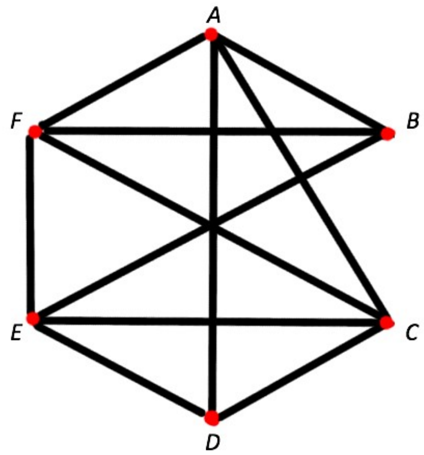
Second implication:
PF: by contradiction.
Suppose there exists a graph with n nodes and n−1 and have a cycle, by removing edges until it became acyclic, then this graph has nodes until it's acyclic then it has fewer thay n−1 edges, contradicting the original assumsion.

Using the bijection, then the statement is true.

b) P(n): any connected graph with n and n−1 vertices edges has a spanning tree.

Base Case: a graph with 1 node is a spanning tree.
Induction Case:
assume we have a graph with n+1 nodes and edges > n, we start removing edges that creates a cycle until the number of edges is n from part 1, then it's a cycle.

Problem 5.
a)



b) the diameter of the graph is 2, its the largest distance between {1,5}, {6,4},{2,3}.

c) cycle is 1-6-3-4-5-2-1, it contain every vertex and it's the largest cycle.

d) 3 colors, since the max degree is 4 and every 4 degree node it's adjacent nodes connected to maximum of 2 nodes of it's neighbor. So it's 3-colorable.

Problem 6.

(a) in the longest walk the we visit every vertex twice (since we enter the node and then leave it) and since the starting node degree is odd then this can't be the finish point $(e.i.\, w \neq v)$.

(b) since this edge is the finish edge, this means that every time we traverses this edge the degree is added by two but since at the last edge of the walk since we just enter the node and don’t leave this leave us with a total number of odd number degree.