1.
Proof by induction:
first we define the bipartite graph, A bipartite graph is a graph together with a partition of its vertices into two sets, L and R, such that every edge is incident to a vertex in L and to a vertex in R.

I.H : P(n) is a graph G =(V,E) with n vertices, matching M_1, M_2 , then $\grave{G} = (V, M_1 \cup M_2)$ is a bipartite.

Base case: a graph with one vertex is bipartite.

Iduction case:
assume that I.H is true for n and try to proof its true for n+1 and if we remove vertex v then we consider three cases:

FIRST: if v is not incident to either a edge in M_1 or M_2 then anding v to either side of the graph will result in a bipartite as the definition.

SECOND: if v is in either of M_1 or M_2 , WLOG on either of the matchings, adding v to $\grave{G}$ and connecting it to its end point either v will result in a bipartite graph.

THIRD: if v is in both M_1 and M_2 , this will be in the form of v-x and v-y and in that case either x and y are in the same set then $\grave{G}$ is a bipartite.

If they are in the same set and they are

Problem 2.

(a)every edge denotes a degree for both of its vertices so for every edge there are double the amount of degrees.

(b)expressing the students as nodes and student shaking hand with another student as edge and using the formula from (a) to calculate that $2|E|$ is even and 17 is odd.

(c) $2|E| = \sum_{v \in V} deg(v) = \frac{n(n-1)}{2}$

Problem 3.

(a) it’s preserved under isomorphism considering the function f that maps every vertex of a graph G to a vertex f(v) of a graph $\grave{G}$ that is isomorphic to graph G, then the number of vertices of both graphs are the same and the plarity are the same.

(b) it’s not preserved under isomorphism, acually the names of the vertices is not important for the isomorphism.

(c) it’s preserved under isomorphism using the argument of the function that there exists a function that maps every vertex then using the same principle there we note that this property is invariant.

(d) it’s preserved under isomorphism using the same agrument as the previous.

(b) G_1, G_3, G_4 are isomorphic graphs all of them has the same number of vertices, all of their vertices has a degree of 3 and also has the same numer of edges.

Niether of them is isomorphic to G_2 ,because g2 have two 4 degree vertices. And neither of them has a vertex that is 4 degree, which is necessary for the isomorphism.

Problem 4.

(a) considering the simple graph G= ({a,b,c,d}, {{a,b}, {b,c}, {c,a}}) (i.e. a trigle with a dot any where on the plan) this graph is not K-colorable because it needs 3 color for every vertics(i.e. 3 colors).

(b)
the flaw is in the part where the proof says “first remove the vertex v to produce a graph, G_n , with n vertices. Removing v reduces the degree of all vertices adjacent to v by 1.” but this implies that every node is connected, which can easily depated be a graph in the form of “Star of David with a dot in the middel” where removing any vertex with degree of 2 doesn’t make it K-colorable.

Problem 5.

P(n): “for some $n \geq 3$ (n boys and n girls, for a total of 2n people), there exists a set of boys and girls preferences such that every dating arrangement is stable.”
Disproof by contradiction:
assume there exists a couple that they are in each other top choice and both of them are with another partner, so they form a rogue couple which contradict the hypothesis.

Problem 6.
a) first starting with the linear graph, we can color this graph using two colors as follows: nodes associated with 2n is colored with the first color, nodes associated with 2n+1 is colored with the second color. Now to color the graph we notice that the maximum degree for a vertex is 4 and this makes the graph 4-colorable#.

b) using the idea from before