1.

(a)

lets consider two cases:

1. $a_3 > a_2$ then $a_3 > a_2 > a_1$ form a 3-chain
2. $a_3 < a_2$ we consider a_4 in two cases:
   1. : if $a_4 > a_3$ then $a_3 > a_4 > a_2$ forms a 3-chain
   2. : if $a_3 > a_4$ then $a_4 > a_3 > a_4$ forms a 3-chain

so a_3 has to be less than a_1 not to form a 3-chain.

(b)

considering there is no 3-chain then we have some cases to consider:

1. $a_4 < a_3$ then $a_2 > a_3 > a_4$ forms a 3-chain.
2. $a_3 < a_2 \wedge a_4 > a_3$ then $a_4 > a_2 > a_1$ forms a 3-chain.

Then $a_2 > a_4 > a_3$ must be to not form a three chain.

(c)

let’s consider a_5 in half of the sequence if $a_4 > a_5$ then $a_2 > a_4 > a_5$ forms a three chain, and if $a_5 > a_4$ then whereever a_5 is $a_4 > a_3 > a_1$ forms a three chain anyway.

(d)

P(n): any sequence of five distinct integers must contain a 3-chain.

PF: by contradiction.
Form the first proof, implimenting any three vaules of the in the first form will result in a no 3-chain, but we have one more value to consider, and this also can lead to a no 3-chain, in some cases. But using the forth lemma we see that the fifth value cover lemma 1 and lemma 2 flaws and result always in a three chain. Leading to a contradiction.

2. Proof using induction:

the induction hypothesis: P(n) = $\sum_{i=0}^{n} i^3 = (\frac{n(n+1)}{2})^2$

Base case: P(0) = L.H.S = 0, R.H.S = $(\frac{0(0+1)}{2})^2 = 0$. P(0) is true.

Inductive step: assum P(n) = $\sum_{i=0}^{n} i^3 = (\frac{n(n+1)}{2})^2$ is true,

P(n+1) = $\sum_{i=0}^{n+1} (i)^3 = (\frac{n(n+1)}{2})^2 + (n+1)^3 = \frac{n^2(n+1)^2 + 4(n+1)^3}{4}$

$= \frac{(n+1)^2 + (n^2 + 4(n+1))}{4} = \frac{(n+1)^2 + (n^2 + 4n + 4)}{4} = \frac{(n+1)^2 + (n+2)^2}{4} = (\frac{(n+1)(n+2)}{2})^2$

4.

the flaw is that we proved the base case and proved that p(n) is true for n>0, but we didn't prove it's true for P(0), and implementing that $P(0)=\frac{a^1 . a^1}{a^{-1}}$ and we can't know for sure what a^{-1} is for sure.

5.

the induction hypothesis: let P(n) is $G_n=3^n-2^n$

base case: $P(0)=3^0-2^0=0=G_0$ $\quad P(1)=3^1-2^1=1=G_1$

induction step: for n>=2 suppose P(n) is True. Then $G_{n+1}=5G_n-6G_{n-1}$ is true.

Where $G_n=3^n-2^n$ and $G_{n-1}=3^{n-1}-2^{n-1}$.

$$G_{n+1}=5(3^n-2^n)-6(3^{n-1}-2^{n-1})$$
$$G_{n+1}=5(3*3^{n-1}-2*2^{n-1})-6(3^{n-1}-2^{n-1})$$
$$G_{n+1}=15*3^{n-1}-10*2^{n-1}-6*3^{n-1}+6*2^{n-1}$$
$$G_{n+1}=9*3^{n-1}-4*2^{n-1}$$
$$G_{n+1}=3^2*3^{n-1}-2^2*2^{n-1}$$
$$G_{n+1}=3^{n+1}-2^{n+1} \quad \#$$

6.

a) a row move moves the i_{th} card to i+1 or to i-1, where ever the space is avilable, and it doesn't change the relative order to any pair.

b) A column move does the next:
moving the k letter to the adjecent square below result in change in the relative order of 3 pairs.

A	B	C	D
E	F	G	H
I	J	K	L
M	O		**N**

$\rightarrow$

A	B	C	D
E	F	G	H
I	J		L
M	O	**K**	N

c) A row move doesn't change the the relative order of any pair, so the number of inversion doesn't change.

d) Column move change the number of inversion by ± 3 , in the starting position we start with exactly one inversion and every column move increases/ decreases the value of inversion by 3. so every move changes the parity of inversion by every move from odd to even and vice versa.

e) To get letters back in order we need the blank square to be but back at the 16th box, but from obbservation, every move up and down the rows changes the parity of the to the opposite, which differ from the row's parity containing the blank square.

row 1	A	B	C	D
row 2	E	F	G	H
row 3	I	J	K	L
row 4	M	O	N	

In the desired position the parity of the last row is odd and the parity of the inversions is even. But in the starting position the parity of the inversions is odd, and every row move change the parity of the inversions is to even and from even, and change the last row parity as will, which remain invariant that the parity of invariant and parity of the last row is the same .

PF: in the target configuration on the right, the the parity of the last row is odd and the parity of the inversions is even, and there is no legal moves that change the starting configuration to the desired one by lemma e.

7.

let the hypothesis :
p(n) = there is atleast number of B-lings as Z-lings in generation n.

Base Case: p(0) or p(1) is truee because there is 800 B’s which is enough for Z’s.

Inductive case: assume p(n) is true. And try to prove p(n+1) is true.
Let z be the number of Z-lings, and b be the number of B-lings.
In p(n+1), there is $b \geq z$, then we have cases to consider:

1) b=z, then we have offspring equal of each type.

2) b>z, then we consume all B-Z pairs, and the remaining b pairs can be one of two cases:
    - odd number of B-lings can produse two B-lings and one Z-lings for every pair and the remaining z dies.
    - Even number of B-lings can produse two B-lings and one Z-lings for every pair.

In all cases b is here is atleast number of B-lings as Z-lings. #