P1.

a. $\exists x \in S(S(x) \wedge A(x))$

b. $\forall x \in X((T(x) \wedge S(x)) \rightarrow A(x))$

c. $\forall x \in X(-T(s) \wedge A(s))$

d. $\exists x, y, z \in X\, A(x) \wedge -S(x) \wedge A(y) \wedge -S(y) \wedge A(z) \wedge -S(z) \wedge E(x,y) \wedge E(x,z) \wedge E(z,y)$

P2.

a)

p	r	q	$q \wedge r$	$-(p \vee (q \wedge r))$	-p	-r	-q	$-q \vee -r$	$(-p) \wedge (-q \vee -r)$
F	F	F	F	T	T	T	T	T	T
F	F	T	F	T	T	T	F	T	T
F	T	F	F	T	T	F	T	T	T
F	T	T	T	F	T	F	F	F	F
T	F	F	F	F	F	T	T	T	F
T	F	T	F	F	F	T	F	T	F
T	T	F	F	F	F	F	T	T	F
T	T	T	T	F	F	F	F	F	F

b)

NOT the same Truth Table!

p	r	q	$q \vee r$	$-(p \wedge (q \vee r))$	-p	-r	-q	$-q \vee -r$	$(-p) \vee (-q \vee -r)$
F	F	F	F	T	T	T	T	F	T
F	F	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	T	T	T
F	T	T	T	T	T	F	F	T	T
T	F	F	F	T	F	T	T	F	F
T	F	T	T	F	F	T	F	T	T
T	T	F	T	F	F	F	T	T	T
T	T	T	T	F	F	F	F	T	T

P3.

a)
- i. -(A nand B)
- ii. -A or -B
- iii. -A and -B

b) A nand A

c)
- i. Evaluating to true : (A nand (A nand A))
- ii. Evaluating to false : (A nand (A nand A)) nand (A nand (A nand A))

P4.

we divide the 12 coins into 3 group’s of 4 coins and this can be done in 3 weightings. Lets call the 3 groups a, b and c.

First. weighting we weight a and b, if they are equal: then we divide c into two groups and then divide the less heavier one into two groups and we have the less weighted coin.

And if a and b are not equal we do the same approach and divide the less weighted group into half and so on.

P5.

Proof by contrapositive: if $r^{\frac{1}{5}}$ is rational, then it can be written as $r^{\frac{1}{5}}=\frac{a}{b}$, where a and b are integers, $r^{\frac{1}{5}}=\frac{a}{b}$ can be written as $r=\frac{a^5}{b^5}$, and since a and b are integers then r is rational. $-q(x)\rightarrow -p(x)\equiv p(x)\rightarrow q(x)$.

P6.

W-even	x-even	y-even	Z-even
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

case1 (all odd): $(2i+1)^2+(2j+1)^2+(2k+1)^2$ = odd.
case2 (one even): $(2i+1)^2+(2j+1)^2+(2k)^2$ = $4(i^2+j^2+k^2j+i)+2$.
so $z^2=4(i^2+j^2+k^2j+i)+2$, $z=\sqrt{(4(i^2+j^2+k^2j+i)+2)}$ which is a contradiction because if z is even, then it’s a multiple of 4 and z here is not a multiple of 4, then z is not odd. Same for case.

Case3 (two even) : $(2i)^2+(2j)^2+(2k+1)^2$ = $4(i^2+j^2+k^2+k)+1$ = odd

case4 (three evens) : $(2i)^2+(2j)^2+(2k)^2=4(i^2+j^2+k^2)$ a multiple of 4 → even.