5.

case1 : $x>0$, $x>0$ then $|x+y| = x+y$
and $|x|+|y| = x+y$

case2 : $x<0$, $y<0$ then $|x + y| = -(x+y) = -x + (-y)$
and$|-x| + |-y| = -(x) + (-y)$

case3 : $x>0$, $y<0$ then $|x + (-y)| = x +(-y)$
$|x|+|y| = x + (-y)$

case4 :WLOG $x<0$, $y>0$ follows the same prinsiple.

Case5: $x =0$, $y =0$ then $|x+y| = 0$ and $|x|+|y| = 0$

since we have proved all cases, hence $|x|+|y| = x+y$

17.

if n is odd, then it can be written as $2a+1$, a is an integer.

$2a+1 = (k-2)(k+3) = 2k+3-2 = 2k+1$

assume there exists b integer and we are trying to disprove that it forms the odd number $2b+1$ different from n

$2b+1=x$
$2k+1=n$

if $n = x$,then $2b+1= 2k+1$, leads to $b = k$, then there exists unique number k for $(k-2)(k+3)$ forms only odd number.

20.

given:

(x+1/x)^2 >=0 (squaring a number is always non negative)

then:
x^2+2(1/x)*x +1/x^2 >=0

x^2 + 1/x^2 >=2 #

21.

geometric means = sqr(x+y)
harmonic mean = 2xy /(x + y)

we can prove this conjucture using a series of steps as follows:

(x-y)^2 >=0

x^2 -2xy -y^2 >= 0

x^2 +2xy – y^2 >= 4xy   +4xy

(x+y)^2 >= 4xy   *xy

xy(x+y)^2 >= 4x^2 y^2

xy >= 4x^2 y^2 / (x+y)^2

sqr(xy) >= 2xy/x+y #

22.

after computing some vaules the conjucture is
sqr((x^2 +y^2)/2) >=1/2(x+y)

sqr((x^2 +y^2)/2) >=1/2(x+y)   ^2

(x^2 +y^2)/2 >=1/4(x+y)^2   *2

(x^2 +y^2) >= 1/2(x+y)^2

2(x^2 +y^2)>= x^2 + 2xy + y^2

x^2 +y^2 -2xy >=0

(x-y)>=0 ###

25.
conjecture: last digit of x^4 is always one of this set(0,1,6,5)

proof:
for x can be written in the form of 10a+b, then

x^4 = (10a+b)^4

(10a+b)^4 =
(10a)^4 + 4(10a)^3 b + 6(10a)^2 b^2 + 4(10a)b^3+b^4

last digit is determined by b^4

b	0	1	2	3	4	5	6	7	8	9
b^4	0	1	16	81	256	625	1296	2401	4096	6561

conjecture proved!

27.
for $n^3 < 100$ when $n = 0, 1, 2, 3, 4$
there exists no n^2 that satisfy the equation.

28.
$(2x)^2 + (5y)^2 = 14$

for x^2 values that under 14 are when $x = 0, 1, 2, 3$
for y^2 values that under 14 are when $y = 0, 1, 2, 3$

from the equation y can’t execeeds the value 0 to satisfy the equation.

For $x = 0$ $(0)^2 + (0)^2 \neq 14$
For $x = 1$ $(1)^2 + (0)^2 \neq 14$
For $x = 2$ $(2)^2 + (0)^2 \neq 14$
For $x = 3$ $(3)^2 + (0)^2 \neq 14$

therefore there is no solution to the equation.

30.
$x = m^2 - n^2$
$y = 2mn$
$z = m^2 + n^2$

$x^2 = m^4 - 2n^2 m^2 + n^4$
$y^2 = 4m^2 n^2$
$z^2 = m^4 + 2n^2 m^2 + n^4$

$x^2 + y^2 = m^4 - 2n^2 m^2 + n^4 + 4m^2 n^2$
$= m^4 + 2n^2 m^2 + n^4$
$= z^2$ #2
since there is infinte pairs of n and m, then there are infinte many solutions