1.

let the first number be in the form of $p = 2k+1$ where k is an integer, and the second number be in the form of$q = 2l+1$

$$
\begin{aligned}
q + p &= (2k+1) + (2l+1) \\
&= 2k + 2l + 2 \\
&= 2(k + l + 1) \rightarrow \text{is even}
\end{aligned}
$$

2.

let the first number be in the form of $p = 2k$ where k is an integer, and the second number be in the form of
$q = 2l$

$$
\begin{aligned}
p + k &= 2k + 2q \\
&= 2(k+q) \rightarrow \text{is even}
\end{aligned}
$$

11.

we can disprove it by counterexample of value sqr(2), where sqr(2) * sqr(2) = 2.

12.

a : nonzero rational number
b : irrational number
$P \rightarrow Q$
$a*b \rightarrow$ irrational

- proof by contradition :
  we assume that ab is rational and trying to disprove it
  $a = x/y$

  $ab = z/w$

  $b(x/y) = z/w$

  $b = (y/x) * (z/w)$ which is rational

  $-Q \rightarrow -P$ that means that that the product of two irrational numbers is rational.

15.

if x+y >= 2 then x >=1 and y >=1

x+y >= 2 ⟶ x >=1 ∧ y >=1

proving using contrapositive $p \to q \equiv -q \to -p$

-q = x < 1 ∧ y < 1

x < 1 ∧ y < 1 → x+y<2 #

16.

if mn is even, then m is even and n is even

- we can prove this using contrapositive:
  -q → -p
  assuming m is an odd number and n is an odd number
  m = 2k+1 , n = 2l+1
  m*n = (2k+1)*(2l+1)
  = 4lk + 2k + 2l +1
  = 2(2lk+k+l)+1
  = 2a+1 , m = 2lk+k+l m is an integer
  2a+1, m = 2lk+k+l is odd number

so using contrapositive we proved that – q → -p ,then p → q

17.

n^3 + 5 is odd, then n is even

p → q

a) using contrapositive:

n is odd can be written as n = 2t+1

n^3 + 5

= (2t+1)^3 + 5

= (2t)^3 + 3(2t)^2 + 3(2t) + 1 + 5

= 8t^3 + 12t^2 + 6t + 1 + 5

= 2(4t^3 + 6t^2 + 6t + 3) → is even

-q → -p == p → q

b)

assuming that n^3 + 5 is odd and n is odd

goal : p ∧ -q → false

since n^3 + 5 is odd and n^3 is odd, n^3 + 5 – n^3 is odd which is false because odd – odd = even

p ∧ -q → false == p → q

18.

3n + 2 is even, then n is even

$p \to q$

a)

if n is odd $n = 2k+1$

$3n + 2 = 3(2k+1) + 2 = 6k+3+2 = 6k+5 = 2(3k+2)+1$ is odd

$-q \to -p == p \to q$

b)

if 3n + 2 is even and n is odd $n = 2k+1$

$3(2k+1)+2 = 6k+3+2 = 6k+5 = 2(3k+2)+1$ is odd which is a contradiction to the assumption we made.

$p \wedge -q \to \text{false} == p \to q$

21.

for n = 1 the proposition is $(a + b)^1 >= a^1 + b^1$

$== (a + b) >= a + b$

$(a + b) = a + b$ #

22.

p : if you pick three socks from a drawer containing just blue socks and black socks.

q : you must get either a pair of blue socks or a pair of black socks.

$p \to q$

using contradiction $p \wedge -q$

if you pick three socks from a drawer containing just blue socks and black socks, but u didn’t get neithera pair of blue socks nor a pair of black socks, which is a contradiction to our statement, because there are only two color not 3 colors in the drawer. $p \to q$

24.

choosing 3 different dates from 25 dates different from each other is immpossible because the year has only 12 months, such that 25 dates is 2 dates from the same month is equal to 24 + 1 last date from any other month.

25.

theorm : r irrational such that $r^3 + r + 1 = O$, where $r = a/b$

$$(a/b)^3 + (a/b) + 1 = 0$$

if a and b both odds, then the L.H.S is odd $!= 0$
if a is odd and b is even, then the L.H.S is odd $!= 0$
if a is even and b is odd, then the L.H.S is odd $!= 0$

because a and b are in the simplist form, both cant be even at the same time. Leading to there is no such root exists

34.

this is NOT a valid solution.
it’s a solution for $(2X)^2 - 1 = x^2$ NOT $sqr((2x)^2 -1)= x$

35.

this is NOT a valid solution
its valid for $x +3 = (x -3)^2$ not for $sqr(x+3)= x - 3$

36.

$p1 \rightarrow p2$
$p2 \rightarrow p3$
$p3 \rightarrow p4$
$p4 \rightarrow p1$

to prove $p4 \leftrightarrow p1$ we need to prove that $p4 \rightarrow p1$
$(p2 \rightarrow p3 \wedge p3 \rightarrow p4 \wedge p4 \rightarrow p1) \rightarrow (p4 \rightarrow p1)$, then we can say that p1 and p4 are logically equivalence and same for the rest

37. reverse the method before.

38. 2 is a counter example and 4.