

7.

- a) Jan is not rich or not happy
- b) Carlos will not bicycle and will not run tomorrow.
- c) Mei never walks and never takes the bus to class.
- d) Ibrahim is not smart or not a hard working

8.

- a) Kwame will not take a job in industry and will not go to graduate school.
- b) Yoshiko dont know Java or Yoshiko dont know calculus.
- c) James is not young or not strong.
- d) Rita will not move to Oregon and not to Washington.

9.

a)

p	q	$(P \wedge q)$	$(P \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b)

p	q	$(P \vee q)$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c)

p	q	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow (p \rightarrow q)$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

d)

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e)

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f)

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

11.

- a) $(p \wedge q) \rightarrow p$
 $\neg(p \wedge q) \vee p$
 $(\neg p \vee \neg q) \vee p$
 $\neg p \vee \neg q \vee p \rightarrow \text{always true}$

- b) $p \rightarrow (p \vee q)$
 $\neg p \vee (p \vee q)$
 $\neg p \vee p \vee q \rightarrow \text{always true}$

c) $\neg p \rightarrow (p \rightarrow q)$
 $\neg(\neg p) \vee (p \rightarrow q)$
 $p \vee (\neg p \vee q)$
 $p \vee \neg q \vee q \rightarrow \text{always true}$

d) $(p \wedge q) \rightarrow (p \rightarrow q)$
 $\neg(p \wedge q) \vee (p \rightarrow q)$
 $\neg(p \wedge q) \vee (\neg p \vee q)$
 $\neg p \vee \neg q \vee (\neg p \vee q)$
 $\neg p \vee \neg q \vee \neg p \vee q \rightarrow \text{always true}$

e) $\neg(p \rightarrow q) \rightarrow p$
 $\neg(\neg p \vee q) \rightarrow p$
 $p \wedge \neg q \rightarrow p$
 $\neg(p \wedge \neg q) \vee p$
 $\neg p \vee \neg q \vee p \rightarrow \text{always true}$

f) $\neg(p \rightarrow q) \rightarrow \neg q$
 $\neg(\neg p \vee q) \rightarrow \neg q$
 $p \wedge \neg q \rightarrow \neg q$
 $\neg(p \wedge \neg q) \vee \neg q$
 $\neg p \vee q \vee \neg q \rightarrow \text{always true}$

16.

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \wedge q)$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	T	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

$p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are indeed equivalent.

17.

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg q$	$p \leftrightarrow \neg q$
T	T	T	T	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	T	F	T	F

$\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are indeed equivalent.

22.

r	p	q	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	T
F	T	T	T	F	F	F	F
F	T	F	F	F	F	F	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are indeed equivalent

23.

r	p	q	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$p \vee q$	$(p \vee q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	T
F	T	T	F	F	F	T	F
F	T	F	F	T	F	T	F
F	F	T	T	F	F	T	F
F	F	F	T	T	T	F	T

$(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are indeed equivalent

24.

r	p	q	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \vee (p \rightarrow r)$	$q \vee r$	$p \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	F	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

$(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are indeed equivalent.

25.

r	p	q	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$(p \wedge q)$	$(p \wedge q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	F	F	T	F
F	T	F	F	T	T	F	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	F	T

$(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are indeed equivalent.

40.

$p \wedge q \wedge \neg r$

41.

$(p \wedge q \wedge \neg r) \vee (p \wedge r \wedge \neg q) \vee (q \wedge r \wedge \neg p)$

42.

p	q	$p \rightarrow q$	Case
T	T	T	$(p \wedge q)$
T	F	F	
F	T	T	$(\neg p \wedge q)$
F	F	T	$(\neg p \wedge \neg q)$

The proposition is true when p is true and q is true OR p is false and q is true OR p is false and q is false.

so the disjunctive normal form would be: $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

43.

if we write a proposition in the form of a disjunctive normal form it shows it only consists of the NOT, AND and OR operators (functionally complete set).

44.

if we use the example before we can reduce it using only NOT and AND operators.

first to prove that $(p \vee q)$ is equivalent to $\neg(\neg p \wedge \neg q)$

$$\neg(\neg(p \vee q))$$

$$\neg(\neg p \wedge \neg q) \equiv \neg(\neg p \wedge \neg q)$$

then : $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ could be written as

$$\neg(\neg((p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)))$$

$$\neg(\neg(p \wedge q) \wedge \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)) \quad \#$$

55.

the number of compound propositions for 2 variables is

$$2^n = 2^2 = 4$$

56.

if p and q are logically equivalent that means that their truth table is the same, and if q and r are logically equivalent that means that their truth table is the same, which logically leads to that p and r are logically equivalent.

for example: $(p \leftrightarrow r) \wedge (q \rightarrow r) \rightarrow p \leftrightarrow r$

r	p	q	$p \leftrightarrow q$	$q \leftrightarrow r$	$(p \leftrightarrow r) \wedge (q \rightarrow r)$	$p \leftrightarrow r$	$(p \leftrightarrow r) \wedge (q \rightarrow r) \rightarrow p \leftrightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T
T	F	T	F	T	F	F	T
T	F	F	T	F	F	F	T
F	T	T	T	F	F	F	T
F	T	F	F	T	F	F	T
F	F	T	F	F	F	T	T
F	F	F	T	T	T	T	T

is a tautology.

57.

if the directory database is opened, then either the system is not in its initial state or the monitor is put in a closed state.

58.

r	p	q	$p \vee \neg q$	$\neg p \vee q$	$q \vee r$	$q \vee \neg r$	$\neg q \vee \neg r$	No of simultaneously true
T	T	T	T	T	T	T	F	4
T	T	F	T	T	T	F	T	3
T	F	T	F	F	T	T	F	2
T	F	F	T	T	T	F	T	3
F	T	T	T	T	T	T	T	5
F	T	F	T	T	F	T	T	2
F	F	T	F	F	T	T	T	3
F	F	F	T	T	F	T	T	2