

1.
  - a) True              b) True      c) False
2.
  - a) True              b) False              c) False              d) True
5.
  - a) There exists at least one student who spends more than five hours every weekday in class.
  - b) Every student spends more than five hours every weekday in class.
  - c) There exists at least one student who doesn't spend more than five hours every weekday in class.
  - d) Every student doesn't spend more than five hours every weekday in class.
6.
  - a) There is atleast one student who has visited North dakota.
  - b) Every student has visited North dakota.
  - c) There is no student who has visited North dakota.
  - d) There is atleast one student who hasn't visited North dakota.
  - e) There is no student who has visited North dakota.
  - f) Every student hasn't visited North dakota.
7.
  - a) Every comedian is funny.
  - b) Everyone is comedian and funny.
  - c) There is one person, if he is a comedian, then he is funny.
  - d) There is one person who's a comedian and funny.
8.
  - a) Every rabbits hops.
  - b) All rabbits hops.
  - c) There is atleast one animal if he is a rabbit, then he hops.
  - d) There is an animal who's a rabbit and hops.
9.
  - a)  $\exists x (P(x) \wedge Q(x))$
  - b)  $\exists x (P(x) \wedge \neg Q(x))$
  - c)  $\forall x (P(x) \vee Q(x))$
  - d)  $\exists x (\neg P(x) \wedge \neg Q(x))$

10.

- a)  $\exists x (C(x) \wedge Q(x) \wedge F(x))$
- b)  $\forall x (C(x) \vee Q(x) \vee F(x))$
- c)  $\exists x (C(x) \wedge Q(x) \wedge \neg F(x))$
- d)  $\forall x \neg (C(x) \vee Q(x) \vee F(x))$
- e)  $\exists x C(x) \wedge \exists x Q(x) \wedge \exists x F(x)$

15.

- a) True
- b) False
- c) True
- d) False

16.

- a) True
- b) False
- c) True
- d) False

17.

- a)  $P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$
- b)  $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$
- c)  $\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$
- d)  $\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$
- e)  $\neg(P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$
- f)  $\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

18.

- a)  $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$
- b)  $P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$
- c)  $\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$
- d)  $\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$
- e)  $\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$
- f)  $\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

21.

- a) True for CS students, False for every human.
- b) True for every person whose age is bigger than 21, False for every person whose age is smaller than 21.
- c) True for Brothers, False for Non brothers.
- d) True for non cousins, False for every Person.

22.

- a) True for Indians, False for people of the world.
- b) True for People of the world, False for kindergarten children.
- c) True for Ahmed Mahmoud and Ahmed Hossam, False for all the people.
- d) True for famous people, False for all the people.

23.

$C(x)$  : "x in your class"

a)  $P(x)$  : "x can speak Hindi"

- 1.  $\exists x P(x)$
- 2.  $\exists x (C(x) \wedge P(x))$

b)  $P(x)$  : "x in your class is friendly"

- 1.  $\forall x P(x)$
- 2.  $\exists x (C(x) \rightarrow P(x))$

c)  $P(x)$  : "x was born in California"

- 1.  $\exists x \neg P(x)$
- 2.  $\exists x (C(x) \wedge \neg P(x))$

d)  $P(x)$  : "x has been in a movie"

- 1.  $\exists x P(x)$
- 2.  $\exists x (C(x) \wedge P(x))$

e)  $P(x)$  : "x has taken a course in logic programming. "

- 1.  $\forall x \neg P(x)$
- 2.  $\exists x (C(x) \rightarrow \neg P(x))$

30.

- a)  $P(1, 3) \vee P(2, 3) \vee P(3, 3)$
- b)  $P(1, 1) \wedge P(1, 2) \wedge P(1, 3)$
- c)  $\neg(P(2, 1) \vee P(2, 2) \vee P(2, 3))$
- d)  $\neg(P(1, 2) \wedge P(2, 2) \wedge P(3, 2))$

35.

- a) No Counterexample
- b)  $x = 0$
- c) all integers except one

36.

- a)  $x = 0$  ,  $x = 1$
- b)  $\text{sqr}(2)$
- c)  $x = 0$

41.

- a) "At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space."

$M(x)$  : "x is a mail message"

$E(x)$  : "x in the nonempty set of messages"

$S(x)$  : "x can be saved"

$F(x)$  : "disk x has more than 10 kilobytes of free space."

$$( \exists x E(M(x)) \leftrightarrow \exists x F(x) ) \rightarrow S(x)$$

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- b) "Whenever there is an active alert, all queued messages are transmitted."

$A(x)$  : "x is an active alert"

$M(x)$  : "x is a queued message that is transmitted."

$$\exists x A(x) \rightarrow \forall x M(x)$$

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- c) "The diagnostic monitor tracks the status of all systems except the main console."

$T(x)$  : "The diagnostic monitor tracks the status of system x"

$C(x)$  : "x is the main console"

$$\neg C(x) \rightarrow T(x)$$

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- d) "Each participant on the conference call whom the host of the call did not put on a special list was billed."

$C(x)$  : "participant on the conference call."

$H(x)$  : "the host x of the call put on a special list."

$B(x)$  : "x is billed"

$$( \forall x C(x) \leftrightarrow \neg H(x) ) \rightarrow B(x)$$

42.

a) "Every user has access to an electronic mailbox"

$U(x)$  : "x has access to an electronic mailbox"

$$\forall x U(x)$$

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b) "The system mailbox can be accessed by everyone in the group if the file system is locked."

$G(x)$  : "x in the group"

$M(x)$  : "x can access the system mailbox"

$L$  : "The file system is locked."

$$L \rightarrow ( G(x) \rightarrow M(x) )$$

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c) "The firewall is in a diagnostic state only if the proxy server is in a diagnostic state."

$F$  : "The firewall is in a diagnostic state"

$P$  : "The proxy server is in a diagnostic state."

$$F \leftrightarrow P$$

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d) "At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode."

e)

$R(x)$  : "x is router that functions normally"

$T(x)$  : "the throughput x is between 100 kbps and 500 kbps"

$P$  : "The proxy server is not in diagnostic mode"

$$T(x) \wedge P \rightarrow \exists x R(x)$$

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43. "Determine whether  $\forall x (P(x) \rightarrow Q(x))$  and  $\forall x P(x) \rightarrow \forall x Q(x)$  are logically equivalent."

Sufficiency:

By definition  $\forall x(P(x) \rightarrow Q(x))$  is always True no matter the values of the  $x$  are.

Necessity:

$\forall x P(x) \rightarrow \forall x Q(x)$  can be False in the Case that  $Q(x)$  is False and  $P(x)$  is True which is not necessary for  $\forall x(P(x) \rightarrow Q(x))$  to be True.

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44. "Determine whether  $\forall x (P(x) \leftrightarrow Q(x))$  and  $\forall x P(x) \leftrightarrow \forall x Q(x)$  are logically equivalent."

Sufficiency:

By definition  $\forall x(P(x) \leftrightarrow Q(x))$  is always True which is sufficient for  $\forall x P(x) \leftrightarrow \forall x Q(x)$ .

Necessity:

$\forall x P(x) \leftrightarrow \forall x Q(x)$  on the other hand can be:

$x = a$  where  $P(a) \neq Q(a)$  which makes  $\forall x P(x) \leftrightarrow \forall x Q(x)$  which is not necessary for  $\forall x (P(x) \leftrightarrow Q(x))$ .

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49.

a)  $\forall x (P(x) \rightarrow A) \equiv \exists x P(x) \rightarrow A$

$$\forall x (\neg P(x) \vee A)$$

$$\forall x \neg P(x) \vee \forall x A$$

$$\neg \exists x P(x) \vee A$$

$$\exists x P(x) \rightarrow A \#$$

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$$b) \exists x (P(x) \rightarrow A) \equiv \forall x P(x) \rightarrow A$$

$$\exists x (P(x) \rightarrow A)$$

$$\exists x (\neg P(x) \vee A)$$

$$\exists x \neg P(x) \vee \exists x A$$

$$\neg \exists x P(x) \vee A$$

$$\forall x P(x) \rightarrow A$$

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$$50. \forall x (P(x) \vee Q(x)) \not\equiv \forall x P(x) \vee \forall x Q(x)$$

using counterexample:

assume:

$P(x)$  : "x is Tall"

$Q(x)$  : "x is Small"

for the first part it says :

**Every person is Tall or Small.**

the second part says:

**Every person is Tall or Every person is Small.**

Which is logically not correct (False).

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$$51. \exists x (P(x) \wedge Q(x)) \not\equiv \exists x P(x) \wedge \exists x Q(x)$$

Sufficiency:

for  $\exists x (P(x) \wedge Q(x))$  to be True,  $P(x)$  must be true AND  $Q(x)$  must be true, assuming  $x = a \dots P(a) \wedge Q(a)$  is True.

Necessity:

for  $\exists x P(x) \wedge \exists x Q(x)$  to be True  $\exists x P(x)$  must be true AND  $\exists x Q(x)$  must be true, assuming  $x = a$   $P(a)$  must be True and assuming  $x = b$   $Q(x)$  must be also True but  $x$  here doesn't have the same value. so we can't say that they are logically equivalent.