

1.  
a)  $f(0)$  is not defined  
b)  $f(x)$  where  $x$  is less than 0 is not defined  
c) maps to two different values for the same  $x$

2.  
a) not a function  
b) a function  
c) not a function

8.  
a) 1  
b) 2  
c) -1  
d) 0  
e) 3  
f) -2  
g) 1  
h) 2

9.  
a) 1  
b) 0  
c) 0  
d) -1  
e) 3  
f) -1  
g) 2  
h) 1

10.  
a) one-to-one  
b) NOT one-to-one  
c) NOT one-to-one

11.  
a) onto  
b) NOT onto  
c) NOT onto

14.  
a) onto  
b) NOT onto  
c) onto  
d) onto  
e) not onto

15.  
a) onto  
b) NOT onto  
c) onto  
d) NOT onto  
e) onto

16.  
a)  $f(x) = x^2$   
b) the ceil function  
c)  $f(x) = \sqrt{((2*x)/2)^2}$   
d)  $f(x) = x*0$

17.  
a)  $f(x) = 2x+1$   
b)  $f(x) = |x|$   
c)  $f(x) = x$   
d)  $f(x) = \sqrt{(x)}$

18.

- a) bijection
- b) NOT a bijection
- c) NOT a bijection
- d) bijection

19.

- a) bijection
- b) NOT a bijection
- c) bijection
- d) NOT a bijection

24.

for  $f(x)=e^x$  where  $\mathbf{R}$  maps to  $\mathbf{R}$ , maps every  $\mathbf{R}$  to  $\mathbf{R}^+$  only. So its not invertible, to be invertible the function have to be bijection function. If the codomain changed to  $\mathbf{R}^+$ .

injection :  $f(x_1)=e^{(x_1)}, f(x_2)=e^{(x_2)}$  if  $f(x_1) = f(x_2)$ , then  $e^{(x_1)}=e^{(x_2)} \rightarrow e^{(x_1-x_2)}=1$  leading to  $x_1=x_2$ .

serjuction :  $f(x)=e^x$  maps to every  $\mathbf{R}^+$  from  $\mathbf{R}$ .

then  $f(x)=e^x$ ,  $\mathbf{R}$  to  $\mathbf{R}^+$  is invertible.

25.

for  $f(x) = |x|$  is not one to one. Because  $f(-2)$  and  $f(2)$  has the same result, changing the domain to nonnegative real numbers.

Injection :  $f(x) = |x|$ ,  $f(y) = |y|$ , if  $f(x) = f(y)$ , then  $x = y$ .

serjuction:  $f(x)$  maps every real number to it self.

27.

- a)  $f(s)=\{4/3, 1/3, 0, 3\}$   $|f(x)| = \{0, 1, 3\}$
- b)  $\{0, 1, 3, 5, 8\}$
- c)  $\{0, 8, 16, 40\}$
- d)  $\{1, 12, 33, 65\}$

28.

- a)  $\{..., -4, -2, 0, 2, 4, 6, ...\}$
- b)  $\{0, 2, 4, 6, ...\}$
- c)  $\{\mathbf{R}\}$

29.

a)  $f \circ g = f(g(x))$  and since  $f(x)$  is a one-to-one function it maps every value of  $x$  to a distinct element same for  $g(x)$ .

b)  $f \circ g = f(g(x))$  is just mapping  $x$  using the  $g$  function and since it covers all the codomain and it maps  $x$  to  $n$ , then  $f(n)$  maps the  $n$  to the codomain. Leading to  $f \circ g = f(g(x))$  is a onto function.

30. yes, for  $g(x), g(y)$  let both are equal, then  $f(g(x))$  and  $f(g(y))$  is equal since  $f \circ g$  is one-to-one.

$g(x)$  and  $g(y)$  cant be equal if and only if  $x = y$ . aka  $g$  is a one-to-one function.

31. no, for  $g(x)$  can miscover every element in  $b$  but  $f$  still cover every element in  $c$ .

35.

injection:  $f(i) = a*i+b$ ,  $f(j) = a*j+b$ ,

if  $f(i) = f(j)$ , then  $a*i+b = a*j+b$  leading th  $j = i$ .

Serjuction: let  $i$  in the codomain  $f(x) = i$

$ax+b = i$ , then  $x = (i-b)/a$  for every codomain, then  $ax+b$  is invertible, and the reverse is  $f(x)^{-1}=g(y)=\frac{y-b}{a}$  .

36.

a) for the set let  $j=S \cup T$  ,  $f(j)$  maps  $j$  to a subset of  $B$  let it be  $O$ , for  $f(S)$  maps  $S$  to  $K$  which is a subset of  $B$ , and  $f(T)$  maps  $T$  to  $G$  which is a subset of  $B$ ,  $K \cup G = O$  .

b) for  $x \in S \cap T$  ,  $f(x) = I$ ,  $I$  is a subset of  $B$ , for  $f(S) = u$ ,  $f(T)=v$ ,  $f(S) \cap f(T)$  is equal to  $y$ , a subset of  $B$ , for  $I$  is a subset of the union of  $u$  and  $v$ .

44.

if  $x$  is not integer  $x=\lfloor x \rfloor - \varepsilon_1$  and  $x=\lfloor x \rfloor + \varepsilon_2$  from the definition,  $\varepsilon_1 + \varepsilon_2 = 1$  .

49.

if  $n$  is even, let it be  $2a$   $\lfloor 2\frac{a}{2} \rfloor = a$  , , where  $a = n/2$ . If  $n$  is even,  $n = 2b+1$

$$\lfloor \frac{2b+1}{2} \rfloor = b \text{ , where } b = \frac{(n-1)}{2} .$$

52.

since  $a \leq n$  that happens when  $\lfloor a \rfloor \leq n$  and for  $b$ :  $n \leq b$  that happens when  $n \leq \lfloor b \rfloor$  leading to  $a \leq n \leq b$  .

53.

since  $a < n$  that happens when  $\lfloor a \rfloor < n$  and for  $b$ :  $n < b$  that happens when  $n < \lfloor b \rfloor$  leading to  $a < n < b$  .

69.

a) is True because  $\lfloor x \rfloor$  is already an integer and the ceil of any integer  $x$  is the  $x$  itself.

b) not true, because for example  $\lfloor 2 * 1.1 \rfloor = \lfloor 2.2 \rfloor = 2$   
for  $2 * \lfloor 0.1 \rfloor = 2 * 0 = 0$

c) seems true.cant prove it

d) not true for  $\lfloor 1.1 * 1.1 \rfloor = \lfloor 1.2 \rfloor = 1$  , and  $\lfloor 1.1 \rfloor * \lfloor 1.1 \rfloor = 1 * 1 = 1$

e) false for  $x = 0.5$  ,  $\lfloor \frac{0.5}{2} \rfloor = 0$ , and  $\lfloor \frac{0.5+1}{2} \rfloor = 0$