1.

a) for every real number x, there exist atleast one number y where x is less than y.
b) for every real number x and y, if x is greater than or equal to zero and y is greater than or equal to zero, then x multiplied by y is greater than or equal to zero.
c) for every real numer x and y there exists x such that x multiplied by y equal to z.

2.

a) there exists a real number x, for every real number y such that x multiplied by y equal to y.
b) for every real number y and x, if x is nonnegative and y is negative, then x - y is a positive number.
c) for every real numer x and y there exists x such that x is the result of the sum of both y + z.

13.

a) -M(Chou, Koko)
b) -M(Arlene, Sarah) /\ -T(Arlene, Sarah)
c) -M(Deborah, Jose)
d) ∀x M(x, Ken)
e) ∀x -P(x, Nina)
f) ∀x M(x, Avi) \/ P(x, Avi)
g) ∃x ∀y (x != y -> M(x, y))
h) ∃x ∀y (x!=y -> (M(x, y) \/ T(x, y)))
i) ∃x ∃y (x != y ->(M(x, y) /\ M(y, x)))
j) ∃x M(x, x)
k) ∃x ∀y (x!=y -> (-M(y,x) /\ -T(y,x)))
l) ∀x ∃y (x!=y ->(M(y,x) \/ T(y,x)))
m) ∃x ∃y (x!=y -> M(x, y) /\ T(y,x))
n) ∃x ∃y ∀z (x!=y -> (M(x,y) \/ M(x,y) \/ T(x,z) \/ T(y,z)))

15.

a) C(x) : “x needs a course in discrete mathematics.”
S : computer science students
∀x εS C(x)

b) $P(x)$: “x owns a personal computer.”
   S : students in the class
   $\forall x \, \varepsilon S \, P(x)$

c) $C(x, y)$: “x has taken y”
   x : student in the class
   y : computer science course
   $\forall x \; \exists y \; C(x, y)$

d) $C(x, y)$: “x has taken y”
   x : student in the class
   y : computer science course
   $\exists x \; \exists y \; C(x, y)$

e) $B(x,y)$: “ x has been in y”
   x : student in the class
   y : building on campus
   $\forall x \; \forall y \; B(x, y)$

f) $B(x,y, z)$: “ x has been in y of z”
   x : student in the class
   y : room
   z : building on campus
   $\exists x \; \exists y \; \forall z \; B(x, y, z)$

g) $B(x,y, z)$: “ x has been in y of z”
   x : student in the class
   y : room
   z : building on campus
   $\forall x \; \exists y \; \forall z \; B(x, y, z)$

16.

a) $C(x)$: “x in the class who is a junior”
   $\exists x \; C(x)$->True

b) C(x) : “x in the class is a computer science major”
∀x C(x) ->False

c) C(x) : “x in the class”
M(x) : “x studies mathematics major”
J(x) : “x is a junior”
∃x (C(x) /\ -J(x) /\ -M(x)) ->False

d) C(x) : “x in the class”
S(x) : “x is a sophomore”
M(x) : “x studies computer science major”
∀x (C(x) /\ (S(x) \/ M(x))) -> False

e) M(x, y, z): “x major such that y a student in the class in z year of study with that major. ”
∃x ∃y ∀z M(x, y, z) -> True

17.

a) U(x, 1) : “x has access to y mailbox”
∀x ∃y (U(x, y) /\ ∀z(z!=x ->(x, z)))

b) K(x) : “the kernel of process x is working correctly”
P(x, y) : “process x continues to run during y error conditions”
∃x ∀y P(x, y) <-> K(x)

c) N(x, y) : “x user on the campus network can access y website”
W(x, u) : “the website w url has u extension.”
∀x ∀y N(x, y) <-> W(y, .edu)

d) ∃x ∃y (x != y /\ ∀z(∀s M(z, s) <-> (z = x /\ z = y)))
there is system x and there is system y, such that both monitor all remote servers.

18.

a) C(x, y) : “x console must be accessible during y fault condition”
∃x ∀y C(x,y)

b) M(x) : “e-mail address of x is on the system”
R(x) : “e-mail address x can be retrieved”
S(x) : “message sent by x”
A(x) : “the archive contains x”
$\forall x\ (M(x) \rightarrow (\forall y\ (y \neq x \land A(S(x))) \rightarrow R(x)))$

c)
x : security breach
M(y, x) : “mechanism y can detect breach x”
P(x) : “process x has been compromised”
$\forall x\ \exists y\ M(y, x) \leftrightarrow \exists x\ -P(x)$

d) $\exists x\ \exists y\ (x \neq y \land \forall z\ \forall w(z \neq w \rightarrow C(x, y, z , w)))$
there exist path x ,there exist path y, for every point z , for every point w if z != w then path x , y connect z, w
C(x, y, z , w) : “path and path z connect point z , point w”

e) K(x, p) : “x know password p”
A(x) : “x is system administrator”

$\forall x\ \forall p\ (A(x) \rightarrow K(x, p))$
for every user, for every password if the user is the system administrator then the user knows every password.

19.
a) $\forall x\ \forall y\ ((x<0) \land (y<0) \rightarrow x+y<0)$
b) $\forall x\ \forall y\ (x<y \rightarrow x-y<0)$
c) $\forall x\ \forall y\ (x^2 + y^2 >= (x+y) ^ 2)$
d) $\forall x\ \forall y\ (|x*y|= |x| * |y|)$

20.
a) $\forall x\ \forall y\ ((x<0) \land (y<0) \rightarrow x*y >0)$
b) $\forall x\ \forall y\ ((x>0) \land (y>0) \rightarrow (x+y)/2 >0)$
c) $\forall x\ \forall y\ ((x<0) \land (y<0) (x>y) \rightarrow x-y >0)$
d) $\forall x\ \forall y\ -(|x+y| > |x| + |y|)$

24.

a) There exists a real number x, for every real number y such that thier sum is equal to y.
b) for every real number x, for every real number y, such that if x is a non negative number and y is a nonpositive number, then x - y is a positive number.
c) there exists a real number x, there exists a real number y, such that if x is a nonpositive number and y is a nonpositive number then, the difference between them can be positive.
d) for every real number x, for every real number y, the product of x and y !=0 if and only if x != 0 and y != 0

25.

a) there exist a real number x, for every real number y such that thier product is equal to y
b) for every real number x, for every real number y, if x is nonpositive and y is nonpositive and y is non nonpositive then the product of x and y is nonnegative
c) there exists a real number x, there exists a real number y, x squared is bigger than y and x is less than y
d) for every real number x, for every real number y, there exists z such that the sum of x and y is equal to z.

27.

a) True
b) True
c) True
d) True
e) True
f) False
g) False
h) True
i) False

28.

a) True
b) False

c) True
d) False
e) True
f) False
g) True
h) False
i) True
j) True

34.

$\forall x\ \forall y$ ((x!=y) -> $\forall z$ ((z=x) \/ (z =y)))

True for a set of 2 numbers where z is in the set, False for every set else.

35.

$\forall x\ \forall y\ \forall z\ \exists w$ ((w!=x) /\ (w!=y) /\ (w!=z))

True for a unique set of numbers False for else

39.

a) x = 3 , y = -3
b) x = -1
c) x = -2 , y = 2

40.

a) x = 0
b) x = - 100
c) x = 1 , y = 1

48.

To prove $\forall x$ P(x) \/ $\forall x$ Q(x) and $\forall x\ \forall y$ (P(x) \/ Q(y)) are logically equivalent we need to prove that $\forall x$ P(x) $\forall x$ Q(x) <-> $\forall x\ \forall y$ (P(x) \/ Q(y))

sol

sufficiency:

$\forall x\ P(x) \lor \forall x\ Q(x)$ to be True P(x) is required to be True assuming that x = a , then P(a) is True, OR Q(x) is True by assuming that x = b, then Q(b) is True, then it can be written as $\forall x\ \forall y\ (P(x) \lor Q(y))$

necessity:

$\forall x\ \forall y\ (P(x) \lor Q(y))$ is True if P(x) is True OR Q(y) is True which can be written as $\forall x\ P(x) \lor \forall y\ Q(y)$

49.

$\forall x\ P(x) \land \exists x\ Q(x)$ and $\forall x\ \exists y\ (P(x) \land Q(y))$

$\forall x\ P(x) \land \exists x\ Q(x)$ is True if $\forall x\ P(x)$ is True by assuming x = a then P(a) is True AND $\exists x\ Q(x)$ is True by assuming x = b then Q(b) is True. which we can shift $\forall x\ \exists y\ (P(x) \land Q(y))$ using the fact that Ax for Q(y) is like a null quantification and same for $\exists y$ to P(x) and then the statement can be written as $\forall x\ P(x) \land \exists x\ Q(x)$.

50.

a) $\exists x\ \exists y(P(x) \lor Q(x) \lor A)$

b) $(\exists x\ P(x) \land \exists x\ Q(x))$
$\exists x\ \exists y\ (P(x) \land Q(x))$#

c) $\exists x\ P(x) \rightarrow \exists x\ Q(x)$
$-(\exists x\ P(x)) \lor \exists x\ Q(x)$
$\forall x\ P(x) \lor \exists x\ Q(x)$
$\forall x\ \exists x\ (-P(x) \lor Q(x))$
$\forall x\ \exists x\ (P(x) \rightarrow Q(x))$ #

52. $\exists x!\ P(x)$
$= \exists x(P(x) \land \forall y\ ((y!=x) \rightarrow P(x)))$
$= \exists x(P(x) \land \forall y\ (P(x) \rightarrow y = x))$