3.

a) addition
b) simplification
c) modus ponens
d) modus tollens
e) hypothetical syllogism

4.

a) simplification
b) disjunctive syllogism
c) hypothetical syllogism
d) addition
e) hypothetical syllogism

13.

a)

S(x) : “x is a student at the class”
J(x) : “x knows how to write programs in JAVA”
H(x) : “get a high-paying job”

S(Doug)
J(Doug)
∀x (J(x) → H(x))
conclusion :∃x (S(x) ∧ J(x))

1. ∀x (J(x) → H(x))
2. J(Doug) using modus ponens
3. H(Doug)
4. S(Doug) ∧ H(Doug) conjunction 4 and given
5. ∃x (S(x) ∧ H(x)) using existential generalization

b)

C(x) : “x in the class”
W(x) : “enjoys whale watching”
P(x) : “cares about ocean pollution”

∃x (C(x) ∧ W(x))
∀x (W(x) → P(x))
conclusion : ∃x(C(x) ∧ P(x))

1. ∃x (C(x) ∧ W(x))
2. C(a) ∧ W(a) existential instantiation
3. C(a) simplification
4. ∀x (W(x) → P(x))
5. W(a) → P(a) existential instantiation
6. W(a)
7. P(a) simplification
8. C(x) ∧ W(a) conjunction from 3 and 7
9. ∃x(C(x) ∧ P(x)) exitential generalization

c)

S(x) : “x student in the class”
C(x) : “x owns a personal computer”
W(x) : “x use a word processing program”

$\forall x\ (S(x) \rightarrow C(x))$
$\forall x\ (C(x) \rightarrow W(x))$

conclusion : $S(zake) \rightarrow W(zake)$

1. $\forall x\ (S(x) \rightarrow C(x))$
2. $S(zake) \rightarrow C(zake)$ using universal instantiation
3. $S(zake)$ given
4. $C(zake)$ modus ponens
5. $\forall x\ (C(x) \rightarrow W(x))$
6. $C(zake) \rightarrow W(zake)$ using universal instantiation
7. $C(zake)$ modus ponens
8. $W(zake)$ from 4
9. $S(zake) \rightarrow W(zake)$ hypothetical syllogism

d)

J(x) : “x lives in New Jersey”
O(x) : “x lives within 50 miles of the ocean”
N(x) : “x have seen the ocean”

$\forall x\ (J(x) \rightarrow O(x))$
$\exists x(J(x) \wedge -N(x))$

conclusion : $\exists x(O(x)\wedge -N(x))$

1. $\forall x\ (J(x) \rightarrow O(x))$
2. $J(a) \rightarrow O(a)$ using universal instantiation
3. $J(a)$
4. $O(a)$ modus ponens
5. $\exists x(J(x) \wedge -N(x))$
6. $J(a) \wedge -N(a)$ using exitential instantiation
7. $-N(a)$ simplification
8. $O(a) \wedge -N(a)$ using conjunction
9. $\exists x\ (O(x) \wedge -N(x))$ exitential generalization

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13.

a)

S(x) : “x is a student in the class”
R(x) : “x owns a red convertible”
T(x) : “x has gotten a speeding ticket”

$s(x) \wedge R(x)$
$\forall x\ (R(x) \rightarrow T(x))$
R(Linda)

conclusion : $\exists x(s(x) \land T(x))$

1. $S(Linda) \land R(Linda)$
2. $S(Linda)$ simplification
3. $\forall x\ (R(x) \to T(x))$
4. $R(Linda) \to T(Linda)$ universal instantiation
5. $R(Linda)$ given
6. $T(Linda)$ modus ponens
7. $T(Linda) \land S(Linda)$ from 2 using conjunction
8. $\exists x\ (T(x) \land S(x))$ exitential generalization

b)

$R(x)$: “x is a roommate”
$S(x)$: “x is a student”
$D(x)$: “x has taken a course in discrete mathematics”
$A(x)$: “x can take a course in algorithms”

$\forall x\ (S(x) \land D(x))$
$\forall x\ (D(x) \to A(x))$
$R(x) \to S(x)$

conclusion : $\forall x\ (R(x) \to A(x))$

1. $R(Aaron) \to S(Aaron)$
2. $R(Aaron)$
3. $S(Aaron)$ modus ponens
4. $\forall x\ (S(x) \land D(x))$
5. $S(Aaron) \land D(Aaron)$ universal instantiation
6. $D(Aaron)$ simplification
7. $\forall x\ (D(x) \to A(x))$
8. $D(Aaron) \to A(Aaron)$ universal instantiation
9. $R(Aaron) \to A(Aaron)$ from 1 and 8 using hypothetical syllogism
10. $\forall x\ (R(x) \to A(x))$ universal generalization

c)

$M(x)$: “x is a movies produced by John Sayles”
$W(x)$: “x is a wonderful movie”
$C(x)$: “x is a movie about coal miners”

$\forall x(M(x) \to W(x))$
$\exists x(M(x) \land C(x))$

conclusion : $\exists x(W(x) \land C(x))$

1. $\forall x(M(x) \to W(x))$
2. $M(c) \to W(c)$ universal instantiation
3. $\exists x(M(x) \land C(x))$
4. $M(c) \land C(c)$ existential instantiation
5. $M(c)$ simplification
6. $M(c) \to W(c)$

7. $W(c)$ modus ponens
8. $W(c) \land C(c)$ conjunction from 7 and 4 by simplification
9. $\exists x(W(x) \land C(x))$ existential generalization

d)

$S(x)$: “x in the class”
$F(x)$: “x has been to France”
$L(x)$: “x had visited the Louvre”

$\exists x(S(x) \land F(x))$
$\forall x(F(x) \rightarrow L(x))$

conclusion : $\exists x(S(x) \land L(x))$

1. $(S(x) \land F(x)$
2. $S(a) \land F(a)$ existential instantiation
3. $S(a)$ simplification
4. $\forall x(F(x) \rightarrow L(x))$
5. $F(a) \rightarrow L(a)$ universal instantiation
6. $F(a)$ from 2 by simplification
7. $L(a)$ modus ponens
8. $S(a) \land L(a)$ conjunction 7 and 3
9. $\exists x\ (S(x) \land L(x))$ existential generalization

19.

a) NOT a valid argument $p \rightarrow q$ doesn’t mean that I can say $q \rightarrow p$ (misusing of the converse)

b) valid argument using modus tollens

c) NOT a valid argument $p \rightarrow q$ doesn’t mean that I can say $-p \rightarrow -q$ (misusing of the inverse)

20.

a) not a valid argument (misusing of converse)

b) Valid argument (universal instaitation)

23.

error in step 5, we are assuming that the same c can be true for both $P(x)$ and $Q(x)$ which maybe NOT! We can rewrite the argument as follows:

5. $Q(a)$ Existential instantiation
6. $P(c) \land Q(a)$ Conjunction
7. $\exists x\ \exists y\ (P(x) \land Q(y))$ Existential generalization

24.

in step 2 c should be refering to an arbitrary value from the domain not a specific value.

error in step 3, we can simplify only from statement having ∧, same for step 5.
another error in step 7, using conjunction is using the AND operator as follows :
$\forall x\ P(x) \lor \forall x\ Q(x)$.

26.
$\forall x\ (P(x) \rightarrow Q(x))$
$\forall x\ (Q(x) \rightarrow R(x))$

conclusion : $\forall x\ (P(x) \rightarrow R(x))$

1. $\forall x\ (P(x) \rightarrow Q(x))$
2. $P(a) \rightarrow Q(a)$ universal instantiation (a is an arbitrary value)
3. $\forall x\ (Q(x) \rightarrow R(x))$
4. $Q(a) \rightarrow R(a)$ universal instantiation (a is an arbitrary value)
5. $P(a) \rightarrow R(a)$ Hypothetical syllogism from 2 and 4
6. $\forall x\ (P(x) \rightarrow R(x))$ universal generalization

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27.

if $\forall x(P(x) \rightarrow (Q(x) \land S(x)))$ and $\forall x(P(x) \land R(x))$ then $\forall x(R(x) \land S(x))$

1. $\forall x(P(x) \rightarrow (Q(x) \land S(x)))$
2. $P(a) \rightarrow (Q(a) \land S(a))$ universal instanitation (a is arbitary value)
3. $\forall x(P(x) \land R(x))$
4. $P(a) \land R(a)$ universal instanitation (a is arbitary value)
5. $P(a)$ simplification
6. $Q(a) \land S(a)$ modus ponens from 5 and 2
7. $S(a)$ simplification
8. $S(a) \land R(a)$ conjunction and simplification from 4
9. $\forall x(R(x) \land S(x))$ universal generalization