

3.

- a) $a_0=2, a_1=3, a_2=5, a_3=9$
- b) $a_0=1, a_1=4, a_2=27, a_3=256$
- c) $a_0=0, a_1=0, a_2=1, a_3=1$
- d) $a_0=0, a_1=1, a_2=2, a_3=3$

4.

- a) $a_0=1, a_1=-2, a_2=4, a_3=-8$
- b) $a_0=3, a_1=3, a_2=3, a_3=3$
- c) $a_0=8, a_1=11, a_2=33, a_3=71$
- d) $a_0=2, a_1=0, a_2=16, a_3=0$

5.

- a) {2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32}
- b) {1, 1, 1, 2, 2, 2, 3, 3, 3, 4}
- c) {1, 1, 3, 3, 5, 5, 7, 7, 9, 9}
- d) {0, -1, -2, -2, 8, 88, 656, 4912, 40064, 362368}
- e) {6, 12, 24, 48, 96, 192, 384, 768, 1536, 3072}
- f) {1, 1, 2, 3, 5, 8, 13, 21, 34, 55}
- g) {1, 2, 2, 3, 3, 3, 3, 4, 4}
- h) {3, 3, 5, 4, 4, 3, 5, 5, 4, 3}

7.

- 1- starting from one, the next element is double the value of the previous number.
- 2-starting with one and two, the next element equals the sum of the two numbers before plus one.
- 3-the sequence is the numbers non divisible by 3.

8.

- 1-even numbers starting from 33
- 2-odd prime numbers.
- 3-starting with 3, each element equals the element before +3.

9.

- a) one one and one zero, two one's two zero's,...
- b) the set of natural numbers, where every even number is repeated twice.
- c) every 2^i , else is zero
- d) $3 \cdot 2n - 1$
- e) starting with 15, every number equals the pervious number - 7
- f) starting with 3, the nth number = $(i_{n-1}) + (2+i)$
- g) $2n^3$, h) $n! + 1$

10.

- a) $2 + n^2$, 123, 146, 171
- b) $4n + 3$, 50, 57, 64
- c) conversation to binary, 1100, 1101, 1110

- d) sequence of numbers nondivisible by 4, and every number is repeated by odd numbers, 6, 6, 6
 e) $a_n = 3^{n-1} - 1$
 f) ...can't find a pattern
 g) starting with one one, then two zeros's, then three one's, and so on, 0, 0, 0
 h) every number is the square of the number before it.

13.

- a) 20
 b) 11
 c) 30
 d) 511

14.

- a) 16
 b) 84
 c) 1.6761904
 d) 4

21.

from 19:

$$\sum_{i=1}^n (a_i - a_{i-1}) = a_n - a_0$$

and since $k^2 - (k-1)^2 = 2k - 1$

using the property before $a_n = k^2$

$$\sum_{k=1}^n (a_k - a_{k-1}) = a_n - a_0$$

$$= k^2 - 0$$

$$= k^2 \quad \#$$

$$\text{for } 2k-1 \quad \sum_{k=1}^n (2k-1) = n^2$$

$$2 \sum_{k=1}^n (k) - \sum_{k=1}^n (1) = n^2, \quad 2 \sum_{k=1}^n (k) - n = n^2$$

$$2 \sum_{k=1}^n (k) = n^2 + n = \sum_{k=1}^n (k) = \frac{n^2 + n}{2} \quad \#$$

22.

noting that $k^3 - (k-1)^3 = 3k^2 - 3k + 1$

$$3 \sum_{k=1}^n (k^2) - 3 \sum_{k=1}^n (k) + \sum_{k=1}^n (1) = 3 \sum_{k=1}^n (k^2) - 3 \left(\frac{n^2 + n}{2} \right) + n = n^3$$

$$\sum_{k=1}^n (k^2) = 3 \left(\frac{n^2 + n}{2} \right) - n + n^3 \quad \sim \sim \text{leading to} \quad \sum_{k=1}^n (k^2) = \frac{(n^2 + n)(2n + 1)}{6}$$

27.

- a) 0
- b) 1680
- c) -1
- d) 1024

28.

$$\prod_n$$

31.

- a) countable -1, -2, -3, ...
- b) countable 2, 4, 6, ...
- c) uncountable
- d) countable 7, 14, 21, 38, 45

32.

- a) countable 11, 12, 13, ...
- b) countable 1, 3, 5, ...
- c) uncountable
- d) countable 10, 20, 30, ...

36.

suppose we have a set A and b is a subset of A , A elements can be listed as a_1, a_2, a_3, \dots for b $|b| \leq |A|$ and since b is countable, then mapping b to A is doable since the $|b| \leq \aleph_0$ because its countable, thus A is countable.

37.

for $A \subseteq B$ since listing the elements of A is uncountable, and $|A| \leq |B|$ leading to that $|A| \leq \aleph_0$, thus A is impossible to count.