**Experiment No. 1**

**Problem Statement - Implement Union, Intersection, Complement and Difference operations on fuzzy sets. Also create fuzzy relations by Cartesian product of any two fuzzy sets and perform max-min composition on any two fuzzy relations.**

**Aim:** Fuzzy set Operations & Relations.

**Objective:**

Implement Union, Intersection, Complement and Difference operations on fuzzy sets. Also create fuzzy relations by Cartesian product of any two fuzzy sets and perform max-min composition on any two fuzzy relations.

**Theory:**

Fuzzy Set Operations

1. Define Fuzzy Sets:

   Choose a universe of discourse (U) representing the range of possible values for your variable. Define two fuzzy sets, P and Q, on the universe U using membership functions. Membership functions map elements in U to a degree of membership between 0 and 1. Common functions include triangular, trapezoidal, and Gaussian functions.

2. Equal fuzzy sets:

   Two fuzzy sets A(x) and B(x) are said to be equal, if μA(x) = μB(x) for all x ∈ X. It is expressed as follows A(x) = B(x), if μA(x)= μB(x). Two fuzzy sets A(x) and B(x) are said to be **unequal,** if μA(x) ≠ μB(x) for at least x ∈ X.

   Example:
   A(x) = {(x1,0.1),(x2,0.2),(x3,0.3),(x4,0.4)}
   B(x) = {(x1,0.1),(x2,0.5),(x3,0.3),(x4,0.6)}
   As μA(x) ≠ μB(x) for different x ∈ X, A(x) ≠ B(x)

3. Complement (NOT)

   The complement is the opposite of the set. The complement of a fuzzy set is denoted by Ā(x) and is defined with respect to the universal set X as follows:
   Ā(x) = 1- A(x) for all x ϵ X

   1. Implement a function that takes a fuzzy set (A) and its membership function (μ_A) as input.
   2. For each element x in U, calculate the degree of membership in the complement $(\mu\_A^c)(x)$ by inverting the membership of A:
      **$\mu\_A^c(x) = 1 - \mu\_A(x)$**

4. Union (OR):

   Union of fuzzy sets consists of every element that falls into either set. The value of the membership value is will be the largest membership value of the element in either set Let A(x) and B(x) are two fuzzy sets for all x ∈ X, Union of fuzzy sets is denoted by **(AUB)(x)** and the membership function value is determined as follows

**μ (AUB)(x)= max{μA(x),μB(x)}**

1. Implement a function that takes two fuzzy sets (A, B) and their membership functions (μ_A, μ_B) as input.
2. For each element x in U, calculate the degree of membership in the union (μ_P U B)(x) using the maximum of individual memberships:

   μ_A U B(x) = max(μ_A(x), μ_B(x))

Example:
A(x)={(x1,0.7),(x2,0.3),(x3,0.9),(x4,0.1)}
B(x)={(x1,0.2),(x2,0.5),(x3,0.7),(x4,0.4)}

μ (AUB)(x1)= max{μA(x1),μB(x1)} = max{0.7,0.2} = 0.7
μ (AUB)(x2)= max{μA(x2),μB(x2)} = max{0.3,0.5} = 0.5
μ (AUB)(x3)= max{μA(x3),μB(x3)} = max{0.9,0.7} = 0.9
μ (AUB)(x4)= max{μA(x4),μB(x4)} = max{0.1,0.4} = 0.4

5. Intersection (AND):

Inter section of a fuzzy sets define how much of the element belongs to both sets. May have different degrees of membership in each set. The degree of membership is the lower membership in both sets of each element. Let A(x) and B(x) are two fuzzy sets, the intersection of is denoted by **(A∩B)(x)** and the membership function value is given as follows **μ (A∩B)(x)= min{μA(x),μB(x)}**

Intersection is analogous to logical AND operation

1. Implement a function that takes two fuzzy sets (A, B) and their membership functions (μ_A, μ_B) as input.
2. For each element x in U, calculate the degree of membership in the intersection
3. (μ_A ∩ B)(x) using the minimum of individual memberships:

   **μ_A ∩ B(x) = min(μ_A(x), μ_B(x))**

4. Difference (A except B):
   The Algebraic deference of two fuzzy sets A(x) and B(x) for all x ∈ X, is denoted by A(x) -B(x) and defined as follows A(x) - B(x)= {(x,μA-B(x), x ϵ X } Where μA-B(x) = μA∩B̅(x)

   1. Implement a function that takes two fuzzy sets (A, B) and their membership functions (μ_A, μ_B) as input.
   2. For each element x in U, calculate the degree of membership in the difference (μ_A \ B)(x) using the membership of A and the complement of B:

      **μ_A \ B(x) = min(μ_A(x), μ_B$^c$(x))**

Example:

A(x)={(x1,0.1),(x2,0.2),(x3,0.3),(x4,0.4)}
B(x)={(x1,0.5),(x2,0.7),(x3,0.8),(x4,0.9)}
B̅(x) = {(x1,0.5),(x2,0.3),(x3,0.2),(x4,0.1)}
A(x) - B(x) = {(x1,0.1),(x2,0..2),(x3,0.2),(x4,0.1)}

Fuzzy Relations

1. Cartesian Product:
   - Define two fuzzy sets, A and B, on universes U and V respectively.
   - The Cartesian product of A and B creates a new fuzzy relation R on the universe U x V (cartesian product of U and V).
   - The membership function of R, μ_R((u, v)), represents the degree of relationship between element u in U and element v in V.
   - Implement a function that takes two fuzzy sets (A, B) and their membership functions (μ_A, μ_B) as input.

2. Max-Min Composition:
   - Define two fuzzy relations, R and S, on universes (U x V) and (V x W) respectively.
   - Max-min composition combines these relations to create a new fuzzy relation
   - T on universe U x W.
   - The membership function of T, μ_T((u, w)), represents the composed relationship between element u in U and element w in W, considering the relationship between u and all elements v in V.
   - Implement a function that takes two fuzzy relations (R, S) and their membership functions (μ_R, μ_S) as input.
   - For each pair (u, w) in U x W, iterate through all elements v in V and calculate the intermediate values:
   - z_uv = min(μ_R((u, v)), μ_S((v, w)))
   - Use the maximum of these intermediate values as the degree of membership in the composed relation

$X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, and $Z = \{z_1, z_2, z_3\}$.

**Conclusion:** Thus, we have successfully implemented a program for Fussy set operations& its relations.

**Outcome:** Understand the concept of how fuzzy logic is used to define uncertainty.

**Questions:**

1.Explain Fuzzy System? Give Example for the same?
2.What is fuzzy set? State the applications of fuzzy sets
3.Differentiate Fuzzy Set and Crisp set
4.Write applications of Fuzzy system
5.Explain fuzzy operations with suitable example

**Code and Output:**

# Experiment 1

In [1]:
```
import numpy as np
```

In [2]:
```
# Function to perform Union operation on fuzzy sets
def fuzzy_union(A, B):
    return np.maximum(A, B)

# Function to perform Intersection operation on fuzzy sets
def fuzzy_intersection(A, B):
    return np.minimum(A, B)

# Function to perform Complement operation on a fuzzy set
def fuzzy_complement(A):
    return 1 - A

# Function to perform Difference operation on fuzzy sets
def fuzzy_difference(A, B):
    return np.maximum(A, 1 - B)

# Function to create fuzzy relation by Cartesian product of two fuzzy sets
def cartesian_product(A, B):
    return np.outer(A, B)

# Function to perform Max-Min composition on two fuzzy relations
def max_min_composition(R, S):
    return np.max(np.minimum.outer(R, S), axis=1)
```

In [3]:
```
A = np.array([0.2, 0.4, 0.6, 0.8])  # Fuzzy set A
B = np.array([0.3, 0.5, 0.7, 0.9])  # Fuzzy set B

# Operations on fuzzy sets
union_result = fuzzy_union(A, B)
intersection_result = fuzzy_intersection(A, B)
complement_A = fuzzy_complement(A)
difference_result = fuzzy_difference(A, B)

print("Fuzzy Set A:", A)
print("Fuzzy Set B:", B)

print("\nUnion:", union_result)
print("Intersection:", intersection_result)
print("Complement of A:", complement_A)
print("Difference:", difference_result)
```

```
Fuzzy Set A: [0.2 0.4 0.6 0.8]
Fuzzy Set B: [0.3 0.5 0.7 0.9]

Union: [0.3 0.5 0.7 0.9]
Intersection: [0.2 0.4 0.6 0.8]
Complement of A: [0.8 0.6 0.4 0.2]
Difference: [0.7 0.5 0.6 0.8]
```

In [4]:
```python
# Fuzzy relations
R = np.array([0.2, 0.5, 0.4])  # Fuzzy relation R
S = np.array([0.6, 0.3, 0.7])  # Fuzzy relation S

# Cartesian product of fuzzy relations
cartesian_result = cartesian_product(R, S)

# Max-Min composition of fuzzy relations
composition_result = max_min_composition(R, S)

print("Fuzzy Relation R:", R)
print("Fuzzy Relation S:", S)

print("\nCartesian product of R and S:")
print(cartesian_result)

print("\nMax-Min composition of R and S:")
print(composition_result)
```

```
Fuzzy Relation R: [0.2 0.5 0.4]
Fuzzy Relation S: [0.6 0.3 0.7]

Cartesian product of R and S:
[[0.12 0.06 0.14]
 [0.3  0.15 0.35]
 [0.24 0.12 0.28]]

Max-Min composition of R and S:
[0.2 0.5 0.4]
```