Assignment #2

Autonomous Navigation: AE640
Deadline 07 November 2020, 11:59PM

Q1. Consider the localization problem given in Fig.1. The unit vectors from the vehicle to beacon and respective ranges are observed as follows:

Beacon no	Unit Vector	Range	Landmark location
1	$[0.50 \;\; 0.87]^T$	12.2	$[16.0 \;\; 25.4]^T$
2	$[-0.77 \;\; 0.64]^T$	6.9	$[4.6 \;\; 19.5]^T$
3	$[-0.50 \;\; -0.87]^T$	14.2	$[3.0 \;\; 2.9]^T$
4	$[0.34 \;\; -0.94]^T$	8.9	$[13.0 \;\; 6.5]^T$

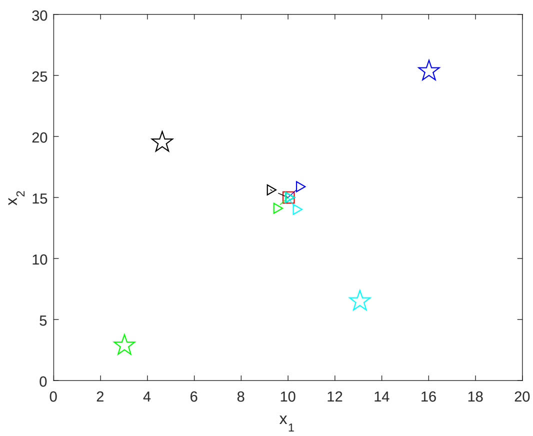

Figure 1: Localization problem with four beacons

Determine the position of vehicle using least-squares approach. 20 marks

Q2. Assume that in the above problem the unit vectors are not available. The locations of beacons are now available, formulate nonlinear least-square problem and determine the solution. Assume that each sensor has the following accuracy: Sensor 1 -95%, Sensor 2 -98%, Sensor 3 -90%, Sensor 4 -92%), how would use this information in your solutions ? 20 marks

Q3. Prove $P(A|B,C) = \frac{P(B|A,C)P(A|C)}{P(B|C)}$. 10 marks

Q4. Prove $P(x_t|z_t, x_{t-1}, u_{t-1}) = \frac{P(z_t|x_t,x_{t-1},u_{t-1})P(x_t|x_{t-1},u_{t-1})}{P(z_t|x_{t-1},u_{t-1})}$. 10 marks

Q5. Prove $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$. 10 marks

Q6. Prove $\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} x e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \mu$.

Q7. Find $E[x_{t+1}]$ given $x_{t+1} = Fx_t$ and $x_t \sim \mathcal{N}(\hat{x}_t, \Sigma_{x_t})$. 10 marks

Q6. Find $E[(x_{t+1} - E[x_{t+1}])(x_{t+1} - E[x_{t+1}])^T]$ using the above given system. 10 marks

Q8. Find $E[(x_{t+1} - E[x_{t+1}])(x_{t+1} - E[x_{t+1}])^T]$ given $x_{t+1} = Fx_t + Gu_t$ and $x_t \sim \mathcal{N}(\hat{x}_t, \Sigma_t)$, $u_t = u + \eta_t$, $\eta_t \sim \mathcal{N}(0, Q_t)$ 10 marks

Q9. Show that Kalman Filter is a minimum mean-square error (MMES) estimator. You can use error covariance to show this. 20 marks

Q10. Consider the below polar to rectangular coordinate transformation: 40 marks

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

where r is the range and θ is the angle or bearing and can be measured using sensors. The x and y coordinates are the function of range and angle. In a generic form, the above equations can be written as:

$$f = h(r, \theta)$$

1. The range and bearings can be further written as

   $$\begin{aligned} r &= \bar{r} + r_e \\ \theta &= \theta + \theta_e \end{aligned}$$

   where the true means of the range ($\bar{r}$) and bearing ($\bar{\theta}$) are 1 m and $\frac{\pi}{2}$ degrees respectively, and r_e and θ_e are the zero-mean deviations from their means. Derive the expression for mean and covariance for the 'f'.

   MATLAB (Python) exercise: Generate 1000 random range and bearing points which are uniformly distributed between ± 0.01 m and ± 10 degrees. Plot the mean and covariance ellipse using the above derived expression.

2. Derive the expression for mean and covariance for the nonlinear function 'f' using the concepts of first order linearization (EKF).

   MATLAB (Python) exercise: Generate 1000 random range and bearing points which are uniformly distributed between ± 0.01 m and ± 10 degrees. Plot the mean and covariance ellipse using the expressions obtained from first order linearization.

3. Derive the unscented sigma points and weights for the nonlinear function 'f'.

   MATLAB (Python) exercise: Generate 1000 random range and bearing points which are uniformly distributed between ± 0.01 m and ± 10 degrees. Plot the mean and covariance ellipse of 'f' using unscented transformation.

4. Compare the mean and covariance for true, first order approximation and unscented transformations.