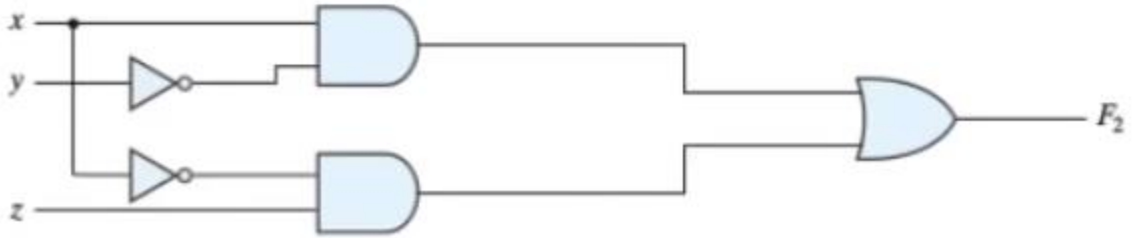
Simplify the function to simplify the circuit; why?

$$
\begin{aligned}
F_2 &= x'y'z + x'yz + xy' \\
&= x'z(y' + y) + xy' \\
&= x'z + xy',
\end{aligned}
$$

which has only 4 literals*, where a literal is a single variable (complemented or un-complemented).*



Less gates (HW) means:

- *low cost.*
- *low power consumption.*
- *simpler implementation.*

Simplifying functions is by:

- *algebraic manipulation (this chapter)*
- *K-map (next chapter)*
- *computer programs for many-input functions*

The Map Method

- K-map (named after Karnaugh) is a visual method, utilizing **visual power**.
- Difficult for more than 5 variables.
- Produces always SOP or POS expressions.

Standard Forms

- **<u>Standard Sum-of-Products (SOP) form:</u> equations are written as an OR of AND terms**
- **<u>Standard Product-of-Sums (POS) form:</u> equations are written as an AND of OR terms**
- **Examples:**
  - **SOP:**
  - **POS:**
- **These "mixed" forms are <u>neither SOP nor POS</u>**

Two-Variable Map

0	0
0	1
1	0
1	1

m_0	m_1
m_2	m_3

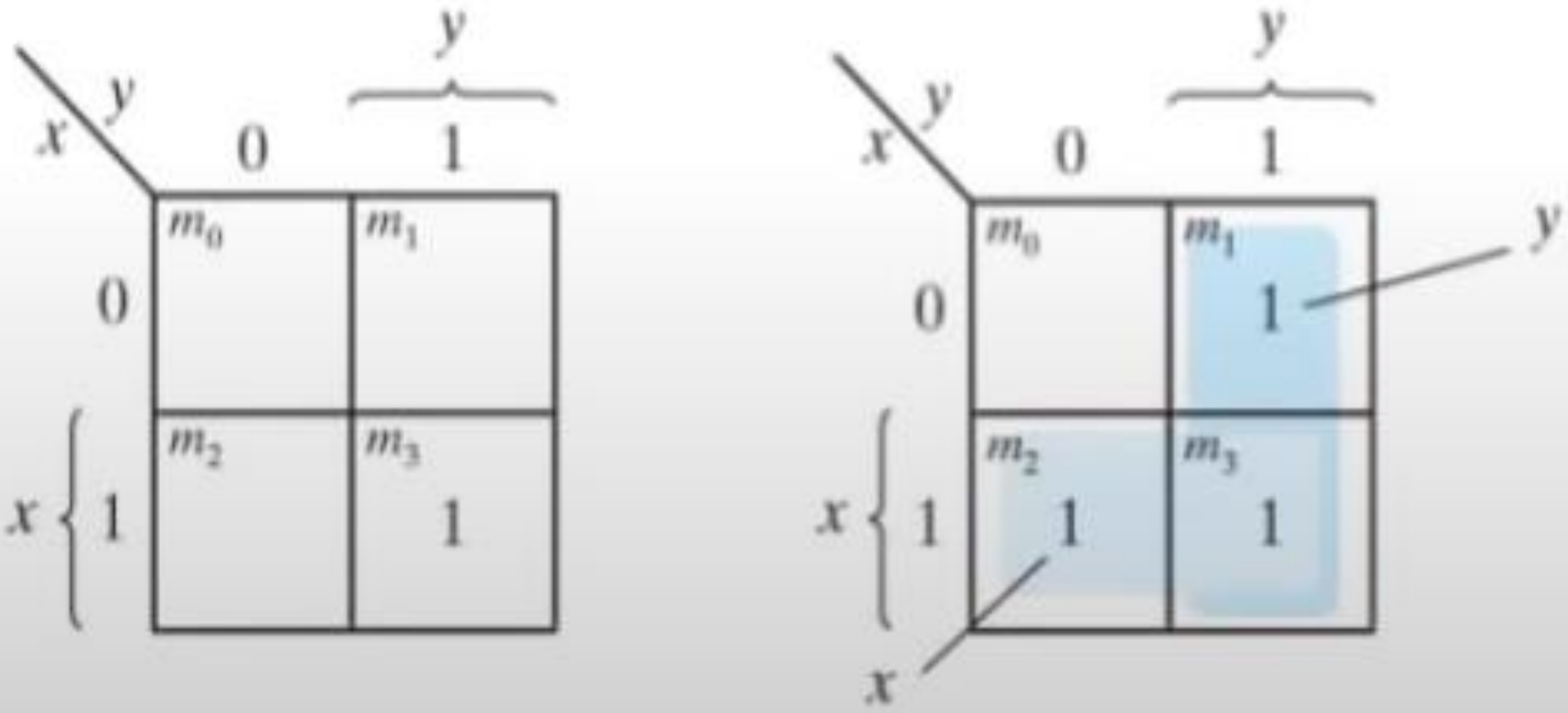
$x \backslash y$	0	1
0	m_0 $x'y'$	m_1 $x'y$
1	m_2 xy'	m_3 xy

Simplify the following functions both algebraically and with K-Map and observe the visual power:

$$F_1 = m_1 + m_2 + m_3 \quad = x'y + xy' + xy \quad = xy + x'y + xy' + xy = y + x.$$

$$F_2 = m_3 \quad = xy.$$

$$F_3 = m_0 + m_3 \quad = x'y' + xy.$$



- Only one-bit (variable) change for any two adjacent squares!

Description of Kmaps and Terminology

- For example, the minterms for a function having the inputs x and y are: $\bar{x}\bar{y}, \bar{x}y, x\bar{y}$, and xy
- Consider the Boolean function, $F(x,y) = xy + x\bar{y}$
- Its minterms are:

Minterm	X	Y
$\bar{X}\bar{Y}$	0	0
$\bar{X}Y$	0	1
$X\bar{Y}$	1	0
XY	1	1

- Similarly, a function having three inputs, has the minterms that are shown in this diagram.

Minterm	X	Y	Z
$\bar{X}\bar{Y}\bar{Z}$	0	0	0
$\bar{X}\bar{Y}Z$	0	0	1
$\bar{X}Y\bar{Z}$	0	1	0
$\bar{X}YZ$	0	1	1
$X\bar{Y}\bar{Z}$	1	0	0
$X\bar{Y}Z$	1	0	1
$XY\bar{Z}$	1	1	0
XYZ	1	1	1

- A Kmap has a cell for each minterm.
- This means that it has a cell for each line for the truth table of a function.
- The truth table for the function *F(x,y) = xy* is shown at the right along with its corresponding Kmap.

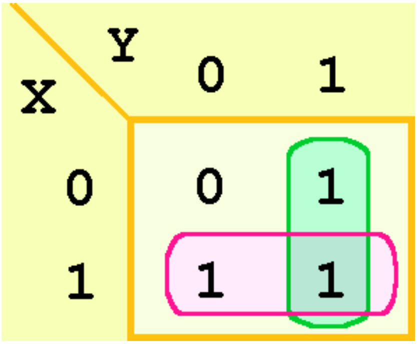
F(X,Y) = XY

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X \ Y	0	1
0	0	0
1	0	1

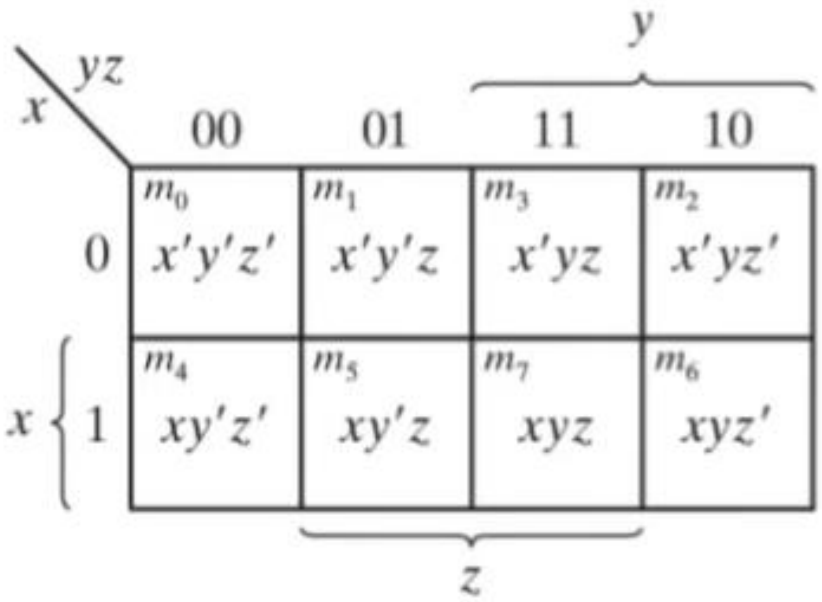
$$F(x,y) = x + y = \bar{x}y + x\bar{y} + xy$$

- The best way of selecting two groups of 1s form our simple Kmap is shown below.
- We see that both groups are powers of two and that the groups overlap.
- The next slide gives guidance for selecting Kmap groups.



Three-Variable Map

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



X \ YZ	00	01	11	10
0	$\overline{X}\,\overline{Y}\,\overline{Z}$	$\overline{X}\,\overline{Y}Z$	$\overline{X}YZ$	$\overline{X}Y\overline{Z}$
1	$X\overline{Y}\,\overline{Z}$	$X\overline{Y}Z$	XYZ	$XY\overline{Z}$

- Only one-bit (variable) change for any two adjacent squares! Therefore, e.g.,

$$F = m_5 + m_7 = xy'z + xyz = xz(y' + y) = xz$$

- Make sure of number of **variables** (e.g., $m_2 + m_3$ for 2 or 3 variables) and their **order**

Consider the function: •

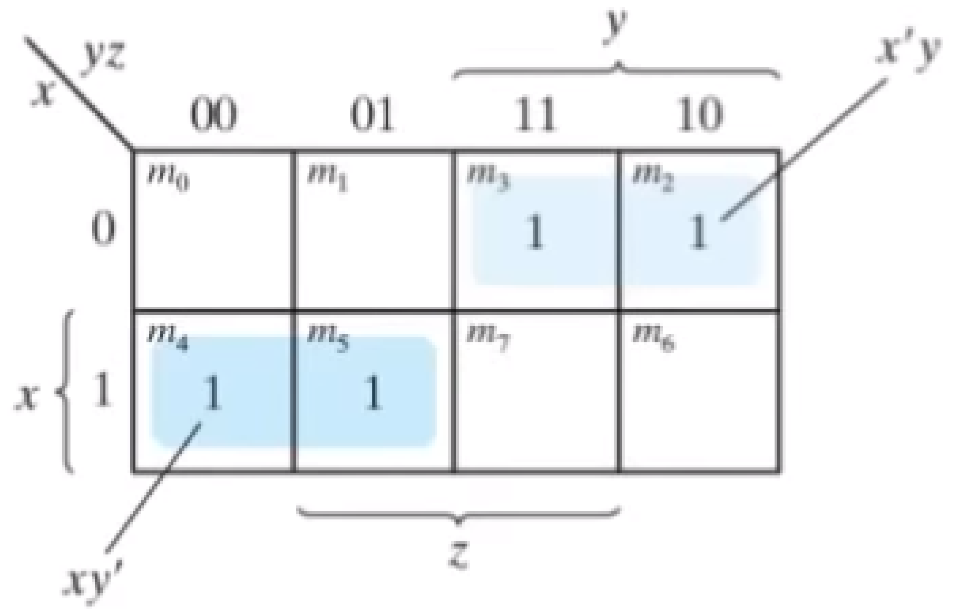
Its Kmap is given below. •

$$F(X,Y) = \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}Z + XYZ$$

What is the largest group of 1s that is a power of 2? •

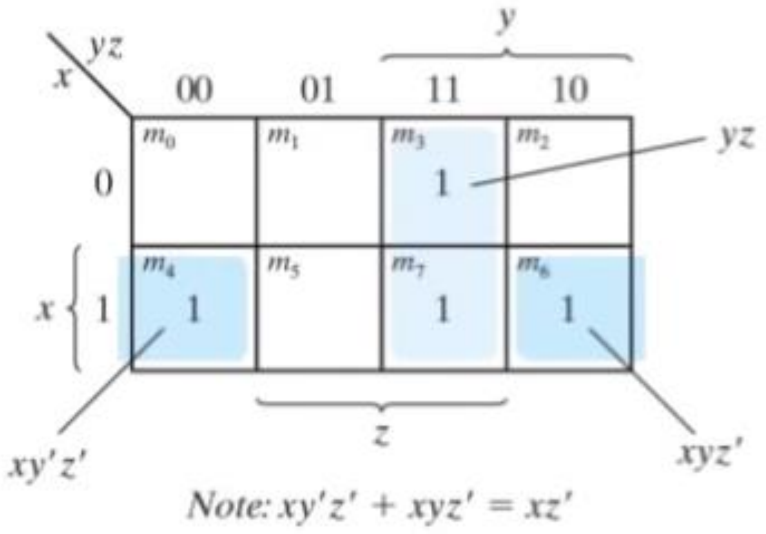
X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

Example : *Simplify the function* $F(x, y, z) = \sum(2, 3, 4, 5)$.



$$F = x'y + xy' = x \oplus y.$$

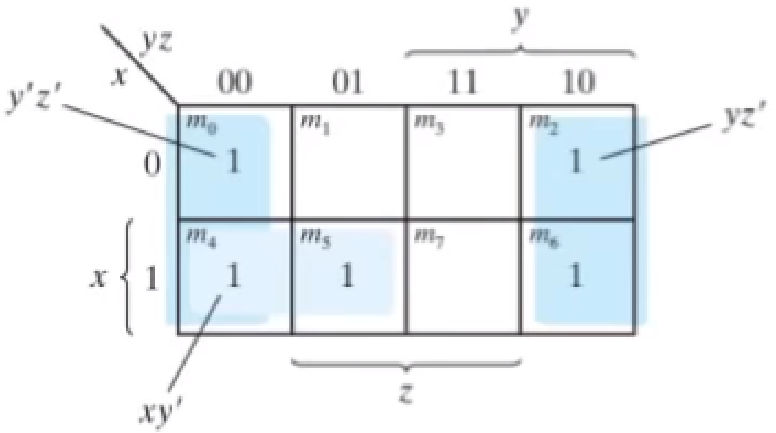
Example *Simplify* $F(x, y, z) = \sum(3, 4, 6, 7)$.



$$F = xz' + yz.$$

Example : *Consider the function* $F(x, y, z) = \sum(0, 2, 4, 5, 6)$.

1. *Simplify* F. (***Hint:*** *a group should have* (2^m) *ones and its resulting SOP has* $(n-m)$ *literals.*
2. *Implement the function using* `AND`, `OR`, `NOT`.
3. *Observe the number of* `AND` *and* `OR` *gates (ignore inverters for now).*



Note: $y'z' + yz' = z'$

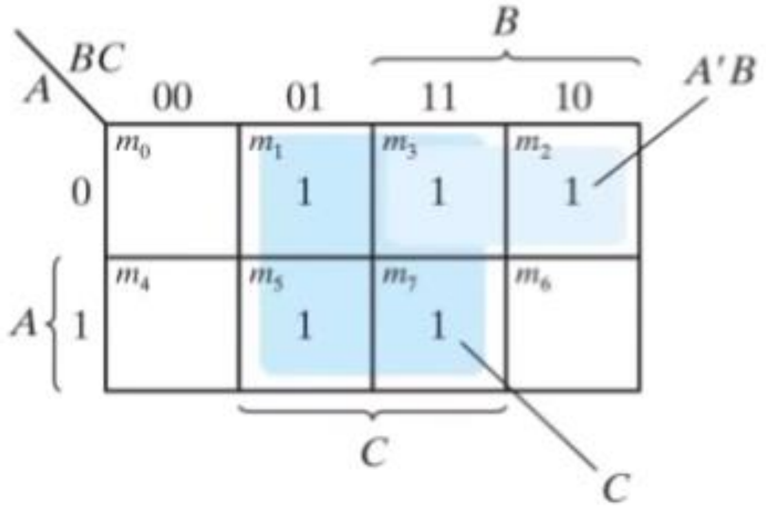
$$
\begin{aligned}
m_0 + m_2 + m_4 + m_6 &= x'y'z' + x'yz' + xy'z' + xyz' && (z' \text{ with the 4 combinations of } x, y)\\
&= (x'y' + x'y + xy' + xy)z' && (z' \cdot \textstyle\sum_{\text{all Minterms of } x,\ y} = z')\\
&= (x'(y' + y) + x(y' + y))z' = (x' + x)z' = z'.
\end{aligned}
$$

$F = xy' + z'$.

Example *Consider the function* $F = A'C + A'B + AB'C + BC.$

1. *Express the function as a sum of Minterms.*
2. *Find the minimal SOP expression.*

Hint: *each SOP term missing m literals will be expanded by* 2^m *Minterms.*



$$F(A,B,C) = \sum(1,2,3,5,7)$$
$$F = C + A'B.$$

Golden Rules to Remember:

- Only one-bit (variable) change for any two adjacent squares!
- The boundaries are adjacent as well.
- A group of ones should be 2^m, where m is the number of removed variables in this group (SOP), and the SOP will have $n - m$ literals.
- Therefore, maximize the number of 1s in each group to minimize the number of literals in the SOP.
- Minimize the number of groups (the SOP terms).
- Therefore, start with the most isolated 1s.
- Make sure of number of **variables** and their **order**
- The number of literals in each group (SOP) is the number of inputs to its `AND` gate.
- The number of groups is the number of SOP terms is the number of `AND` gates is the number of inputs to the `OR` gate.
- All Minterms are covered.

Four-Variable Map

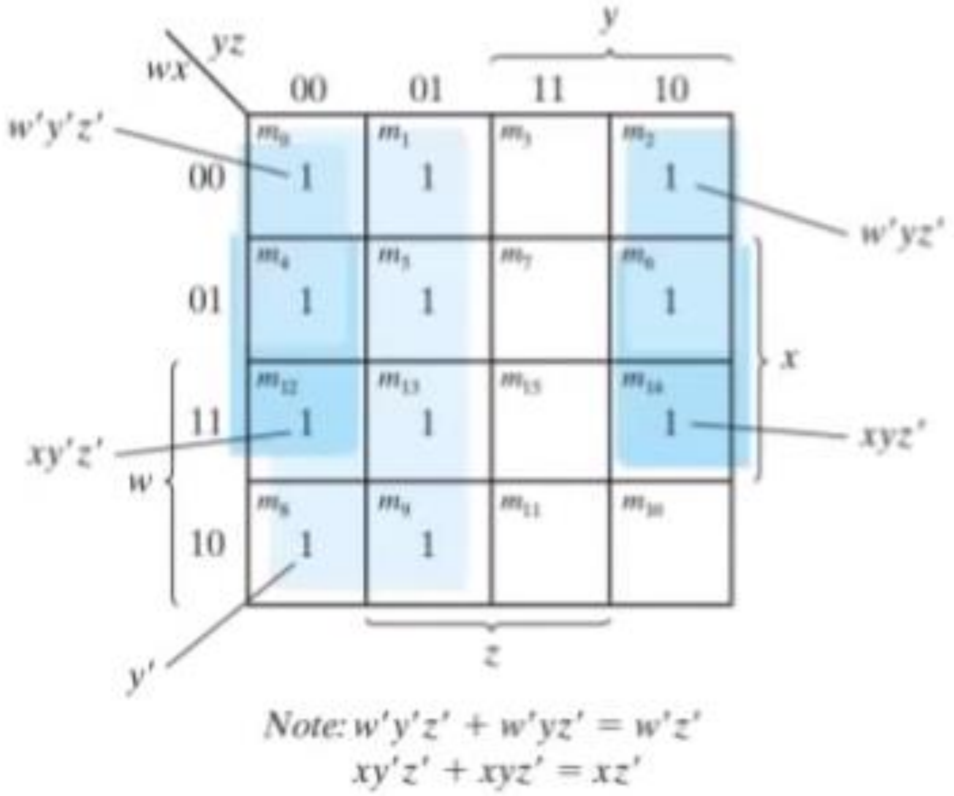
WX \ YZ	00	01	11	10
00	$\overline{W}\overline{X}\overline{Y}\overline{Z}$	$\overline{W}\overline{X}\overline{Y}Z$	$\overline{W}\overline{X}YZ$	$\overline{W}\overline{X}Y\overline{Z}$
01	$\overline{W}X\overline{Y}\overline{Z}$	$\overline{W}X\overline{Y}Z$	$\overline{W}XYZ$	$\overline{W}XY\overline{Z}$
11	$WX\overline{Y}\overline{Z}$	$WX\overline{Y}Z$	$WXYZ$	$WXY\overline{Z}$
10	$W\overline{X}\overline{Y}\overline{Z}$	$W\overline{X}\overline{Y}Z$	$W\overline{X}YZ$	$W\overline{X}Y\overline{Z}$

wx \ yz	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$

- Adjacency from top-bottom, right-left, and corners.
- **Corners:** w and y took their 4 combinations at $x = 0,\ z = 0$: $\Longrightarrow$

$$\begin{aligned} m_0 + m_2 + m_8 + m_{10} &= w'x'y'z' + w'x'yz' + wx'y'z' + wx'yz' \\ &= (w'y' + w'y + wy' + wy)x'z' \\ &= x'z'. \end{aligned}$$

Example *Simplify the function* $F(w, x, y, z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$

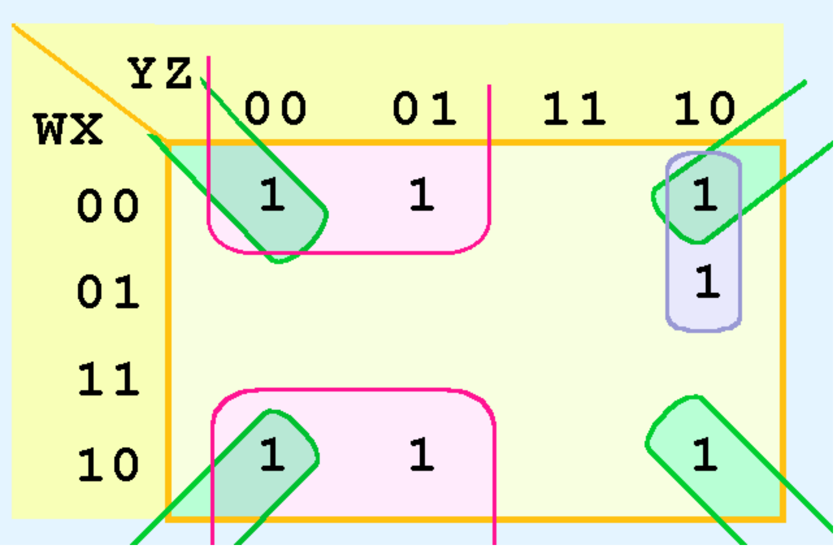


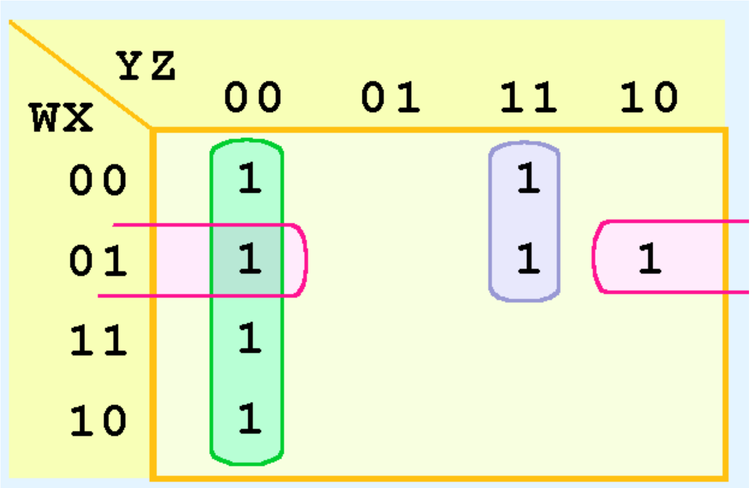
$$F = y' + w'z' + xz'.$$

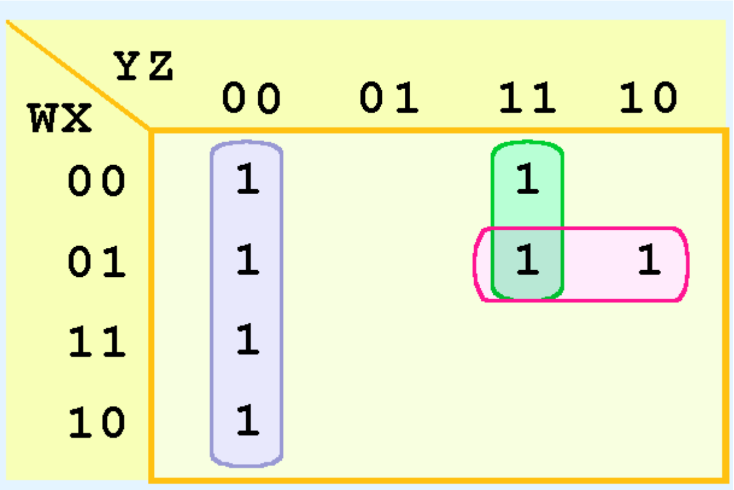
WX \ YZ	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$$F(W,X,Y,Z) = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z} + \overline{W}\,\overline{X}\,\overline{Y}Z + \overline{W}\,\overline{X}Y\overline{Z} \\ + \overline{W}XY\overline{Z} + W\overline{X}\,\overline{Y}\,\overline{Z} + W\overline{X}\,\overline{Y}Z + W\overline{X}Y\overline{Z}$$

$$F(W,X,Y,Z) = \overline{W}\,\overline{Y} + \overline{X}\,\overline{Z} + \overline{W}Y\overline{Z}$$







$$F(W,X,Y,Z) = \overline{W}\,\overline{Y} + YZ$$

$$F(W,X,Y,Z) = \overline{W}Z + YZ$$