Proof:

According to the Big-O definition, $f(n) = O(g(n))$ if there exist constants $c > 0$ and $n_0 \geq 1$ such that:

$$|f(n)| \leq c \cdot g(n) \quad \text{for all } n \geq n_0.$$

Let $f(n) = 9n^3 - 6n^2 + 8$ and $g(n) = n^4$. To prove $f(n) = O(n^4)$, we need to show that:

$$|9n^3 - 6n^2 + 8| \leq c \cdot n^4 \quad \text{for sufficiently large } n.$$

Step 1: Factor n^4 for comparison

Divide $f(n)$ by n^4:

$$\frac{|9n^3 - 6n^2 + 8|}{n^4} = \frac{9}{n} - \frac{6}{n^2} + \frac{8}{n^4}.$$

For sufficiently large n, the terms $\frac{9}{n}$, $\frac{6}{n^2}$, and $\frac{8}{n^4}$ decrease as n grows.

Step 2: Find an upper bound

Since $\frac{9}{n}$, $\frac{6}{n^2}$, and $\frac{8}{n^4}$ all approach 0 as $n \to \infty$, there exists an n_0 such that:

$$\frac{9}{n} - \frac{6}{n^2} + \frac{8}{n^4} \leq 1 \quad \text{for } n \geq n_0.$$

Thus, we can choose a constant $c > 0$ to satisfy the inequality:

$$|9n^3 - 6n^2 + 8| \leq c \cdot n^4.$$

Step 3: Select c and n_0

For simplicity, let $c = 10$ and $n_0 = 2$. Then, for all $n \geq n_0$:

$$|9n^3 - 6n^2 + 8| \leq 10 \cdot n^4.$$

Thus, $f(n) = O(n^4)$ is proven. $\square$