

# 深圳大学实验报告

课程名称: 电磁场与电磁波

实验名称: 电磁场电磁波仿真实验 1

学 院: 电子与信息工程学院

专 业: 电子信息工程

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## Problem one

### Theoretical Calculation

Question: Assuming that charges of density  $\rho_0$  are uniformly distributed inside a sphere of radius  $a$ , find the electric field strength at any point.

Uniformly distributed 1C charge. The charge density is  $\rho_0$ . Radius is  $a$ . Assume dielectric constant inside the sphere is  $\epsilon_1$ . The vacuum is  $\epsilon_0$ .

$\therefore$  According to Gauss's law  $q = \rho_0 \frac{4}{3} \pi r^3$

① Inside the sphere  $r \leq a$ :

$$\int E \cdot ds = \frac{q}{\epsilon_1 \epsilon_0} \quad \therefore E \cdot 4\pi r^2 = \frac{\rho_0 \frac{4}{3} \pi r^3}{\epsilon_1 \epsilon_0} \quad \therefore E = \frac{\rho_0 r}{3 \epsilon_0 \epsilon_1}$$

② Outside the sphere  $r > a$ :

$$\int E \cdot ds = \frac{q}{\epsilon_0} \quad \therefore E \cdot 4\pi r^2 = \frac{\rho_0 \frac{4}{3} \pi a^3}{\epsilon_0} \quad \therefore E = \frac{\rho_0 a^3}{3 \epsilon_0 r^2}$$

Fig 1.1 Theoretical calculation

### Simulation Model

Simulation: A sphere of radius 10 mm and dielectric constant  $(1 + 0.3)$  is uniformly distributed with a charge of 1C, when the center of the sphere is at the origin of the coordinates, find the electric field  $E(x)$  on the x-axis.

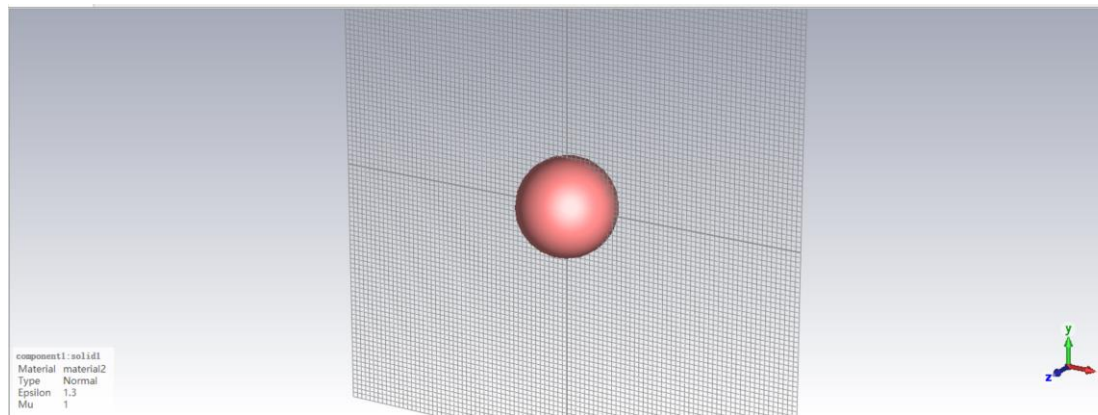


Fig 1.2 Simulation model(1.3 Epsilon)

### Simulation Result(matlab)

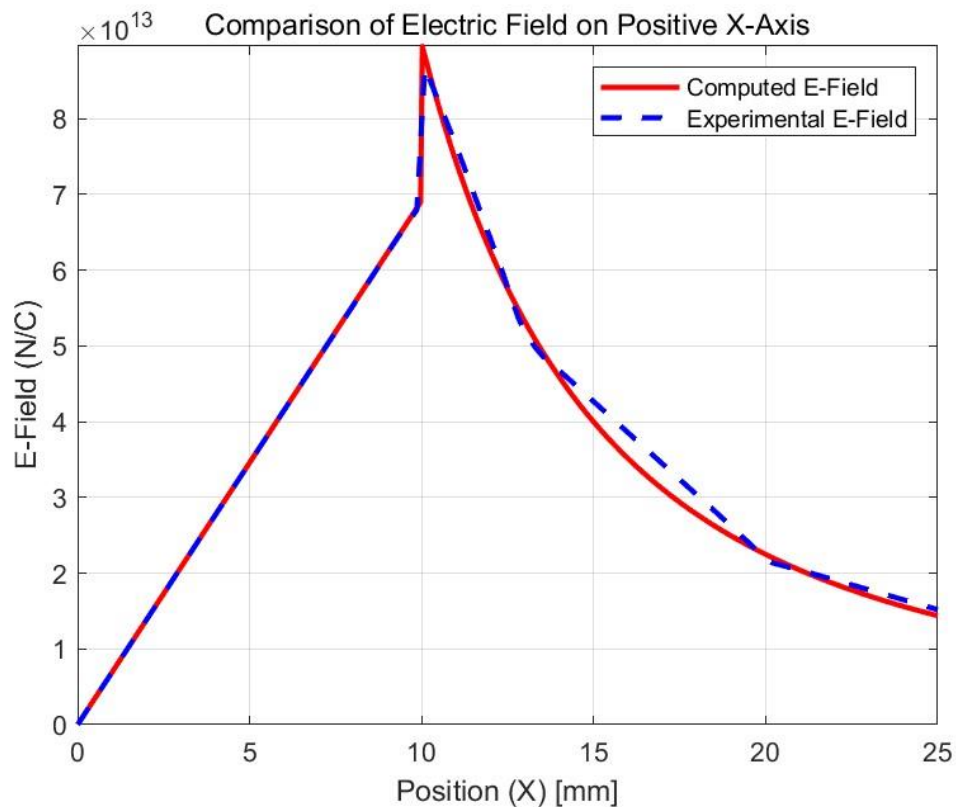


Fig 1.3 Simulation Result(matlab)

#### Comparison and Analysis:

Due to the symmetry, I just show the range of  $x > 0$ . The value of  $x < 0$  is the same as the  $x > 0$ .

We first observe the change tendency of the E distribution for calculation and simulation value. When  $R \leq 10\text{mm}$ , the E distribution increases by Linear growth. When the dielectric constant inside the sphere is greater than the dielectric constant outside the sphere, the image will show a sudden increase, which can be easily understood from the calculation of the equation. When  $R > 10\text{mm}$ , the E distribution decreases by inverse proportional function.

When  $R < 10\text{mm}$ , the calculation and simulation value are coincide. However, when  $R > 10\text{mm}$ , the coincidence degree of curves is not high, and the experimental results show that the curves are not smooth enough

The different may causes by the boundary condition. In the experiment, I set the boundary condition equal to 15 which is relatively small compared to the real situation. Although there exists tiny error, it is acceptable since it is impossible to absolutely simulate the real world condition. When I set the background to 500 when the output is as follows, it can be clearly compared to the original 15 to see that the sampling rate of 500 is higher, the curve is smoother, but it will bring a serious problem is that the calculation time is too long, set 500 when the calculation of 7 minutes.

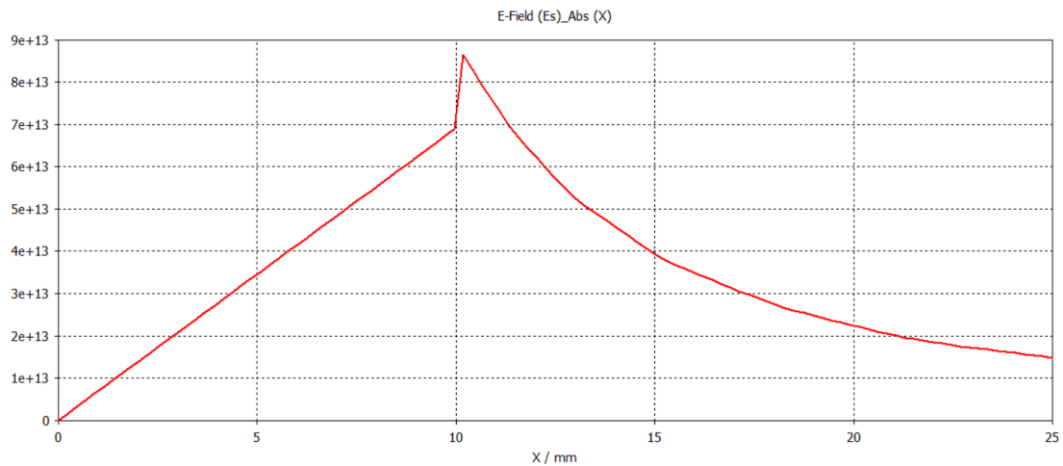


Fig 1.4 Background 500 curve

## Problem Two

### Theoretical Calculation

Question: Find the strength of the electric field on the z-axis of a charged disk of radius a and center at the origin of the coordinates, located in the xoy plane, with a surface charge density of  $\rho_s$ .

$$\begin{aligned} \text{area: } dA &= 2\pi r dr, \quad dq = \rho_s \cdot dA = \rho_s \cdot 2\pi r dr, \quad R = \sqrt{z^2 + r^2}, \quad \cos \theta = \frac{z}{\sqrt{z^2 + r^2}} \\ \therefore d\phi &= \frac{\rho_s 2\pi r dr}{4\pi \epsilon \sqrt{z^2 + r^2}} \quad \therefore \phi = \frac{\rho_s}{2\epsilon_0} (\sqrt{z^2 + a^2} - z) \\ \therefore E &= -\nabla \phi = \frac{\rho_s}{2\epsilon_0} \left( 1 - \frac{z^2}{\sqrt{z^2 + a^2}} \right) \end{aligned}$$

Fig 2.1 Theoretical Calculation

### Simulation Model

Simulation: located in the xoy plane, radius of 10mm, dielectric constant of 1 small disk uniformly distributed  $(1 + 0.3)$  C charge, when the center of the circle at the origin of the coordinates, find the z-axis of the electric field  $E(z)$ .

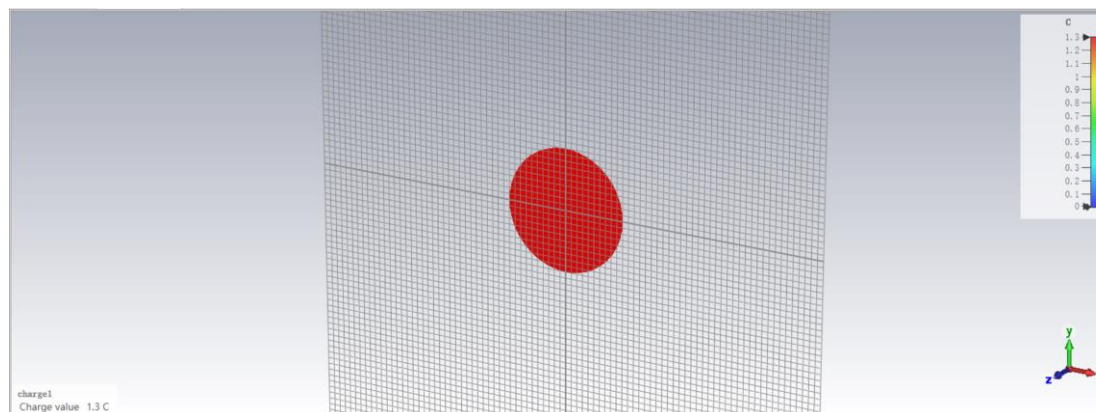


Fig 2.2 Simulation Model(1.3C charge)

### Simulation Result(matlab)

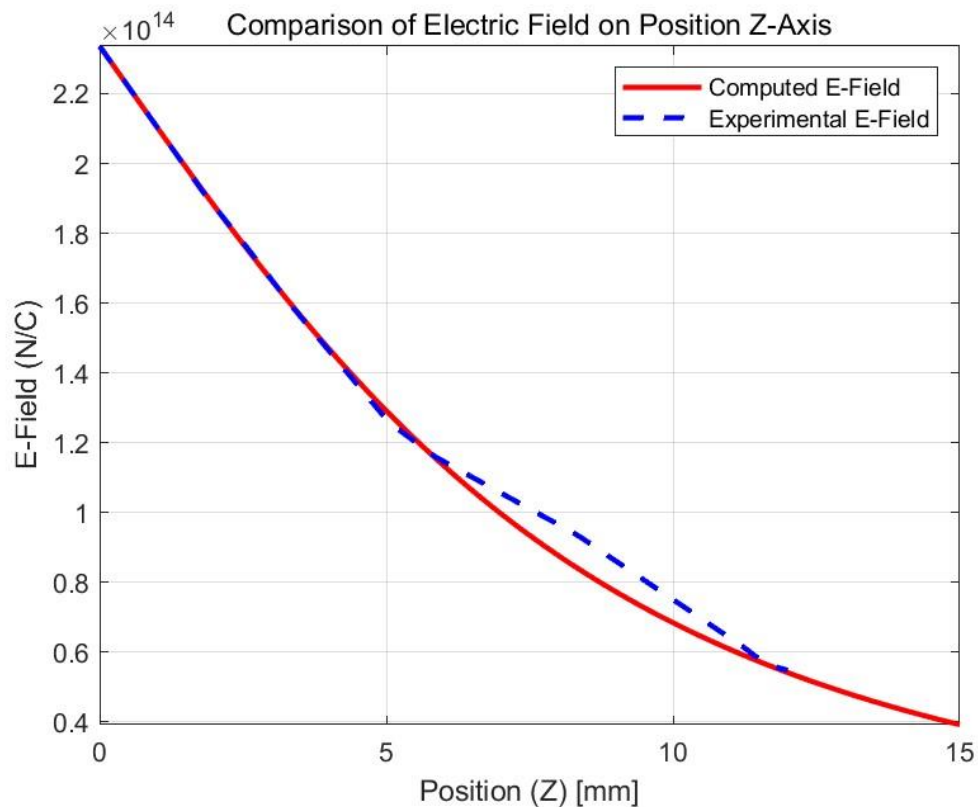


Fig 2.3 Simulation Result(matlab)

### Comparison and Analysis

Due to the symmetry, I just show the range of  $z > 0$ . The value of  $z < 0$  is the same as the  $z > 0$ .

We first observe the change tendency of the E distribution for calculation and simulation value. When  $z \geq 0$  mm, the E distribution gradually decreases by inverse proportional function and finally stable around zero. From the figure, we observe that the calculation and simulation result is basic equal.

## Problem Three

### Theoretical Calculation

Question: A conductor sphere of radius  $a$ , charged with  $Q$ , is coated with a concentric dielectric shell of outer radius  $b$ . Outside the shell is air, as shown in the figure. Find  $D$  and  $E$  at any point in space.

$$\begin{aligned}
 \textcircled{1} \quad r < a \quad \int E_1 \cdot ds &= 0 \quad E_1 = 0 \quad D_1 = \epsilon E_1 = 0 \\
 \textcircled{2} \quad a < r \leq b \quad \int E_2 \cdot ds &= \frac{Q}{\epsilon_0 \epsilon_1} \quad \therefore E_2 = \frac{Q}{4\pi \epsilon_0 \epsilon_1 r^2} \quad D_2 = \epsilon_1 E_2 \quad D_2 = \frac{Q}{4\pi r^2} \\
 \textcircled{3} \quad b < r \quad \int E_3 \cdot ds &= \frac{Q}{\epsilon_0} \quad \therefore E_3 = \frac{Q}{4\pi r^2 \epsilon_0} \quad D_3 = \epsilon_0 E_3 \quad D_3 = \frac{Q}{4\pi r^2}
 \end{aligned}$$

Fig 3.1 Theoretical Calculation

### Simulation Model

Simulation: An ideal conductor sphere of radius 10mm has a uniformly distributed charge of 1C, with the center of the sphere at the origin of the coordinates. A concentric dielectric spherical shell of relative permittivity 4 of radius  $(20+0.3 \times 10)$  mm is attached to the outer shell, find the electric field  $E(x)$  and the potential shift vector  $D(x)$  on the x-axis.

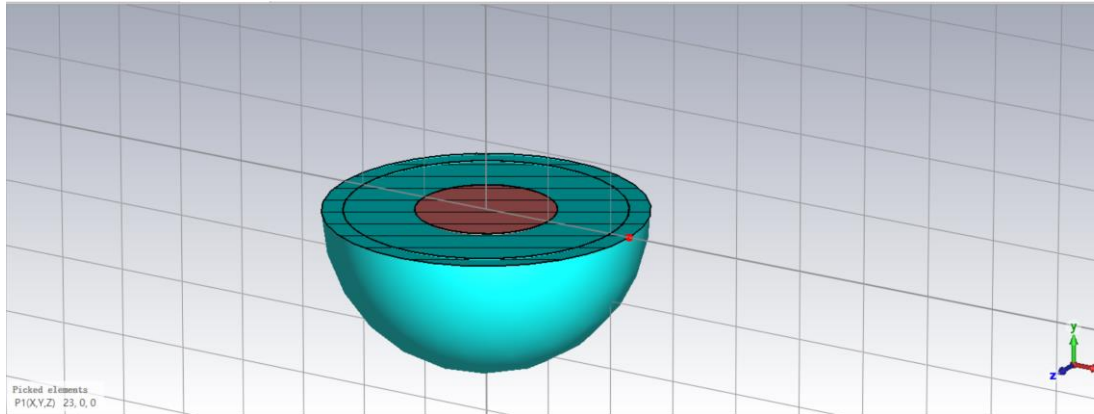


Fig 3.2 Simulation Model(radius = 23)

#### Simulation Result(matlab)

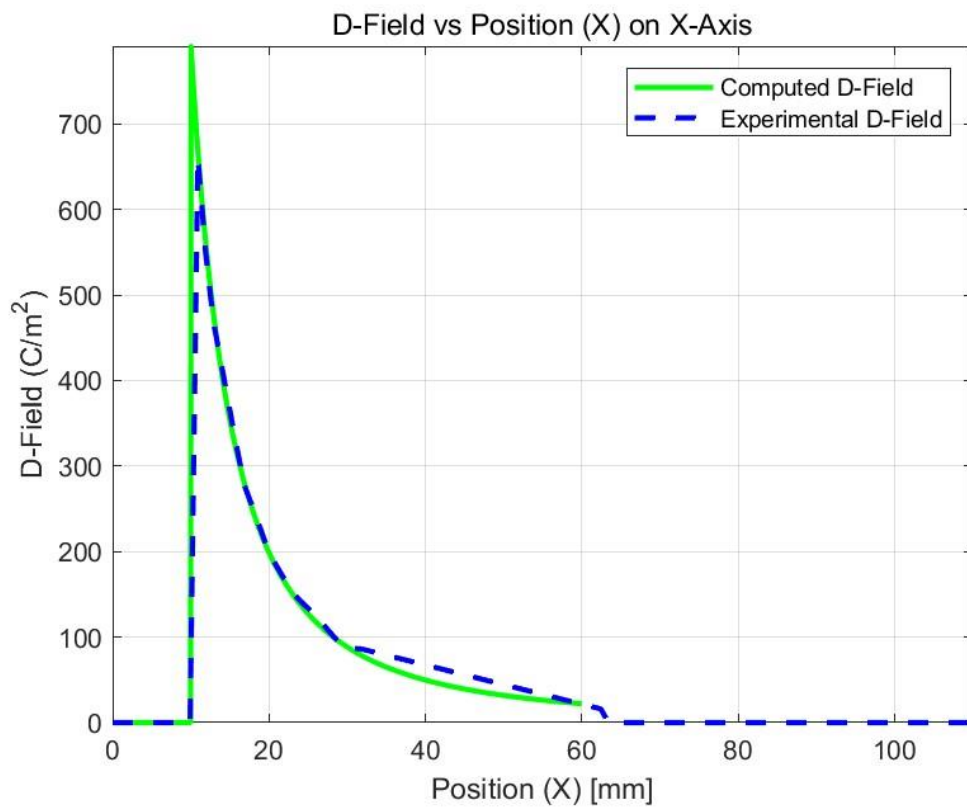


Fig 3.3 Simulation Result(matlab)

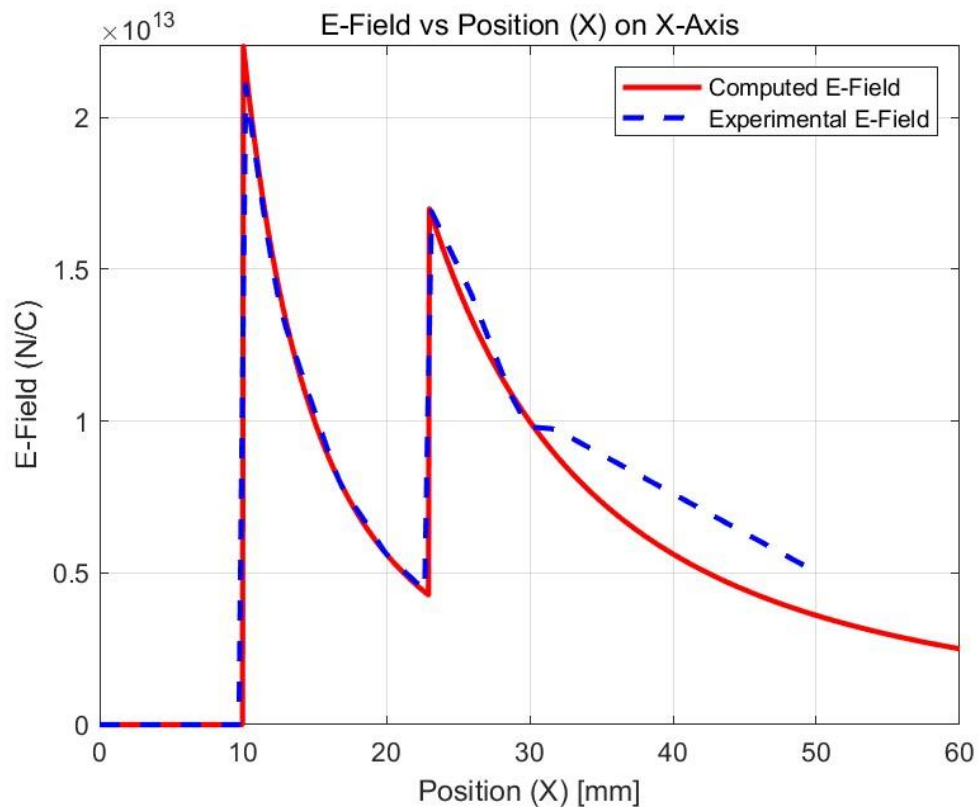


Fig 3.4 Simulation Result(matlab)

### Comparison and Analysis

Due to the symmetry, I just show the range of  $x > 0$ . The value of  $x < 0$  is the same as the  $x > 0$ .

We first observe the change tendency of the E distribution and D distribution for calculation and simulation value. For E distribution, when  $x < 10\text{mm}$ , the E is equal to 0 since the inner sphere has no charge. When  $x = 10\text{mm}$ , there happens a sudden change due to the surface charge of the inner sphere. When  $10 < x < 20$ , E gradually decreases by inverse proportional function. When  $x = 23\text{mm}$ , there happens another sudden change due to the change of epsilon and E increased around four time. When  $x > 23\text{mm}$ , E gradually decreases by inverse proportional function and finally stable around zero. For D distribution, when  $x < 10\text{mm}$ , the D is equal to 0 since the inner sphere has no charge. When  $x = 10$ , there happen a sudden change due to the surface charge of the inner sphere. When  $x > 10\text{mm}$ , D gradually decreases by inverse proportional function and finally closes to zero. It is worth to say that D is irrelevant to epsilon, so there may not change in  $x = 23\text{mm}$ .

We observe there exists some error between calculation and simulation when  $x = 10\text{mm}$ . For the error of  $x = 10\text{mm}$ , one possible reason is that the material of the sphere. In the experiment, we use PEC for conduct sphere, however, in real world the material can not be that perfect. In conclusion, it is inevitable to avoid this tiny errors which are acceptable.

## Problem Four

### Theoretical Calculation

Question: A concentric spherical capacitor has an inner conductor of radius  $a$ , an outer

conductor of inner radius  $b$ , and is filled with two types of dielectrics, the dielectric constant of the upper part being  $\epsilon_1$  and that of the lower part being  $\epsilon_2$ , as shown in the figure. Let the inner and outer conductors be charged with  $q$  and  $-q$ , respectively, and find the potential shift vector and the electric field strength in each part.

On the dividing surface of the two media :  $\vec{E}_1 = \vec{E}_2 = \vec{E}_r$

Due to the symmetry of the field distribution:  $\epsilon_1 \vec{E}_r 2\pi r^2 + \epsilon_2 \vec{E}_r 2\pi r^2 = q$

$$\therefore \vec{E}_r = \frac{q}{2\pi r^2 (\epsilon_1 + \epsilon_2)}$$

$$\therefore \text{Top half : } D_1 = \epsilon_1 \vec{E}_r = \frac{q \epsilon_1}{2\pi r^2 (\epsilon_1 + \epsilon_2)}$$

$$\text{Bottom half : } D_2 = \epsilon_2 \vec{E}_r = \frac{q \epsilon_2}{2\pi r^2 (\epsilon_1 + \epsilon_2)}$$

Fig 4.1 Theoretical Calculation

### Simulation Model

Simulation: A concentric sphere capacitor has an inner conductor with a radius of 10 mm and an outer conductor with an inner radius of 20 mm, which is filled with two types of dielectrics, the relative permittivity of the upper half of which is  $(2 + 0.3)$  and the lower half of which is 4. Given that the inner and outer conductors are charged with 1C and -1C, respectively, find the electric field  $E(x)$  and the vector of potential shifts  $D(x)$  on the x-axis. (The x-axis passes through the vertices of the two dielectric hemispheres and the centers of the concentric spheres, pointing toward the hemisphere with the smaller dielectric constant)

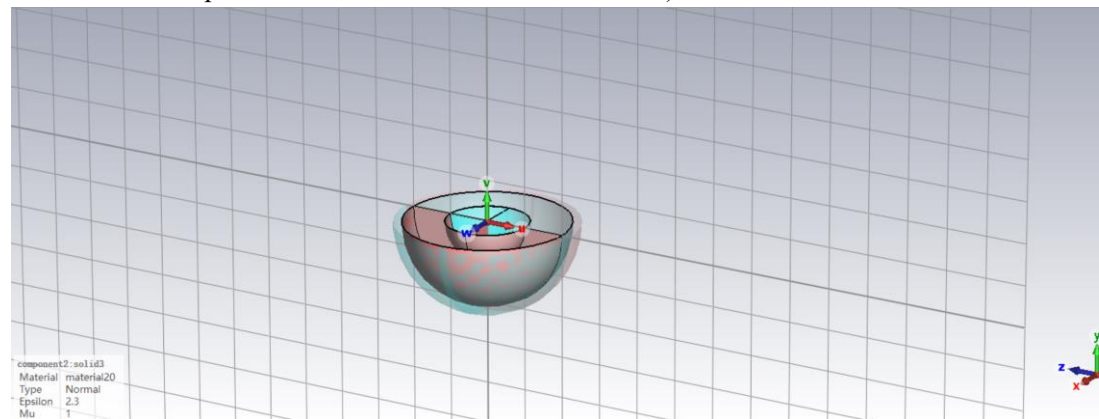


Fig 4.2 Simulation Model(x forward Epsilon = 2.3)

### Simulation Result(matlab)



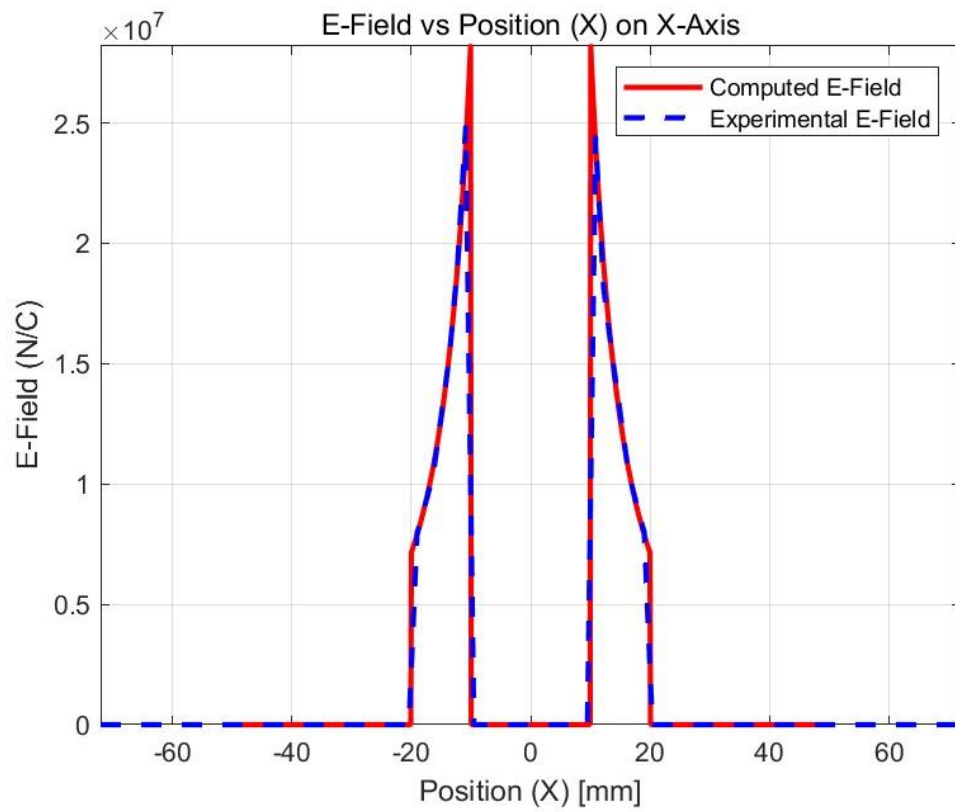


Fig 4.3 Simulation Result(matlab)

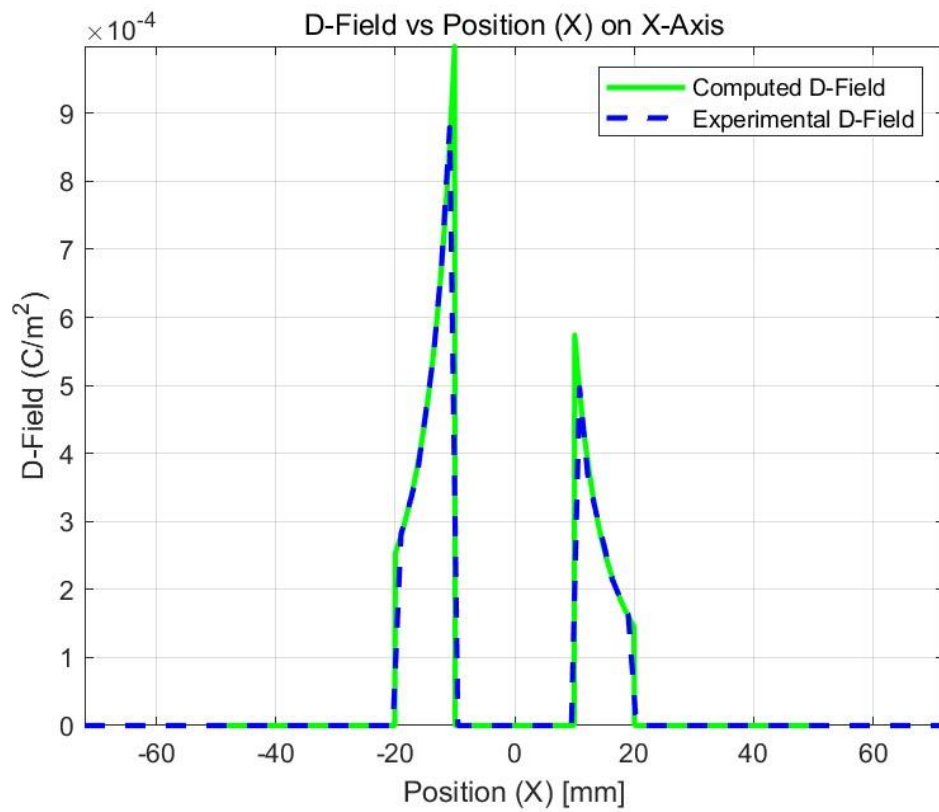


Fig 4.4 Simulation Result(matlab)

### Comparison and Analysis

I plot the whole x axis for the E distribution and D distribution since there may be different around the  $x = 0\text{mm}$ . We first observe the change tendency of the E distribution and D distribution for calculation and simulation value. For E distribution, we observe that E is symmetry to  $x = 0\text{mm}$  due to the boundary condition of E theorem, so we just look the range of  $x > 0\text{mm}$ . When  $0 < x < 10\text{mm}$ , E is equal to 0 since there is no free charge in the inner sphere. When  $x = 10\text{mm}$ , there happens a sudden change due to the surface charge of the inner sphere. When  $10 < x < 20$ , E gradually decreases by inverse proportional function. When  $x = 20\text{mm}$ , E is equal to 0 since conduct sphere isolates the outside area. For D distribution, when  $x < -20$ , D is equal to 0 since conduct sphere isolates the outside area. When  $x = -20\text{mm}$ , there happens a sudden change due to the surface charge of the inner sphere. When  $-20 < x < -10$ , D increases by inverse proportional function. When  $-10 \leq x \leq 10\text{mm}$ , D is equal to 0 due the characteristic of conduct sphere. When  $x = 10\text{mm}$ , there happen another change due to the surface charge of the inner sphere. When  $10 < x \leq 20\text{mm}$ , D is decreased by inverse proportional function. When  $x > 20\text{mm}$ , D is equal to 0 since the outer conduct sphere isolates the outside area. It is worth to say that when  $-20 < x < -10$  and  $10 < x < 20$ , the change magnitudes are different since the epsilon is different in these two specific area.

We observe there exist some error between calculation and simulation result when  $x = -10\text{mm}$  and  $x = 10\text{mm}$ . When  $x = -10$  and  $x = 10\text{mm}$ , it cross the shell of the conduct sphere and its material is ideal PEC which may be different with the real word situation so it may cause this error, however, the tiny error is acceptable.

### Problem Five

#### Theoretical Calculation

Question: A uniform body charge of density  $\rho$  is distributed between two spherical surfaces of radius  $a$  and  $b$  ( $a > b$ ) and center distance  $c$  ( $c < a - b$ ). Let  $c$  be along the x-axis in the positive direction.

$$\begin{aligned} \text{As for the radius } a \text{'s sphere: } E_1 &= \frac{\rho r_1}{3\epsilon_0} \vec{e}_{r_1} \\ \text{As for the radius } b \text{'s sphere: } E_2 &= \frac{\rho r_2}{3\epsilon_0} \vec{e}_{r_2} \\ \because r_1 \vec{e}_{r_1} - r_2 \vec{e}_{r_2} &= c \vec{e}_x = c \quad \therefore \vec{E} = \frac{\rho c}{3\epsilon_0} \vec{e}_x \quad \text{a constant} \end{aligned}$$

Fig 5.1 Theoretical Calculation

#### Simulation Model

Simulation: There is a 1C uniform body charge distribution between two spherical surfaces of radius 20mm and 8mm, corresponding to the spherical center coordinates (0,0,0) and (10mm,0,0), respectively, and find the electric field  $E(x)$  at [2mm,18mm] on the x-axis.

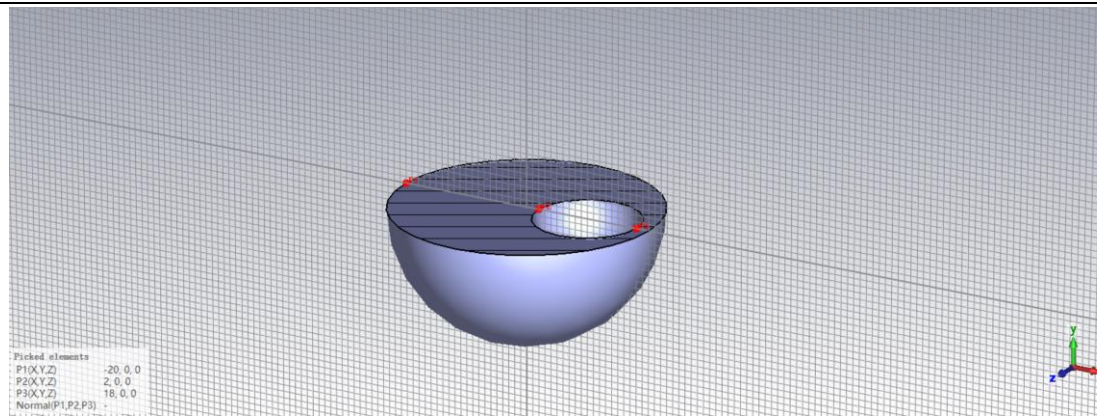


Fig 5.2 Simulation Model

### Simulation Result(matlab)

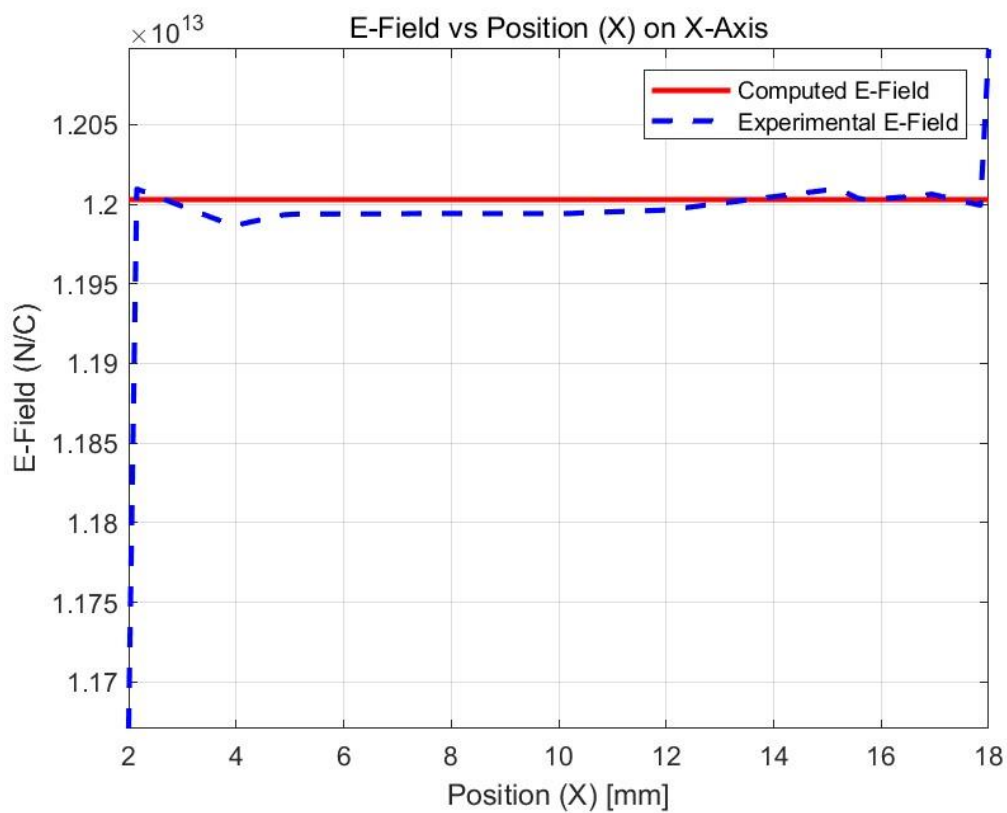


Fig 5.3 Simulation Result(matlab)

### Comparison and Analysis

For the calculation result, it confirms that E is a constant value. However, according to the simulation results, in the range of 2mm to 18mm, the experimental values of E fluctuate. From the overall results, it can be seen that the experimental graph and the calculation graph show a similar shape, but the values fluctuate in the experimental graph. This may be related to the selection of a too small background value during the experiment, resulting in fewer sampling points in the final experiment. The curve obtained from the experiment is not smooth and is rather jagged, which is a problem in all previous experiments. Generally, setting the background to about twice the size of the graph is sufficient. Setting it too large can produce a smooth curve but will lead to significant computational overhead.

五、指导教师批阅意见：

成绩评定：

指导老师签名：  
年 月 日