深 圳 大 学 实 验 报 告

课程名称：　随机信号处理

实验项目名称：　**Experiment Report three**

学院：　电子与信息工程学院

专业：　电子信息工程

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教务处制

Description of format:

- Use Times New Roman, 12 pt, single column, single line spacing.
- When inserting figures and tables, title of the figures and tables must be included.
- Do not change '1、Purposes of the experiment' and '2、Design task and detail requirement'.

1、Purposes of the experiment

1) learn the periodogram and Correlogram method to estimate power spectrum.
2) Use Matlab to sample a chirp signal and learn the matched filter.
3) Analyze the results and draw reasonable conclusions

2、Design task and detail requirement

See 'Appendix 1 – Task and requirement for experimental report 3.doc'.

3、The result and Analysis

- **Part 1: Basic 1 (40 points)**

You should submit your codes that can generate the figures in 3). The codes should be runnable!

1) Plot the Periodogram with different window (rectangular and hamming), and compare the results, describe the differences.

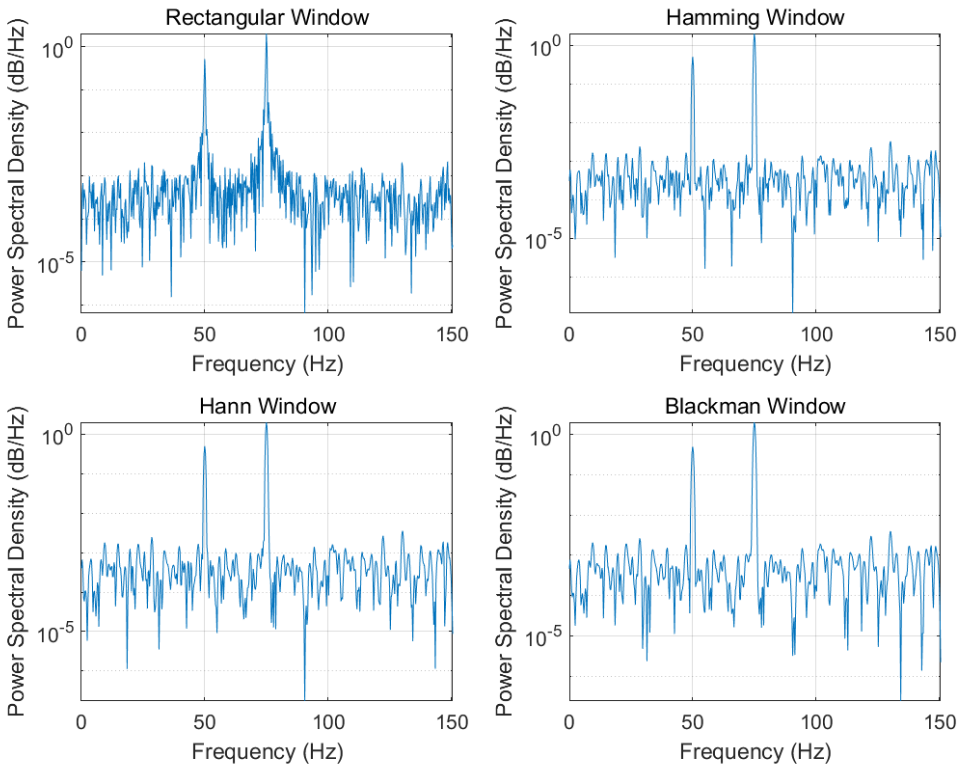


Fig 1 Periodogram with different window

The objective of this experiment is to analyze the effects of different window functions on the periodogram of a signal composed of sinusoidal and cosinusoidal components with added white Gaussian noise. The **parameters** of the experiment are defined as follows:

Signal Composition: $X(t) = \sin(\omega_1 t) + 2cos(\omega_2 t) + N(t)$ and ω_1=100π, ω_2=150π, $N(t)$ is zero mean White Gaussian Noise

Noise Variance: σ^2=0.1

Signal Duration: 2 seconds

Sampling Rate: To satisfy the Nyquist criterion, the sample rate is set slightly above twice the highest frequency in the signal, calculated as $2\omega_2/\pi+1$

Analysis of Periodograms with Different Windows:

Rectangular Window: The PSD plot shows the most significant leakage among the four windows, leading to a less clear distinction of the primary frequency components at 50 Hz and 75 Hz. However, it offers the best resolution, which is evident from the relatively narrow main lobes.

Hamming Window: This window reduces spectral leakage better than the rectangular window, as seen by the decreased amplitude of side lobes. The main lobes are slightly broader, which may affect the resolution of closely spaced frequencies but enhances the clarity of peak frequencies by reducing noise around them.

Hann Window: The Hann window shows further reduction in side lobe amplitude compared to the Hamming window. This characteristic is beneficial for applications where suppression of side lobe levels is crucial, although it comes with even broader main lobes, slightly compromising frequency resolution.

Blackman Window: This window exhibits the greatest reduction in side lobes among the four types, making it highly effective for applications requiring minimal spectral leakage. The trade-off is the broadest main lobes, significantly impacting the ability to resolve close frequency components.

Each window function offers a trade-off between resolution and spectral leakage. The choice of window depends on the specific requirements of the analysis. For applications needing precise frequency resolution, the Rectangular window might be preferable despite its high leakage. When minimizing leakage is a priority, especially in noisy environments, the Blackman window would be ideal. The Hamming and Hann windows provide a balanced approach, suitable for general purposes where moderate resolution and leakage suppression are needed.

2) Change the sampling rate, signal length, FFT length and the value of σ^2, use the Periodogram to do the spectrum estimation. Show your results (you can use figures and/or figures), and give analysis.

In this experiment, we analyzed the power spectral density of a signal composed of a sine and a cosine function with varying parameters like sampling rate, signal length, noise variance, and FFT length. The objective was to understand how changes in these parameters influence the spectrum estimation of the signal, given by the equation:

$X(t) = \sin(\omega_1 t) + 2cos(\omega_2 t) + N(t)$ and ω_1=100π, ω_2=150π, $N(t)$ is zero mean White Gaussian Noise

Analysis by Parameters:

Sampling Rate:

300 Hz: Shows clear peaks at 50 Hz and 75 Hz but with considerable noise.

400 Hz: Similar clarity in peaks with slight improvements in noise reduction.

600 Hz: Best performance with the clearest peaks and reduced noise, showing the benefit of higher sampling rates for better frequency resolution and noise handling.

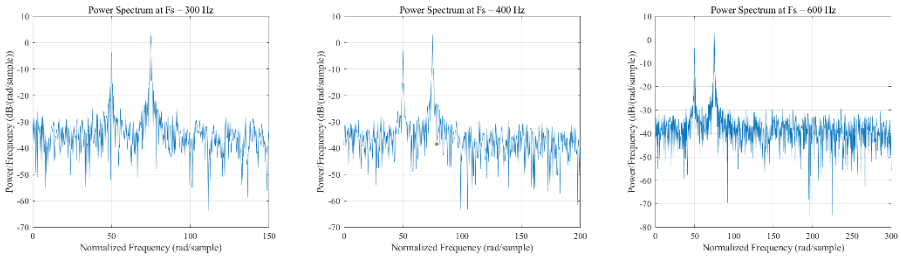


Fig 2 Different sampling rate

Signal Length:

1 second: The spectrum has wider peaks, indicating less precise frequency estimation due to shorter data duration.

2 seconds: Improves the resolution with narrower peaks.

4 seconds: Offers the best frequency resolution with very sharp and distinct peaks, demonstrating how longer signal lengths provide more accurate spectral estimates.

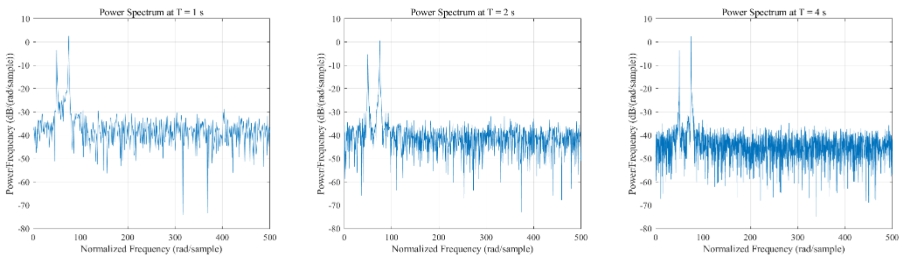


Fig 3 Different signal length

Noise Variance:

0.01: Low noise level leading to very clear and distinct peaks.

0.1: Moderate noise, still showing clear peaks but with some spectral smearing.

1.0: High noise level significantly affects clarity, showing the impact of increased noise on spectral purity and peak detection.

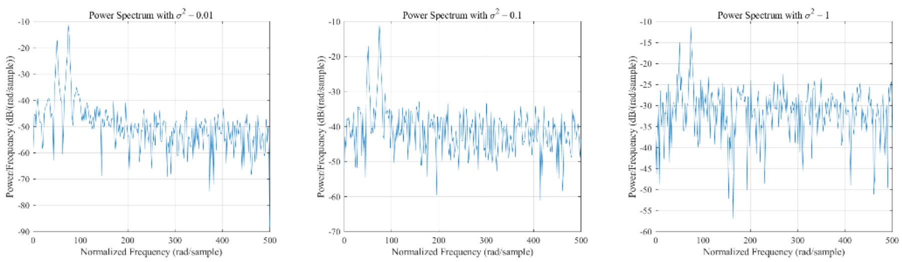


Fig 4 Different noise variance

FFT Length:

256: Provides the coarsest frequency resolution, evident from broader peaks.

512: Offers a balance between resolution and computational efficiency.

1024: Shows the highest resolution with very narrow and distinct peaks, highlighting how larger FFT lengths improve frequency resolution at the cost of increased computational demand.

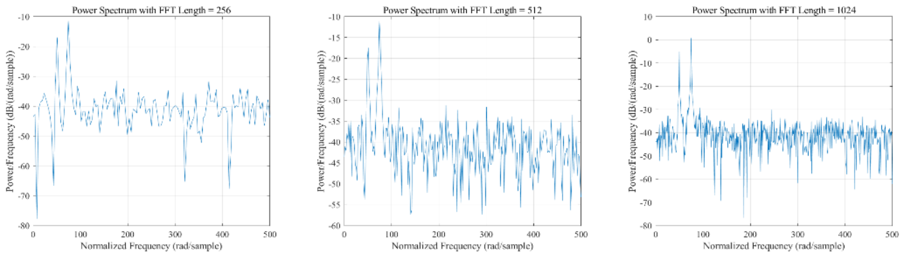


Fig 5 Different FFT length

In conclusion Sampling Rate: Increasing the sampling rate improves the spectral resolution and the ability to handle noise, crucial for accurate frequency detection. Signal Length: Longer signal durations enhance the spectral resolution, allowing for more precise frequency estimations. Noise Variance: Lower noise variances yield clearer spectral peaks, which are essential for accurate signal analysis. FFT Length: Larger FFT lengths increase the frequency resolution, beneficial for detailed spectral analysis but require more computational resources.

3) plot the figures/tables in 2) using your own Periodogram and Correlogram again, and show the comparison between your own Periodogram and Correlogram function and the default Periodogram function used in 2)

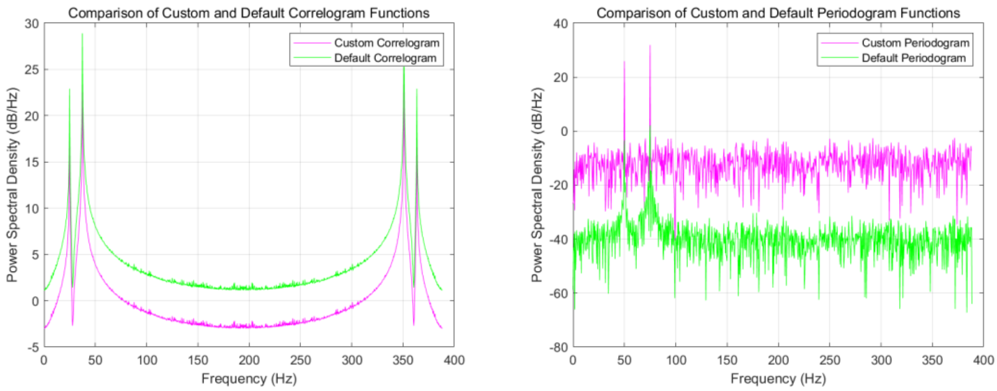


Fig 6 Periodogram and Correlogram

In this experiment, we explore the spectral estimation techniques using both standard and custom implementations of Periodogram and Correlogram functions, applied to a synthetic signal. The signal is composed of a sine and cosine function with added white Gaussian noise:

$X(t) = \sin(\omega_1 t) + 2cos(\omega_2 t) + N(t)$ and ω_1=100π, ω_2=150π, $N(t)$ is zero mean White Gaussian Noise. A signal sampled at 1000 Hz over 2 seconds.

Spectral Analysis:

- **Custom Periodogram**: Implemented using the fast Fourier transform (FFT) and normalized power computation.
- **Custom Correlogram**: Based on the biased autocorrelation of the signal and its FFT.

- **Default Periodogram and Correlogram**: MATLAB’s built-in periodogram and xcorr functions are used for comparison.

Periodogram Comparison:

The custom Periodogram closely matches the default in terms of peak locations, which are primarily observed at 50 Hz and 75 Hz—the frequencies corresponding to $\omega_1/2\pi$and $\omega_2/2\pi$respectively. The custom implementation shows slightly different power levels due to potential differences in windowing or normalization methods used in MATLAB's default implementation.

Correlogram Comparison:

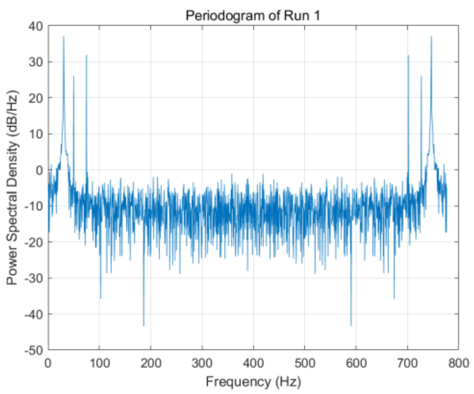
The custom Correlogram shows similar spectral characteristics as the default, but with smoother transitions between the spectral components. This could indicate differences in how autocorrelation values are handled or normalized. Both correlograms correctly identify the main frequency components, although the custom version may apply a different scaling or normalization, affecting the absolute power values but not the frequency locations.

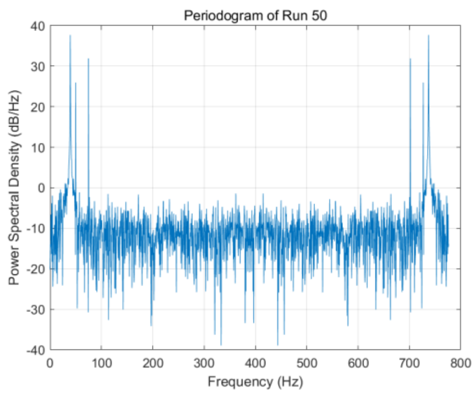
In conclusion. Both custom functions demonstrate the ability to accurately identify significant spectral components, which validates their reliability for basic spectral analysis tasks. Minor differences in the power spectral density values between custom and default functions suggest variations in underlying implementations, such as window functions, FFT length, and normalization techniques. These factors can influence the resolution and leakage characteristics of the spectral estimates.

- **Part 2: Basic 2 (40 points)**

You should submit your codes that can generate the figures in 1). The codes should be runnable!

1) Plot the periodogram of the 1st, 50nd, 100nd run and the power spectrum. (there are totally four figures, show your figures here only, analysis can be given in 2) below)





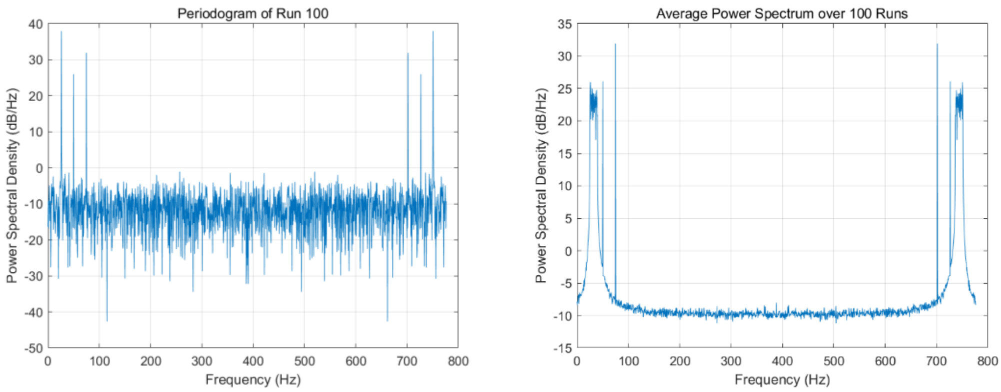


Fig 7 Periodogram of run and the power spectrum

In this experiment, we evaluate the spectral characteristics of a composite signal containing both deterministic and random components, with an added interference term. The signal is given by:

$$X(t) = \sin(\omega_1 t) + 2cos(\omega_2 t) + 4cos(\omega_I t) + N(t)$$

where ω_1=100π rad/s, ω_2=150π rad/s, N(t) represents zero-mean white Gaussian noise with a variance σ2=0.1, and ω_I is a random frequency uniformly distributed between 50π and 80π rad/s. The task involves generating this signal for 100 runs, each for a duration of 2 seconds, and analyzing its frequency content using a custom periodogram function.

For each of the 100 runs, ω_I is varied randomly within the specified range, introducing variability in the interference frequency across trials. A custom periodogram function is used to compute the power spectral density of the signal in each trial.

Analysis:

Each periodogram shows significant peaks at frequencies corresponding to $\omega_1/2\pi \approx 50$ Hz and $\omega_2/2\pi \approx 75$ Hz, reflecting the sine and cosine components of the signal. Additional peaks appear due to the random interference frequency $\omega_I/2\pi$, which falls between 25 Hz and 40 Hz. These peaks vary between runs due to the randomness in ω_I.

The average power spectrum consolidates the spectral content from all runs, smoothing out variations due to noise and emphasizing consistent frequency components. The deterministic components at 50 Hz and 75 Hz show prominently due to their presence in every run. The effect of the interference is manifested as a broadened peak area spanning 25 Hz to 40 Hz, rather than distinct peaks, due to averaging over various ω_I values.

2) Show the power spectrum result for different σ^2 and provide analysis.

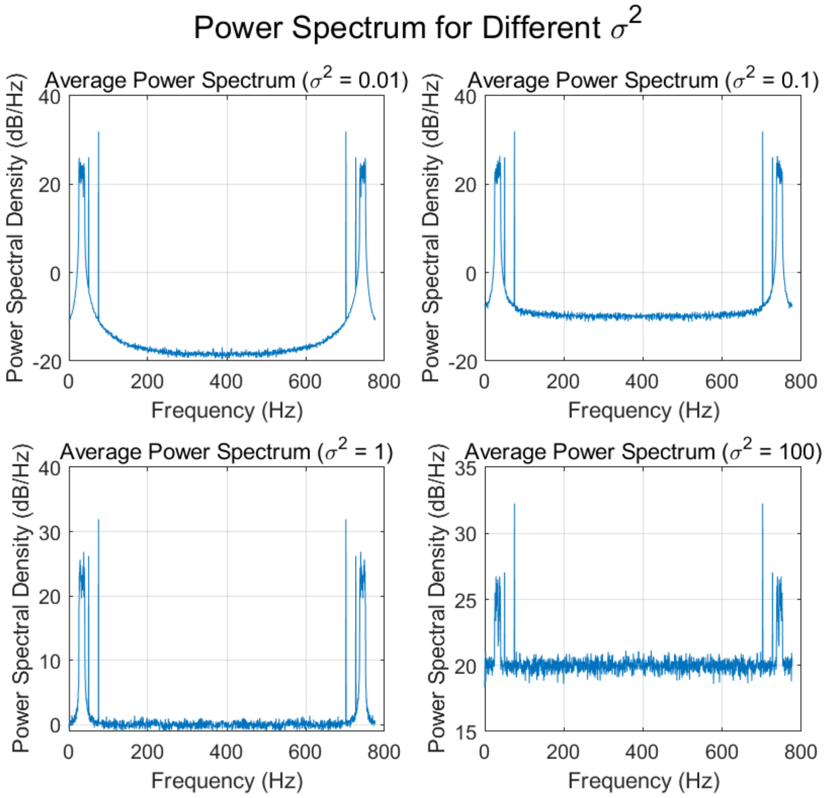


Fig 8 Power Spectrum for different σ^2

The experiment was designed to evaluate the impact of varying noise variances (σ^2) on the power spectrum of a signal composed of deterministic sinusoidalcomponents and a randomly varying interference term. The power spectra were calculated for different noise levels to observe how increasing noise affects the visibility of the signal's frequency components.

$X(t) = \sin(\omega_1 t) + 2cos(\omega_2 t) + 4cos(\omega_I t) + N(t) \quad \sigma^2$=0.01,0.1,1,100

Analysis:

Low Noise Variance: (σ^2=0.01 and 0.1)Clarity: The deterministic components at 50 Hz and 75 Hz are clearly visible with sharp peaks, indicating minimal distortion from noise. Interference Visibility: The interference component appears as distinct peaks within the range of 25 Hz to 40 Hz. The low noise level allows for the clear observation of these components.

Moderate Noise Variance: (σ^2=1) Impact on Visibility: The primary frequencies are still identifiable, but the peaks are less sharp compared to lower noise levels. This suggests some masking effect of the noise, but not to a degree that significantly obscures the signal's components.

High Noise Variance: (σ^2=100) Significant Masking: The increased noise significantly affects the clarity of all frequency components. Peaks at 50 Hz and 75 Hz are still visible but are no longer distinct and sharp. The interference range becomes harder to distinguish from the noise floor, demonstrating how high noise levels can obscure the underlying signal characteristics.

In conclusion Noise Sensitivity: The results illustrate the signal's sensitivity to noise. Lower values of σ2\sigma^2σ2 maintain the integrity of the spectral features, while higher values lead to spectral blurring and loss of detail. Spectral Resolution: The spectral resolution remains consistent across different noise levels due to the fixed sample rate and periodogram computation method. However, noise influences the amplitude accuracy and visibility of the spectral components. Practical Implications: In practical scenarios where noise levels can be high, it is crucial to implement noise reduction techniques or increase the power of the signal components (if possible) to ensure critical frequencies are detectable.

- **Part 3: Advance (40 points)**

1) You are required to submit your code, and your code should directly give all the tables or figures in 1.2).

1.1) Plot your system flow chart. You can provide necessary explanations.

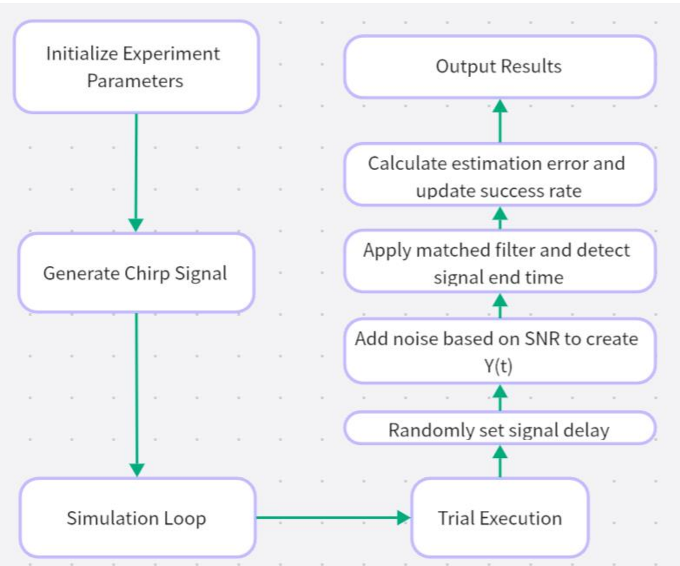
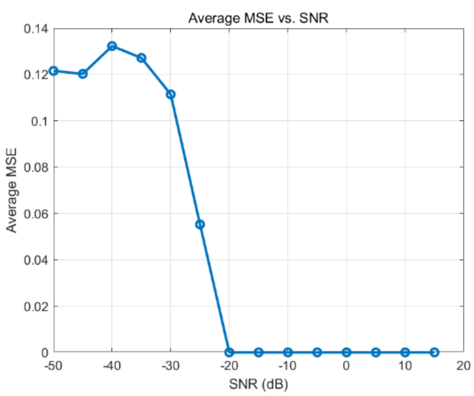
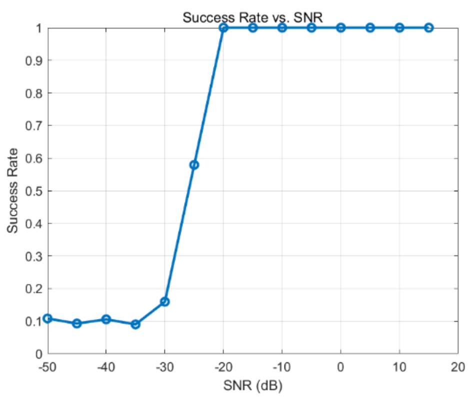


Fig 9 System Flow Chart of advance 1

1.2) Give your MSE and success rate results, and analysis, under different SNR. (Hint: use table or figure, and you should choose an SNR range that can at least see '100% success' and '100% fail')





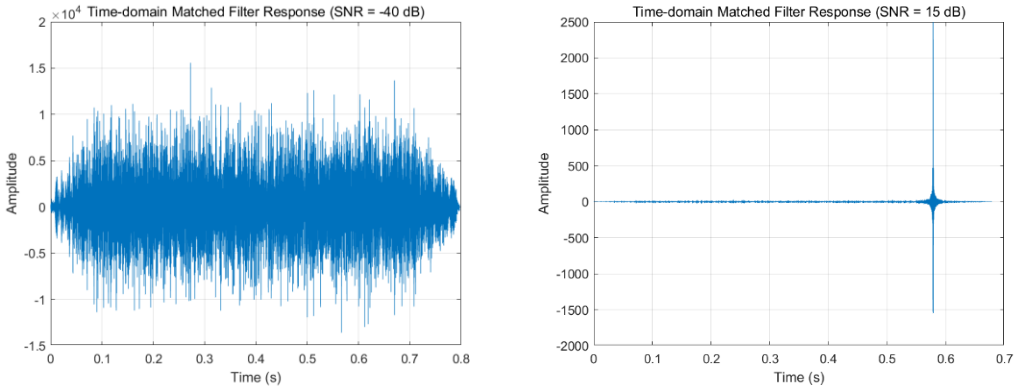


Fig 10 Under SNR=-40dB and SNR=15dB

The experiment aimed to evaluate the performance of a matched filter designed for a chirp signal under various signal-to-noise ratio (SNR) conditions. The primary goal was to detect the presence and estimate the end time of a chirp signal when mixed with noise. This setup allows us to understand the effectiveness of matched filtering in a radar signal processing scenario, here is the experiment parameters:

A chirp signal $X(t) = \cos[2\pi(f_0 t + kt^2)]$, where f_0=1000Hz, k=12000Hz, and the signal starts from 0 to 0.1s ($t \in [t_{\min} = 0,\ t_{\max} = 0.1]$) Sampling Frequency: 50000 Hz The reflected signal $X_1(t)$ is assumed to end randomly between 0.11 s and 1 s The received signal is modeled as $Y(t)=X_1(t)+N(t)$, where $N(t)$ represents Gaussian noise. SNR Levels: Tested across a range from -50 dB to 15 dB, calculated as $10\cdot\log_{10}(Ps/\sigma^2)$

Analysis:

At SNR = -40 dB: The response is noisy, indicating difficulty in signal detection.At SNR = 15 dB: Clear detection with a sharp peak, showcasing effective signal extraction. Success rates are extremely low at SNRs below -20 dB, with a significant improvement observed as SNR increases, stabilizing at nearly 100% for SNRs above -10 dB. High MSE values at low SNRs reflect poor estimation accuracy, which sharply improves and stabilizes at minimal levels beyond -10 dB SNR.

In conclusion. The matched filter effectively detects the chirp signal and accurately estimates its end time in high SNR scenarios. The performance in low SNR conditions highlights the challenges in practical radar systems, particularly when targets are weak or near the noise level. Future work should focus on enhancing filter designs or employing advanced signal processing techniques to improve detection reliability in low SNR environments. The results indicate that maintaining an operational SNR above -10 dB is crucial for the reliable functioning of radar systems using matched filtering.

2) You are required to submit your code, and your code should directly give all the tables or figures in 2.2).

1.1) Plot your algorithm flow chart. You can provide necessary explanations.

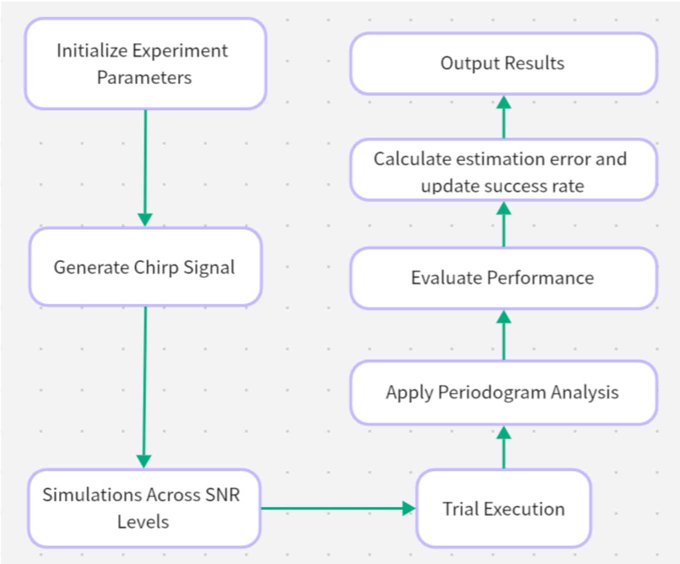


Fig 11 System Flow Chart of advance 2

1.2) Give your MSE and success rate results, and analysis, under different SNR, and compare the results with 1). (Hint: use table or figure, and you should choose an SNR range that can at least see '100% success' and '100% fail')

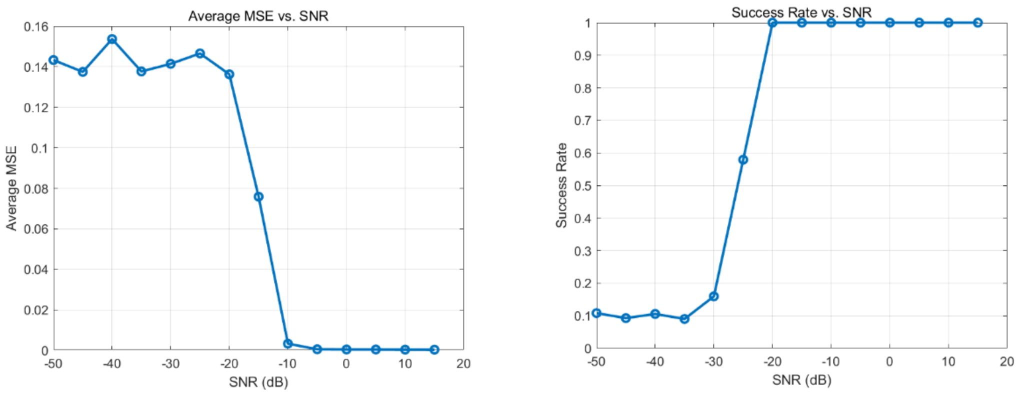


Fig 12 MSE and Success Rate

The new experiment scenario involves detecting the end time of a chirp signal, assumed to be emitted by a target and not known a priori, using a periodogram for spectral analysis. The task evaluates the system's performance under different signal-to-noise ratios (SNR) and measures the accuracy in estimating the signal's end time using Mean Squared Error (MSE) and success rate metrics. Here comes experiment parameters:

A chirp signal $X(t) = \cos[2\pi(f_0 t + kt^2)]$, where f_0=1000Hz, k=12000Hz, and the signal starts from 0 to 0.1s. 50000 Hz sampling frequency. -50 dB to 15 dB SNR levels The reflected signal $X_1(t)$ is assumed to end randomly between 0.11 s and 1 s

Analysis:

Success Rate vs. SNR: Demonstrated a critical threshold around -10 dB, below which the system rarely succeeds, and above which success rates improve dramatically, approaching 100%. **Average MSE vs. SNR:** Showed high errors at low SNR values, which decrease sharply as SNR improves, indicating better accuracy in timing estimation with higher SNR. Both success rates and MSE are highly sensitive to SNR changes, with significant improvements noted as the noise level decreases. When compared to the previous method using a matched filter, the periodogram-based approach shows a similar sensitivity to SNR but might be slightly less efficient in noise resilience, suggesting the matched filter’s superiority in direct signal detection scenarios.

In conclusion The experiment underscores the critical role of SNR in the effectiveness of spectral analysis techniques like periodograms in signal end-time detection. While achieving high success rates at favorable SNR levels, the approach requires careful consideration of noise impacts, suggesting enhancements or alternative methods might be necessary for robust performance in lower SNR conditions. The insights gained point to the importance of optimizing signal processing techniques according to the noise characteristics of the operational environment.

指导教师批阅意见：   成绩评定：   指导教师签字： 年　　月　　日
备注：

注：1、报告内的项目或内容设置，可根据实际情况加以调整和补充。

2、教师批改学生实验报告时间应在学生提交实验报告时间后 10 日内。