Министерство образования Республики Беларусь

Учреждение образования

«Белорусский государственный университет информатики и радиоэлектроники»

Лабораторная работа №3

Приближение функций

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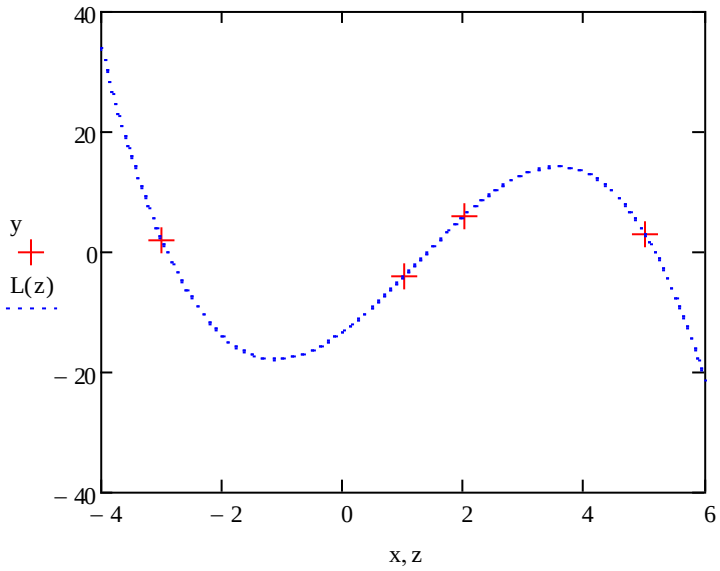
I.

1) Интерполяционный полином Лагранжа

$$x := \begin{pmatrix} -3 \\ 2 \\ 1 \\ 5 \end{pmatrix} \qquad y := \begin{pmatrix} 2 \\ 6 \\ -4 \\ 3 \end{pmatrix}$$

$$L(z) := y_0 \cdot \left[\frac{(z-x_1)\cdot(z-x_2)\cdot(z-x_3)}{(x_0-x_1)\cdot(x_0-x_2)\cdot(x_0-x_3)}\right] + y_1 \cdot \left[\frac{(z-x_0)\cdot(z-x_2)\cdot(z-x_3)}{(x_1-x_0)\cdot(x_1-x_2)\cdot(x_1-x_3)}\right] + y_2 \cdot \left[\frac{(z-x_1)\cdot(z-x_0)\cdot(z-x_3)}{(x_2-x_1)\cdot(x_2-x_0)\cdot(x_2-x_3)}\right] + y_3 \cdot \left[\frac{(z-x_1)\cdot(z-x_2)\cdot(z-x_0)}{(x_3-x_1)\cdot(x_3-x_2)\cdot(x_3-x_0)}\right]$$

$z := -4, -3.99 .. 6$



2) Метод наименьших квадратов (МНК)

$a0 := 0 \qquad a1 := 0 \qquad a2 := 0 \qquad a3 := 0$

Given

$$\left[2 \cdot \sum_{k=0}^{3} \left[\left[a0 + a1 \cdot x_k + a2 \cdot (x_k)^2 + a3 \cdot (x_k)^3\right] - y_k\right]\right] = 0$$

$$\left[2 \cdot \sum_{k=0}^{3} \left[\left[a0 + a1 \cdot x_k + a2 \cdot (x_k)^2 + a3 \cdot (x_k)^3\right] - y_k\right] \cdot x_k\right] = 0$$

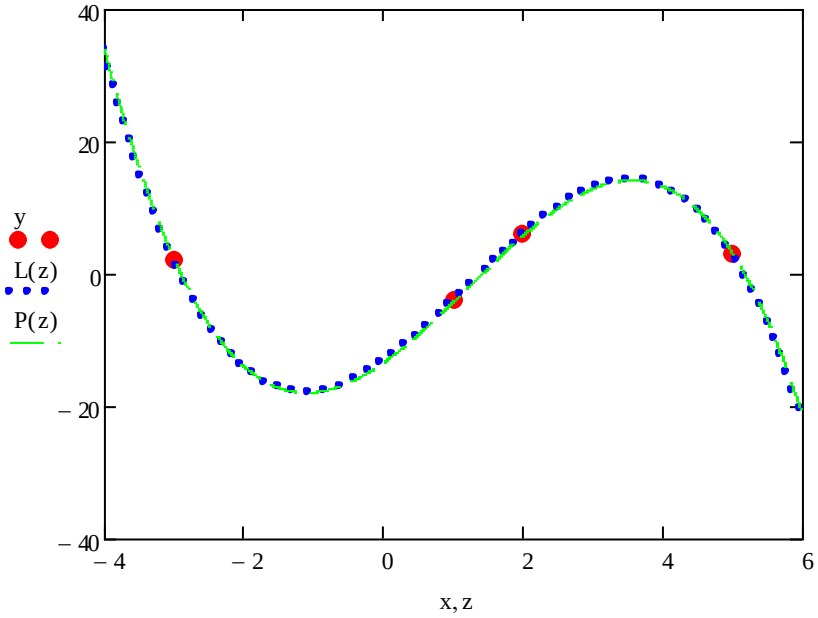
$$\left[2 \cdot \sum_{k=0}^{3} \left[\left[a0 + a1 \cdot x_k + a2 \cdot (x_k)^2 + a3 \cdot (x_k)^3\right] - y_k\right] \cdot (x_k)^2\right] = 0$$

$$\left[2 \cdot \sum_{k=0}^{3} \left[\left[a0 + a1 \cdot x_k + a2 \cdot (x_k)^2 + a3 \cdot (x_k)^3\right] - y_k\right] \cdot (x_k)^3\right] = 0$$

$M := \text{Find}(a0, a1, a2, a3)$

$$M = \begin{pmatrix} -13.187 \\ 7.519 \\ 2.3 \\ -0.631 \end{pmatrix}$$

$P(z) := M_0 + M_1 z + M_2 z^2 + M_3 z^3$



3) К у б и ч е с к и й с п л а й н (К С)

$a1 := y_0$ $\quad b1 := 0$ $\quad c1 := 0$ $\quad d1 := 0$

$a2 := y_1$ $\quad b2 := 0$ $\quad c2 := 0$ $\quad d2 := 0$

$a3 := y_2$ $\quad b3 := 0$ $\quad c3 := 0$ $\quad d3 := 0$

Given

$$a1 + b1\cdot(x_1 - x_0) + c1\cdot(x_1 - x_0)^2 + d1\cdot(x_1 - x_0)^3 = y_1$$

$$a2 + b2\cdot(x_2 - x_1) + c2\cdot(x_2 - x_1)^2 + d2\cdot(x_2 - x_1)^3 = y_2$$

$$a3 + b3\cdot(x_3 - x_2) + c3\cdot(x_3 - x_2)^2 + d3\cdot(x_3 - x_2)^3 = y_3$$

$$b1 + 2\cdot c1\cdot(x_1 - x_0) + 3\cdot d1\cdot(x_1 - x_0)^2 = b2$$

$$b2 + 2\cdot c2\cdot(x_2 - x_1) + 3\cdot d2\cdot(x_2 - x_1)^2 = b3$$

$$2\cdot c2 + 6\cdot d2\cdot(x_2 - x_1) = 2\cdot c3$$

$$2\cdot c1 + 6\cdot d1\cdot(x_1 - x_0) = 2\cdot c2$$

$$2\cdot c3 + 6\cdot d3\cdot(x_3 - x_2) = 0$$

$$2\cdot c1 + 6\cdot d1\cdot(x_0 - x_0) = 0$$

$S := \text{Find}(b1, b2, b3, c1, c2, c3, d1, d2, d3)$

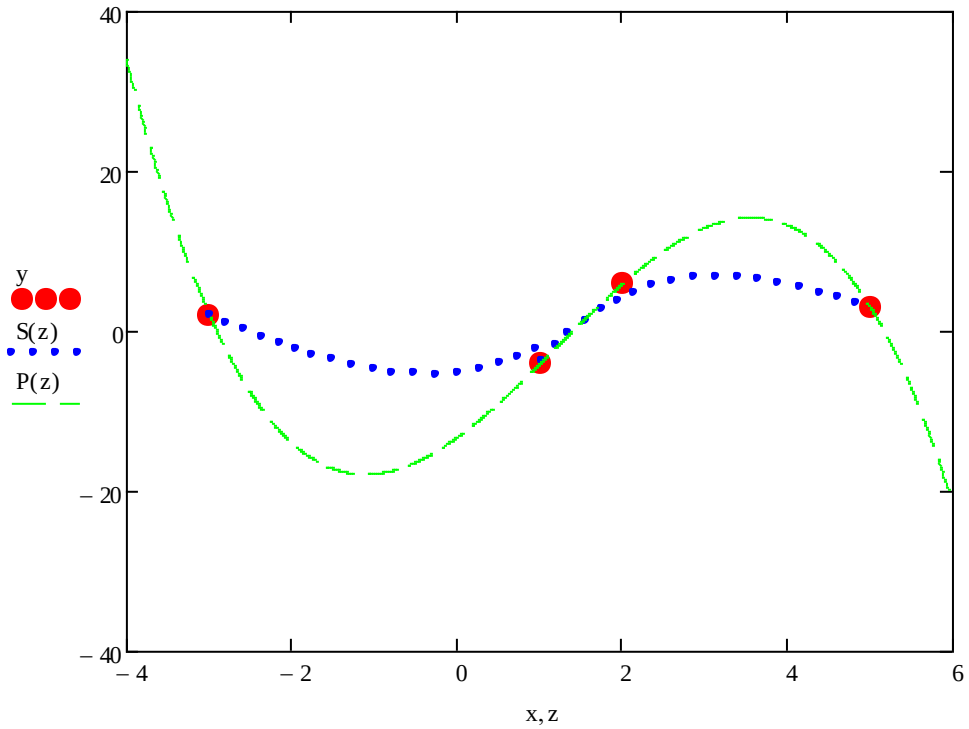
$$S = \begin{pmatrix} -4.195 \\ 10.789 \\ 11.418 \\ 0 \\ 2.997 \\ -3.626 \\ 0.2 \\ 2.207 \\ 0.302 \end{pmatrix}$$

$b1 := S_0$ $\quad c1 := S_3$ $\quad d1 := S_6$

$b2 := S_1$ $\quad c2 := S_4$ $\quad d2 := S_7$

$b3 := S_2$ $\quad c3 := S_5$ $\quad d3 := S_8$

$$S(z) := \begin{cases} a1 + b1 \cdot (z - x_0) + c1 \cdot (z - x_0)^2 + d1 \cdot (z - x_0)^3 & \text{if } x_0 \le z \le x_1 \\ a2 + b2 \cdot (z - x_1) + c2 \cdot (z - x_1)^2 + d2 \cdot (z - x_1)^3 & \text{if } x_1 \le z \le x_2 \\ a3 + b3 \cdot (z - x_2) + c3 \cdot (z - x_2)^2 + d3 \cdot (z - x_2)^3 & \text{if } x_2 \le z \le x_3 \end{cases}$$



II.

$g(x) := \sin(x)\cdot\cos\left(\frac{x}{3}\right)$ $N := 8$ $i := 0..N-1$

$a := 0$ $b := 2\cdot\pi$ $h := \frac{(b-a)}{N-1}$

$X1_i := a + i\cdot h$ $x := a, 0.01..b$

$Y1_i := g(X1_i)$

$$X1 = \begin{pmatrix} 0 \\ 0.898 \\ 1.795 \\ 2.693 \\ 3.59 \\ 4.488 \\ 5.386 \\ 6.283 \end{pmatrix} \qquad Y1 = \begin{pmatrix} 0 \\ 0.747 \\ 0.806 \\ 0.271 \\ -0.159 \\ -0.073 \\ 0.174 \\ 0 \end{pmatrix}$$

a) $f(z) := \text{linterp}(X1, Y1, z)$

$VS1 := \text{lspline}(X1, Y1)$

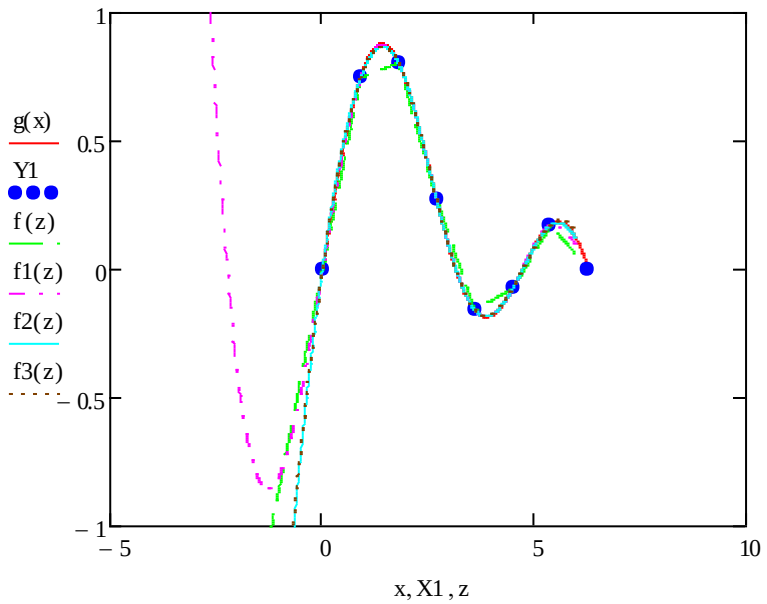
$VS2 := \text{pspline}(X1, Y1)$

$VS3 := \text{cspline}(X1, Y1)$

$f1(z) := \text{interp}(VS1, X1, Y1, z)$

$f2(z) := \text{interp}(VS2, X1, Y1, z)$

$f3(z) := \text{interp}(VS3, X1, Y1, z)$



$$u1 := \begin{pmatrix} -1 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \qquad u2 := \begin{pmatrix} -1 \\ 1 \\ 2 \\ 2.8 \\ 4 \\ 5.5 \\ 6.5 \end{pmatrix} \qquad u3 := \begin{pmatrix} -1 \\ 1.5 \\ 3 \\ 3.5 \\ 4.5 \\ 6.5 \end{pmatrix}$$

$n1 := 1 \qquad n2 := 2 \qquad n3 := 3$

VS4 := bspline (X1, Y1, u1, n1)

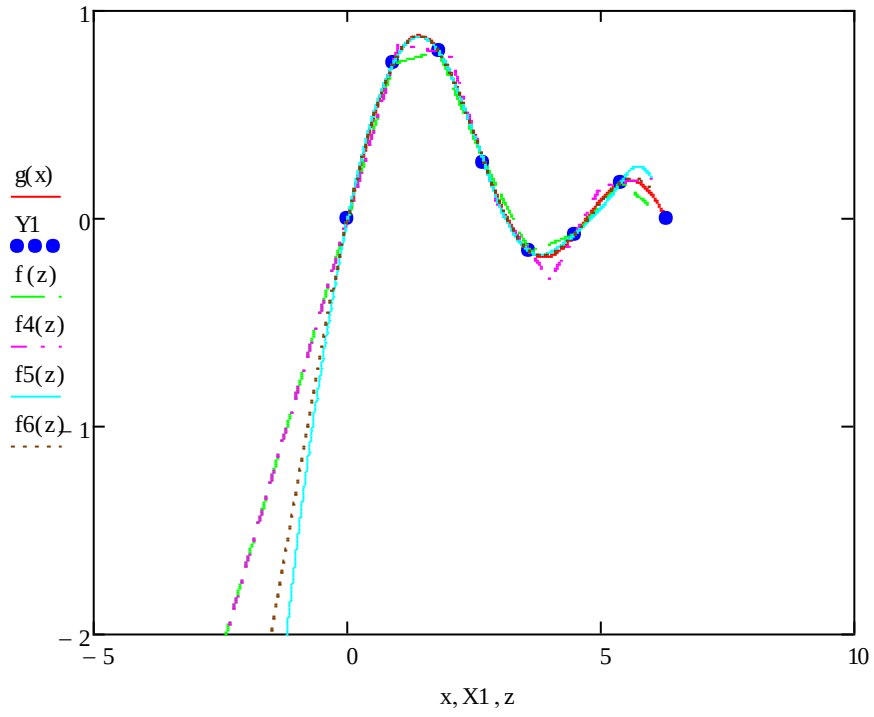
VS5 := bspline (X1, Y1, u2, n2)

VS6 := bspline (X1, Y1, u3, n3)

f4(z) := interp(VS4, X1, Y1, z)

f5(z) := interp(VS5, X1, Y1, z)

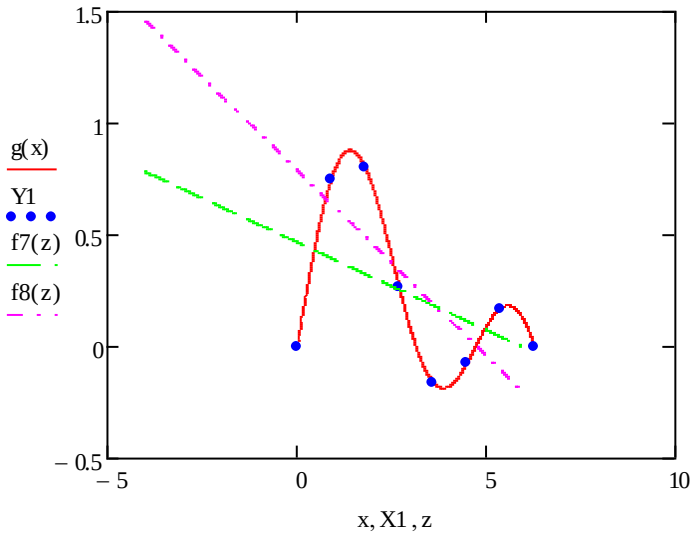
f6(z) := interp(VS6, X1, Y1, z)



б)

$r1 := 2 \qquad r2 := 4 \qquad r3 := 7$

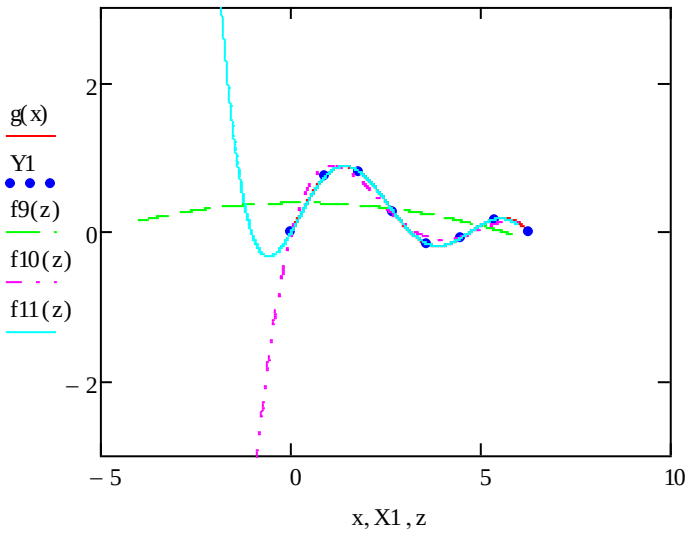
$R := line(X1, Y1)$ $\qquad f7(z) := R_1 \cdot z + R_0$



$R2 := regress(X1, Y1, r1)$ $f9(z) := interp(R2, X1, Y1, z)$

$R3 := regress(X1, Y1, r2)$ $f10(z) := interp(R3, X1, Y1, z)$

$R4 := regress(X1, Y1, r3)$ $f11(z) := interp(R4, X1, Y1, z)$

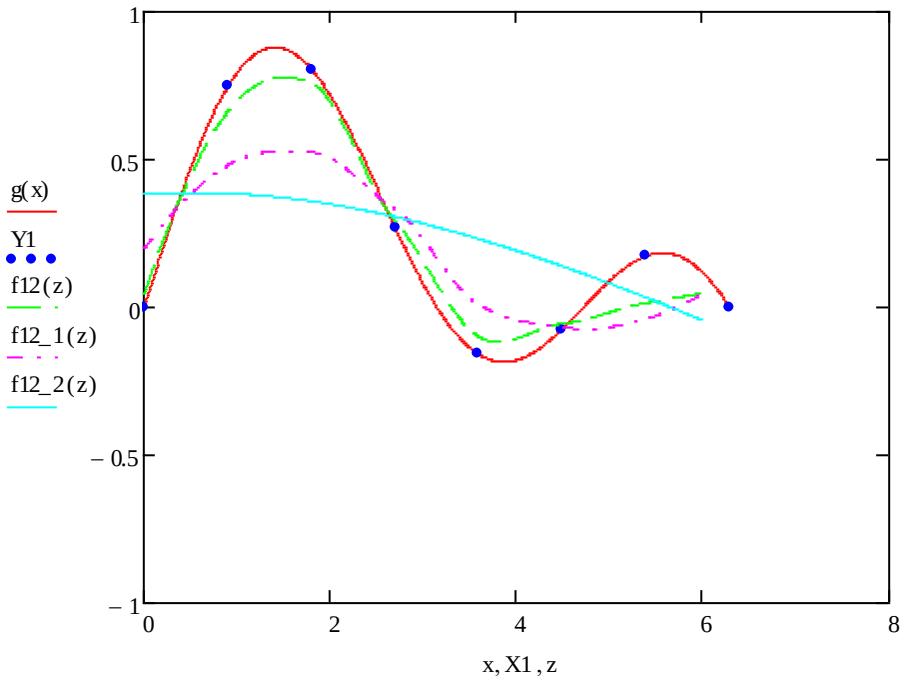


$span1 := 0.8$ $span2 := 1$ $span3 := 5$

$R5 := loess(X1, Y1, span1)$ $f12(z) := interp(R5, X1, Y1, z)$

$R5\_1 := loess(X1, Y1, span2)$ $f12\_1(z) := interp(R5\_1, X1, Y1, z)$

$R5\_2 := loess(X1, Y1, span3)$ $f12\_2(z) := interp(R5\_2, X1, Y1, z)$



$R6 := expfit(X1, Y1)$

$f13(z) := R6_0 \cdot e^{R6_1 \cdot z} + R6_2$

$q1 := \begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ $q2 := \begin{pmatrix} 0.75 \\ 0.1 \\ 0.2 \end{pmatrix}$

$R6\_1 := expfit(X1, Y1, q1)$

$f13\_1(z) := R6\_1_0 \cdot e^{R6\_1_1 \cdot z} + R6\_1_2$

$R6\_2 := expfit(X1, Y1, q2)$

$f13\_2(z) := R6\_2_0 \cdot e^{R6\_2_1 \cdot z} + R6\_2_2$