Q5. Discuss your findings from comparing the performance of all 6 portfolios and from observing the graphs from questions 3 & 4.

By determining the returns of 6 portfolios on \$1000 investment, we got the minimum variance portfolio returns, maximum return portfolio returns, and optimal Sharpe ratio portfolio returns around \$28.22, -\$584.59 and \$279.59 respectively on the classical efficient frontier. The returns of the minimum variance portfolio, the maximum return portfolio, and the optimal Sharpe ratio portfolio on the resampled efficient frontier are \$65.18, -\$86.69 and \$225.18 respectively. Here, I applied a 100-times repetition to resample the data from the origin. Based on the return we've calculated, we can see the maximum return portfolio performs the worst under both classical and resampled efficient frontier, while the optimal Sharpe ratio portfolio performs the best under both frontiers.

From Figure 1, we can see many of individual stocks are lie on the efficient frontier. The classical efficient frontier (blue) has the similar expected return around 5% with the resampled efficient frontier (red) when the standard deviation at 0.1. As the volatility increases, the expected return on classical efficient frontier is greater than the resampled efficient frontier. However, when the volatility is greater than 0.25, the expected return doesn't fluctuate too much, but with an increasing volatility. Also, we get to know that minimum variance portfolio has both the lowest expected return and the lowest volatility among minimum variance portfolio, Sharpe ratio portfolio and maximum return portfolio. Therefore, it has a small return with large certainty as determined in the previous paragraph. Moreover, maximum return portfolio has both the highest expected return and the highest volatility among three types of portfolios. However, due to the high volatility, it may happen to have a loss as previously showed.

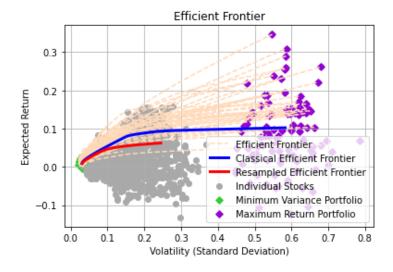


Figure 1. Classical and Resampled Efficient Frontiers

The composition plot shows the mean-variance optimization frontier with increasing target return from left to right and ranging from minimum variance on the left to maximum return on the right. Based on Figure 2, we can see the portfolio on classical frontier is mostly weighted on stocks AMD, TSLA, GLD, AAPL and MULN. However, the percentage change of stocks AMD, TSLA, GLD, AAPL and MULN in 2021 (out-of-sample) are 68%, 33%, -0.95% and -58% respectively. The result is not the same as the portfolio return from 2018 to 2020 (in-sample). In Figure 3, we can see the weight of each stock in the portfolio is more balanced and frontier is smoother with resampled optimization. The portfolio weights stabilize on a final answer. However, in the investment case, more simulations are usually performed. It usually takes 5,000 simulations to ensure that the weights of the portfolio have converged sufficiently to eliminate the noise in the portfolio weights.

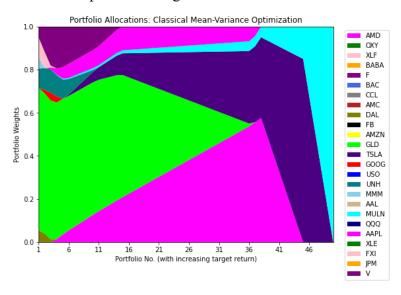


Figure 2. Portfolio Compositions for Classical Frontiers

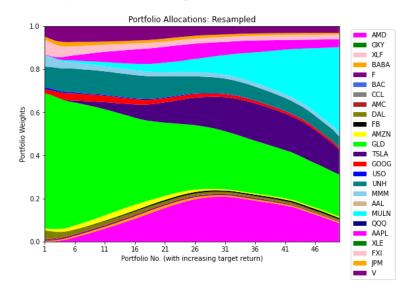


Figure 3. Portfolio Compositions for Resampled Frontiers