

Some Thoughts on Engine failures

In attempts to find a good cumulative distribution function I've come up to such a construction:

$$F(t) = \frac{2}{\pi} * \text{atan}((kt)^r * 10^{3-r})$$

It means that an engine's chance to fail by time t is $F(t)$. r represents overall tech progress and usage experience. When increased, it makes early failures less likely to occur. k represents how much resources are spent on the engine. Dividing it by n makes the engine last n times longer. Now we are going to find a function that returns failure time based on "luck". The multiplier 10^{3-r} ensures that increasing r makes the curve steeper and does not increase the cumulative function at the same time.

$$F * \frac{\pi}{2} = \text{atan}((kt)^r * 10^{3-r})$$

$$\tan\left(F * \frac{\pi}{2}\right) * 10^{r-3} = (kt)^r$$

$$\sqrt[r]{\tan\left(F * \frac{\pi}{2}\right) * 10^{1-\frac{3}{r}}} = kt$$

$$t = \frac{\sqrt[r]{\tan\left(F * \frac{\pi}{2}\right) * 10^{1-\frac{3}{r}}}}{k}$$

An important note: we use random values from $[0; 1)$, thus we SHOULD not get any issues. Probably we should use something $[0; 0.99)$ in order to avoid weird durations with enormous burn durations. Let us bind k to r in a way that makes a certain t_1 cause $F(t_1) = 0.01$ when $r = 8$.

$$\frac{2}{\pi} * \text{atan}((kt_1)^8 * 10^{-5}) = 0.01$$

$$\text{atan}((kt_1)^8 * 10^{-5}) = 0.005 * \pi$$

$$(kt_1)^8 * 10^{-5} = \tan(0.005 * \pi)$$

$$(kt_1)^8 = \tan(0.005 * \pi) * 10^5$$

$$kt_1 = \sqrt[8]{\tan(0.005 * \pi) * 10^{\frac{5}{8}}}$$

$$k = \frac{\sqrt[8]{\tan(0.005 * \pi) * 10^{\frac{5}{8}}}}{t_1}$$