GATE 2020
PSUs 2020

MADE EASY
Publications

Revised & Updated Edition

POSTAL
STUDY PACKAGE

COMPUTER SCIENCE & IT
Discrete & Engineering Mathematics

Objective Practice Sets

POSTAL Study Package 2020

Computer Science & IT

Objective Practice Sets

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Contents

MADE EASY Publications

1 CHAPTER

Propositional Logic

1. Argument: $([P \rightarrow (q \vee r)] \wedge \bar{q} \wedge \bar{r}) \rightarrow \bar{P}$ is
(a) valid argument (b) invalid argument
(c) unknown (d) none of these

2. $\neg\forall_x \forall_y[(x<y) \rightarrow (x^2<y^2)]$ is equivalent to
(a) $\exists_x \exists_y[(x<y) \wedge (x^2 \geq y^2)]$
(b) $\exists_x \exists_y[\neg(x<y) \wedge \neg(x^2<y^2)]$
(c) $\exists_x \exists_y[\neg(x<y) \vee \neg(x^2<y^2)]$
(d) $\exists_x \exists_y[(x<y) \vee (x^2 \geq y^2)]$

3. $P(x, y): x+y=x-y$
If the universe is the set of integers which of the following are true
(i) $P(1, 1)$ (ii) $P(3, 0)$
(iii) $\exists x P(x, 2)$ (iv) $\exists x \forall y P(x, y)$
(v) $\exists y \forall x P(x, y)$ (vi) $\forall x \exists x P(x, y)$
(a) (ii) and (v) only (b) (ii), (v) & (iv) only
(c) (ii) only (d) (v) and (vi) only

4. Which of the following is true?
(i) $\exists x\{P(x) \wedge Q(x)\} \equiv \exists x P(x) \wedge \exists x Q(x)$
(ii) $\exists x\{P(x) \wedge Q(x)\} \Rightarrow \exists x P(x) \wedge \exists x Q(x)$
(iii) $\exists x P(x) \wedge \exists x Q(x) \equiv \exists x P(x) \wedge \exists y Q(y)$
(a) (i) only (b) (ii) and (iii) only
(c) (ii) only (d) None of these

5. Consider the following predicates with the domain as set of integers for all variables.
$P(s)$: $s^2 \geq 0$
$Q(s) = s^2 - 5s + 6 = 0$
$R(s, t) = s^2 = t$
Find which of the following is true?
(a) $\forall s\, P(s) \wedge \exists t\, Q(t)$ (b) $\forall s\, [P(s) \wedge Q(s)]$
(c) $\forall t \forall s\, [P(s) \rightarrow R(s, t)]$ (d) None of these

6. $\neg(X \wedge Y) \rightarrow (\neg X \vee (\neg X \vee Y))$ is logically equivalent to
(a) $\sim(X \vee Y)$ (b) $\sim X \vee Y$
(c) $\sim X \wedge Y$ (d) $\sim(X \wedge Y)$

7. Let P and Q are two functions over the variables x and y and the domains of x and y are from set of integers. Consider the following two functions.
$P(x, y): x = y^2$
$Q(x, y): x + 5 \geq 2 - y$
Find which one of the following gives truth value as FALSE?
(a) $P(3, -5) \vee Q(1, -3)$
(b) $\exists_y P(4, y)$
(c) $\forall_x \exists_y Q(x, y)$
(d) $\exists_x \forall_y Q(x, y)$

8. Consider the following compound proposition:
$[(A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D)] \rightarrow (D \vee C)$
Which of the following is true for the above proposition?
(a) Satisfiable (b) Contradiction
(c) Tautology (d) None of these

9. Consider the following predicates.
$S(x)$: "x is student"
GATE (x, y) : "x has written gate in every stream".
Find the equivalent predicate logic for the following statement:
"There does not exist a student who has written a GATE in every stream."
(a) $\exists y \exists x\, [S(x) \wedge \sim \text{GATE}(x, y)]$
(b) $\forall y \exists x [\sim S(x) \vee \sim \text{GATE}(x, y)]$
(c) $\exists y \forall x [\sim S(x) \vee \sim \text{GATE}(x, y)]$
(d) $\exists y \exists x [\sim S(x) \wedge \sim \text{GATE}(x, y)]$

10. "If GATE rank is needed, i will not write the GATE exam, if i do not join MADEEASY. GATE rank is needed and i will join MADEEASY. Therefore i will write the GATE exam."
Identify the above inference?
(a) Valid
(b) Invalid
(c) One argument is missed
(d) More than one argument is missed.

11. Consider the following statement:
"Every bird can fly"
The negation of the above statement in simple english.
(a) No bird cannot fly
(b) There is a bird that can fly
(c) Every bird cannot fly
(d) There is a bird that cannot fly

12. Which of the following statement is the negation of the statement 4 is odd or –5 is not negative.
(a) 4 is odd or –5 is negative
(b) 4 is even or –5 is negative
(c) 4 is even and –5 is negative
(d) –4 is even and +5 is negative

13. $(p \wedge q) \wedge \sim (p \vee q)$ is a negation of
(a) tautology (b) fallacy
(c) Both (a) and (b) (d) None of these

14. Let A be "she looks pretty" and let B is "she is tall". The statement "It is false that she looks ugly and She is tall" is:
(a) $\sim (A \vee B)$ (b) $\sim (\sim A \wedge B)$
(c) $\sim (\sim A \vee B)$ (d) $A \wedge \sim B$

15. Choose the correct statement
(a) $\sim(p \leftrightarrow q)$ and $((p \wedge \sim q) \vee (q \wedge \sim p))$ is equivalent
(b) $\sim(p \leftrightarrow q)$ and $(p \to \sim q \wedge q \to \sim p)$ is equivalent
(c) Both (a) and (b)
(d) None of these

16. Consider the following formula
$$\neg(\neg p \vee q) \vee (r \to \neg s)$$
Which of the following is equivalent to the above CNF formula.
(a) $(p \wedge \neg r \vee \neg s) \wedge (\neg q \vee \neg r \wedge \neg s)$
(b) $(p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$
(c) $(p \wedge \neg r \wedge \neg s) \vee (\neg q \wedge \neg r \wedge \neg s)$
(d) None of these

17. Consider the following statements:
P_1: $\exists x \exists y (x \neq y \wedge \forall z (\text{Apple}(z) \leftrightarrow ((z = x) \vee (z = y)))$
P_2: $\exists x \exists y (\text{Apple}(x) \wedge \text{Apple}(y) \wedge x \neq y) \wedge \forall x \forall y \forall z ((\text{Apple}(x) \wedge \text{Apple}(y) \wedge \text{Apple}(z)) \to (x = y \vee x = z \vee y = z))$
P_3: $\exists x \exists y (\text{Apple}(x) \wedge \text{Apple}(y) \wedge (x \neq y) \wedge \forall z (\text{Apple}(z) \to (z = x) \vee (z = y)))$
Which of the predicate logic statements represent the following statement.
"there are exactly two apples".
(a) P_1 and P_2 only (b) P_1 and P_3 only
(c) P_2 and P_3 only (d) P_1, P_2 and P_3

18. $(\exists ! x)\, P(x)$ is equivalent to ________.
(a) $\forall x \exists y (P(x) \wedge [P(y) \to (x != y)])$
(b) $\exists y (\forall x P(x) \wedge [P(y) \to (x = y)])$
(c) $\exists x (P(x) \wedge \forall y [P(y) \to (x = y)])$
(d) None of these

19. The predicate statement
$\neg \forall z [P(z) \to (\neg Q(z) \to P(z))]$ is ________.
(a) Satisfiable
(b) Tautology
(c) Contradiction
(d) None of these

20. Consider the following statement over the domain of natural number.
"No prime number except 7 is divisible by 7"
Find the equivalent predicate logic for the above statement.
(a) $\forall x \in N\, [x \neq 7 \wedge \text{Prime}(x) \to (\neg \text{Divisibleby7}(x))]$
(b) $\neg \exists x \in N\, [x \neq 7 \wedge \text{Prime}(x) \wedge \text{Divisibleby7}(x)]$
(c) Both (a) and (b)
(d) None of these

21. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

A. Everybody loves Modi
B. Everybody loves somebody
C. There is somebody whom everybody loves
D. There is somebody whom no one loves

List-II

1. $\forall x$ Loves $(x, modi)$
2. $\forall x\, \exists y$ Loves (x, y)
3. $\exists y\, \forall x$ Loves (x, y)
4. $\exists y\, \forall x\, \neg$ Loves (x, y)

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	1	4	3	2
(d)	1	2	4	3

22. Which of the following is negation of $\forall x \forall y[((x>0) \wedge (y>0)) \rightarrow (x+y>0)]$?

(a) $\exists x\, \exists y\, [(x>0) \wedge (y>0) \wedge (x+y \leq 0)]$

(b) $\exists x\, \exists y\, [((x>0) \wedge (y>0)) \wedge (x+y \leqq 0)]$

(c) $\exists x\, \exists y\, [\neg((x>0) \wedge (y>0)) \vee (x+y \leq 0)]$

(d) None of the above

23. What is the predicate logic for the following statement?

There are atmost two males in the class.

(a) $\forall x \forall y((\text{Male}(x) \wedge \text{Male}(y)) \rightarrow (x=y \vee y=x))$

(b) $\exists x \exists y\, (\text{Male}(x) \wedge \text{Male}(y) \wedge x \neq y \wedge \forall z(\text{Male}(z) \rightarrow (z=x \vee z=y)))$

(c) $\forall x \forall y \forall z\, ((\text{Male}(x) \wedge \text{Male}(y) \wedge \text{Male}(z)) \rightarrow (x=y \vee x=z \vee y=z))$

(d) None of these

24. Let $a(x, y)$, $b(x, y)$ and $c(x, y)$ be three statements with variables x and y chosen from some universe. Consider the following statement:

$$(\exists x)(\forall y)[(a(x, y)) \wedge (b(x, y)) \wedge \neg c(x, y)]$$

Which one of the following is equivalent to the above statement?

(a) $(\forall x)(\exists y)[(a(x, y) \vee (b(x, y)) \rightarrow c(x, y)]$

(b) $(\exists x)(\forall y)[(a(x, y) \vee b(x, y)) \wedge \neg c(x, y)]$

(c) $\neg[(\forall x)(\exists y)[(a(x, y) \wedge b(x, y)) \rightarrow c(x, y)]]$

(d) $\neg(\forall x)(\exists y)[(a(x, y) \vee b(x, y)) \rightarrow c(x, y)]$

Answers Propositional Logic

1.	(a)	2.	(a)	3.	(b)	4.	(b)	5.	(a)	6.	(b)	7.	(d)	8.	(c)	9.	(c)
10.	(b)	11.	(d)	12.	(c)	13.	(a)	14.	(b)	15.	(c)	16.	(b)	17.	(d)	18.	(c)
19.	(c)	20.	(c)	21.	(a)	22.	(b)	23.	(c)	24.	(c)						

Explanations Propositional Logic

1. (a)

$([P \to (q \vee r)] \wedge \bar{q} \wedge \bar{r}) \to \bar{P}$ is tautology hence it is valid argument.

2. (a)

$$\neg\forall x\forall y\left[(x<y)\to(x^2<y^2)\right]$$

$$\equiv \exists x\exists y\,\neg\left[(x<y)\to(x^2<y^2)\right]$$

$$\equiv \exists x\exists y\,\neg\left[\neg(x<y)\vee(x^2<y^2)\right]$$

$$\equiv \exists x\exists y\left[(x<y)\wedge\neg(x^2<y^2)\right]$$

$$\equiv \exists x\exists y\left[(x<y)\wedge(x^2\geq y^2)\right]$$

3. (b)

$P(1, 1) : 1 + 1 = 1 - 1$ is false

$P(3, 0) : 3 + 0 = 3 - 0$ is true

$\exists x P(x, 2) : \exists x(x+2=x-2)$ is false as there is no solution

$\exists x\forall y P(x,y) : \exists x\forall y(x+y=x-y)$ is false since this can be made true only if y = 0

$\exists y\forall x P(x,y) : \exists y\forall x(x+y=x-y)$ is true since for y = 0 the equation is true for all x

$\forall x\exists y P(x, y) : \forall x\exists y(x+y=x-y)$ is true since for all x, $y = 0$ will satisfy the equation.

4. (b)

III is true since once a variable is bound to a qualifier it's name does not matter.

So $\exists x Q(x)$ is same $\exists y Q(y)$ and so II is true since LHS and RHS is same.

I is false, since

LHS: some value of x satisfies both P and Q

RHS: some values satisfies P and some value satisfies Q, but these 2 values need not be same.

II is true, since

If the same value satisfies both P and Q surely some value satisfies P and some values satisfies Q.

In other words LHS of implication is stronger than RHS and hence implication will be true.

5. (a)

$\forall s\, P(s) \wedge \exists t\, Q(t)$

$\forall s\, P(s)$: Square of every integer is always ≥ 0, so $\forall s\, P(s)$ is true.

$\exists t\, Q(t)$: If $t = 2$ then $2^2 - 5 \times 2 + 6 = 0$, so $\exists t\, Q(t)$ is also true.

$\therefore$ $\forall s\, P(s) \wedge \exists t\, Q(t)$ is true.

So option (a) is correct.

6. (b)

$\sim(X \wedge Y) \to (\sim X \vee (\sim X \vee Y))$

$= \sim(X \wedge Y) \to (\sim X \vee Y)$

$= (X \wedge Y) \vee (\sim X \vee Y)$

$= \underline{(X \wedge Y) \vee \sim X} \vee Y$

$= (X \vee \sim X) \wedge (Y \vee \sim X) \vee Y$

$= T \wedge (Y \vee \sim X) \vee Y$

$= Y \vee \sim X$

$= \sim X \vee Y$

So option (b) is correct.

7. (d)

(a) $P(3, -5) \vee Q(1, -3) = F \vee T = T$

(b) $\exists y\, P(4, y) = T$

$P(4, y)$: $4 = y^2 \Rightarrow 4 = 2^2$ for $y = 2$

(c) $\forall x\, \exists y\, Q(x, y)$ is true

(d) $\exists x\, \forall y\, Q(x, y)$ is false

(same x can not hold for all values of y)

$\therefore$ Option (d) is correct.

8. (c)

$[(A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D)] \rightarrow (D \vee C)$

$= \neg[(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D)] \vee (D \vee C)$

$= \neg(A \vee B) \vee \neg(\neg A \vee C) \vee \neg(\neg B \vee D) \vee D \vee C$

$= (\neg A \wedge \neg B) \vee (A \wedge \neg C) \vee \underline{(B \wedge \neg D) \vee D} \vee C$

$= (\neg A \wedge \neg B) \vee \underline{(A \wedge \neg C) \vee B \vee D \vee C}$

$= \underline{(\neg A \wedge \neg B) \vee A \vee B} \vee D \vee C$

$= \underline{\neg A \vee A} \vee B \vee D \vee C$

$= T \vee B \vee D \vee C$

= true

∴ It is tautology

9. (c)

$\sim[\exists x\, S(x) \wedge \forall_y \text{GATE}(x, y)]$

$= \sim\forall y \exists x\, [S(x) \wedge \text{GATE}(x, y)]$

$= \exists y \forall x\, [\sim S(x) \vee \sim GATE(x, y)]$

10. (b)

Let p : GATE rank is needed

q : I will write the GATE exam

r : I will join in MADEEASY.

Given arguments:

1. If GATE rank is needed, i will not write GATE exam, if i do not join MADEEASY.
2. GATE rank is needed : p
3. I will join MADEEASY : r
4. I will write the GATE exam : q

Inference is: $(p \wedge \neg r) \rightarrow \neg q$

$$\begin{array}{c} p \\ \underline{r} \\ \underline{q} \end{array}$$

We can also write the above inference as following: $(p \wedge \neg r)$

$$([(p \wedge \neg r) \rightarrow \neg q] \wedge p \wedge r) \rightarrow q$$

If above proposition is tautology then given inference is valid.

Above given arguments are independent of q, because p and r are true.

∴ Given inference is invalid and we cannot judge about missing arguments.

11. (d)

The statement "every bird can fly" can be expressed as $\forall x\, P(x)$. Supposing $P(x)$: x can fly.

So negation will be $\exists x \in P(x)$ which can be expressed as "there is a bird that cannot fly".

12. (c)

A or B

Negation = $\sim A$ and $\sim B$

Hence C is the correct option.

13. (a)

After drawing truth table of the statement it is found that it is fallacy hence negation of fallacy = tautology.

14. (b)

She looks ugly $\equiv \sim A$

She is tall $\equiv \sim B$

Now she looks ugly and she is tall $\equiv \sim A \wedge B$ and given it is false $\Rightarrow \sim(\sim A \vee B)$.

15. (c)

$\sim(p \leftrightarrow q) \equiv \sim(p \rightarrow q \wedge q \rightarrow p)$ option (b)

$\equiv \sim((\sim p \vee q) \wedge (\sim q \vee p))$

$\equiv ((p \wedge \sim q) \vee (q \wedge \sim p))$ option (a)

Hence both option (a) and (b) are correct.

16. (b)

$\neg(\neg p \vee q) \vee (r \rightarrow \neg s) \cong (p \wedge \neg q) \vee (\neg r \vee \neg s) \equiv (p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$

17. (d)

P_1, P_2 and P_3 are equivalent.

All are representing the same statement: "there are exactly two apples".

25. (c)

$(\exists!\, x)\, P(x)$ is true if $P(x)$ is true for exactly one x [Assume there are many x's in domain]. $(\exists!\, x)\, P(x)$ is called unique existential quantification of a predicate $P(x)$ over a domain.

$(\exists!\, x)\, P(x) \Leftrightarrow \exists x\, [P(x) \wedge \forall y\, [P(y) \rightarrow x = y]]$

Here x and y are from same domain.

Every element y in domain ensures only one element which makes $P(x)$ is true i.e., $x = y$.

For unique existence of x, $P(x)$ is true.

19. (c)

$\neg\forall z[P(z) \to (\neg Q(z) \to P(z))]$

$\equiv \neg\forall z[\neg P(z) \vee Q(z) \vee P(z)]$

$\equiv \neg\forall z[T \vee Q(z)]$

$\equiv \neg T$

$\equiv F$

20. (c)

$\forall x \in N\,[x \neq 7 \wedge \text{Prime}(x) \to \neg \text{Divisibleby7}(x)]$

$\cong \forall x \in N\,[x = 7 \vee \neg \text{Prime}(x) \vee \neg \text{Divisibleby7}(x)]$

$\cong \neg\exists x \in N\,[x \neq 7 \wedge \text{Prime}(x) \wedge \text{Divisibleby7}(x)]$

All represents that "no prime except 7 is divisible by 7".

21. (a)

Everybody loves Modi: $\forall x$ Loves (x, Modi).

Everybody loves somebody: $\forall x \exists y$ Loves (x, y).

There is somebody whom everybody loves: $\exists y \forall x$ Loves (x, y).

There is somebody whom no one loves: $\exists y \forall x \neg$ Love (x, y).

22. (b)

$\forall x \forall y[((x > 0) \wedge (y > 0)) \to (x + y > 0)]$

Negation of above logic statement.

$\neg[\forall x \forall y[((x > 0) \wedge (y > 0)) \to (x + y > 0)]$

$\exists x \exists y \neg[((x > 0) \wedge (y > 0)) \to (x + y > 0)]$

$\exists x \exists y \neg[\neg((x > 0) \wedge (y > 0)) \vee (x + y > 0)]$

$\exists x \exists y[((x > 0) \wedge (y > 0)) \wedge (x + y \leq 0)]$

23. (c)

$\forall x \forall y((\text{Male}(x) \wedge \text{Male}(y)) \to (x = y \vee y = x))$

There is atmost one male in the class.

$\exists x \exists y(\text{Male}(x) \wedge \text{Male}(y) \wedge x \neq y \wedge \forall z(\text{Male}(z) \to (z = x \vee z = y)))$

There are exactly two males in the class.

$\forall x \forall y \forall z((\text{Male}(x) \wedge \text{Male}(y) \wedge \text{Male}(z)) \to (x = y \vee x = z \vee y = z))$

There are atmost two males in the class.

24. (c)

$(\exists x)(\forall y)[(a(x, y)) \wedge (b(x, y)) \wedge \neg c(x, y)]$

is equivalent as

$\neg[(\forall x)(\exists y)[(a(x, y) \wedge b(x, y)) \to c(x, y)]]$.

■■■■

Combinatorics

1. The number of distinguishable permutations of the letters in the word ELEPHANT are
(a) 40320 (b) 20160
(c) 20610 (d) 40230

2. How many words can be formed out of the letters of the word "TECHNOLOGY" starting with T and ending with Y
(a) 40320 (b) 20160
(c) 10080 (d) 1814400

3. Total number of words of 4 letters can be formed with letters *a, b, c, d, e, f, g* and *h*, when *e* and *f* are excluded and *a* occurs twice.
(a) 360 (b) 180
(c) 15 (d) None of these

4. Find the coefficient of x^{20} in $(1 + x + x^2 + ...)^2$ ________?

5. In how many ways can 11 men and 8 women sit in a row if all the men sit together and all the women sit together.
(a) 11 ! 8 ! (b) $\frac{11!8!}{2}$
(c) 2.11 ! 8! (d) None of these

6. Suppose that a state's license plates consist of 3 letters followed by 3 digit then total number of different places can be formed (no repetitions allowed) ________?

7. Find $\sum_{k=0}^{n}\binom{n}{k}\cdot 3^k$ =?
(a) $3^n + 1$ (b) $4^n + 1$
(c) 3^n (d) 4^n

8. Find the number of arrangement that can be made from the letters of the word MADEEASY, without changing the place of vowels in the word?
(a) 6 (b) 24
(c) 120 (d) None of these

9. Find the solution of $x_n = 2x_{n-1} + 3x_{n-2}$, $n \geq 2$ with $x_0 = 3$ and $x_1 = 1$.
(a) $2(-1)^n + 3^n$ (b) $2 + 3^n$
(c) 3^n (d) None of these

10. How many numbers [Each with different digits] between 1000 and 10000 can be formed with 7, 6, 5, 4, 3, 2?
(a) 60 (b) 360
(c) 720 (d) None of these

11. The number of ways of distributing 52 cards to 4 people with a hand of 5 cards each is
(a) $\frac{52!}{(5!)^4\,4!}$ (b) $\frac{52!}{(5!)^4}$
(c) $\frac{52!}{(5!)^4\,(32)!}$ (d) $\frac{52!}{(5!)^4\,(32)!\,4!}$

12. How many ways can 10 balls be chosen from an urn containing 10 identical green balls, 5 identical yellow balls and 3 identical blue balls.
(a) 10 (b) 15
(c) 18 (d) 24

13. The number of triangles that can be formed from a '*n*' sided regular polygon, none of whose sides comes from the side of the polygon is
(a) ${}^nC_3$ (b) ${}^nC_3 - \{n(n-2\}$
(c) ${}^nC_3 - \{n(n-4) + n\}$ (d) ${}^nC_3 - n$

14. Let $a_n = -4a_{n-1} + 12 \cdot a_{n-2}$. Then find a_n?
(a) $a_n = A(6)^n + B \cdot (3)^n$
(b) $a_n = A(3)^n + B \cdot (-2)^n$
(c) $a_n = A(-3)^n + B \cdot (2)^n$
(d) $a_n = A(-6)^n + B \cdot (2)^n$

15. Find the number of possible 5 character passwords, if all characters must be lower case letters and distinct. Assume characters are only from a to z.

(a) $(26)^5$ (b) $\frac{26!}{16!}$

(c) $\frac{26!}{21!}$ (d) ${}^{26}C_5$

16. In how many ways can 4 girls and 5 boys be arranged in a row so that all the 4 girls are together?

(a) 1440 (b) 4320

(c) 17280 (d) 86400

17. In how many ways can we distribute 5 distinct balls, $B_1, B_2, \ldots, B_5$ in 5 distinct cells, $C_1, C_2, \ldots, C_5$ such that ball B_i is not in cell C_i, $\forall i = 1, 2, \ldots, 5$ and each cell contains exactly one ball?

Answers Combinatorics

1.	(b)	2.	(b)	3.	(b)	5.	(c)	7.	(d)	8.	(b)	9.	(a)	10.	(b)	11.	(c)
12.	(d)	13.	(c)	14.	(d)	15.	(c)	16.	(c)								

Explanations Combinatorics

1. (b)

Total number of distinguishable permutation of word ELEPHANT

$$= \frac{\lfloor 8}{\lfloor 2} = 8\times7\times6\times5\times4\times3 = 20160$$

2. (b)

TECHNLOLOGY

Excluding T and Y there are 8 letters left. Among these 8 letters two O's present.

So total permutation

$$= \frac{8!}{2!} = 8\times7\times5\times6\times5\times4\times3 = 20160$$

3. (b)

Excluding e and f total number of letters is 6. Among this 6, select 4 letters then arrange among themselves (there are two a's).

$$= \frac{{}^6P_4}{2!} = \frac{6\times5\times4\times3}{2} = 180$$

4. (21)

Let $f(x) = (1 + x + x + x^3 + \ldots + x^n)^2$

$$= \left\{\frac{1}{1-x}\right\}^2 = (1-x)^2$$

the coefficient of x^{20} is equal to

$$= {}^{2+20-1}C_{20} = {}^{21}C_{20} = \frac{21!}{20! * 1!} = 21.$$

5. (c)

Men permutation $\Rightarrow$ 11 ! ways

Women permutation $\Rightarrow$ 8 ! ways

There are two choices

$$\Rightarrow \left.\begin{array}{c}\text{Men follow the women}\\ \text{or}\\ \text{Women follow the men}\end{array}\right\} \Rightarrow 2\times11!\times8!$$

6. (11232000)

$${}^{26}P_3 * {}^{10}P_3$$
$$= 26 * 25 * 24 * 10 * 8 * 9$$
$$= 11232000$$

7. (d)

$$\sum_{k=0}^{n}\binom{n}{k}x^{n-k}\cdot y^k = (x+y)^n$$

$$\sum_{k=0}^{n}\binom{n}{k}(1)^{n-k}\cdot 3^k = (1+3)^n = 4^n$$

So option (d) is correct.

8. (b)

M $\boxed{A}$ D $\boxed{EEA}$ SY

4 positions are fixed.

Remaining 4 positions can be arranged in ${}^4P_4$

$= 4! = 24$ ways

So option (b) is correct.

9. (a)

$(x^2 - 2x - 3) = 0$

$\Rightarrow \quad x = -1, 3$

$x_n = A(-1)^n + B(3)^n$

$\Rightarrow \quad x_0 = 3 \quad \Rightarrow 3 = A + B \qquad$ ...(i)

$\Rightarrow \quad x_1 = 1 \quad \Rightarrow \quad 1 = -A + 3B \qquad$ ...(ii)

From (i) and (ii)

$A = 2$ and $B = 1$

$\therefore \quad x_n = 2(-1)^n + 3^n$

So option (a) is correct.

10. (b)

Each number contains 4 digits and all digits are distinct,

$\begin{bmatrix}\text{Given one of 6 digits to} \\ \text{place in each position}\end{bmatrix} \Rightarrow \underset{6}{\square} \times \underset{5}{\square} \times \underset{4}{\square} \times \underset{3}{\square} = 360$

So option (b) is correct.

11. (c)

The problem is ordered partitions of type

$$(52; 5, 5, 5, 5, 32) = \frac{52!}{(5!)^4\, 32!}$$

12. (d)

The problem corresponds to the number of non negative integral solutions to

$x_1 + x_2 + x_3 = 10$ with the conditions,

$0 \le x_1 \le 10$

$0 \le x_2 \le 5$

$0 \le x_3 \le 3$

Generating functions are required, since the variables have an upper constraint

The generating function is

$(1 + x + x^2 ...)(1 + x + x^2 + x^3 ... + x^5)(1 + x + ... x^3)$

$$= \left(\frac{1}{1-x}\right)\left(\frac{1-x^6}{1-x}\right)\left(\frac{1-x^4}{1-x}\right)$$

$$= \frac{(1-x^6)(1-x^4)}{(1-x)^3}$$

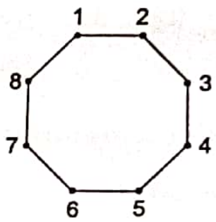
$$= \left(1 - x^4 - x^6 + x^{10}\right)\sum_{r=0}^{\infty} {}^{3-1+r}C_r x^r$$

$$= \left(1 - x^4 - x^6 + x^{10}\right)\sum_{r=0}^{\infty} {}^{r+2}C_r x^r$$

The coefficient of x^{10} in above generating function is ${}^{12}C_{10} - {}^8C_6 - {}^6C_4 + {}^2C_0 = 24$.

13. (c)

For example take $n = 8$ (octagon)



${}^8C_3$ is the total number of triangles using vertices of the octagon.

We need to subtract those triangles formed using exactly one edge of octagon first (say (1, 2) which is (1, 2, 4), (1, 2, 5), (1, 2, 6) and (1, 2, 7).

Since there are 8 such edges, no. of triangles with exactly one edge common with edge of octagon is 4×8.

Then with 2 edges in common is (1, 2, 3), (2, 3, 4), (3, 4, 5).... (8, 1, 2).

Totally 8 of them.

So answer is ${}^8C_3 - (4 \times 8 + 8)$

(Note: 3 edges common is not possible)

Similarly for n sides polygon answer is

${}^nC_3 - \{(n-4)n + n\}$

14. (d)

$a_n = -4a_{n-1} + 12\, a_{n-2}$

$a_n + 4a_{n-1} - 12\, a_{n-2} = 0$

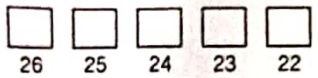
$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x = -6, x = 2$

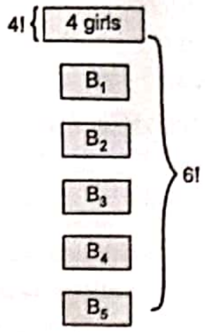
$\therefore a_n = A(-6)^n + B \cdot (2)^n$

15. (c)



$$= 26 \times 25 \times 24 \times 23 \times 22 = \frac{26!}{21!}$$

16. (c)



$\therefore$ Total ways = 6! × 4! = 17280

17. (44)

Given problem is dearrangement problem

$$D_n = \sum_{r=2}^{n} (-1)^r \frac{n!}{r!}$$

$$D_5 = \sum_{r=2}^{5} (-1)^r \frac{5!}{r!}$$

$$= \left(\frac{5!}{2!} - \frac{5!}{3!} + \frac{5!}{4!} - \frac{5!}{5!}\right)$$

$$= (60 - 20 + 5 - 1) = 44$$

■■■■

Set Theory and Algebra

1. Let $R_1 = \{(a, b), (a, c), (c, a), (a, a)\}$ and $R_2 = \{(c, c), (c, b), (a, d), (d, b), (c, d)\}$.
Where R_1 and R_2 are relations on set $A = \{a, b, c, d\}$. Then $R_1 \cup R_2$ is
(a) reflexive (b) symmetric
(c) transitive (d) assymetric

2. Let f is a function from set of integers to the set of integers. Find which of the following function is neither one to one nor onto function.
(a) $f(x) = x + 5$ (b) $f(x) = 3x - 100$
(c) $f(x) = x^3 + 1$ (d) $f(x) = x^2 + 1$

3. Let $|A| = 56$, $|A| = 57$, $|C| = 59$, $|A \cap C| = 46$, $|B \cap C| = 44$, $|A \cup B \cup C| = 80$ and $|A \cap B \cap C| = 41$. Then find $|A \cup B|$?
(a) 50 (b) 60
(c) 70 (d) None of these

4. Consider $(S, *)$ is a semigroup where $S = \{a, b, c, d\}$. Assume $a * b = c$, $b * b = a$ and $d * a = b$. Then $d * c =$
(a) a (b) b
(c) c (d) None of these

5. The function $f : Z \times Z \to Z$ defined as $f(m, n) = m - n$ is
(a) one-to-one
(b) onto
(c) bijective
(d) neither one-to-one nor onto

6. Let N be the set of natural numbers and G is a set defined as:
$$G = \{x | x \in N \text{ and } \forall_y \in N, x = y \text{ modulo } 5\}$$
Relation $R \subseteq G \times G$ which is defined as:
$$R = \{(a, b) \mid |a - b| = 1\}.$$
Consider the following relations:
(i) Reflexive (ii) Symmetric and
(iii) Transitive
Find the relation R?
(a) (i) only (b) (ii) only
(c) (i), (ii) and (iii) (d) None of these

7. Let $(S, .)$ be a semigroup where:
$S = \{g, a, t, e, c, s\}$ and $a^2 = s$, $g \cdot a = t$, $e \cdot g = a$.
Find the value of $e \cdot t$?
(a) t (b) s
(c) g (d) None of these

8. Let $X = \{\{\ \}, \{1\}\}$. Then find $P(x)$?
(a) $\{\{\ \}, \{1\}, \{\{\ \}\}, \{\{\ \}, \{1\}\}\}$
(b) $\{\{\ \}, \{1\}, \{\{\ \}\}, \{\{\ \}, \{1\}\}\}$
(c) $\{\{\ \}, \{\{1\}\}, \{1\}, \{\{\ \}, \{1\}\}\}$
(d) $\{\{\ \}, \{\{\ \}\}, \{\{1\}\}, \{\{\ \}, \{1\}\}\}$

9. Let f be a function from set of integers to set of integers (i.e., $f : Z \to Z$). Find which of the following function f is onto function?
(a) $f(x) = x^3 - 1$ (b) $f(x) = x^4 + 1$
(c) $f(x) = 7x - 5$ (d) $f(x) = x - 6$

10. Let $f : A \to B$ and $g : B \to C$ denote two functions. If the function $g \circ f : A \to C$ is a surjection and g is an injection, then function f is ______.
(a) Injection (b) Surjection
(c) Bijection (d) None of these

11. In a survey of 85 people it is found that 31 speak Hindi, 43 speak English and 39 speak Bangla. Also 13 speak both Hindi and English and 20 speak both English and Bangla and 15 speak none. Find out number of people who speak all the three language.

(a) 20 (b) 25
(c) 5 (d) 10

12. The relation {(1, 2), (1, 3), (3, 1), (2, 1), (2, 2), (3, 2), (3, 1), (1, 4), (4, 2), (3, 3)}
(a) Reflexive (b) Symmetric
(c) Transitive (d) Asymmetric

13. Let $A = \{p, q, r\}$
$B = \{1, 2, 3, 4\}$
$C = \{a, b, c, d, e\}$
then the difference of element in $(A \times B) \times C$ and $(A \times B) \times B$ is ________? [Neglect the sign]

14. A relation "congruence modulo m" ($\equiv$) over the set of positive integers is given by $\equiv\; = \{(x, y) \mid x - y$ is divisble by $m\}$ then relation is
(a) Reflexive relation
(b) Symmetric relation
(c) Transitive relation
(d) Equivalence relation

15. Let $R = \{(x, y) \mid x, y \in \mathbb{N}, x = 10 - y\}$. Then R is ________.
(a) Reflexive, symmetric, transitive
(b) Symmetric, reflexive, not transitive
(c) symmetric, not reflexive, not transitive
(d) not reflexive, not symmetric, not transitive

16. A survey of 150 college students reveals that 83 own cars, 97 own bikes, 28 own motorcycles, 53 own a car and a bike, 14 own car and a motorcycle, 7 own a bike and a motorcycle and 2 own all three. How many students own a bike and nothing else?

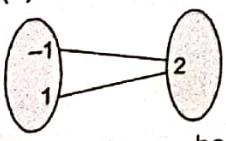
Answers Set Theory and Algebra

1.	(c)	2.	(d)	3.	(c)	4.	(a)	5.	(b)	6.	(b)	7.	(b)	8.	(d)
9.	(d)	10.	(b)	11.	(c)	12.	(c)	14.	(d)	15.	(c)				

Explanations Set Theory and Algebra

1. (c)
$X = R_1 \cup R_2$
$= \{(a, b), (a, c), (c, a), (a, a), (c, c), (c, b), (a, d), (d, b)\}$
X is not reflexive, $(b, b) \notin X$
X is not symmetric, $(a, b) \in X$ but $(b, a) \notin X$.
X is not asymmetric, $(a, c) \in X$ but $(c, a) \in X$
X is transitive.

2. (d)
(a) f is both one-one and onto.
(b) f is one-one, and not onto.
(c) f is one-one, and not onto.
(d) f is not one-one and not onto
$f(x) = x^2 + 1$
$f(-1) = f(1) = 2$, not one to one



and zero has no preimage, hence not onto.

3. (c)
$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|} \rightarrow (1)$$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cup B|$
$- |A \cap C| - |B \cap C| + |A \cap B \cap C|$
$80 = 56 + 57 + 59 - |A \cap B| - 46 - 44 + 41$
$|A \cap B| = 43$
$|A \cup B| = |A| + |B| - |A \cap B|$
$= 56 + 57 - 43 = 70$

4. (a)
$d * c = d * (a * b)$ [Given, $c = a * b$]
$= (d * a) * b$
[Associative holds in semigroup]
$= b * b$ [Given, $b * b = a$] $= a$

5. (b)
One-to-one:
We need to check if
$f(x_1, x_2) = f(y_1, y_2)$

$\Rightarrow \quad (x_1, x_2) = (y_1, y_2)$ ...(*i*)

LHS: $f(x_1, x_2) = f(y_1, y_2)$

$\Rightarrow \quad x_1 - x_2 = y_1 - y_2$

which does not imply that $x_1 = y_1$ and $x_2 = y_2$

So in eq. (*i*) LHS $\nRightarrow$ RHS

So, *f* is not one-to-one

Onto:

For every number say $k \in z$

Since we have $f(k, 0) = k - 0 = k$

$\exists$ a pre-image $(k, 0) \in Z \times Z$ such that

$f(k, 0) = k$

So it is onto.

6. (b)

$$G = \{x \mid x \in N \text{ and } x = y \text{ modulo } 5, \forall_y \in N\}$$

$\Rightarrow G = \{0, 1, 2, 3, 4\}$

Given

$$R = \{(a, b) \mid |a - b| = 1\}, R \subseteq G \times G$$

$$\Rightarrow R = \{(0,1), (1,0), (1,2), (2,1), (2,3), (3,2), (3,4), (4,3)\}$$

(i) R is not reflexive ; $(a, a) \notin R$

(ii) R is symmetric ; if $(a, b) \in R$

$\Rightarrow (b, a) \in R$

(iii) R is not transitive; $(0, 1)$ and $(1, 0) \in R$ but $(0, 0) \notin R$

$\therefore$ Option (b) is correct.

7. (b)

$e \cdot t = e \cdot (g \cdot a) \quad [\because t = g \cdot a]$

$= (e \cdot g) \cdot a \quad [\because \text{Associativity}]$

$= a \cdot a \quad [\because e \cdot g = a]$

$= a^2 = s$

So option (b) is correct.

8. (d)

Given $X = \{\{\,\}, \{1\}\} = \{a, b\}$

[Assume $a = \{\,\}, b = \{1\}$]

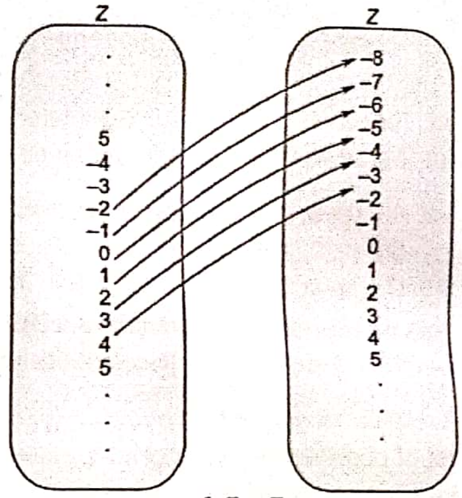
$P(x) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$= \{\{\,\}, \{\{\,\}\}, \{\{1\}\}, \{\{\,\}, \{1\}\}\}$

Note: If X has 2 elements then its power set contains $2^2 = 4$ elements.

$\therefore$ Option (d) is correct.

9. (d)



$f: Z \to Z$

f is both one to one and onto function

(a) $f(x) = x^3 - 1$ is not onto

$\left.\begin{array}{l} f(-2) = -7 \\ f(-1) = 0 \end{array}\right\}$ in between 0 and -7 elements have no preimage

(b) $f(x) = x^4 + 1$ is also not onto

$\left.\begin{array}{l} f(1) = 2 \\ f(-1) = 2 \end{array}\right\}$ *same* element (2) has two preimages

(c) $\left.\begin{array}{l} f(-1) = -12 \\ f(0) = -5 \end{array}\right\}$ not all elements are mapped. $\therefore f(x) = 7x - 5$ is not onto

So option (d) is correct.

10. (b)

$f: A \to B$

$g: B \to C$ is injection: $\forall b \in B, g(b) = c$ distinct images in C.

$g \circ f: A \to C$ is surjection

$g(f(a)) = c$

$\Rightarrow g(f(a)) = g(b)$

$\exists a \in A$

$\therefore \quad f(a) = b$

So, $f: A \to B$ is surjection.

11. (c)

Suppose P, Q and R is set of people who speak Hindi, English and Bangla respectively.

$$\Rightarrow \quad |P| = 31$$
$$|Q| = 43$$
$$|R| = 39$$
$$|P\cap Q| = 15$$
$$|P\cap R| = 13$$
$$|Q\cap R| = 20$$
$$\Rightarrow |P\cup Q\cup R| = 85 - 15 = 70$$

therefore $|P\cap Q\cap R|$

$$= 70 + (15+13+20) - (31+43+39)$$
$$= 70 + 48 - (113) = 118 - 113 = 8$$

12. (c)

Since (4, 4) is not in relation hence not reflexive.

If (4, 1) is there then, in relation but (4, 1) is not in symmetric relation.

If follows all the transitive property.

13. (0)

$(A\times B) \equiv 3\times 4 = 12$ elements

$(A\times B)\times C \equiv$ contain $12\times 5 = 60$ elements

Now $(A\times C)$ contains $3\times 5 = 15$ elements

$(A\times C)\times B$ contains $15\times 4 = 60$ elements

So, difference $= 60 - 60 = 0$.

14. (d)

$a - a = 0$ is divisible by m i.e., $a \equiv a \pmod m$

So $\equiv$ is reflexive.

Let

$$\Rightarrow \quad a \equiv b + km \text{ where } k\in z$$
$$b = a - km$$
$$b = a + (-k)\,m$$
$$b = a \pmod m$$

Hence $\equiv$ is symmetric.

Again, let $a \equiv b \pmod m$, $b \equiv c \pmod m$

$\Rightarrow a - b = km$ and $b - c = K'm$, where $k, K' \in z$

$\Rightarrow a - b + b - c = km + K'm$

$\Rightarrow a - c = (k + K')\,m$

$\Rightarrow \quad a \equiv c \pmod m$

So, $\equiv$ is transitive.

Hence, $\equiv$ is an equivalence relation.

15. (c)

$x = 10 - y$ can be written as $x + y = 10$

$x + x \neq 10 \Rightarrow (x, x) \notin R$ [Not reflexive]

If $x + y = 10$ then $y + x = 10$ [Symmetric]

If $x + y = 10$ and $y + z = 10$ then $x + z \neq 10$ [Not transitive]

$\therefore$ R is symmetric only.

16. (39)

$A \rightarrow$ The set of students owning cars

$B \rightarrow$ The set of students owning bikes

$C \rightarrow$ The set of students owning motorcycles.

$|A| = 83$, $|B| = 97$, $|C| = 28$, $|A\cap B| = 53$

$|A\cap C| = 14$, $|B\cap C| = 7$, $|A\cap B\cap C| = 2$

$|A\cup B\cup C| = 150$.

"$B - (A\cup C)$" denotes the set of students who own a bike and nothing else.

$|B - (A\cup C)| = |B| - |B\cap (A\cup C)|$

$(\because |X - Y| = |X| - |X\cap Y|)$

$= |B| - (|(B\cap A)\cup (B\cap C)|)$

$= |B| - (|B\cap A| + |B\cap C| - |B\cap A\cap B\cap C|)$

$= |B| - (|B\cap A| + |B\cap C| - |A\cap B\cap C|)$

$= 97 - (53 + 7 - 2) = 39$

■■■■

4
CHAPTER

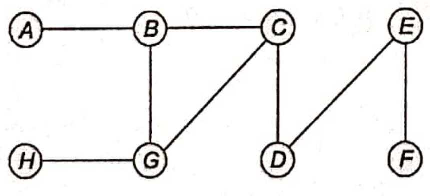
Graph Theory

1. Find number of edges in an undirected graph with 2 vertics of degree 5, 4 vertices of degree 3 and remaining 4 vertices of degree 4?
   (a) 15 (b) 19
   (c) 22 (d) 17

2. If the graph has more than one component ($K > 1$) then find the Euler formula? [Assume 'n' vertices 'e' edges, 'r' regions]
   (a) $n - e + r = 2$ (b) $n - r + e = 2$
   (c) $n - e + r = K + 1$ (d) $n - r + e = K = 1$

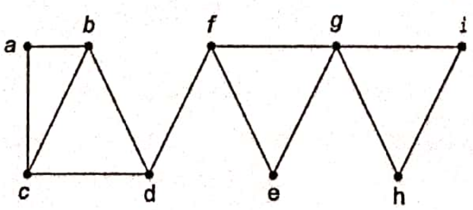
3. Identify the correct statement. (Assume a graph is Hamiltonian graph if it contains Hamiltonian cycle, and a graph is Euler graph if it contains Euler circuit)
   (a) Every Hamiltonian is Euler graph
   (b) Every Euler graph is Hamiltonian graph
   (c) Both (a) and (b)
   (d) None of these

4. Find number of possible cut vertices in the following graph?



   (a) 3 (b) 4
   (c) 5 (d) None of these

5. Find chromatic number for the following graph?



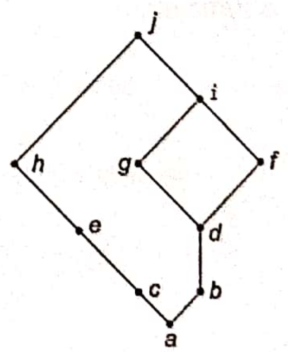
   (a) 2 (b) 3
   (c) 4 (d) 5

6. Let a graph (G) has 4 vertices and there are two vertices are of degree 2, one vertex of degree 1 and remaining vertex of degree 3. Then complement of G is
   (a) connected (b) disconnected
   (c) either (a) or (b) (d) none of these

7. A graph 'G' is self complementary graph and G has n vertices then find number of edges in self complementary graph.
   (a) $n/2$ (b) ${}^{n}C_2$
   (c) ${}^{n}C_2/2$ (d) n

8. Assume undirected graph G is connected. G has n-vertices and m edges. Find the minimum number of edges whose deletion from graph G is **always** guarantee that it will become disconnected.
   (a) ${}^{n}C_2 + 1 + m$ (b) ${}^{(n-1)}C_2 + 1 - n$
   (c) ${}^{n}C_2 - n + 2$ (d) $m - n + 2$

9. Consider the following lattice



   Find the number of complements of h?
   (a) 3 (b) 4
   (c) 5 (d) None of these

10. Consider the following graph G.

Find the chromatic number for the above graph.

(a) 4 (b) 5
(c) 6 (d) None of these

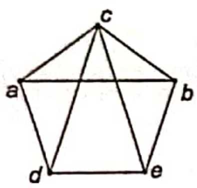
11. Assume a full binary tree contain l-levels. Find the number of cut vertices in the tree.

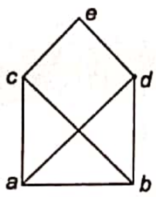
(a) 2^l (b) 2^{l-1}
(c) $2^l - 1$ (d) $2^{l-1} - 1$

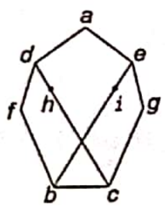
12. Suppose a graph G has the degree sequence (1, 1, 2, 2, 3, 3, 3, 3, 4, 5, 5). Then find the degree sequence of the complement of G.

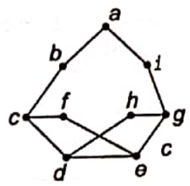
(a) (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)
(b) (5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 9)
(c) (5, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9)
(d) None of these

13. Which of the following graphs have hamiltonian cycle?

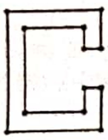
(a) 

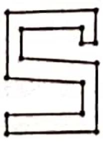
(b) 

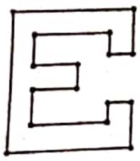
(c) 

(d) 

14. Consider the following graphs.

(1) 

(2) 

(3) 

Find the order of each graph in decreasing order of their covering number?

(a) 2, 1, 3 (b) 2, 3, 1
(c) 3, 1, 2 (d) 3, 2, 1

15. Let G be a finite group. If A and B are subgroups of G with orders 4 and 5 respectively then $|A \cap B|$ = ________.

16. Which of the following properties of algebraic system is/are not defined by lattices?

(a) Join and meet opeations are commutative.
(b) For any a and b in lattice A,
$a \vee (a \wedge b) = a$
$a \wedge (a \wedge b) = a$
(c) Both join and meet operations are not associative
(d) For every a in A, $a \vee a = a$ and $a \wedge a = a$

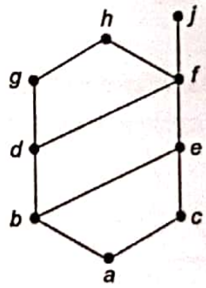
17. Which of the following graph has both eulerian and hamiltonian circuit?

(a) K_{14} (b) K_{13}
(c) Both (a) and (b) (d) None of these

18. What is the number of n-vertex labelled graph whose vertex set can be partitioned into two parts P and Q with P being clique and Q being a set of isolated vertices?

(a) 2^n (b) $2^n - n$
(c) $2^n - 1$ (d) None of these

19. Consider the hasse diagram of a poset given below. What is the number of lower bounds of the subset {g, h, f, d}?

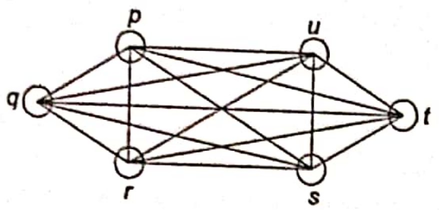


(a) 2 (b) 3
(c) 4 (d) 5

20. Let G be a directed graph where vertex set is the set of numbers from 1 to 100. There is an edge from a vertex i to a vertex j iff either $j = i + 1$ or $j = 3i$. The minimum number of edges in a path in G from vertex 1 to vertex 100 is:

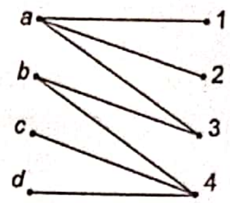
(a) 4 (b) 7
(c) 23 (d) 99

21. Consider the graph $G(V, E)$:



What is the colouring number of the graph?

22. Consider the following bipartite graph.



Identify the maximum matching for the above graph?

(a) {{b, 4},{c, 4},{d, 4}} (b) {{a, 1},{b, 3},{c, 4}}
(c) {{a, 1}, {b, 4}} (d) None of these

Answers Graph Theory

1.	(b)	2.	(c)	3.	(d)	4.	(c)	5.	(b)	6.	(b)	7.	(c)	8.	(d)	9.	(c)
10.	(a)	11.	(d)	12.	(c)	13.	(d)	14.	(d)	16.	(c)	17.	(b)	18.	(b)	19.	(b)
20.	(c)	22.	(b)														

Explanations Graph Theory

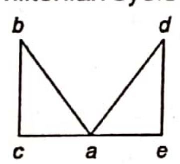
1. (b)

$$\Sigma d(V_i) = 2 * |E|$$
$$2 * 5 + 4 * 3 + 4 * 4 = 2 * |E|$$
$$10 + 12 + 16 = 2 * |E|$$
$$\Rightarrow \quad |E| = \frac{38}{2} = 19 \text{ edges}$$

3. (d)

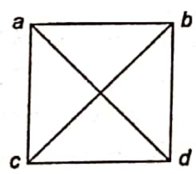
If graph contain Euler circuit, it need not contain Hamiltonian cycle and converse also true.

1.



[$a-b-e-b-c-a$ is Euler circuit
no hamiltonian cycle.]

2.



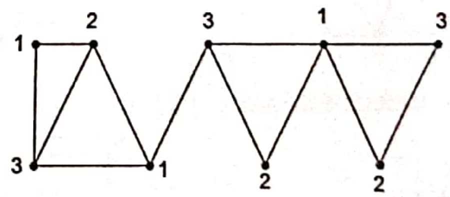
[No Euler circuit (all vertices must even degree)
$a-b-d-c-a$ is hamiltonian cycle.]

(1) is Euler but not Hamiltonian graph.
(2) is Hamiltonian but not Euler graph.

4. (c)

Delete B, graph can be disconnected.
By deleting any of B, G, C, D, E the graph will be disconnected. Hence 5 possible cut vertices in the graph.

5. (b)



Three colors required to color the above graph.
∴ Chromatic number = 3.

6. (b)

G has 4 vertices

$$\text{Maximum \# of edges} = \frac{4(4-1)}{2} = 6 \text{ Edges}$$
$$2 * 2 + 1 + 3 = 2|E|$$
$$\Rightarrow 4 + 1 + 3 = 2|E|$$
$$\Rightarrow |E| = 4$$

G has 4 edges

$\bar{G}$ has ${}^4C_2 - 4 = 6 - 4 = 2$ Edges

With 4 vertices and 2 edges, the graph is always disconnected.

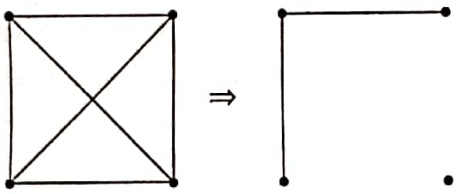
7. (c)

If G is self complementary, then $G \cong \bar{G}$ and G and $\bar{G}$ has same number of edges only when G has ${}^nC_2/2$ edge.

$$\boxed{G \cup \bar{G} = K_n}$$

8. (d)

Complete graph has ${}^nC_2$ edges (worst case). To make a connected graph, atmost $(n-1)$ edges required. To make it disconnected, graph should contain $(n-2)$ edges.



$$m = {}^nC_2 = {}^4C_2 = 6 \text{ edges}$$

${}^nC_2 - n + 2 = 6 - 4 + 2 = 4$ edges deleted

$\therefore$ $(m-n+2)$ edges deletion always guarantee that any graph will become-disconnected.

9. (c)

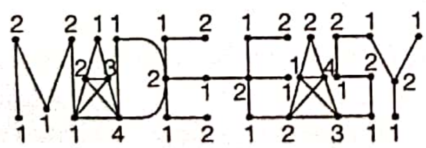
h^c = 'g' or 'i' or 'f' or 'd' or 'b'

x and y are complement to each other

iff $\quad x \vee y = L \cup B$

and $\quad x \wedge y = \text{GLB}$

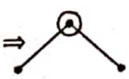
10. (a)



$\therefore$ 4 colours are required

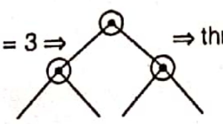
11. (d)

$l = 2 \Rightarrow$



$\Rightarrow$ one cut vertex

$l = 3 \Rightarrow$



$\Rightarrow$ three cut vertices

$\therefore$ # cut vertices $= 2^{l-1} - 1$

12. (c)

Number of vertices = 11

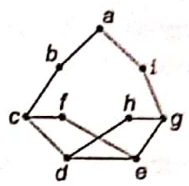
Each vertex can contain maximum degree 10.

In complement of G:

(10-1, 10-1, 10-2, 10-2, 10-3, 10-3, 10-3, 10-3, 10-4, 10-5, 10-5) = (9, 9, 8, 8, 7, 7, 7, 7, 6, 5, 5) is degree sequence.

So option (c) is correct.

13. (d)



$f-c-b-a-i-g-h-d-e$

It covers all vertices in cycle.

So option (d) is correct.

14. (d)

Graph 1 covering number = 6

Graph 2 covering number = 7

Graph 3 covering number = 8

15. (1)

$A \cap B = \{e\}$ where e is an identity of A and B.

$A \cap B$ is a subgroup of each of A and B.

So, order of $A \cap B$ must divide each of 4 and 5.

$\therefore \quad |A \cap B| = 1$

16. (c)

Both join and meet are associative for the given lattice.

17. (b)

K_{14} doesn't have Euler circuit.

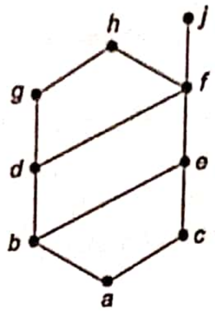
18. (b)

- One graph in which $|P| < 2$ i.e. there is no edge in the graph
- Second is ${}^nC_{|P|}$ where $|P| \geq 2$ where all vertex make complete graph. So, total number of such graphs are

$$= 1 + \sum_{k=2}^{n} {}^nC_K = 1 + \sum_{k=2}^{n} \left({}^nC_K\right) - 1 - n$$

$$= 2^n - n$$

19. (b)



The lower bound of subset {g, h, f, d} are d, b, a i.e. 3.

20. (c)
There is an edge from a vertex i to a vertex j iff either $j = i + 1$ or $j = 3i$.

So, possible set of edge are:

$$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 9 \to \ldots\ldots \to 100$$

So, minimum number of edges in a path or from vertex 1 to vertex 100 is

$$\underbrace{1 \to 3 \to 9 \to 27 \to 81}_{4} \to \underbrace{82 \to \ldots\ldots \to 99 \to 100}_{19}$$

So, total = 4 + 19 = 23.

21. (6)
For complete graph colouring number is equal to the number of vertices.

22. (b)
{{a, 1}, {b, 3}, {c, 4}} is maximum matching.

■■■■

5
CHAPTER

Probability

1. A fair six sided die is thrown twice. If the sum of the face values of these two tosses is 5 then what is the probability that the face value of the first toss is less than that of second toss?

2. There were three candidates for the position of the chairman of a college Mr. X, Mr. Y and Mr. Z, whose chances of getting the appointment are in the ratio 4 : 2 : 3 respectively. The probability that Mr. X if selected would introduce computer education in the college is 0.3. The probabilities of Mr. Y and Mr. Z doing the same are respectively 0.5 and 0.8. What is the probability that there was computer education in the college ______.

3. Fifty-two percent of the residents of New York city are in favor of out lawing cigarette smoking in publicaly owned areas. Approximation of the probability that greater than 50 percent of a random sample of n people from New York are in favor of this prohibition when
(a) $n = 11$
(b) $n = 101$
(c) $n = 10001$ are .55, .65 and .89 respectively.
How large would n have to be to make this probability exceed 0.95 ________ atleast.

4. A six card hand is dealt from an ordinary deck of cards. Find the probability that there are 3 cards of one suit and 3 of another suit.
(a) $\frac{^{13}C_6}{^{52}C_6}$
(b) $\frac{(^{13}C_3)^2}{^{52}C_6}$
(c) $\frac{2(^{13}C_3)^2}{^{52}C_6}$
(d) $\frac{6(^{13}C_3)^2}{^{52}C_6}$

5. Find the probability of getting number of tails more than 1 time in n flips ($n > 3$) of a fair coin?
(a) $2^n + 1$
(b) $2^n - n$
(c) $2^n - n - 1$
(d) $2^n - n + 1$

6. Let X be a real valued random variable-taking value in the interval [–1, 1] with a density function of the form $f(x) = x^2 + a$ for $-1 \le x \le 1$. Find the value of a?
(a) $\frac{1}{3}$
(b) $\frac{1}{6}$
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$

7. A typical page in a book contains one typo per page. What is the probability that there are exactly 8 typos in a given 10-page chapter?
(a) $e^{-10} . \frac{10^8}{8!}$
(b) $e^{-8} \frac{8^{10}}{10!}$
(c) $e^{-8} \frac{10^8}{8!}$
(d) none of these

8. Suppose a fair six sided dice is rolled 5 times. What is the probability to roll 4 atleast 1 time, but not more than 3 times?
(a) $\frac{23}{36}\left(\frac{5}{6}\right)^3$
(b) $\frac{37}{36}\left(\frac{5}{6}\right)^3$
(c) $\frac{1}{18}\left(\frac{5}{6}\right)^3$
(d) None of these

9. If the coin is tossed for even number of times then find the probability?
(a) $\frac{2}{3}$
(b) $\frac{3}{4}$
(c) $\frac{4}{5}$
(d) None of these

10. Suppose a random variable X has normal distribution with mean 0. If

$$P(X<-2)=\frac{1}{\sqrt{2\pi}}\int_{\sqrt{2}}^{\infty}\exp\left(-\frac{x^2}{2}\right)dx$$

Then the variance of X is

(a) 1 (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

11. Let A, B and C be three independent events each having the same probability p. Let $\bar{C}$ denote the complement of C. Then the conditional probability $P\{\bar{C}/A\cap(B\cup C)\}$ equals

(a) $\frac{1-p}{2-p}$ (b) $\frac{p}{1+p}$

(c) $1-p$ (d) $\frac{1}{1+p}$

12. A bag contains 40 tickets numbered 1, 2, 3, . . ., upto 40. Among them 4 are drawn at random and arranged in ascending order $t_1 < t_2 < t_3 < t_4$. The probability of t_3 being 25 (upto 2 decimal places) is ________.

13. Three horses A, B, C are in a race. A is twice as likely to win as B and B is twice as likely to win as C. Then probability that A wins is $P(A)$. Then value of $100P(A)$ is ________.

14. A and B throwing an unbiased dice alternatively. Whoever throws a number ≥ 5 first wins. If A throws the dice first then probability of B winning the game is ________.

15. Fifteen coupons are numbered, 1, 2, 3, ... 15. Five coupons are selected at random one at a time with replacement. The probability that the largest number appearing on the selected coupon is 8, is

(a) $\frac{8^5}{15^5}$ (b) $\frac{7^5}{15^5}$

(c) $\frac{8^5-7^5}{15^5}$ (d) None of these

16. A can hit a target 3 times in 5 shots, B hits target 2 times in 5 shots, C hits target 3 times in 4 shots. Find the probability of the target being hit when all of them try.

17. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3 (b) 3/7

(c) 1/2 (d) 4/7

18. A jar-1 contains m red and n green marbles. Jar-2 contains n red and m green marbles. If a marble is taken from 1st jar and put into 2nd jar, and then a marble is randomly picked from 2nd jar, what is the probability that the marble selected is red?

(a) $\frac{n}{m+n}$ (b) $\frac{m(n+1)+n^2}{(m+n)(m+n+1)}$

(c) $\frac{2mn}{(m+n)^2}$ (d) None of these

19. In a box of 20 pens, five are defective. Two pens are drawn at random without replacement. The probability of both pens being non-defective is ________.

(a) $\frac{21}{40}$ (b) $\frac{21}{38}$

(c) $\frac{45}{76}$ (d) $\frac{9}{16}$

20. A problem in statistics is given to four students A, B, C and D and their chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{3}$, and $\frac{1}{4}$ respectively. The probability that the problem will be solved is ________.

Answers Probability

4. (d) 5. (c) 6. (b) 7. (a) 8. (b) 9. (a) 10. (c) 11. (a) 15. (c)
17. (c) 18. (b) 19. (b)

Explanations Probability

1. (0.5)

Total number of possible pairs = 36

$\{(a,b) | 1 \le a \le 6, 1 \le b \le 6\}$

(*i*) Sum of face values = 5

$\{(1,4), (2,3), (3,2), (4,1)\}$

(*ii*) First toss is less than that of second toss.

$\{(1,4), (2,3)\}$

$\therefore$ Probability $= \frac{2}{4} = \frac{1}{2}$

2. (0.51)

E: Introduction of computer education.

Let A_1 : Mr. X is selected as chairman.

A_2 : Mr. Y is selected as chairman.

A_3 : Mr. Z is selected as chairman.

$$P(E) = P[(E \cap A_1) \cup (E \cap A_2) \cup (E \cap A_3)]$$

$$= \frac{4}{9}\cdot\frac{3}{10} + \frac{2}{9}\cdot\frac{5}{10} + \frac{3}{9}\cdot\frac{8}{10}$$

$$= \frac{23}{45} = 0.51$$

3. (1692)

Let S_n, the count of people in the sample who are in the favor of smoking prohibition.

$$P\{S > 0.5n\} = P\left\{\frac{S_n - 0.5n}{\sqrt{n(0.52)(0.48)}} > \frac{0.5n - 0.52n}{\sqrt{n(0.52)(0.48)}}\right\}$$

$$\equiv (0.04\sqrt{n})$$

So $0.04\sqrt{n} > 1.645$ (as $\phi(1.645) = 0.95$)

$n \ge 1691.266$

$\Rightarrow$ $n > 1692$

4. (d)

Two suits can be chosen: ${}^4C_2$ ways

3 cards can be picked from same suit: ${}^{13}C_3$ ways

$$\text{Probability} = \frac{{}^4C_2({}^{13}C_3 \cdot {}^{13}C_3)}{{}^{52}C_6}$$

$$= \frac{6\cdot({}^{13}C_3)^2}{{}^{52}C_6}$$

So option (d) is correct.

5. (c)

P(number of tails ≤ 1)

$= P(\# T's = 0) + P(\# T's = 1)$

$= 1 + {}^nC_1$

$= 1 + n$

$P(\# T's > 1) = 2^n - (\# T's \le 1)$

$= 2^n - (1 + n)$

$= 2^n - n - 1$

$\therefore$ Option (c) is correct.

6. (b)

Given $f(x) = x^2 + a$ for $-1 \le x \le 1$

Density function:

$$\int_{-1}^{1} f(x)\cdot dx = 1$$

$$\Rightarrow \int_{-1}^{1} (x^2 + a) = 1$$

$$\Rightarrow \left[\frac{x^3}{3} + ax\right]_{-1}^{1} = 1$$

$$\Rightarrow \frac{2}{3} + 2a = 1$$

$$\Rightarrow a = \frac{1}{6}$$

So option (b) is correct.

7. (a)

Poisson distribution $= e^{-\lambda} \cdot \frac{\lambda^x}{x!}$

Expected typos on one page is 1

$\Rightarrow$ Expected typos in 10 page is 10

$$\lambda = 10$$

$$\therefore \quad \text{Probability} = e^{-10}.\frac{10^8}{8!}.$$

So option (a) is correct.

8. (b)

$$\text{One } 4 \Rightarrow \binom{5}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^5$$

Two 4's $\Rightarrow$

$$\binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{1}{3}\left(\frac{5}{6}\right)^4$$

Three 4's $\Rightarrow$

$$\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = \frac{1}{18}\left(\frac{5}{6}\right)^3$$

$$\text{Probability} = \sum_{i=1}^{3}\binom{5}{i}\left(\frac{1}{6}\right)^i\left(\frac{5}{6}\right)^{5-i}$$

$$= \left(\frac{5}{6}\right)^5 + \frac{1}{3}\left(\frac{5}{6}\right)^4 + \frac{1}{18}\left(\frac{5}{6}\right)^3$$

$$= \left(\frac{5}{6}\right)^3\left[\frac{25}{36} + \frac{5}{18} + \frac{1}{18}\right]$$

$$= \left(\frac{5}{6}\right)^3\left[\frac{25+10+2}{36}\right]$$

$$= \frac{37}{36}\left(\frac{5}{6}\right)^3$$

So option (b) is correct.

9. (a)

Coin is tossed n times:

$$\left.\begin{matrix} HTHT\cdots HH \\ \text{(or)} \\ THTH\cdots TT \end{matrix}\right\} n \text{ is even number}$$

$$\text{Probability} = \frac{1}{2^n} + \frac{1}{2^n} = \frac{1}{2^{n-1}}$$

Probability for even number of times

= 2 times + 4 times + ...

$$= \frac{1}{2^{2-1}} + \frac{1}{2^{4-1}} + \frac{1}{2^{6-1}} + \cdots$$

$$= \frac{1}{2^1} + \frac{1}{2^3} + \frac{1}{2^5} + \cdots$$

$$= \frac{1}{2}\left(1 + \frac{1}{4} + \frac{1}{4^2} + \cdots\right)$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{2}{3}$$

$\therefore$ Option (a) is correct.

10. (c)

$$P(X < -2) = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{2}}^{\infty} e^{-\frac{x^2}{2}}dx$$

Changing X to Z on RHS,

$$\text{RHS} = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{2}}^{\infty} e^{-\frac{z^2}{2}}dz$$

$$= P(Z > \sqrt{2})$$

$$= P(Z < -\sqrt{2})$$

$$\text{Now } P(X < -2) = P\left(Z < \frac{-2-\mu_x}{\sigma_x}\right)$$

$$\text{So, } P\left(Z < \frac{-2-\mu_x}{\sigma_x}\right) = P(Z < -\sqrt{2})$$

$$\text{So } \frac{-2-\mu_x}{\sigma_x} = -\sqrt{2}$$

Here $\mu_x = 0$

$$\text{So, } \frac{-2-0}{\sigma_x} = -\sqrt{2}$$

$$\Rightarrow \quad \sigma_x = \sqrt{2}$$

So variance $= \sigma_x^2 = 2$

11. (a)
Given: A, B, C are independent with

$$P(A) = P(B) = P(C) = p$$

Now

$$P\{\bar{C} | A \cap (B \cap C)\} = \frac{P\{\bar{C} \cap (A \cap (B \cup C))\}}{P\{A \cap (B \cup C)\}}$$

Using boolean algebra we can simplify

$$\bar{C} \cap \{A \cap (B \cup C)\} = C'\{A(B+C)\}$$
$$= C'(AB + AC)$$
$$= C'AB + 0 = C'AB$$
$$= \bar{C} \cap A \cap B$$

So requires probability

$$= \frac{P(\bar{C} \cap A \cap B)}{P\{A \cap (B \cup C)\}}$$

$$= \frac{P(\bar{C}) \times P(A) \times P(B)}{P(A) \times \{P(B) + P(C) - P(B) \cap P(C)\}}$$

$$= \frac{(1-p)p^2}{p(p+p-p \times p)} = \frac{1-p}{2-p}$$

12. (0.04)
4 cards can be choosen from 40 cards in ${}^{40}C_4$ ways.

$\therefore$ Total number of outcomes = ${}^{40}C_4$

Assume t_3 is 25. There are 24 cards preceding 25. t_1 and t_2 can be choosen from these 24 cards in ${}^{24}C_2$ ways.

t_4 should be greater than 25.

Number of such cards = 15

$\therefore$ Number of ways of choosing $t_4 = {}^{15}C_1$

$\therefore$ Favourable outcomes = ${}^{24}C_2 \times {}^{15}C_1$

$\therefore$ Required probability = $\dfrac{{}^{24}C_2 \times {}^{15}C_1}{{}^{40}C_4}$

$$= \frac{414}{9139} = 0.045$$

13. (57.14)
Let $P(C) = p$

$P(B) = 2p$

$P(A) = 4p$

$P(A) + P(B) + P(C) = 1$

$p + 2p + 4p = 1$

$$p = \frac{1}{7}$$

$$\Rightarrow \quad P(A) = \frac{4}{7}$$

$$100P(A) = \frac{400}{7} = 57.14$$

14. (0.4)
A_W – A throws number greater than or equal to 5

$$P(A_W) = \frac{2}{6} = \frac{1}{3}$$

A_L – A throws number less than 5

$$P(A_L) = \frac{4}{6} = \frac{2}{3}$$

similarly $B_W = \dfrac{1}{3}$, $B_L = \dfrac{2}{3}$

For B to win = $A_L \cdot B_W + A_L B_L A_L B_W$ .....

$$= \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} \ldots\ldots$$

$$= \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}$$

$$a = \frac{2}{9} \quad r = \frac{4}{9}$$

The sum is infinite GP

Probability of B winning is = $\dfrac{\frac{2}{9}}{1-\frac{4}{9}} = \dfrac{2}{5} = 0.4$

15. (c)
Total ways = 15^5

For favorable ways, we must 5 coupons numbered from 1 to 8 so that '8' is selected atleast one. Thus total number of favorable ways are, $8^5 - 7^5$.

$\Rightarrow$ Required probability = $\dfrac{8^5 - 7^5}{15^5}$.

16. (0.94)

Let $P(A)$ be the probability of A hitting the target, $P(B)$ be the probability of B hitting the target, and $P(C)$ be the probability of C hitting the target.

Given $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$, $P(C) = \frac{3}{4}$

$\Rightarrow \quad P(\bar{A}) = \frac{2}{5}$, $P(\bar{B}) = \frac{3}{5}$, $P(\bar{C}) = \frac{1}{4}$

The probability that none of A, B, C hits the target

$$= P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot (\bar{B}) \cdot (\bar{C})$$

$$= \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{1}{4} = \frac{3}{50}$$

$\therefore$ Required probability $= P(A \cap B \cap C)$
= The probability of atleast one of A, B, C hitting the target.

$$= P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - \frac{3}{50} = \frac{47}{50} \simeq 0.94$$

17. (c)

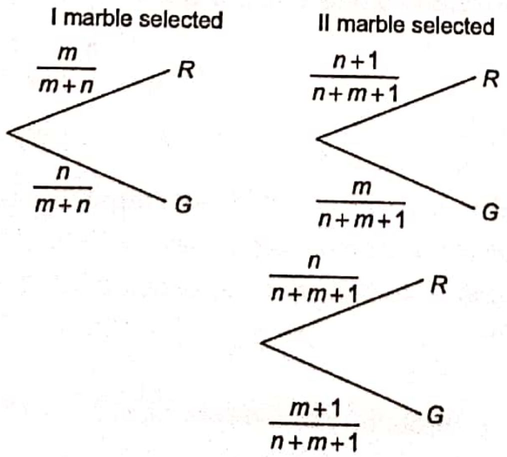
$$p(2^{nd} \text{ is red} \mid 1^{st} \text{ is white}) = \frac{p(2^{nd} \text{ is red and } 1^{st} \text{ is white})}{p(1^{st} \text{ is white})}$$

$$= \frac{p(1^{st} \text{ is white and } 2^{nd} \text{ is red})}{p(1^{st} \text{ is white})}$$

$$= \frac{\frac{4}{7} \times \frac{3}{6}}{\frac{4}{7}} = \frac{3}{6} = \frac{1}{2}$$

18. (b)

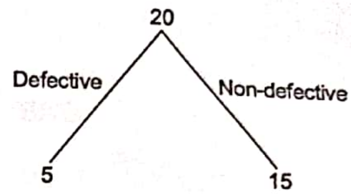
The tree diagram for problem is



$$p(R) = \frac{m}{m+n} \times \frac{n \times 1}{n+m+1} + \frac{n}{m+n} \times \frac{n}{n+m+1}$$

$$= \frac{m(n+1)+n^2}{(m+n)(m+n+1)}$$

19. (b)



$$\frac{{}^{15}C_1 \cdot {}^{14}C_1}{{}^{20}C_1 \cdot {}^{19}C_1} = \frac{15 \times 14}{20 \times 19} = \frac{3}{4} \cdot \frac{14}{19} = \frac{3 \times 7}{2 \times 19} = \frac{21}{38}$$

20. (0.75)

Probability that A fails to solve the problem is

$$1 - \frac{1}{3} = \frac{2}{3}$$

Probability that B fails to solve the problem is

$$1 - \frac{1}{4} = \frac{3}{4}$$

Probability that C fails to solve the problem is

$$1 - \frac{1}{3} = \frac{2}{3}$$

Probability that D fails to solve the problem is

$$1 - \frac{1}{4} = \frac{3}{4}$$

Since the events are independent, the probability that all the four students fail to solve the problem

is $\frac{2}{3} \times \frac{3}{4} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4} = 0.25$

$\therefore$ The probability that the problem will be solved $1 - 0.25 = 0.75$.

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6 CHAPTER

Linear Algebra

1. Consider the difference equation: $4a_r - 20a_{r-1} + 17a_{r-2} - 4a_{r-3} = 0$ with characteristic roots are $\frac{1}{2}, \frac{1}{2}, 4$. Then homogeneous solution is
(a) $a_r(h) = (A_1r + A_2)(1/2)^r + A_3(4)^r$
(b) $a_r(h) = (A_1r^2 + A_2r + A_3)(1/2)^r + A_4(4)^r$
(c) $a_r(h) = (A_1r + A_3)(1/2)^r + A_2(4)^r$
(d) None of these

2. For the linear system:

$$y + z = 2$$
$$2x + 3x = 5$$
$$x + y + z = 3$$

Solution gives value of z as ________.

3. Number of eigen value for $A_2 = \begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ are ________.

4. Find the value of x in the matrix A^{-1} for the following matrix A and A^{-1}.

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & \frac{1}{2} & x \\ 0 & 0 & 1 \end{pmatrix}$$

5. Let $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}_{3\times 2}$. If AB = I

then element f = ________.
(a) a
(b) $a + 1$
(c) b
(d) $1 - b$

6. Consider the following system of equations.

$$x + y = 2$$
$$x + py = 2$$

Where p is constant. Find the value of 'p' such that the system has more than two solutions.

7. Which of the following is a scalar matrix?
(a) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

8. Consider the following matrix A.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$$

For which value of x is the matrix A not invertible?

9. Find an eigen vector corresponding to largest eigen value of matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$
(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
(b) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$
(c) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$

10. Let $A = \begin{bmatrix} w & 2 & x \\ 1 & -3 & 0 \\ y & -1 & z \end{bmatrix}$, where x, y, z and w are unknown. Suppose $\begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$ is an eigen vector of A, then find the corresponding eigen value.

11. Let $M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$. Then find which of the following is not an eigen value of M.
(a) 2
(b) 3
(c) 4
(d) None of these

12. Assume the following set is linearly dependent.

$$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \left\{ \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ a \end{bmatrix} \right\}$$

Find the value of h?
(a) –5
(b) –6
(c) –7
(d) –8

13. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$.

Find the value of x, if the matrix A is not invertible?
(a) 9/2
(b) 7/2
(c) 10/2
(d) 11/2

14. Consider the system:

$$x_1 + 2x_2 = a$$
$$4x_1 + bx_2 = 5$$

If the above system has infinitely many solutions, find the values of 'a' and 'b'?

(a) $a = \frac{5}{4}, b \neq 8$
(b) $a \neq \frac{5}{4}, b = 8$
(c) $a = \frac{5}{4}, b = 8$
(d) $a \neq \frac{5}{4}, b \neq 8$

15. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$. Find A^{-1}?

(a) $\begin{bmatrix} 3 & 3 & -2 \\ -2 & 3 & 2 \\ -2 & 2 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 3 & -3 & 2 \\ -2 & 3 & -2 \\ 2 & 2 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 3 & -3 & 2 \\ -2 & 3 & -2 \\ -2 & 2 & -1 \end{bmatrix}$
(d) $\begin{bmatrix} 3 & 3 & -2 \\ -2 & 3 & -2 \\ -2 & 2 & -1 \end{bmatrix}$

16. Compute the determinant of the following matrix A.

$$A = \begin{pmatrix} 3 & -4 & 7 & 9 & 5 \\ 4 & 0 & 0 & 0 & 0 \\ 17 & 21 & -5 & 11 & 6 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \end{pmatrix}$$

(a) 0
(b) –14
(c) –24
(d) None of these

17. Consider the matrix $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$. Consider the following statements
I. A is Hermitian
II. A is Skew-Hermitian
III. A is Unitary

Which of the above statements are true?
(a) I only
(b) II only
(c) III only
(d) I and III only

18. If a matrix A is singular, then which of the following is the constant term of the characteristic polynomial of A. Assume λ_i are the eigen values.
(a) $\Sigma\lambda_i$
(b) 0
(c) 1
(d) Negative

19. Let $M = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $i = \sqrt{-1}$. Then $M - i\mathbf{I}$ has rank (where $\mathbf{I}$ is the identity matrix of order 4)
(a) 1
(b) 2
(c) 3
(d) 4

20. A matrix M has eigenvalues 2 and 5. The corresponding eigenvectors are $[1\ \ 2]^T$ and $[1\ \ 7]^T$ respectively. Let $X = [3\ \ 11]^T$. What is MX?
(a) $[9\ \ 43]^T$
(b) $[2\ \ 9]^T$
(c) $[7\ \ 39]^T$
(d) $[5\ \ 35]^T$

21. Which one of the following matrices is LU-decomposable (without interchanging rows or columns) for non-singular L and U?

(a) $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

22. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then the incorrect statement is
(a) $A^2 - 4A - 5I_3 = 0$
(b) $A^{-1} = \frac{1}{5}(A - 4I_3)$
(c) A^3 is not invertible
(d) A^2 is invertible

23. Let A be a 3×3 real matrix. Suppose 1 and -1 are two of the three eigen values of matrix A and 18 is one of the eigen values of $A^2 + 3A$, Then
(a) both A and $A^2 + 3A$ are invertible
(b) $A^2 + 3A$ is invertible but A is not invertible
(c) A is invertible but $A^2 + 3A$ is not invertible
(d) both A and $A^2 + 3A$ are not invertible

24. Matrix $A = \begin{bmatrix} a & 3 & 2 \\ 1 & b & 4 \\ 2 & 2 & c \end{bmatrix}$, if $abc = 70$ and $8a + 4b + 3c = 20$, then $A\,(\text{adj}\ A)$ is equal to

(a) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ (b) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$

(c) $\begin{bmatrix} 78 & 0 & 0 \\ 0 & 78 & 0 \\ 0 & 0 & 78 \end{bmatrix}$ (d) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

25. A 3×3 matrix is defined as

$$A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

The values of x required for which A^{-1} can't be determined will be
(a) 0, 3
(b) 0, 1 , 3
(c) 0, –1, 4
(d) 1, –1, 3

26. If the system of equations:
$x - ky - z = 0,\ kx - y - z = 0,\ x + y - z = 0$
has a non-zero solution, then the possible values of k are: '$-a$' and 'b' then $|a + b|$ = ______.

27. Choose the correct matrix given below, that has an inverse

(a) $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix}$

28. If 2, – 4 are the eigen values of a non-singular matrix A and $|A| = 4$, then the eigen values of Adj A are x and $-y$ then the value of $x + y$ is ______.

29. Consider the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

What are the eigen values of above matrix?
(a) 1, 1, 4
(b) 1, 0, 4
(c) 2, 1, 4
(d) 1, 4, 4

30. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = 2ab, \beta = a^2 + b^2$
(b) $\alpha = a^2 + b^2, \beta = ab$
(c) $\alpha = a^2 + b^2, \beta = 2ab$
(d) $\alpha = a^2 + b^2, \beta = a^2 - b^2$

31. For what values of α and β, the following simultaneous equations have an infinite number of solutions?

$$x + y + z = 5$$
$$x + 3y + 3z = 9$$
$$x + 2y + \alpha z = \beta$$

(a) 2, 7
(b) 3, 8
(c) 8, 3
(d) 7, 2

32. Find the absolute sum of squares of eigen values for the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

33. Let A be a square matrix, whose characteristic polynomial is $p(\lambda) = \lambda^2 - \frac{3}{4}\lambda - \frac{1}{4}$. What is the determinant of A?

(a) $\frac{3}{4}$
(b) $-\frac{3}{4}$
(c) 0
(d) $-\frac{1}{4}$

34. Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 60$ and $8x + 4y + 3z = 20$, then A (adj A) is equal to

(a) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$
(b) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$
(c) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$
(d) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

35. Let A be 2×2 matrix, Suppose $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ are eigen vectors corresponding to eigen values 1 and 2 of A respectively. Then A is ______.

(a) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$

36. Consider the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. The sum of its eigen values is ______.

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Answers Linear Algebra

1.	(a)	5.	(c)	7.	(c)	9.	(a)	11.	(b)	12.	(c)	13.	(d)
14.	(c)	15.	(c)	16.	(c)	17.	(b)	18.	(d)	19.	(d)	20.	(a)
21.	(c)	22.	(c)	23.	(a)	24.	(c)	25.	(a)	27.	(b)	29.	(a)
30.	(c)	31.	(a)	33.	(d)	34.	(c)	35.	(c)				

Explanations Linear Algebra

1. (a)

Characteristic equation for given difference equation is $4\alpha^3 - 20\alpha^2 + 17\alpha - 4 = 0$ and characteristic roots given are $\frac{1}{2}, \frac{1}{2}, 4$. Which gives homogeneous solution as

$$a_r(h) = (A_1 r + A_2)(1/2)^r + A_3(4)^r$$

2. (1)

As matrix will be

$$\begin{bmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 3 & 5 \\ 1 & 1 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 & 3 & 5 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3/2 & 5/2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow \quad z = 1$

3. (3)

Values of eigen values are 0, 1, 2.

4. (−2)

$$(A|I) = \left(\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \leftarrow R_1 - 2R_3$ and $R_2 \leftarrow R_2 - 4R_3$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & 0 & -2 \\ 0 & 2 & 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \leftarrow R_1/3$ and $R_2 \leftarrow R_2/2$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) = (I|A^{-1})$$

$$\therefore \quad A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow x = -2$$

5. (c)

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Given $AB = I$

$$\therefore \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -a+e & -b+f \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-b + f = 0 \Rightarrow f = b$

6. (1)

$$\left.\begin{array}{r} x+y=2 \\ x+py=2 \end{array}\right\} Ax = B$$

where $A = \begin{bmatrix} 1 & 1 \\ 1 & p \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$$(A|B) = \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & p & 2 \end{array}\right)$$

$R_2 \leftarrow R_2 - R_1$

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & p-1 & 0 \end{array}\right)$$

If $p = 1$ rank of A = rank of B and rank is less than 2.

$\therefore$ The system has multiple solutions when $p = 1$

7. (c)

If all diagonal elements of a square matrix are equal then such matrix is scalar matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is scalar matrix.}$$

8. (5.5)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$$

$$|A| = |(6-2x) - 1(0-10) + 1(0-5)|$$
$$= 11 - 2x$$

If $|A| = 0 \Rightarrow A$ is not invertible

$$\therefore \; 11 - 2x = 0 \Rightarrow x = \frac{11}{2}$$

9. (a)

$$|\lambda - AI| = (1-\lambda)(\lambda^2 - 2) + (2-\lambda) - \lambda$$
$$= -\lambda^3 + \lambda^2$$
$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$
$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$
$$\lambda = 0, \lambda = 1$$

The largest eign value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_2 + R_1}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$
$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$$
$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x^3$$

$$\therefore \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

10. (–2)

$$A\vec{V} = \lambda.\vec{V}$$

$$\begin{bmatrix} w & 2 & x \\ 1 & -3 & 0 \\ +y & -1 & z \end{bmatrix}\begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \lambda \cdot \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} p \\ -8 \\ q \end{bmatrix} = \lambda \cdot \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} \text{ (}p\text{ and }q\text{ are unknown)}$$

$$\Rightarrow \quad 4.\lambda = -8$$
$$\Rightarrow \quad \lambda = -2$$

11. (b)

$$\det(\lambda I - M) = \det \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda - 3 & 1 \\ 0 & 1 & \lambda - 3 \end{pmatrix}$$
$$= \lambda[(\lambda - 3)^2 - 1]$$
$$= \lambda(\lambda - 2)(\lambda - 4)$$
$$\Rightarrow \lambda(\lambda - 2)(\lambda - 4) = 0$$
$$\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4$$

$\therefore$ Option (b) is not correct eigen value.

12. (c)

$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent

$\Rightarrow$ $[\vec{v}_1, \vec{v}_2, \vec{v}_3]\vec{x} = 0$ has non trivial solutions

$$\Rightarrow \begin{bmatrix} 1 & -2 & -1 \\ 3 & -4 & 1 \\ -3 & 1 & a \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 2 & 4 \\ 0 & -3 & a+1 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - 3R_1$$
$$R_3 \leftarrow R_3 + R_2$$

$$R_2 \leftarrow -\frac{1}{2}R_2$$

$$R_3 \leftarrow R_3 - \frac{3}{2}R_2$$

$$= \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & a+7 \end{bmatrix}$$

If $a + 7 = 0$, then equation will have free variable. $a = -7$ [vectors are linearly dependent].

13. (d)

Determinant $= 1(6-2x) - 1(0-10) + 1(0-5)$

$= 11 - 2x$

Matrix is not invertible if determinant = 0

$\Rightarrow \quad 11 - 2x = 0$

$$\Rightarrow \quad x = \frac{11}{2}$$

14. (c)

$$\left[\begin{array}{cc|c} 1 & 2 & a \\ 4 & b & 5 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & b-8 & 5-4a \end{array}\right] R_2 \leftarrow R_2 - 4R_1$$

If $b = 8$ and $a = 5/4$, the last row is reduced to zero.

$\therefore$ System has infinite solutions

15. (c)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

$$[A/I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 2 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array}\right]$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} 3 & -3 & 2 \\ -2 & 3 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

16. (c)

Three row changes in the determinant as following.

$$\det = \begin{pmatrix} 3 & -4 & 7 & 9 & 5 \\ 4 & 0 & 0 & 0 & 0 \\ 17 & 21 & -5 & 11 & 6 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$= (-1)3\det \begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 17 & 21 & -5 & 11 & 6 \\ 3 & -4 & 7 & 9 & 5 \end{pmatrix} \begin{matrix} (1)\ R_1 \leftrightarrow R_2 \\ (2)\ R_2 \leftrightarrow R_5 \\ (3)\ R_3 \leftrightarrow R_4 \end{matrix}$$

$$= -\left(4 \cdot \det \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 21 & -5 & 11 & 6 \\ -4 & 7 & 9 & 5 \end{pmatrix} - 0 + 0 - 0 + 0 \right)$$

$$= -4 * 3 \cdot \det \begin{pmatrix} 2 & 0 & 0 \\ -5 & 11 & 6 \\ 7 & 9 & 5 \end{pmatrix} = -12 * 2 \cdot \begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix}$$

$= -24 * (11 * 5 - 6 * 9)$

$= -24 * 1 = -24$

17. (b)

The constant term of characteristic polynomial of any matrix A is always $= \Pi\lambda_i = |A|$.

Here, A is singular and hence $|A| = 0$

The constant term is therefore $= 0$

18. (d)

In this matrix

$$A^{Q} = A^{-t} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$

$$A^{-t} = \frac{1}{i^2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$

Since here $A^{\theta} = A$, the matrix is Hermitian

Since, also $A^{\theta} = A^{-1}$, the matrix is Unitary

So both I & III are true.

19. (d)

$$M - iI = \begin{bmatrix} 1-i & 0 & 2 & 0 \\ 0 & -1-i & 1 & 0 \\ 2 & 1 & 2-i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}$$

By Gauss-Elimination, we reduce this to echelon form and find its rank

$$\begin{bmatrix} 1-i & 0 & 2 & 0 \\ 0 & -1-i & 1 & 0 \\ 2 & 1 & 2-i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \xrightarrow{R_3 - \frac{2}{1-i}R_1}$$

$$\begin{bmatrix} 1-i & 0 & 2 & 0 \\ 0 & -1-i & 1 & 0 \\ 0 & 1 & -3i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \xrightarrow{R_3 + \frac{1}{1+i}R_2}$$

$$\begin{bmatrix} 1-i & 0 & 2 & 0 \\ 0 & -1-i & 1 & 0 \\ 0 & 0 & 1-\frac{7i}{2} & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}$$

The matrix is in echelon form and rank = Number of non-zero rows = 4.

20. (a)

$$M\hat{x} = \lambda\hat{x}$$

where $\hat{x}$ is eigen vector and λ is corresponding eigen values.

So, $M\begin{bmatrix}1\\2\end{bmatrix} = 2\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}$ ...(i)

$M\begin{bmatrix}1\\7\end{bmatrix} = 5\begin{bmatrix}1\\7\end{bmatrix} = \begin{bmatrix}5\\35\end{bmatrix}$ ...(ii)

Now let us express $\begin{bmatrix}3\\11\end{bmatrix}$ as linear combination of $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}1\\7\end{bmatrix}$ as follows:

$$\begin{bmatrix}3\\11\end{bmatrix} = 2\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}1\\7\end{bmatrix}$$

So $MX = M\begin{bmatrix}3\\11\end{bmatrix} = 2M\begin{bmatrix}1\\2\end{bmatrix} + M\begin{bmatrix}1\\7\end{bmatrix}$

Now, substituting from equation (i) and (ii), we get,

$$MX = 2\begin{bmatrix}2\\4\end{bmatrix} + \begin{bmatrix}5\\35\end{bmatrix} = \begin{bmatrix}4\\8\end{bmatrix} + \begin{bmatrix}5\\35\end{bmatrix} = \begin{bmatrix}9\\43\end{bmatrix}$$

21. (c)

The condition for LU decomposability is that all the principle minors should be non-zero.

In choice (a) and (b),

$a_{11} = 0 \Rightarrow$ first principle minor = 0

So the matrices in (a) and (b) are not LU decomposable.

In choice (c)

The principle minors are

$$|1| = 1 \neq 0, \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = -8 \neq 0,$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 132 \neq 0$$

So the matrix in choice (c) is LU-decomposable.

In choice (d) the third principle minor $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$ and hence the matrix in choice (d) is not LU decomposable.

22. (c)

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

We have $A^2 - 4A - 5I_3$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad 5I_3 = A^2 - 4A = A(A - 4I_3)$$

$$\Rightarrow \quad I_3 = A\left[\frac{1}{5}(A - 4I_3)\right]$$

$$\Rightarrow \quad A^{-1} = \frac{1}{5}(A - 4I_3)$$

Note that $|A| = 5$.

Since $|A^3| = |A|^3 = 5^3 \neq 0$, A^3 is invertible.

Similarly, A^2 is invertible.

23. (a)

$\lambda, 1, -1$ are eigen values of A then

$\lambda + 3, 4, 2$ are eigens of $A + 3I$

$\lambda^2 + 3\lambda, 4, -2$ are eigen of $A^2 + 3A$

Since $\lambda^2 + 3\lambda = 18$

$\therefore \ |A^2 + 3A| \neq 0$

Since $\lambda \neq 0$

$\therefore \ |A| \neq 0$

24. (c)

$$A.\text{ adj } A = |A| I = 78I = \begin{vmatrix} 78 & 0 & 0 \\ 0 & 78 & 0 \\ 0 & 0 & 78 \end{vmatrix}$$

Or

$$A.\text{ adj } A = |A| I$$
$$|A| = abc - 8a - 3(c-8) + 2(2-2b)$$
$$|A| = abc - (8a + 3c + 4b) + 28$$
$$\Rightarrow 70 - 20 + 28 = 78$$

25. (a)

For A^{-1} to be non existent, $|A| = 0$

$$|A| = \begin{vmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{vmatrix} = 0$$

$R_2 \to R_2 + R_3$

$$\begin{vmatrix} 3-x & 2 & 2 \\ 0 & -x & -x \\ -2 & -4 & -1-x \end{vmatrix} = 0$$

$$(-x)\begin{vmatrix} 3-x & 2 & 2 \\ 0 & 1 & 1 \\ -2 & -4 & -1-x \end{vmatrix} = 0$$

$R_1 \to R_1 - 2R_2$

$$(-x)\begin{vmatrix} 3-x & 0 & 0 \\ 0 & 1 & 1 \\ -2 & -4 & -1-x \end{vmatrix} = 0$$

$$(-x)(3 - x(-1 - x + 4)) = 0$$
$$(-x)(3-x)(3-x) = 0$$
$$x = 0, 3, 3$$

26. (2)

For the given homogeneous sytem to have non zero solution, determinant of coefficent matrix should be zero; i.e.,

$$= \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 1(1+1) + k(-k+1) - 1(k+1) = 0$$

$$\Rightarrow 2 - k^2 + k - k - 1 = 0 \Rightarrow k^2 = 1$$
$$\Rightarrow k = +1, -1$$
$$a = 1, b = 1 \Rightarrow a + b = 2$$

27. (b)

If $|A|$ is zero, A^{-1} does not exist and the matrix A is said to be singular.

$|A| \neq 0$ then only A^{-1} exists.

Hence (b) is correct answer.

28. (3)

If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigen value of A then the eigen value of adj A are $\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}, \ldots \frac{|A|}{\lambda_n}$; $|A| \neq 0$.

Thus eigen values of adj A are $\frac{4}{2}, -\frac{4}{4}$ i.e. 2 and -1, so, $x = 2$, $y = 1$ then sum $x + y = 3$.

29. (a)

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix}$$
$$= -\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$$

Characteristic equation

$$\lambda^3 - 6\lambda + 9\lambda - 4 = 0$$
$$\lambda = 1, 1, 4$$

30. (c)

$$A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$
$$= \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$
$$\alpha = a^2 + b^2, \beta = 2ab$$

31. (a)

By putting the system of simultaneous equations in the form

$AX = B$

we get, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 2 & \alpha \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 9 \\ \beta \end{bmatrix}$

So, the augmented matrix is $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right]$

$$\begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]$$

$$R_2 \to \frac{1}{2}R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]$$

$$R_3 \to R_3 - R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & \alpha-2 & \beta-7 \end{array}\right]$$

Now for infinite solutions, the last row must be zero.

So, $\alpha - 2 = 0 \Rightarrow \alpha = 2$ and $\beta - 7 = 0$

$\Rightarrow \quad \beta = 7$

32. (2)

$$\lambda I - A = \begin{bmatrix} \lambda & 0 & 0 \\ -1 & \lambda & 1 \\ 0 & -1 & \lambda \end{bmatrix}$$

Note: det $(\lambda I - A) = 0$ is same as det $(A - \lambda I) = 0$.

det $(\lambda I - A) = \lambda.(\lambda^2 + 1)$

$\lambda_1 = 0, \lambda_2 = +i, \lambda_3 = -i$

Sum of squares of eigen values

$= \lambda_1^2 + \lambda_2^2 + \lambda_3^3$

$= (0)^2 + (i)^2 + (-i)^2$

$= 0 + (-1) + (-1) = -2$

$\therefore$ 2 is absolute value.

33. (d)

The constant term in any characteristic polynomial is always $|A|$.

So, $|A| = -\frac{1}{4}$ since constant term of $p(\lambda)$ is $-\frac{1}{4}$.

34. (c)

$A.\text{ adj } A = |A| I$

$|A| = xyz - 8x - 3(z-8) + 2(2-2y)$

$|A| = xyz - (8x + 3z + 4y) + 28$

$\Rightarrow 60 - 20 + 28 = 68$

35. (c)

Let $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$AX_2 = \lambda_2 X_2$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a+0 \\ c+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$AX_1 = \lambda_1 X_1$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 2 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2a-b \\ 2c-d \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\Rightarrow \quad a = 2 \quad b = +2$

$c = 0 \quad d = 1$

$\Rightarrow \quad A = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$

36. (6)

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix}$$

$= -\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$

Characteristic equation

$\lambda^3 - 6\lambda + 9\lambda - 4 = 0$

$\lambda = 1, 1, 4$

■■■■

Calculus

1. Consider the following function.

$$f(x) = \frac{x}{x^2+4}.$$

Find the values of x for which $f(x)$ is increasing?

(a) $-2 \le x \le 2$
(b) $-2 < x < 2$
(c) $-2 > x > 2$
(d) $-2 \ge x \ge 2$

2. If $f(x) = \frac{\sin[x]}{[x]}, \quad [x] \ne 0$

$= 0, \quad [x] = 0$

where $[x]$ is the greatest integer less than or equal to x, then $k \lim_{x \to 0} f(x)$ is

(a) 1
(b) does not exist
(c) 0
(d) –1

3. $\frac{d}{dx}\cos\left(\frac{1}{x}\right)$ is equal to

(a) $\frac{\sin\left(\frac{1}{x}\right)}{x^2}$
(b) $-\frac{\sin\left(\frac{1}{x}\right)}{x^2}$
(c) $\frac{\sin(x)}{x^2}$
(d) $-\frac{\sin(x)}{x^2}$

4. $\int_0^{\pi/2} \sin x \cos x \, dx$ is ________

5. Consider the function $f(x) = x + \ln x$ and f is differentiable on (1, e) and $f(x)$ is continuous on [1, e]. Determine the c value using mean value theorem. [By computing $f'(c) = \frac{f(b)-f(a)}{b-a}$]

(a) e
(b) $e - 1$
(c) $\frac{e}{e-1}$
(d) $\frac{e-1}{e}$

6. Let $z = x \sin y - y \sin x$. The total differential dz = ________.

(a) $(\sin y + y\cos x)\,dx + (x\cos y + \sin x)\,dy$
(b) $(\sin y - y\cos x)\,dx + (x\cos y + \sin x)\,dy$
(c) $(\sin y + y\cos x)\,dx + (x\cos y - \sin x)\,dy$
(d) $(\sin y - y\cos x)\,dx + (x\cos y - \sin x)\,dy$

7. What is the value of the following limit.

$$\lim_{x \to 4} \frac{3-\sqrt{x+5}}{x-4}$$

8. Consider the following function.

$$f(x) = \sqrt{36-4x^2}$$

Find the points at which f has absolute minimum and absolute maximum respectively.

(a) $x = 0, x = 6$
(b) $x = 6, x = 0$
(c) $x = 0, x = 3$
(d) $x = 3, x = 0$

9. Find the x values for which the following function is increasing.

$$f(x) = \frac{x}{x^2+4}$$

(a) $x < 2$
(b) $x > 2$
(c) $x < -2$ and $x > 2$
(d) $x > -2$ and $x < 2$

10. Let $f(x) = x \cdot \int_1^{x^2} \sqrt{y^2+3} \cdot dy$. Find $f'(1)$

11. The output of $\lim_{x \to \pi/2} (\sin x)^{\tan x}$ is ________.

12. Value of $\lim_{x \to 0} \frac{x^2 \sin(1/x)}{\sin x}$ is ________.

13. The value of $\int_0^{2\pi} [|\sin x| + |\cos x|]\,dx$ is equal to

(a) $\frac{\pi}{2}$
(b) π
(c) $\frac{3\pi}{2}$
(d) 2π

14. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at ________.

15. A function $f(x)$ is differentiated twice such that its differential equation $\lambda^2 f(x) - 2\lambda f'(x) + f''(x) = 0$ provides two equal value of λ for all x. If $f(0) = 1$, $f'(0) = 2$, then $f(x)$ at $x = 1$ will be ________.

16. The value of $\lim\limits_{x \to \pi/2} \dfrac{\cos x}{\left(x - \frac{\pi}{2}\right)^3}$

(a) $-\infty$ (b) 0
(c) 1 (d) ∞

17. The function $f(x, y) = 2x^2 + 2xy - y^3$ has
(a) only one stationary point at (0, 0)
(b) two stationary points at (0, 0) and (1/6, 1/3)
(c) two stationary points at (0, 0) and (1, –1)
(d) no stationary point

18. $\int \frac{\sin(\ln x)}{x} dx = -1/a \cos(\ln x) + c$ then value of a is ________.

19. $\underset{x \to \infty}{Lt} \left(\dfrac{x+6}{x+1}\right)^{x+4} =$ ________.

(a) e^5 (b) e^6
(c) e^2 (d) e^{10}

20. For the function $f(x) = x^2 e^{-x}$, the maximum occurs when x is equal to ________.

Answers Calculus

1. (b) 2. (b) 3. (a) 5. (b) 6. (d) 8. (d) 9. (d) 13. (d) 15. (b)
16. (a) 17. (a) 19. (a)

Explanations Calculus

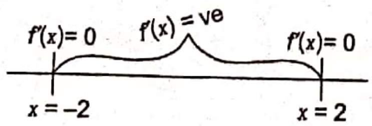
1. (b)

$$f(x) = \frac{x}{x^2+4}$$

$$\Rightarrow \quad f'(x) = \frac{(x^2+4)\cdot(1) - x(2x)}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

$$= \frac{(2-x)(2+x)}{(x^2+4)^2}$$

$\therefore f'(x) = 0 \Rightarrow f$ is increasing for $x > -2$ and $x < 2$



So option (b) is correct

2. (b)

Left limit $= \lim\limits_{h \to 0} f(0-h)$

Since $[0 - h] \neq 0$

$$f(0-h) = \frac{\sin[0-h]}{[0-h]}$$

$$\text{Left limit} = \lim_{h \to 0} \frac{\sin[0-h]}{[0-h]} = \frac{\sin(-1)}{-1} = \sin(1)$$

Right limit $= \lim\limits_{h \to 0} f(0+h)$

Since $[0 + h] = 0$

$f(0 + h) = 0$ (by definition given)

So right limit $= \lim\limits_{h \to 0} 0 = 0$

Since left limit $\neq$ right limit

The limit at $x = 0$ doesn't exist.

3. (a)

as $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ (Let $u = 1/x$)

$$\frac{dy}{du} = -\sin(u)$$

and $\frac{dy}{dx} = -\frac{1}{x^2}$

$$\frac{dy}{dx} = \frac{\sin(1/x)}{x^2}$$

4. (0.5)

Let $u = \sin x$

$du = \cos x\, dx$

So $\int_0^{\pi/2} \sin x \cos x\, dx = \int_0^1 u\,dx$

$$= \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2} = 0.5$$

5. (b)

$$f(x) = x + \ln x$$

$$f'(x) = 1 + \frac{1}{x} \quad \text{...(1)}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(e)-f(1)}{e-1}$$

$$= \frac{e+\ln e-(1+\ln 1)}{e-1}$$

$$= \frac{e+1-1+0}{e-1} = \frac{e}{e-1}$$

$\Rightarrow \quad f'(c) = \frac{e}{e-1}$ [from equation (1)]

$\Rightarrow \quad 1+\frac{1}{c} = \frac{e}{e-1}$

$\Rightarrow \quad \frac{1}{c} = \frac{e}{e-1} - 1$

$\Rightarrow \quad \frac{1}{c} = \frac{e-e+1}{e-1}$

$\Rightarrow \quad c = e - 1$

6. (d)

$$z = x\sin y - y\sin x$$

$$\frac{\partial z}{\partial x} = \sin y - y\cos x$$

$$\frac{\partial z}{\partial y} = x\cos y - \sin x$$

$$dz = \frac{\partial z}{\partial x}\cdot dx + \frac{\partial z}{\partial y}\cdot dy$$

$$= (\sin y - y\cos x)\cdot dx + (x\cos y - \sin x)\cdot dy$$

7. (–0.83)

$$\lim_{x\to 4} \frac{3-\sqrt{x+5}}{x-4}$$

$$= \lim_{x\to 4} \frac{3-\sqrt{x+5}}{x-4} \times \frac{3+\sqrt{x+5}}{3+\sqrt{x+5}}$$

$$= \lim_{x\to 4} \frac{9-(x+5)}{(x-4)(3+\sqrt{x+5})}$$

$$= \frac{-(x-4)}{(x-4)\cdot(3+\sqrt{x+5})}$$

$$= -\frac{5}{3+\sqrt{9}} = -\frac{5}{6} = -0.83$$

8. (d)

$$f(x) = \sqrt{36-4x^2}$$

At $x = 0$, $f(x) = 6$

If $x \neq 0 \Rightarrow f(x) < 6$

$\therefore$ f has absolute maximum at $x = 0$

At $x = 3$, $f(x) = 0$

$\therefore$ f has absolute minimum at $x = 3$.

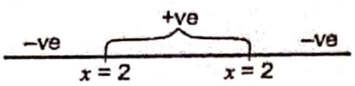
9. (d)

$$f(x) = \frac{x}{x^2+4}$$

$$\Rightarrow \quad f'(x) = \frac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2}$$

$$= \frac{4-x^2}{(x^2+4)^2} = \frac{(2-x)(2+x)}{(x^2+4)^2}$$

$$\therefore \; f'(x) = 0 = \frac{(2-x)(2+x)}{(x^2+4)^2} = 0$$



$\therefore$ f is increasing for $-2 < x < 2$

i.e., $x > -2$ and $x < 2$

Note: f is decreasing for $x < -2$ and $x > 2$.

10. (4)

$$f(x) = x \cdot \int_1^{x^2} \sqrt{y^2+3} \cdot dy$$

$$f'(x) = x \cdot \frac{d}{dx}\left[\int_1^{x^2} \sqrt{y^2+3} \cdot dy\right] + 1 \cdot \int_1^{x^2} \sqrt{y^2+3} \cdot dy$$

$$\Rightarrow f'(x) = x \cdot \left[\sqrt{x^4+3} + \sqrt{4}\right] + \int_1^{x^2} \sqrt{y^2+3} \cdot dy$$

$$\Rightarrow f'(1) = 1 \cdot \left[\sqrt{4} + \sqrt{4}\right] + \int_1^{1} \sqrt{y^2+3} \cdot dy$$

$$= 2 + 2 + 0 = 4$$

11. (1)

$$y = \lim_{x\to\pi/2} (\sin x)^{\tan x}$$

Taking log both side.

$$\log_y = \lim_{x\to\pi/2} \tan x \log(\sin x)$$

$$\log_y = \lim_{x\to\pi/2} \frac{\log(\sin x)}{\cot x} \quad \because \text{(form 0/0)}$$

$$\log_y = \lim_{x\to\pi/2} \frac{(1/\sin x)\cos x}{-\mathrm{cosec}^2 x}$$

$$\log_y = \lim_{x\to\pi/2} \sin x \cos x$$

$$y = e^0$$

$$y = 1$$

12. (0)

$$\lim_{x\to 0} \frac{x^2 \sin(1/x)}{\sin x} = \lim_{x\to 0} \frac{x}{\sin x} \cdot \frac{\sin(1/x)}{1/x}$$

$$= \lim_{x\to 0} \frac{x}{\sin x} \cdot \lim_{x\to 0} x \cdot \lim_{x\to 0} \sin\frac{1}{x}$$

$$= (1) . (0) \text{ (finite value)} = 0$$

13. (d)

Let $f(x) = [|\sin x| + |\cos x|]$

as $|\sin x| + |\cos x| \geq 1$

and $|\sin x| + |\cos x| \leq \sqrt{1^2+1^2}$

$$\Rightarrow \quad 1 \leq [|\sin x| + |\cos x|] \leq \sqrt{2}$$

Thus, $[|\sin x| + |\cos x|] = 1$

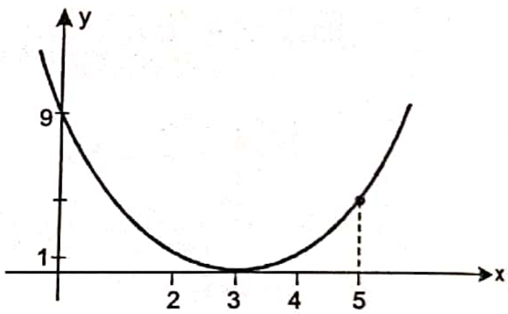
$$\therefore \int_0^{2\pi} [|\sin x| + |\cos x|]\,dx = \int_0^{2\pi} 1.dx = 2\pi$$

14. (5)

$$y = x^2 - 6x + 9 = (x-3)^2$$

$$y(2) = 1$$

$$y(5) = 4$$



$\therefore$ Maximum value of y over the interval 2 to 5 will be at $x = 5$

16. (7.39)

$$f''(x) - 2\lambda f'(x) + \lambda^2 f(x) = 0$$

$$(D^2 - 2\lambda + \lambda^2) = 0$$

$$D = +\lambda, +\lambda$$

$$f(x) = C_1 e^{\lambda x}$$

$$f(0) = 1 \Rightarrow C_1 = 1$$

$$f'(x) = \lambda C_1 e^{\lambda x}$$

$$f'(0) = \lambda C_1 = 2$$

$$\lambda = 2$$

$$\Rightarrow \quad f(x) = e^{2x}$$

$$f(x)|_{x=1} > e^2 = 7.39$$

16. (a)

Let, $$A = \lim_{x\to\pi/2} \frac{\cos x}{\left(x - \frac{\pi}{2}\right)^3}$$

By putting $\left(x-\frac{\pi}{2}\right) = t$

when $x \to \frac{\pi}{2}, t \to 0$

then, $A = \lim_{t\to 0} \dfrac{\cos\left(\frac{\pi}{2}+t\right)}{t^3}$

$= \lim_{t\to 0} \dfrac{-\sin t}{t^3} = \lim_{t\to 0} (-1)\dfrac{\sin t}{t}.\dfrac{1}{t^2}$

$= (-1).1.\dfrac{1}{0} = -\infty$

17. (a)

$f(x, y) = 2x^2 + 2xy - y^3$

$\dfrac{\partial f}{\partial x} = 4x + 2y$

$\dfrac{\partial f}{\partial y} = 2x - 3y^2$

For stationary point of $f(x, y)$

$\dfrac{\partial f}{\partial x} = 0$ and $\dfrac{\partial f}{\partial y} = 0$

$\therefore\ 4x + 2y = 0$

$\Rightarrow\ 2x + y = 0$ ...(i)

and $2x - 3y^2 = 0$ ...(ii)

Solving (i) and (ii)

$-y - 3y^2 = 0$

$\Rightarrow y + 3y^2 = 0$

$\Rightarrow y(1 + 3y) = 0$

$\Rightarrow y = 0, -\frac{1}{3}$ and $x = 0, \frac{1}{6}$

So, the stationary points may be (0, 0) and $\left(\frac{1}{6}, -\frac{1}{3}\right)$.

But point $\left(\frac{1}{6}, -\frac{1}{3}\right)$ doesnot lie on the curve, so there is only one stationary point at origin (0, 0).

18. (1)

Let $\ln x = t$

Then $dt = \frac{1}{x}dx$

Hence,

$I = \int \sin t\, dt = -\cos t + c = -\cos(\ln x) + c$

19. (a)

$$\underset{x\to\infty}{Lt}\left(\frac{x+6}{x+1}\right)^{x+4} = \underset{x\to\infty}{Lt}\left\{\left[1+\frac{5}{x+1}\right]^{\frac{x+1}{5}}\right\}^{5\left(\frac{x+4}{x+1}\right)}$$

[Using $\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^x = e$]

$$\underset{x\to\infty}{Lt}\, e^{5\left(\frac{x+4}{x+1}\right)} = e^{5\lim\limits_{x\to\infty}\left(\frac{1+4/x}{1+1/x}\right)} = e^5$$

20. (2)

$f(x) = x^2 e^{-x}$

$f'(x) = x^2(-e^{-x}) + e^{-x}\cdot 2x$

$= e^{-x}(2x - x^2)$

Putting $f'(x) = 0$

$e^{-x}(2x - x^2) = 0$

$e^{-x}\cdot x\cdot(2 - x) = 0$

$x = 0$ or $x = 2$ are the stationary points.

Now, $f''(x) = e^{-x}(2 - 2x) + (2x - x^2)(-e^{-x})$

$= e^{-x}(2 - 2x - (2x - x^2))$

$= e^{-x}(x^2 - 4x + 2)$

at $x = 0$,

$f''(0) = e^{0}(0 - 0 + 2) = 2$

Since $f''(x) = 2$ is > 0 at $x = 0$ we have a minima.

Now at $x = 2$

$f''(2) = e^{-2}(2^2 - 4\times 2 + 2)$

$= e^{-2}(4 - 8 + 2)$

$= -2e^{-2} < 0$

$\therefore$ at $x = 2$ we have a maxima.

■■■■